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BIG IDEAS MATH®

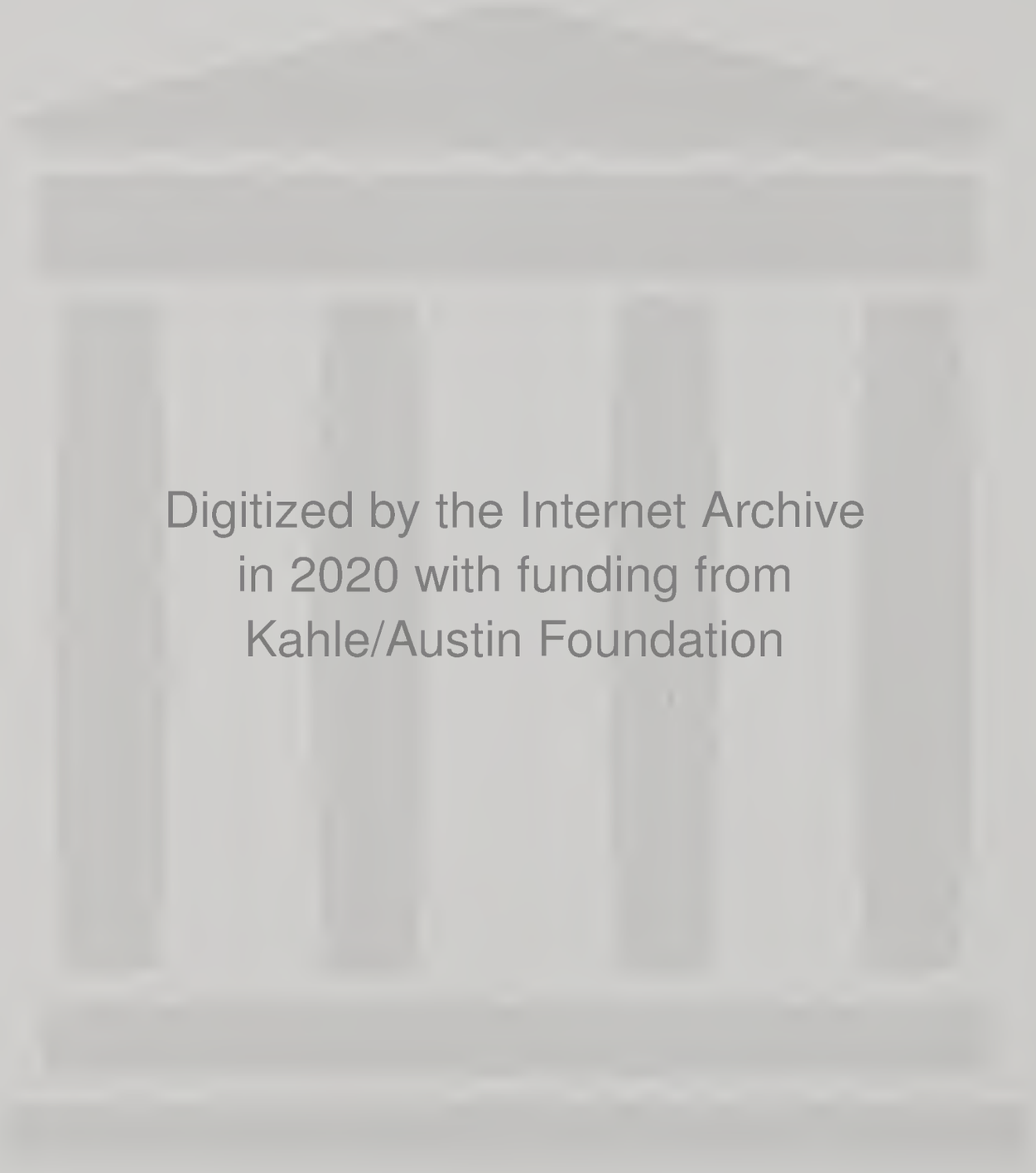
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Algebra 1



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TEACHING EDITION


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BIG IDEAS

MATH®

TEXAS TEACHING EDITION

Algebra 1

Ron Larson and Laurie Boswell



BIG IDEAS LEARNING®

Erie, Pennsylvania

BigIdeasLearning.com



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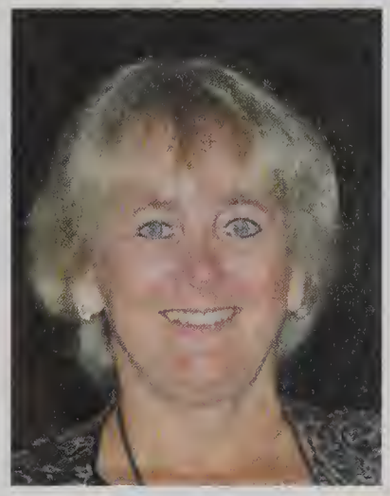
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Ron Larson, Ph.D., is well known as the lead author of a comprehensive program for mathematics that spans middle school, high school, and college courses. He holds the distinction of Professor Emeritus from Penn State Erie, The Behrend College, where he taught for nearly 40 years. He received his Ph.D. in mathematics from the University of Colorado. Dr. Larson's numerous professional activities keep him actively involved in the mathematics education community and allow him to fully understand the needs of students, teachers, supervisors, and administrators.

Ron Larson



Laurie Boswell, Ed.D., is the Head of School and a mathematics teacher at the Riverside School in Lyndonville, Vermont. Dr. Boswell is a recipient of the Presidential Award for Excellence in Mathematics Teaching and has taught mathematics to students at all levels, from elementary through college. Dr. Boswell was a Tandy Technology Scholar and served on the NCTM Board of Directors from 2002 to 2005. She currently serves on the board of NCSM and is a popular national speaker.

Laurie Boswell

Dr. Ron Larson and **Dr. Laurie Boswell** began writing together in 1992. Since that time, they have authored over two dozen textbooks. In their collaboration, Ron is primarily responsible for the student edition while Laurie is primarily responsible for the teaching edition.

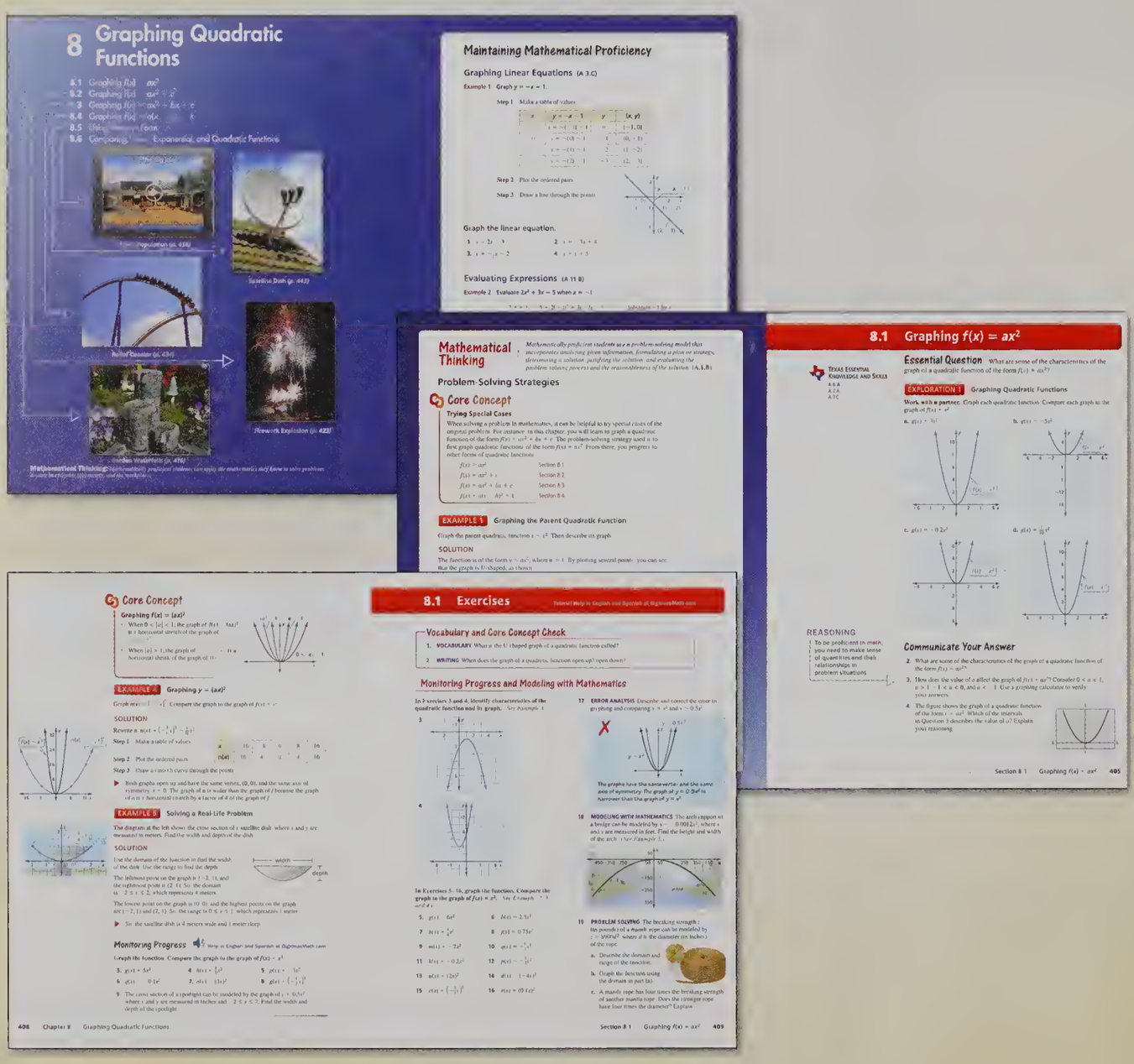
For the Student

Welcome to ***Big Ideas Math Algebra 1***. From start to finish, this program was designed with you, the learner, in mind.

As you work through the chapters in your Algebra 1 course, you will be encouraged to think and to make conjectures while you persevere through challenging problems and exercises. You will make errors—and that is ok! Learning and understanding occur when you make errors and push through mental roadblocks to comprehend and solve new and challenging problems.

In this program, you will also be required to explain your thinking and your analysis of diverse problems and exercises. Being actively involved in learning will help you develop mathematical reasoning and use it to solve math problems and work through other everyday challenges.

We wish you the best of luck as you explore Algebra 1. We are excited to be a part of your preparation for the challenges you will face in the remainder of your high school career and beyond.



Big Ideas Math High School Research

Big Ideas Math Algebra 1, Geometry, and Algebra 2 is a research-based program providing a rigorous, focused, and coherent curriculum for high school students. Ron Larson and Laurie Boswell utilized their expertise as well as the body of knowledge collected by additional expert mathematicians and researchers to develop each course. The pedagogical approach to this program follows the best practices outlined in the most prominent and widely-accepted educational research and standards, including:

Achieve, ACT, and The College Board

Adding It Up: Helping Children Learn Mathematics
National Research Council ©2001

Curriculum Focal Points and the *Principles and Standards for School Mathematics* ©2000
National Council of Teachers of Mathematics (NCTM)

Project Based Learning
The Buck Institute

Rigor/Relevance Framework™
International Center for Leadership in Education

Universal Design for Learning Guidelines
CAST ©2011

We would also like to express our gratitude to the experts who served as consultants for ***Big Ideas Math Algebra 1, Geometry, and Algebra 2***. Their input was an invaluable asset to the development of this program.

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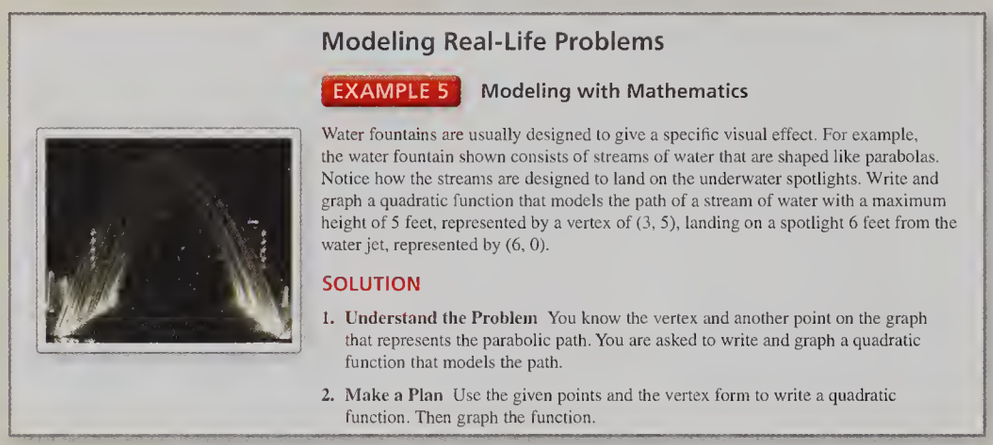
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We would also like to thank all of our reviewers who provided feedback during the final development phases. For a complete list of the ***Big Ideas Math*** program reviewers, please visit *www.BigIdeasLearning.com*.

Texas Mathematical Process Standards

Apply mathematics to problems arising in everyday life, society, and the workplace.

- Real-life scenarios are utilized in *Explorations*, *Examples*, *Exercises*, and *Assessments* so students have opportunities to apply the mathematical concepts they have learned to realistic situations.
- Real-world problems help students use the structure of mathematics to break down and solve more difficult problems.



Modeling Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**

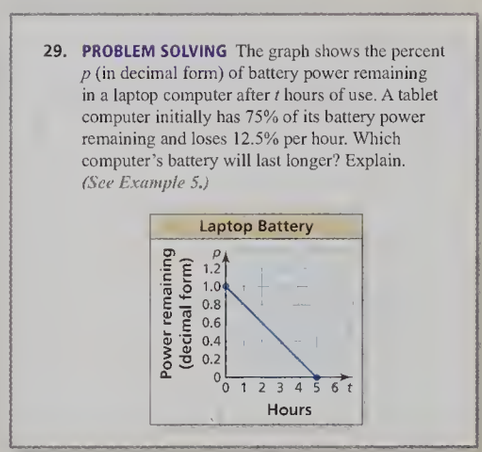
Water fountains are usually designed to give a specific visual effect. For example, the water fountain shown consists of streams of water that are shaped like parabolas. Notice how the streams are designed to land on the underwater spotlights. Write and graph a quadratic function that models the path of a stream of water with a maximum height of 5 feet, represented by a vertex of (3, 5), landing on a spotlight 6 feet from the water jet, represented by (6, 0).

SOLUTION

1. **Understand the Problem** You know the vertex and another point on the graph that represents the parabolic path. You are asked to write and graph a quadratic function that models the path.
2. **Make a Plan** Use the given points and the vertex form to write a quadratic function. Then graph the function.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

- *Reasoning*, *Critical Thinking*, *Abstract Reasoning*, and *Problem Solving* exercises challenge students to apply their acquired knowledge and reasoning skills to solve each problem.
- Students are continually encouraged to evaluate the reasonableness of their solutions and their steps in the problem-solving process.



29. **PROBLEM SOLVING** The graph shows the percent p (in decimal form) of battery power remaining in a laptop computer after t hours of use. A tablet computer initially has 75% of its battery power remaining and loses 12.5% per hour. Which computer's battery will last longer? Explain. *(See Example 5.)*

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

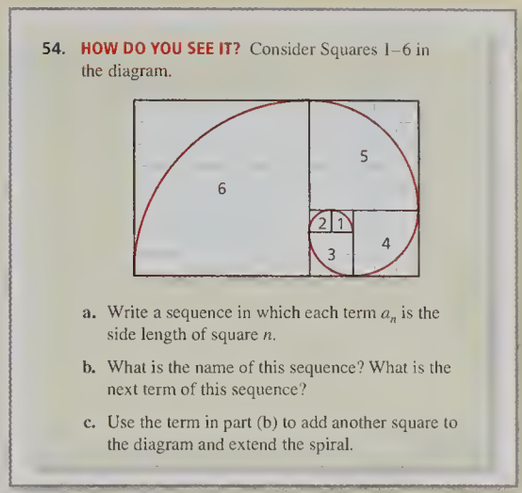
- Students are provided opportunities for selecting and utilizing the appropriate mathematical tool in *Using Tools* exercises. Students work with graphing calculators, dynamic geometry software, models, and more.
- A variety of tool papers and manipulatives are available for students to use in problems strategically, as appropriate.

USING TOOLS **In Exercises 21–26, solve the inequality. Use a graphing calculator to verify your answer.**

21. $36 < 3y$ **22.** $17v \geq 51$

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

- Students are asked to construct arguments, critique the reasoning of others, and evaluate multiple representations of problems in specialized exercises, including *Making an Argument*, *How Do You See It?*, *Drawing Conclusions*, *Reasoning*, *Error Analysis*, *Problem Solving*, and *Writing*.
- Real-life situations are translated into diagrams, tables, equations, and graphs to help students analyze relationships and draw conclusions.

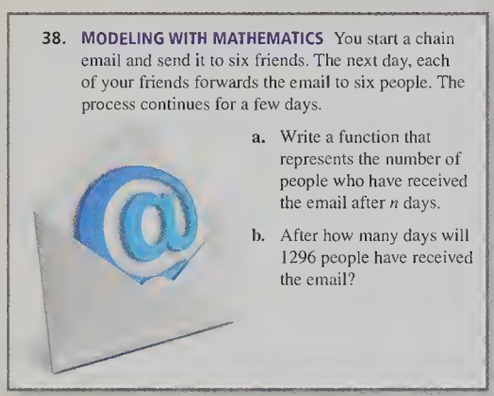


54. **HOW DO YOU SEE IT?** Consider Squares 1–6 in the diagram.

a. Write a sequence in which each term a_n is the side length of square n.

b. What is the name of this sequence? What is the next term of this sequence?

c. Use the term in part (b) to add another square to the diagram and extend the spiral.



38. **MODELING WITH MATHEMATICS** You start a chain email and send it to six friends. The next day, each of your friends forwards the email to six people. The process continues for a few days.

a. Write a function that represents the number of people who have received the email after n days.

b. After how many days will 1296 people have received the email?

Create and use representations to organize, record, and communicate mathematical ideas.

- *Modeling with Mathematics* exercises allow students to interpret a problem in the context of a real-life situation, while utilizing tables, graphs, visual representations, and formulas.
- Multiple representations are presented to help students move from concrete to representative and into abstract thinking.

Analyze mathematical relationships to connect and communicate mathematical ideas.

- *Using Structure* exercises provide students with the opportunity to explore patterns and structure in mathematics.
- Stepped-out *Examples* encourage students to maintain oversight of their problem-solving process and pay attention to the relevant details in each step.

EXAMPLE 3 Inequalities with Special Solutions

Solve (a) $8b - 3 > 4(2b + 3)$ and (b) $2(5w - 1) \leq 7 + 10w$.

SOLUTION

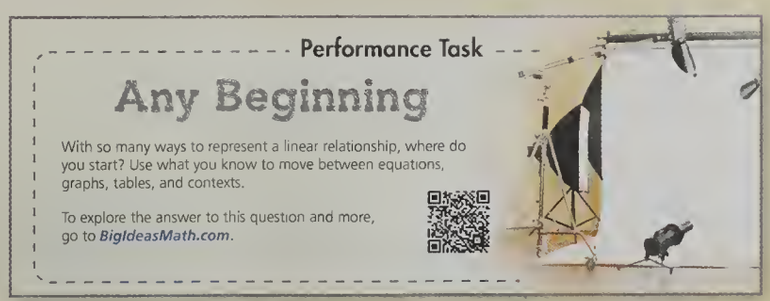
a.

$8b - 3 > 4(2b + 3)$	Write the inequality.
$8b - 3 > 8b + 12$	Distributive Property
$-8b \quad -8b$	Subtract $8b$ from each side.
$-3 > 12$ ✗	Simplify.

▶ The inequality $-3 > 12$ is false. So, there is no solution.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- *Vocabulary and Core Concept Check* exercises require students to use clear, precise mathematical language in their solutions and explanations.
- *Performance Tasks* for every chapter allow students to apply their skills to comprehensive problems and utilize precise mathematical language when analyzing, interpreting, and communicating their answers.



Performance Task

Any Beginning

With so many ways to represent a linear relationship, where do you start? Use what you know to move between equations, graphs, tables, and contexts.

To explore the answer to this question and more, go to *BigIdeasMath.com*.

1 Solving Linear Equations

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Mathematical Thinking 2

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See the Big Idea
Learn how boat navigators use dead reckoning to calculate their distance covered in a single direction.



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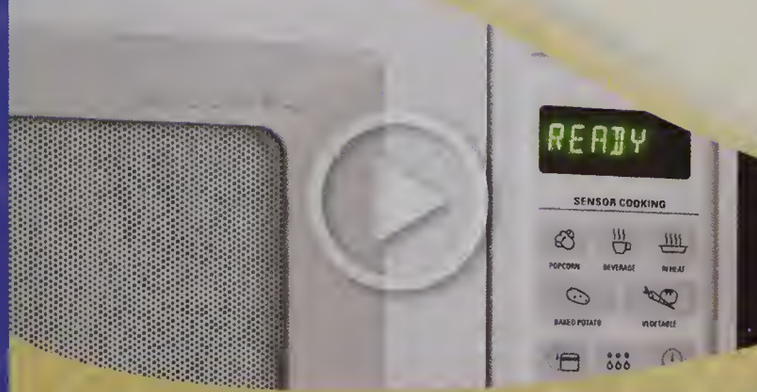
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See the Big Idea

Determine how designers decide on the number of electrical circuits needed in a house.

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See the Big Idea
Discover why unlike almost any other natural phenomenon, light travels at a constant speed.



Writing Linear Functions 4

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See the Big Idea
Explore wind power and discover where the future of wind power will take us.

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See the Big Idea
Learn how fisheries manage their complex ecosystems.

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6

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See the Big Idea
Explore the variety of recursive sequences in language, art, music, nature, and games.

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See the Big Idea

Explore whether seagulls and crows use the optimal height while dropping hard-shelled food to crack it open.



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See the Big Idea
Investigate the link between population growth and the classic exponential pay doubling application.

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See the Big Idea

Explore the Parthenon and investigate how the use of the golden rectangle has evolved since its discovery.



How to Use Your Math Book

Get ready for each chapter by **Maintaining Mathematical Proficiency** and sharpening your **Mathematical Thinking**. Begin each section by working through the **EXPLORATIONS** to **Communicate Your Answer** to the **Essential Question**. Each **Lesson** will explain **What You Will Learn** through **EXAMPLES**, **Core Concepts**, and **Core Vocabulary**. Answer the **Monitoring Progress** questions as you work through each lesson. Look for STUDY TIPS, COMMON ERRORS, and suggestions for looking at a problem ANOTHER WAY throughout the lessons. We will also provide you with guidance for accurate mathematical READING and concept details you should REMEMBER.

Sharpen your newly acquired skills with **Exercises** at the end of every section. Halfway through each chapter you will be asked **What Did You Learn?** and you can use the Mid-Chapter **Quiz** to check your progress. You can also use the **Chapter Review** and **Chapter Test** to review and assess yourself after you have completed a chapter.

Apply what you learned in each chapter to a **Performance Task** and build your confidence for taking standardized tests with each chapter's **Standards Assessment**.

For extra practice in any chapter, use your *Online Resources*, *Skills Review Handbook*, or your *Student Journal*.



Program Overview

Program Philosophy: Rigor and Balance with Real-Life Applications

The ***Big Ideas Math***® program balances conceptual understanding with procedural fluency. Real-life applications help turn mathematical learning into an engaging and meaningful way to see and explore the real world.

Essential Question How can you use function notation to represent a function?

The notation $f(x)$, called **function notation**, is another name for y. This notation is read as "the value of f at x" or "f of x." The parentheses do not imply multiplication. You can use letters other than f to name a function. The letters g, h, j, and k are often used to name functions.

EXPLORATION 1 Matching Functions with Their Graphs

Work with a partner. Match each function with its graph.

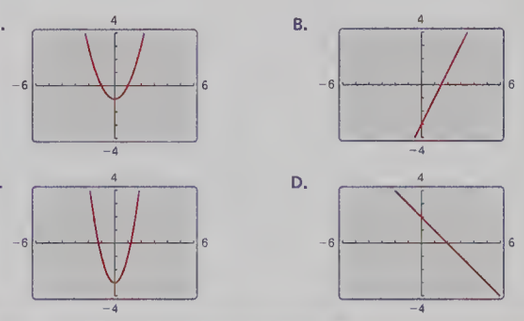
a. $f(x) = 2x - 3$

b. $g(x) = -x + 2$

c. $h(x) = x^2 - 1$

d. $j(x) = 2x^2 - 3$

A. B. C. D.



Explorations and guiding *Essential Questions* encourage **conceptual understanding.**

3.3 Lesson

What You Will Learn

- Use function notation to evaluate an
- Use function notation to solve and g
- Solve real-life problems using function notation.

Core Vocabulary

function notation, *p. 108*

Previous

linear function

quadrant

Using Function Notation to Evaluate and Interpret

You know that a linear function can be written in the form $y = mx + b$. By naming a linear function f, you can also write the function using **function notation**.

$f(x) = mx + b$ Function notation

The notation $f(x)$ is another name for y. If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x. You can use letters other than f to name a function, such as g or h.

EXAMPLE 1 Evaluating a Function

Evaluate $f(x) = -4x + 7$ when $x = 2$ and $x = -2$.

SOLUTION

$f(x) = -4x + 7$	Write the function.	$f(x) = -4x + 7$
$f(2) = -4(2) + 7$	Substitute for x.	$f(-2) = -4(-2) + 7$
$= -8 + 7$	Multiply.	$= 8 + 7$
$= -1$	Add.	$= 15$

READING

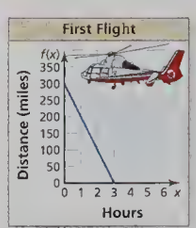
The notation $f(x)$ is read as "the value of f at x" or "f of x." It does not mean "f times x."

Direct instruction lessons allow for **procedural fluency** and provide the opportunity to use clear, precise mathematical language.

Real-life applications provide students with opportunities to connect classroom lessons to realistic scenarios.

Solving Real-Life Problems

EXAMPLE 5 Modeling with Mathematics



The graph shows the number of miles a helicopter is from its destination after x hours on its first flight. On its second flight, the helicopter travels 50 miles farther and increases its speed by 25 miles per hour. The function $f(x) = 350 - 125x$ represents the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

SOLUTION

1. **Understand the Problem** You are given a graph of the first flight and an equation of the second flight. You are asked to compare the flight times to determine which flight takes less time.
2. **Make a Plan** Graph the function that represents the second flight. Compare the graph to the graph of the first flight. The x-value that corresponds to $f(x) = 0$ represents the flight time.
3. **Solve the Problem** Graph $f(x) = 350 - 125x$.

 Step 1 Make an input-output table to find the ordered pairs.

x	0	1	2	3
$f(x)$	350	225	100	−25

Chapter openers focused on **Maintaining Mathematical Proficiency** promote the development of the habits of mind mathematically proficient students demonstrate.

Mathematical process standards are woven into every chapter, including a full page dedicated to practicing **Mathematical Thinking**.

Maintaining Mathematical Proficiency

Using Order of Operations (6.7.A)

Example 1 Evaluate $10^2 \div (30 \div 3) - 4(3 - 9) + 5^1$.

First:	Parentheses	$10^2 \div (30 \div 3) - 4(3 - 9) + 5^1 = 10^2 \div 10 - 4(-6) + 5^1$
Second:	Exponents	$= 100 \div 10 - 4(-6) + 5$
Third:	Multiplication and Division (from left to right)	$= 10 + 24 + 5$
Fourth:	Addition and Subtraction (from left to right)	$= 39$

Evaluate the expression.

1. $12\left(\frac{14}{2}\right) - 3^3 + 15 - 9^2$
2. $5^2 \cdot 8 \div 2^2 + 20 \cdot 3 - 4$
3. $-7 + 16 \div 2^4 + (10 - 4^2)$

Finding Square Roots (A.11.A)

Example 2 Find $-\sqrt{81}$.

▶ $-\sqrt{81}$ represents the negative square root. Because $9^2 = 81$, $-\sqrt{81} = -\sqrt{9^2} = -9$.

Find the square root(s).

4. $\sqrt{64}$
5. $-\sqrt{4}$
6. $-\sqrt{25}$
7. $\pm\sqrt{121}$

Writing Equations for Arithmetic Sequences (A.12.D)

Write an equation for the *n*th term of the arithmetic sequence 5, 15, 25, 35, ...

The first term is 5, and the common difference is 10.

$a_n = a_1 + (n - 1)d$	Equation for an arithmetic sequence
$a_n = 5 + (n - 1)(10)$	Substitute 5 for a_1 and 10 for d.
$a_n = 10n - 5$	Simplify.

n equation for the *n*th term of the arithmetic sequence.

, 16, 18, . . .
9. 6, 3, 0, −3, . . .
10. 22, 15, 8, 1, . . .

RACT REASONING Recall that a perfect square is a number with integers as its square Is the product of two perfect squares always a perfect square? Is the quotient of two t squares always a perfect square? Explain your reasoning.

Mathematical Thinking

Mathematically proficient students use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. **(A.1.B)**

Problem-Solving Strategies

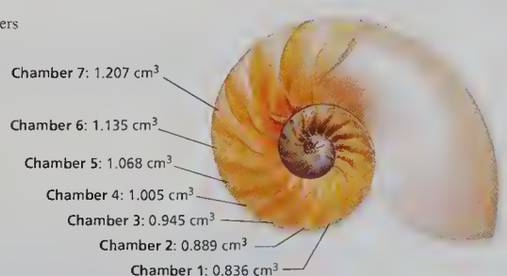
Core Concept

Finding a Pattern

When solving a real-life problem, look for a pattern in the data. The pattern could include repeating items, numbers, or events. After you find the pattern, describe it and use it to solve the problem.

EXAMPLE 1 Using a Problem-Solving Strategy

The volumes of seven chambers of a chambered nautilus are given. Find the volume of Chamber 10.



SOLUTION

To find a pattern, try dividing each volume by the volume of the previous chamber.

$\frac{0.889}{0.836} \approx 1.063$ $\quad \frac{0.945}{0.889} \approx 1.063$ $\quad \frac{1.005}{0.945} \approx 1.063$

$\frac{1.068}{1.005} \approx 1.063$ $\quad \frac{1.135}{1.068} \approx 1.063$ $\quad \frac{1.207}{1.135} \approx 1.063$

From this, you can see that the volume of each chamber is about 6.3% greater than the volume of the previous chamber. To find the volume of Chamber 10, multiply the volume of Chamber 7 by 1.063 three times.

$1.207(1.063) \approx \mathbf{1.283}$ $\quad 1.283(1.063) \approx \mathbf{1.364}$ $\quad 1.364(1.063) \approx \mathbf{1.450}$

volume of Chamber 8 $\quad$ volume of Chamber 9 $\quad$ volume of Chamber 10

▶ The volume of Chamber 10 is about 1.450 cubic centimeters.

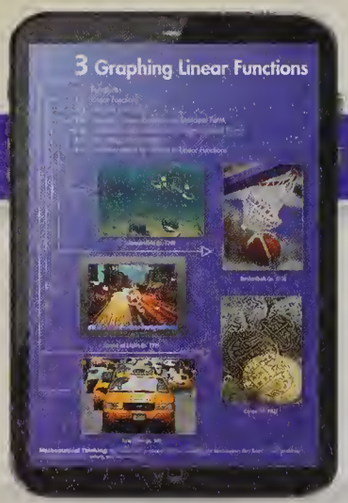
Monitoring Progress

1. A rabbit population over 8 consecutive years is given by 50, 80, 128, 205, 328, 524, 839, 1342. Find the population in the tenth year.
2. The sums of the numbers in the first eight rows of Pascal's Triangle are 1, 2, 4, 8, 16, 32, 64, 128. Find the sum of the numbers in the tenth row.

Monitoring Progress problems allow students to practice and sharpen their skills as they work toward mathematical understanding.

Personalized Learning

The ***Big Ideas Math*** program offers teachers and students many ways to personalize and enrich the learning experience of all levels of learners.



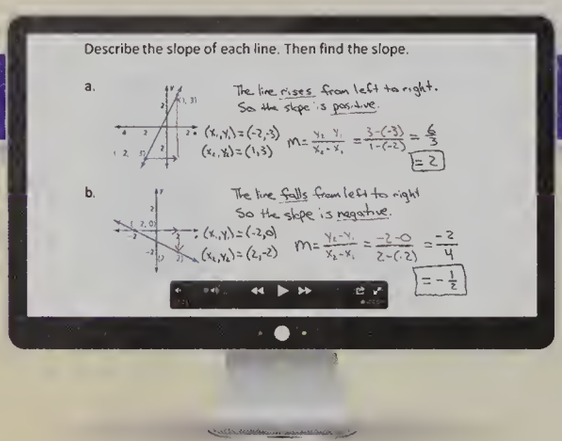
Dynamic Student Edition

This unique tool, available online or as an eBook App, provides students with embedded 21st century learning resources. Students have the opportunity to interact with the underlying mathematics in a number of ways, including engaging tutorials, interactive manipulatives, flashcards, vocabulary support, and games that enhance the learning experience and promote mathematical understanding.

Dynamic Assessment & Progress Monitoring Tool

This tool provides personalized links to remediation at point-of-use on student and teacher reports. Teachers can also create customized practice sets for students struggling with a particular concept. Students will receive immediate feedback on their short-answer and multiple-choice responses.





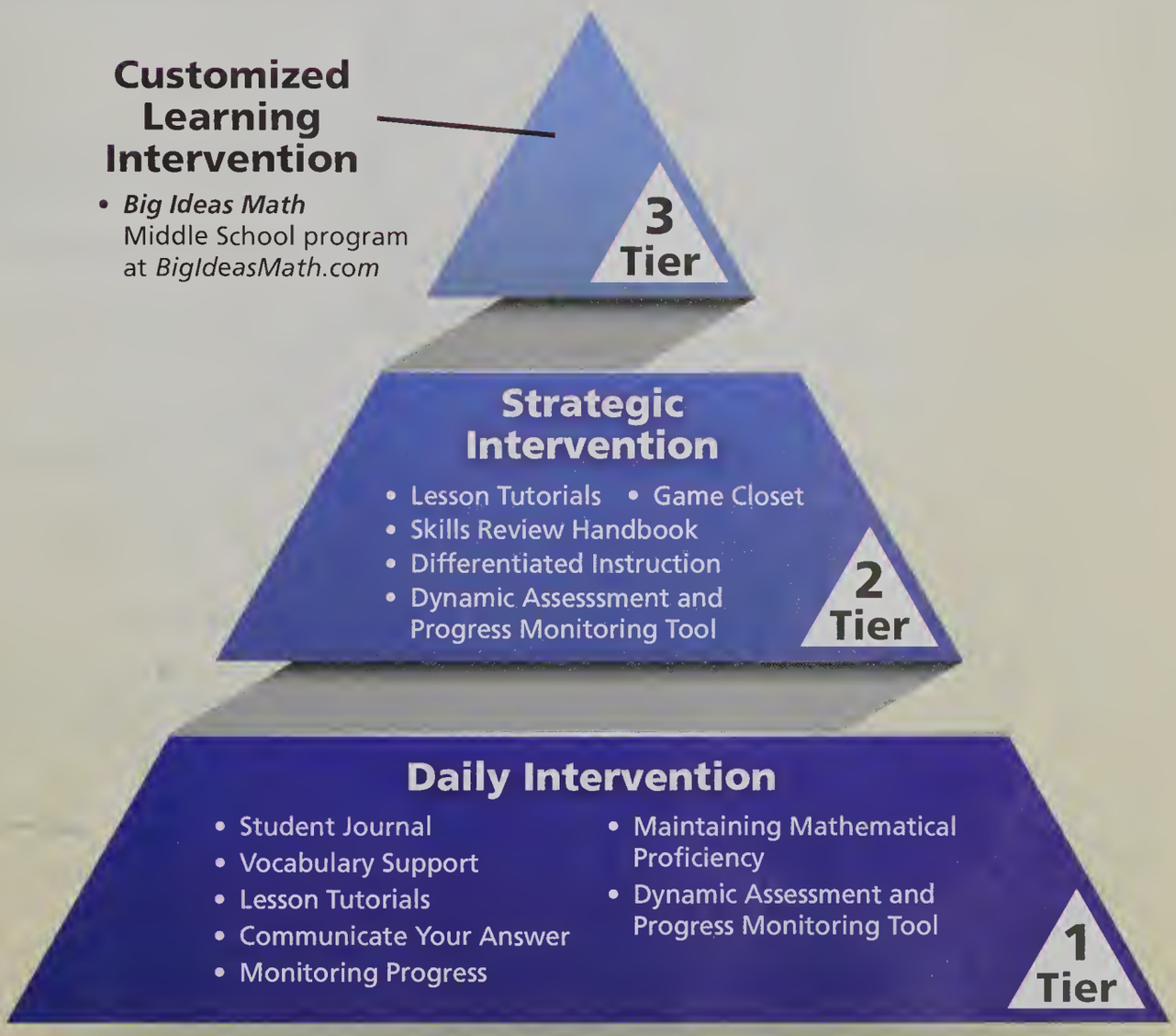
Online Lesson Tutorials

Two- to three-minute lesson tutorial videos provide video and audio support for every example in the textbook. These are valuable for students who miss a class, need a second explanation, or need extra assistance with a homework assignment. Parents can also utilize the tutorials to stay connected or to provide additional help at home.

Differentiated Instruction



Through print and digital resources, the ***Big Ideas Math*** program completely supports the 3-Tier Response to Intervention model. Using research-based strategies, teachers can reach, challenge, and motivate each student with high-quality instruction targeted to individual needs.



Big Ideas Math Dynamic Technology

The Dynamic Assessment and Progress Monitoring Tool

The **Dynamic Assessment and Progress Monitoring Tool** allows teachers to know where their students are and monitor their progress as they guide them to where they need to be. This tool was developed exclusively for ***Big Ideas Math*** by Pacific Metrics, an educational technology powerhouse with extensive experience in providing innovative and research-based solutions for large-scale assessment needs.



Assessment Creation and Scheduling

Assessments can be created by TEKS Standard or ***Big Ideas Math*** content.

Features adaptive testing capabilities.

Direct Ties to Remediation

Includes direct links to Lesson Tutorial Videos and Skills Review Handbook resources.

Appears at point-of-use for students and teachers.

Customizable Reporting

Facilitates progress monitoring by student or by class.

Reports can be generated by assessment or by TEKS Standard.

Assessment Delivery with Embedded Tools

Students have access to scientific calculators, protractors, and other tools when enabled.

Short-answer and multiple-choice questions are automatically graded.

Online System with Secure Message Center

Accounts can be accessed from any location.

Communicate with one student or an entire class.

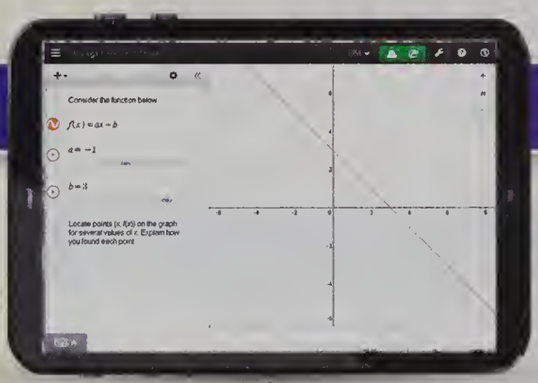
Easy-to-Use Drag-and-Drop Interface

Includes a customizable dashboard and scheduling module.

Dynamic Student Edition



Through the **Dynamic Student Edition** eBook App, students not only have access to the complete textbook, but they can also explore robust interactive digital resources embedded within each lesson. There is audio support for the text and Lesson Tutorial Videos in both English and Spanish. Interactive investigations, direct links to remediation, and additional resources are linked right to the lesson.



Dynamic Investigations

Dynamic Investigations in the ***Big Ideas Math*** program are powered by Desmos® and GeoGebra®. Teachers and students can integrate these investigations into their discovery learning to interact with the explorations in the Student Edition.

Real-Life STEM Videos



Science - Technology - Engineering - Mathematics

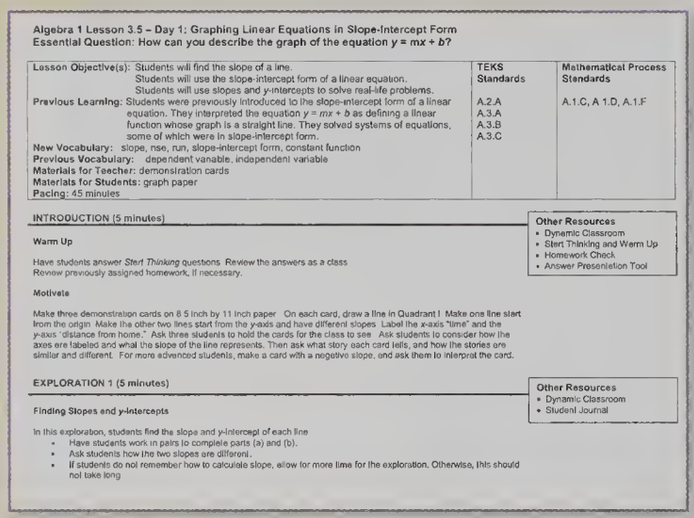
Every chapter in the ***Big Ideas Math*** program contains a Real-Life STEM Video allowing students to further engage with mathematical concepts. Students learn about the speed of light, natural disasters, solar power, and more!

Teacher Support

Lesson Planning Support

Online Lesson Plans

Complete, editable lesson plans are included for every lesson in the program.



Algebra 1 Lesson 3.5 – Day 1: Graphing Linear Equations in Slope-Intercept Form
Essential Question: How can you describe the graph of the equation $y = mx + b$?

	TEKS Standards	Mathematical Process Standards
Lesson Objective(s): Students will find the slope of a line. Students will use the slope-intercept form of a linear equation. Students will use slopes and *y*-intercepts to solve real-life problems.		
Previous Learning: Students were previously introduced to the slope-intercept form of a linear equation. They interpreted the equation $y = mx + b$ as defining a linear function whose graph is a straight line. They solved systems of equations, some of which were in slope-intercept form.	A.2.A A.3.A A.3.B A.3.C	A.1.C, A.1.D, A.1.F

New Vocabulary: slope, rise, run, slope-intercept form, constant function
Previous Vocabulary: dependent variable, independent variable
Materials for Teacher: demonstration cards
Materials for Students: graph paper
Pacing: 45 minutes

INTRODUCTION (5 minutes)

Other Resources
- Dynamic Classroom
- Start Thinking and Warm Up
- Homework Check
- Answer Presentation Tool

Warm Up

Have students answer *Start Thinking* questions. Review the answers as a class. Review previously assigned homework, if necessary.

Motivate

Make three demonstration cards on 8.5 inch by 11 inch paper. On each card, draw a line in Quadrant I. Make one line start from the origin. Make the other two lines start from the *y*-axis and have different slopes. Label the *x*-axis "time" and the *y*-axis "distance from home." Ask three students to hold the cards for the class to see. Ask students to consider how the axes are labeled and what the slope of the line represents. Then ask what story each card tells, and how the stories are similar and different. For more advanced students, make a card with a negative slope, and ask them to interpret the card.

EXPLORATION 1 (5 minutes)

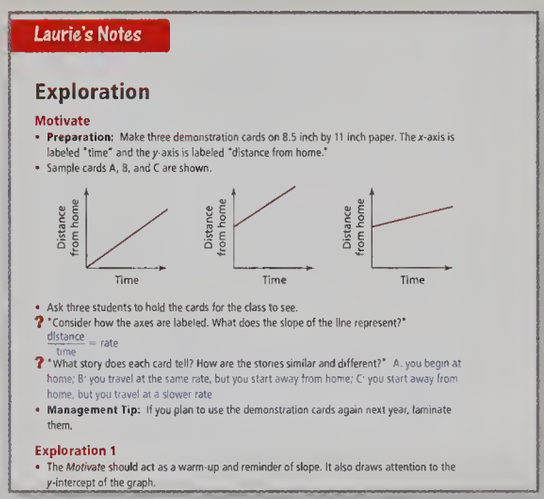
Other Resources
- Dynamic Classroom
- Student Journal

Finding Slopes and *y*-intercepts

In this exploration, students find the slope and *y*-intercept of each line.
- Have students work in pairs to complete parts (a) and (b).
- Ask students how the two slopes are different.
- If students do not remember how to calculate slope, allow for more time for the exploration. Otherwise, this should not take long.

Laurie's Notes

Online teaching support for every chapter is available from master educator Laurie Boswell.



Laurie's Notes

Exploration

Motivate
- **Preparation:** Make three demonstration cards on 8.5 inch by 11 inch paper. The *x*-axis is labeled "time" and the *y*-axis is labeled "distance from home."
- Sample cards A, B, and C are shown.

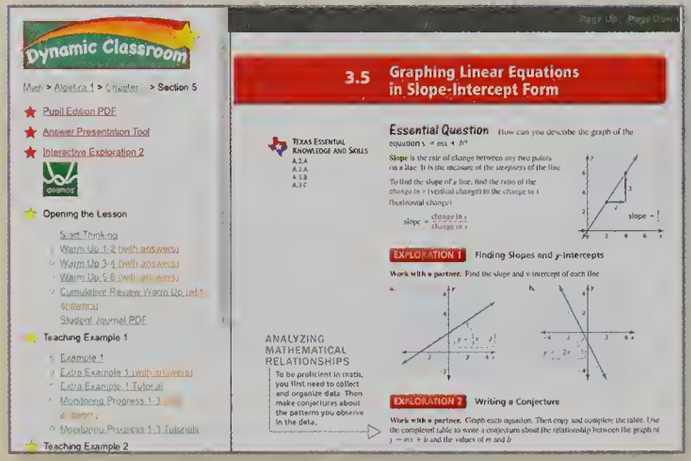
- Ask three students to hold the cards for the class to see.
- ? "Consider how the axes are labeled. What does the slope of the line represent?" $\frac{\text{distance}}{\text{time}} = \text{rate}$
- ? "What story does each card tell? How are the stories similar and different?" A: you begin at home; B: you travel at the same rate, but you start away from home; C: you start away from home, but you travel at a slower rate
- **Management Tip:** If you plan to use the demonstration cards again next year, laminate them.

Exploration 1
- The *Motivate* should act as a warm-up and reminder of slope. It also draws attention to the *y*-intercept of the graph.

Lesson Presentation Support

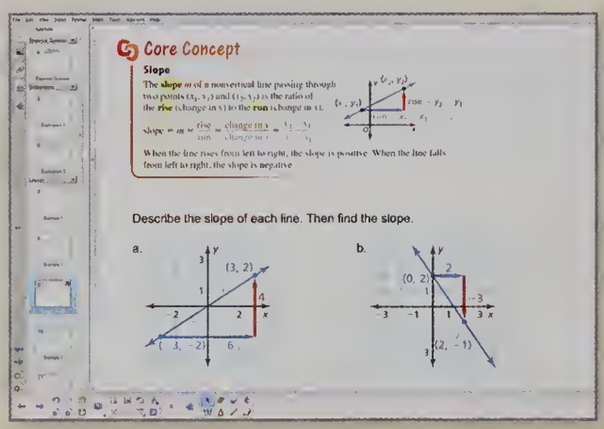
Dynamic Classroom

This online tool contains a collection of interactive resources for presenting lessons.



Interactive Whiteboard Lesson Library

Standard or customizable lessons are provided for SMART®, Promethean®, and Mimio® interactive whiteboards.



Program Resources

Print

Student Edition

Teaching Edition

Student Journal

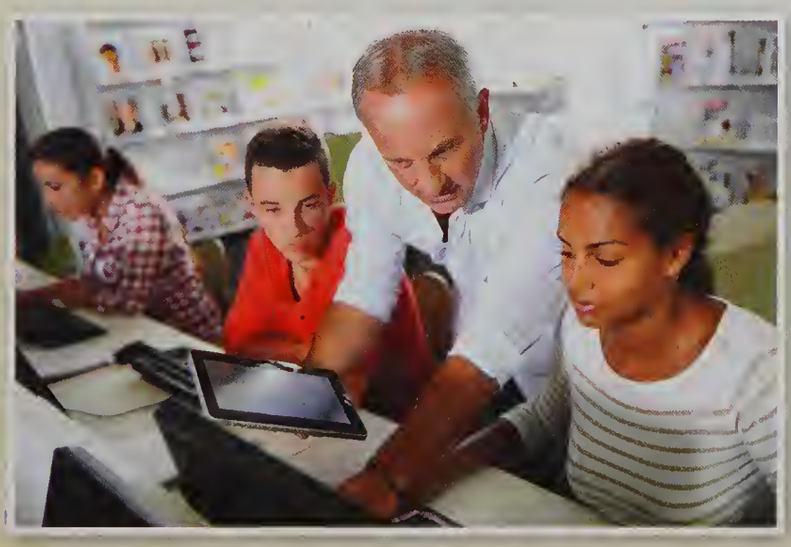
Available in English and Spanish

Resources by Chapter

- Start Thinking
- Warm Up
- Cumulative Review Warm Up
- Practice A and B
- Enrichment and Extension
- Puzzle Time
- Family Communication Letters
 Available in English and Spanish

Assessment Book

- Performance Tasks
- Prerequisite Skills Test with Item Analysis
- Quarterly Standards Based Tests
- Quizzes
- Chapter Tests
- Alternative Assessments with Scoring Rubrics
- Pre-Course Test with Item Analysis
- Post Course Test with Item Analysis



Technology

Student Edition

With complete English and Spanish audio

- Dynamic eBook App
- Online Home Edition
- eReader Format

Dynamic Classroom

- Interactive Manipulatives
- Multi-Language Glossary
- Vocabulary Flash Cards
- Worked-Out Solutions
- Lesson Tutorial Videos

Dynamic Teaching Tools

- Interactive Whiteboard Lesson Library
 - Includes standard and customizable lessons
 - Compatible with SMART®, Promethean®, and Mimio® technology
- Real-Life STEM Videos
- Editable Online Resources
 - Lesson Plans
 - Assessment Book
 - Resources by Chapter
- Answer Presentation Tool

Dynamic Assessment and Progress Monitoring Tool

Teacher Support for TEKS

- Differentiating the Lesson
- Laurie's Notes

Teaching English Language Learners

English language learners (ELLs) and their teachers face significant challenges in the mainstream high school mathematics classroom. In addition to linguistic challenges, ELLs are adjusting to a new culture and may have psychosocial issues that affect their ability to acclimate. Teachers may not be aware of all the linguistic and cultural confusion these students encounter. Teaching and learning across the gap of cultural and linguistic differences can be a challenge. The principles for teaching ELLs presented in the following pages are intended to bridge the gap and help ELLs pursue academic achievement.

High school mathematics teachers in Texas must address forty-three English Language Proficiency Standards (ELPS). One such standard is ELPS 4.F.1.

> **ELPS 4.F.1** **Use visual and contextual support to read grade-appropriate content area text.**

The ***Big Ideas Math*** program provides tools to meet each of the forty-three ELPS required for Algebra 1.

Language Proficiency

English language learners do not always become proficient in a linear fashion. Great gains one week may be offset by significant challenges later. Many ELLs develop passive language skills (listening, reading) more quickly than active language skills (speaking, writing), and understand more than they can express. Some are adept at speaking and listening, but weak in literacy skills. Still others are comfortable reading and writing but are hesitant to speak, or have difficulty understanding spoken English. A student may be an advanced listener, intermediate speaker, and beginning reader all at the same time.

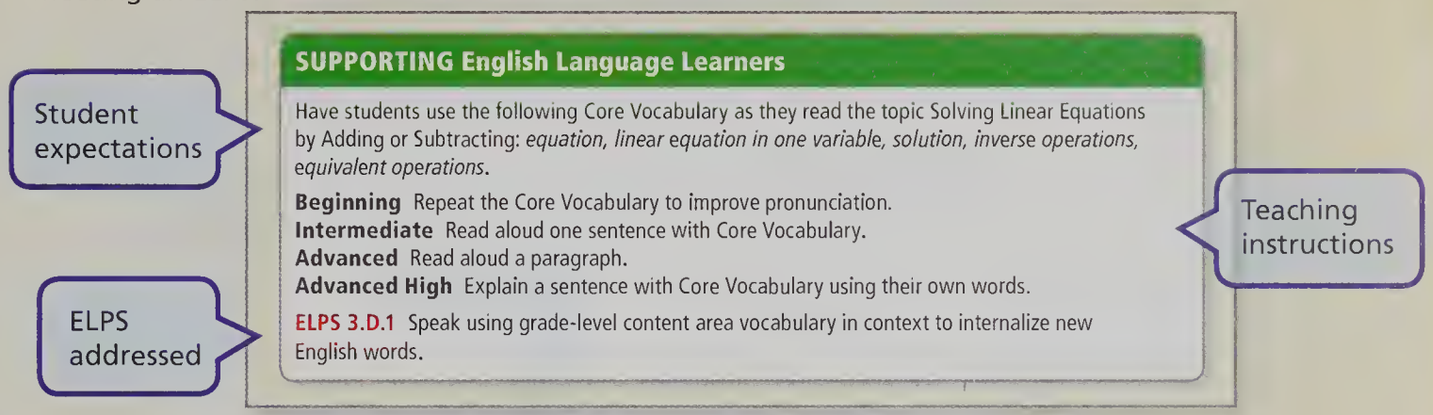
The Texas English Language Proficiency Assessment System (TELPAS) rates ELLs according to four levels of language proficiency under each of the four language domains of listening, speaking, reading, and writing.

- Beginning
- Intermediate
- Advanced
- Advanced High

See pages xxx–xxxi for an abbreviated summary of the ELPS-TELPAS Proficiency Level Descriptors used to rate the proficiency levels of ELLs for the language domains of listening, speaking, and reading. (Note: High school mathematics programs are not required to cover ELPS for the domain of writing.)

Teaching Notes for Supporting English Language Learners

The key ELPS support in the ***Big Ideas Math*** program takes the form of teaching notes entitled SUPPORTING English Language Learners. One such note appears in each section of the book, at the bottom of the appropriate page in the Teaching Edition, to support ELLs while meeting an ELPS.



Notice that student expectations are differentiated according to the language proficiency levels for the language domain of the ELPS addressed.

For example, ELPS 3.D.1 is a speaking proficiency. To meet this standard, a Beginning English speaker is expected to simply repeat the core vocabulary terms, while an Advanced English speaker is expected to read aloud the description of a line.

Three General Principles for Teaching ELLs

The Center for Applied Linguistics outlines three basic principles for improving the achievement of ELLs in a content area classroom. The three principles are to increase the following as you teach.

1. **Comprehension**
2. **Interaction**
3. **Thinking and study skills**

Practicing methods that promote these principles should result in success not only for your ELLs, but for all students in your classroom.

Teaching English Language Learners *(continued)*

1. Comprehension

Providing information that ELLs understand is crucial for their access to both language and content. Both verbal and nonverbal support must be considered to make mathematics more accessible to the student. Visual and experiential clues are especially important to Beginning and Intermediate ELLs. Here are some examples.

- illustrations that relate mathematics to everyday life
- charts and graphs that organize information visually
- experiential activities in which students construct models

Student Edition Exploration

EXPLORATION 2 **Solving by Completing the Square**

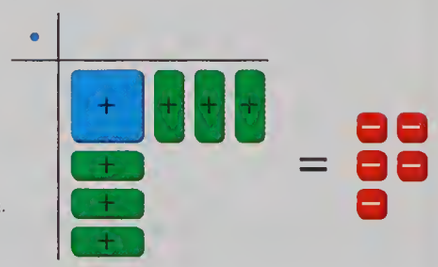
Work with a partner.

a. Write the equation modeled by the algebra tiles.

b. Use algebra tiles to "complete the square."

c. Write the solutions of the equation.

d. Check each solution in the original equation.



- graphics that illustrate processes visually
- providing both verbal and written summaries of processes
- native language reference materials

Having students read information as an introduction to a topic is not very helpful for students with limited language. It is more helpful to immerse them in an experience to help them gain familiarity with the topic and its vocabulary before expecting them to understand the details of the process. For example, students can use visuals or manipulatives as concepts are being introduced.

2. Interaction

Language is learned through communication with others. One negotiates meaning to accomplish real purposes. Students in a discussion will restate, question, explain, and clarify in order to reach a common understanding. This process helps them learn both language and mathematics content. Opportunities for interaction are provided throughout the program in the following ways.

- pair work
- group work
- classroom discussion
- hands on activities

SUPPORTING English Language Learners

Read aloud the Essential Question. Have students work in pairs of mixed language abilities to complete the explorations.

Beginning/Advanced Advanced students read aloud the instructions and help Beginning students by providing alternate language when necessary. Then they work together to complete the explorations. Beginning students read aloud the red side comments. If needed, Advanced students model the pronunciations by reading the side comments aloud.

Intermediate/Advanced High Advanced High students read aloud the instructions and guide the explorations. Intermediate students read aloud and paraphrase the red side comments.

ELPS 3.B.3 Expand and internalize initial English vocabulary by learning and using routine language needed for classroom communication.

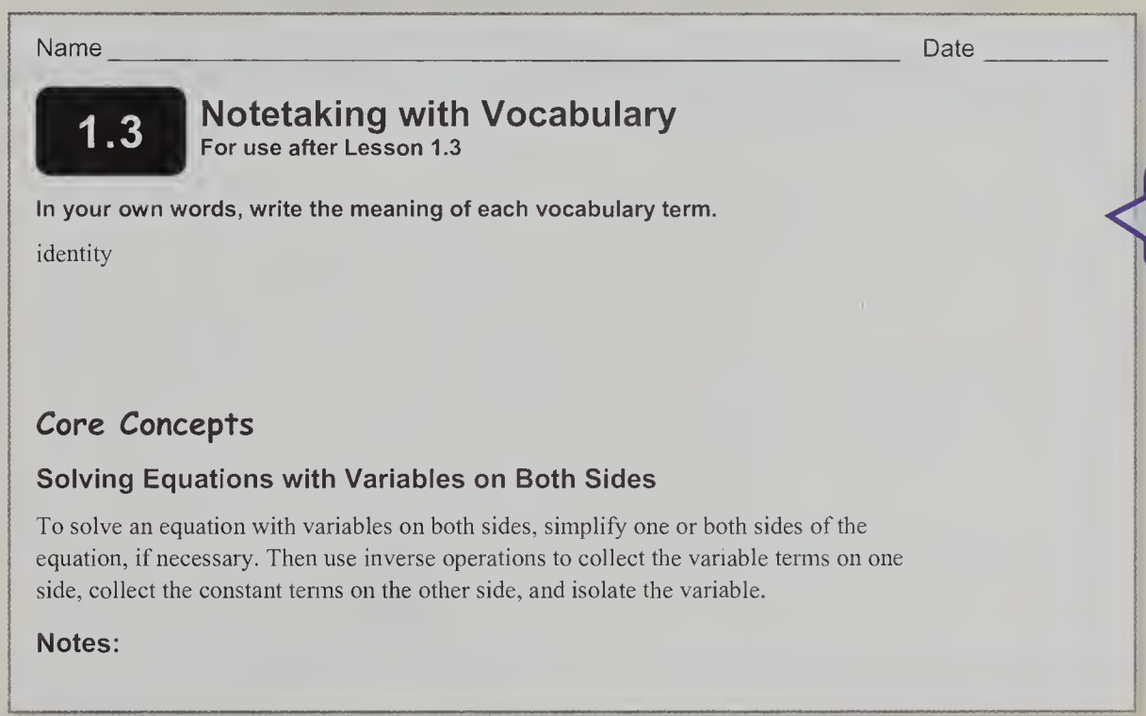
Supporting English Language Learners note from the Teaching Edition

These interactions also allow your students to build relationships that help you to provide a supportive, encouraging environment. This reduces the stress of learning a new language for your ELLs, and promotes real learning.

3. Thinking and Study Skills

Teaching academic skills explicitly helps develop language for ELLs and thinking skills for all students. Students should be given opportunities to perform higher order thinking skills, answer critical-thinking questions, and reinforce study skills. Below are several tools used to support these goals in this program.

- graphic organizers such as charts and tables that help order information
- text features that highlight key terms and concepts
- opportunities for note-taking



Name _____ Date _____

1.3 **Notetaking with Vocabulary**

For use after Lesson 1.3

In your own words, write the meaning of each vocabulary term.

identity

Core Concepts

Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, simplify one or both sides of the equation, if necessary. Then use inverse operations to collect the variable terms on one side, collect the constant terms on the other side, and isolate the variable.

Notes:

Student Journal

- questions that monitor progress
- opportunities to summarize and report information
- test taking practice

The ***Big Ideas Math*** program provides comprehensive support for teaching ELLs and all students. By promoting increased comprehension, interaction, and thinking and study skills as outlined above, you can help your ELLs improve their English language skills, and guide all of your students toward academic success.

Reference

Jameson, J. (1998) "Three Principles for Success: English Language Learners in Mainstream Content Classes," From Theory to Practice 6, Center for Applied Linguistics: Region XIV Comprehensive Center.

ELPS-TELPAS Proficiency Levels

The Texas English Language Proficiency Assessment System (TELPAS) measures the progress of English language learners (ELLs) working toward English Language Proficiency Standards (ELPS). ELPS-TELPAS Proficiency Level Descriptors are used to classify and monitor the language proficiency levels of ELLs as *Beginning*, *Intermediate*, *Advanced*, or *Advanced High* for the language domains of listening, speaking, reading, and writing. (Note: High school mathematics programs are not required to cover ELPS for the domain of writing.) A summary of the descriptors used in the ***Big Ideas Math*** program follows.

Summary of ELPS-TELPAS Proficiency Level Descriptors

	Beginning	Intermediate
Listening	Have little ability to understand spoken English • struggle to understand simple conversations even with familiar topics • struggle to distinguish words and phrases • may not seek clarification; frequently remain silent	Understand simple, high-frequency spoken English • understand simple conversations and short discussions on familiar topics • often distinguish key phrases necessary for general meaning in basic interaction • able to seek clarification by requesting to repeat, slow down, or rephrase
Speaking	Have little ability to speak English • hesitate to speak and use only single words or short, memorized phrases to meet immediate needs • use a limited bank of concrete vocabulary • lack the grammar to speak in sentences unless dealing with highly rehearsed or highly familiar material • exhibit errors that hinder overall communication • pronunciation inhibits communication	Speak in a simple, routine manner • use simple sentences and participate in short conversations; hesitate frequently • speak using basic vocabulary for everyday interactions; rarely use detail • emerging awareness of grammar, speak in simple sentences, mostly present tense • errors inhibit communication when using complex or less familiar English • pronunciation usually understandable by people accustomed to ELLs
Reading	Have little ability to read English • read and understand very limited, familiar English; vocabulary is mostly environmental print, some very high-frequency words, and concrete, visually supported words • read slowly, one word at a time • slight sense of English language structures • comprehend isolated familiar words and phrases, very familiar sentences • depend on visuals and prior knowledge to derive meaning from English text • can apply reading comprehension skills only with English text written for this level	Understand simple, high-frequency English • read and understand vocabulary on a wider range of topics, including everyday language, literal meanings, routine academic terms, and very commonly used abstract language • read slowly in short phrases; may re-read • have a growing understanding of basic, routine English language structures • understand simple sentences in short texts, but need visuals, prior knowledge, pretaught vocabulary, and human support • struggle to read independently • can apply basic comprehension skills when reading linguistically accommodated texts

Complete ELPS-TELPAS Proficiency Level Descriptors are at *www.tea.state.tx.us/student.assessment/ell/telpas*.

	Advanced	Advanced High
Listening	Understand grade-appropriate spoken English with support • usually understand longer conversations on familiar and some unfamiliar topics, but sometimes need support • understand most main points, key details, and some implicit information of basic oral language not modified for ELLs • occasionally need the speaker to repeat, slow down, or rephrase	Understand grade-appropriate spoken English with minimal support • understand long, elaborate conversations on familiar and unfamiliar topics with little dependence on support; some exceptions when complex language is used • understand main points, important details, and implicit information at a level nearly comparable to native English speakers • rarely need the speaker to repeat, slow down, or rephrase
Speaking	Use grade-appropriate English with support • comfortable in most conversations on familiar topics, with some pauses needed • can use content-based terms and common abstract vocabulary to discuss familiar topics; can speak on familiar topics in some detail • grasp basic grammar features, including an ability to use present, past, and future tenses; emerging ability to use complex language structures • errors hinder communication somewhat when using complex grammar, long sentences, and less familiar expressions • pronunciation is usually understandable by people unaccustomed to ELLs	Speak using grade-appropriate English, with minimal support • participate in extended discussions on a variety of topics with only occasional pauses • effective using abstract and content-based vocabulary when low-frequency vocabulary is needed, with some exceptions; comfortable with many native idioms and colloquialisms • use grammar structures and complex sentences to narrate and describe nearly as well as native English speakers • make few errors that hinder communication • mispronounce some words, but rarely interferes with communication
Reading	Read and understand grade-appropriate English with support • read and understand grade-appropriate vocabulary, such as concrete and abstract terms, phrases beyond literal meaning, and multiple meanings of common words • read longer phrases and simple sentences in familiar material at an appropriate speed • starting to comprehend grade-appropriate text through familiarity with English or by applying reading comprehension skills, with some support needed, especially when material is unfamiliar	Read and understand grade appropriate English with minimal support • read and understand at a level nearly comparable to native speakers, with some exceptions for specialized vocabulary • read familiar, grade-appropriate text with appropriate rate, intonation, and expression • can comprehend grade-appropriate text by applying reading comprehension skills with minimal support, or from familiarity with English

Correlation from Algebra 1 to TEKS

Lesson		TEKS
Chapter 1: Solving Linear Equations		
1.1	Solving Simple Equations	A.5.A
1.2	Solving Multi-Step Equations	A.5.A, A.10.D
1.3	Solving Equations with Variables on Both Sides	A.5.A, A.10.D
1.4	Rewriting Equations and Formulas	A.12.E
Chapter 2: Solving Linear Inequalities		
2.1	Writing and Graphing Inequalities	Preparing for A.5.B
2.2	Solving Inequalities Using Addition or Subtraction	A.5.B
2.3	Solving Inequalities Using Multiplication or Division	A.5.B
2.4	Solving Multi-Step Inequalities	A.5.B
2.5	Solving Compound Inequalities	A.5.B
Chapter 3: Graphing Linear Functions		
3.1	Functions	A.12.A
3.2	Linear Functions	A.2.A, A.3.C
3.3	Function Notation	A.3.C, A.12.B
3.4	Graphing Linear Equations in Standard Form	A.3.C
3.5	Graphing Linear Equations in Slope-Intercept Form	A.2.A, A.3.A, A.3.B, A.3.C
3.6	Modeling Direct Variation	A.2.D
3.7	Transformations of Graphs of Linear Functions	A.3.E
Chapter 4: Writing Linear Functions		
4.1	Writing Equations in Slope-Intercept Form	A.2.B, A.2.C, A.3.A
4.2	Writing Equations in Point-Slope Form	A.2.B, A.2.C, A.3.A
4.3	Writing Equations in Standard Form	A.2.B, A.2.C, A.3.A
4.4	Writing Equations of Parallel and Perpendicular Lines	A.2.C, A.2.E, A.2.F, A.2.G, A.3.A
4.5	Scatter Plots and Lines of Fit	A.4.C
4.6	Analyzing Lines of Fit	A.4.A, A.4.B, A.4.C
4.7	Arithmetic Sequences	A.12.D
Chapter 5: Solving Systems of Linear Equations		
5.1	Solving Systems of Linear Equations by Graphing	A.2.I, A.3.F, A.3.G, A.5.C
5.2	Solving Systems of Linear Equations by Substitution	A.2.I, A.5.C
5.3	Solving Systems of Linear Equations by Elimination	A.2.I, A.5.C
5.4	Solving Special Systems of Linear Equations	A.2.I, A.3.F, A.5.C
5.5	Solving Equations by Graphing	A.5.A
5.6	Linear Inequalities in Two Variables	A.2.H, A.3.D
5.7	Systems of Linear Inequalities	A.3.H

Lesson		TEKS
Chapter 6: Exponential Functions and Sequences		
6.1	Properties of Exponents	A.11.B
6.2	Radicals and Rational Exponents	A.11.B
6.3	Exponential Functions	A.9.A, A.9.B, A.9.C, A.9.D, A.9.E
6.4	Exponential Growth and Decay	A.9.B, A.9.C, A.9.D
6.5	Geometric Sequences	A.12.D
6.6	Recursively Defined Sequences	A.12.C
Chapter 7: Polynomial Equations and Factoring		
7.1	Adding and Subtracting Polynomials	A.10.A
7.2	Multiplying Polynomials	A.10.B, A.10.D
7.3	Special Products of Polynomials	A.10.B
7.4	Dividing Polynomials	A.10.C
7.5	Solving Polynomial Equations in Factored Form	A.8.A
7.6	Factoring $x^2 + bx + c$	A.8.A, A.10.E
7.7	Factoring $ax^2 + bx + c$	A.8.A, A.10.D, A.10.E
7.8	Factoring Special Products	A.8.A, A.10.E, A.10.F
7.9	Factoring Polynomials Completely	A.10.D, A.10.E
Chapter 8: Graphing Quadratic Functions		
8.1	Graphing $f(x) = ax^2$	A.6.A, A.7.A, A.7.C
8.2	Graphing $f(x) = ax^2 + c$	A.7.A, A.7.C
8.3	Graphing $f(x) = ax^2 + bx + c$	A.6.A, A.7.A
8.4	Graphing $f(x) = a(x - h)^2 + k$	A.6.B, A.7.A, A.7.C
8.5	Using Intercept Form	A.6.A, A.6.B, A.6.C, A.7.A, A.7.B
8.6	Comparing Linear, Exponential, and Quadratic Functions	A.2.C, A.6.C, A.9.C
Chapter 9: Solving Quadratic Equations		
9.1	Properties of Radicals	A.11.A
9.2	Solving Quadratic Equations by Graphing	A.7.A, A.8.B
9.3	Solving Quadratic Equations Using Square Roots	A.8.A
9.4	Solving Quadratic Equations by Completing the Square	A.8.A
9.5	Solving Quadratic Equations Using the Quadratic Formula	A.8.A

Correlation from TEKS to Algebra 1

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.A** apply mathematics to problems arising in everyday life, society, and the workplace	**A.1.A.i** apply mathematics to problems arising in everyday life	Appears throughout, including: 1.2, p. 14 Example 4 4.7, p. 205 Example 5	Appears throughout, including: 3.1, p. 95 Exercises 22, 33 4.3, p. 178 Exercises 30, 32 5.5, p. 250 Exercises 32, 33
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.A** apply mathematics to problems arising in everyday life, society, and the workplace	**A.1.A.ii** apply mathematics to problems arising in society	Appears throughout, including: 4.1, p. 164 Example 5 5.3, p. 234 Example 3 8.6, p. 450 Example 5	Appears throughout, including: 4.6, pp. 198–200 Exercises 10, 17, 18, 27, 31 6.4, pp. 305–307 Exercises 17, 40, 59
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.A** apply mathematics to problems arising in everyday life, society, and the workplace	**A.1.A.iii** apply mathematics to problems arising in the workplace	Appears throughout, including: 4.4, p. 182 Example 6 5.1, p. 222 Example 3 5.5, p. 247 Example 2	Appears throughout, including: 4.4, p. 184 Exercises 29, 30 5.6, pp. 256, 258 Exercises 17, 46
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.B** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution	**A.1.B.i** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process	Appears throughout, including: 2.2, p. 56 Example 3 3.3, p. 110 Example 5 3.4, p. 119 Example 5	Appears throughout, including: 2.2, p. 57 Exercises 25, 26 3.3, p. 112 Exercises 29, 30
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.B** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution	**A.1.B.ii** use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the reasonableness of the solution	Appears throughout, including: 1.1, pp. 6–7 Examples 3 and 4 1.2, pp. 14–15 Examples 4 and 5 2.3, p. 62 Example 3	Appears throughout, including: 1.1, p. 9 Exercises 42–44, 46, 47 1.2, p. 17 Exercises 35–37
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.C** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems	**A.1.C.i** select tools, including real objects as appropriate, to solve problems	Appears throughout, including: 6.5, p. 311 Exploration 2	Appears throughout, including: 4.5, p. 192 Exercise 19
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.C** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems	**A.1.C.ii** select tools, including manipulatives as appropriate, to solve problems	Appears throughout, including: 7.1, p. 337 Explorations 1 and 2 7.8, p. 383 Explorations 1 and 2 9.4, p. 493 Explorations 1 and 2	Appears throughout, including: 7.6, p. 376 Exercise 42 7.8, p. 388 Exercise 44

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.C** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems	**A.1.C.iii** select tools, including paper and pencil as appropriate, to solve problems	Appears throughout, including: 3.2, p. 102 Example 6 4.5, p. 187 Explorations 1 and 2	Appears throughout, including: 3.2, pp. 105–106 Exercises 39–42, 53, 54 3.5, p. 132 Exercises 47, 48, 49(c), 50(a) 4.4, p. 184 Exercise 38
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.C** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems	**A.1.C.iv** select tools, including technology as appropriate, to solve problems	Appears throughout, including: 4.6, pp. 196–197 Examples 3 and 4(b) 5.5, p. 248 Example 3 6.3, p. 295 Example 7	Appears throughout, including: 4.6, pp. 198–200 Exercises 11–20, 27, 31 5.5, p. 250 Exercises 29, 30
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.C** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems	**A.1.C.v** select techniques, including mental math as appropriate, to solve problems	Appears throughout, including: 1.2, p. 15 Example 5	Appears throughout, including: 1.1, p. 9 Exercise 48
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.C** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems	**A.1.C.vi** select techniques including estimation as appropriate, to solve problems	Appears throughout, including: 4.1, p. 161 Exploration 2(b) 4.6, p. 197 Example 4(b)	Appears throughout, including: 3.1, p. 96 Exercise 34(b), 34(c) 4.1, p. 166 Exercise 36(a) 5.3, p. 236 Exercise 28(a)
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.C** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems	**A.1.C.vii** select techniques, including number sense as appropriate, to solve problems	Appears throughout, including: 1.2, p. 14 Example 4 3.4, p. 119 Example 5 (Step 4)	Appears throughout, including: 1.3, p. 26 Exercise 37 3.4, p. 122 Exercise 59 4.5, p. 191 Exercises 3-6
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.i** communicate mathematical ideas using multiple representations, including symbols as appropriate	Appears throughout, including: 2.1, p. 46 Teaching text, Example 1 2.5, p. 74 Teaching text, Example 1 3.2, p. 99 Concept Summary	Appears throughout, including: 2.1, pp. 50–51 Exercises 5-14, 37–40 2.5, p. 77 Exercises 7–12
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.ii** communicate mathematical ideas using multiple representations, including diagrams as appropriate	Appears throughout, including: 2.4, p. 70 Example 4 3.1, p. 89 Exploration 2 3.2, p. 99 Concept Summary 5.6, p. 254 Core Concept and Table	Appears throughout, including: 1.3, p. 26 Exercise 38

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.iii** communicate mathematical ideas using multiple representations, including graphs as appropriate	Appears throughout, including: 2.1, p. 49 Concept Summary 2.5, p. 74 Teaching text, Example 1 5.4, p. 238 Core Concept	Appears throughout, including: 1.3, p. 26 Exercise 38 2.5, p. 77 Exercises 7–11, 13–20
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.iv** communicate mathematical ideas using multiple representations, including language as appropriate	Appears throughout, including: 2.1, p. 45 Explorations 1 and 2 2.5, p. 74 Teaching text, Example 1 5.6, p. 254 Core Concept and Table	Appears throughout, including: 2.1, pp. 50–51 Exercises 5-14 2.5, p. 77 Exercises 7–12
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.v** communicate mathematical reasoning using multiple representations, including symbols as appropriate	Appears throughout, including: 1.1, p. 7 Example 4 3.7, pp. 140–141 Core Concept, Example 1, Core Concept, Example 2	Appears throughout, including: 3.1, p. 95 Exercise 29 7.8, p. 388 Exercise 46
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.vi** communicate mathematical reasoning using multiple representations, including diagrams as appropriate	Appears throughout, including: 2.2, pp. 54–55 Teaching text before Example 1, Teaching text before Example 2 3.2, p. 100 Example 4(b)	Appears throughout, including: 3.1, p. 95 Exercise 30 7.8, p. 388 Exercise 46
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.vii** communicate mathematical reasoning using multiple representations, including graphs as appropriate	Appears throughout, including: 3.5, p. 125 Concept Summary 3.7, pp. 141–142 Core Concept, Example 2, Core Concept, Example 3	Appears throughout, including: 3.1, p. 95 Exercises 29, 30 3.7, pp. 145–146 Exercises 7–10, 15, 16, 19–22, 25–34, 41–46
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.viii** communicate mathematical reasoning using multiple representations, including language as appropriate	Appears throughout, including: 1.3, p. 24 Example 4 2.2, p. 56 Example 3 3.5, p. 125 Concept Summary	Appears throughout, including: 3.1, p. 95 Exercise 29 3.5, p. 132 Exercise 47
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.ix** communicate [mathematical ideas'] implications using multiple representations, including symbols as appropriate	Appears throughout, including: 3.5, p. 123 Exploration 2 6.6, p. 319 Exploration 1 7.1, p. 337 Explorations 1 and 2	Appears throughout, including: 7.6, p. 376 Exercise 42 7.7, p. 382 Exercise 42

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.x** communicate [mathematical ideas'] implications using multiple representations, including diagrams as appropriate	Appears throughout, including: 4.6, pp. 194–195 Examples 1 and 2 7.3, p. 351 Explorations 1 and 2 7.4, p. 357 Explorations 1 and 2	Appears throughout, including: 7.1, p. 343 Exercises 51–53 7.6, p. 376 Exercise 42
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.xi** communicate [mathematical ideas'] implications using multiple representations, including graphs as appropriate	Appears throughout, including: 3.2, p. 97 Exploration 1 3.5, p. 123 Explorations 1 and 2 4.6, pp. 194–195 Examples 1 and 2	Appears throughout, including: 8.6, p. 453 Exercise 33
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.xii** communicate [mathematical ideas'] implications using multiple representations, including language as appropriate	Appears throughout, including: 3.5, p. 123 Exploration 2 6.6, p. 319 Exploration 1	Appears throughout, including: 7.1, p. 343 Exercises 51–53
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.xiii** communicate [mathematical reasoning's] implications using multiple representations, including symbols as appropriate	Appears throughout, including: 1.4, p. 27 Explorations 1 and 2 4.7, p. 201 Exploration 1 7.1, p. 338 Teaching text	Appears throughout, including: 7.7, p. 382 Exercise 42 7.8, p. 388 Exercise 44
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.xiv** communicate [mathematical reasoning's] implications using multiple representations, including diagrams as appropriate	Appears throughout, including: 2.3, p. 59 Explorations 1 and 2 4.7, p. 203 Example 2 7.3, p. 352 Teaching text	Appears throughout, including: 8.3, p. 423 Exercise 37 9.1, p. 474 Exercises 99, 100
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.xv** communicate [mathematical reasoning's] implications using multiple representations, including graphs as appropriate	Appears throughout, including: 3.4, p. 119 Example 5 4.7, p. 203 Example 2 6.4, p. 303 Example 6	Appears throughout, including: 3.4, p. 121 Exercises 49, 50 8.3, p. 423 Exercise 37
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.D** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	**A.1.D.xvi** communicate [mathematical reasoning's] implications using multiple representations, including language as appropriate	Appears throughout, including: 2.3, p. 59 Explorations 1 and 2 7.1, p. 338 Teaching text 7.3, p. 352 Teaching text	Appears throughout, including: 3.4, p. 121 Exercises 49, 50 7.1, p. 343 Exercises 51–53
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	**A.1.E** create and use representations to organize, record, and communicate mathematical ideas	**A.1.E.i** create representations to organize mathematical ideas	Appears throughout, including: 2.5, p. 74 Example 1 5.5, pp. 247–248 Examples 2 and 3	Appears throughout, including: 4.5, p. 192 Exercises 13, 14, 24 4.6, pp. 198, 200 Exercises 5–8, 31(c)

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.E create and use representations to organize, record, and communicate mathematical ideas	A.1.E.ii create representations to record mathematical ideas	Appears throughout, including: 2.3, p. 59 Explorations 1 and 2 3.2, pp. 100–101 Examples 4(b) and 5(c) 4.6, pp. 194–195 Examples 1 and 2	Appears throughout, including: 3.1, p. 95 Exercise 30(b) 4.5, p. 192 Exercises 13, 14, 19(a), 24
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.E create and use representations to organize, record, and communicate mathematical ideas	A.1.E.iii create representations to communicate mathematical ideas	Appears throughout, including: 3.2, p. 97 Exploration 1 4.5, p. 187 Explorations 1 and 2 4.6, pp. 194–195 Examples 1 and 2	Appears throughout, including: 4.4, p. 184 Exercise 38 4.5, p. 192 Exercises 13, 14, 19(a), 24
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.E create and use representations to organize, record, and communicate mathematical ideas	A.1.E.iv use representations to organize mathematical ideas	Appears throughout, including: 2.1, pp. 46–47 Tables before Examples 1 and 2 3.7, p. 144 Examples 5 and 6 6.6, p. 319 Exploration 1 7.3, p. 352 Teaching text, figure	Appears throughout, including: 2.5, p. 78 Exercise 33
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.E create and use representations to organize, record, and communicate mathematical ideas	A.1.E.v use representations to record mathematical ideas	Appears throughout, including: 3.5, p. 123 Exploration 2 3.7, p. 144 Examples 5 and 6 7.3, p. 354 Example 4	Appears throughout, including: 6.3, p. 297 Exercises 37–40 7.3, pp. 355–356 Exercises 11–14, 33, 35, 38
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.E create and use representations to organize, record, and communicate mathematical ideas	A.1.E.vi use representations to communicate mathematical ideas	Appears throughout, including: 2.2, pp. 54–55 Diagrams before Examples 1 and 2 3.1, p. 92 Core Concept 3.2, p. 99 Concept Summary	Appears throughout, including: 3.1, p. 94 Exercises 5–8, 13–16 7.1, p. 344 Exercises 60, 61
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.F analyze mathematical relationships to connect and communicate mathematical ideas	A.1.F.i analyze mathematical relationships to connect mathematical ideas	Appears throughout, including: 3.1, p. 89 Exploration 2 3.5, p. 123 Exploration 2 5.5, p. 245 Explorations 1 and 2	Appears throughout, including: 3.4, p. 122 Exercise 52 9.1, p. 474 Exercises 99, 100
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.F analyze mathematical relationships to connect and communicate mathematical ideas	A.1.F.ii analyze mathematical relationships to communicate mathematical ideas	Appears throughout, including: 1.1, p. 3 Explorations 1–3 3.2, p. 102 Example 6 6.5, p. 311 Exploration 2	Appears throughout, including: 3.1, p. 96 Exercise 34 3.2, pp. 105–106 Exercises 39–42, 53, 54
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication	A.1.G.i display mathematical ideas using precise mathematical language in written or oral communication	Appears throughout, including: 2.5, p. 74 Teaching text, Example 1 3.7, p. 144 Example 6 4.6, p. 194 Core Concept	Appears throughout, including: 2.5, pp. 77–78 Exercises 7–12, 32 4.4, p. 184 Exercise 38

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication	A.1.G.ii display mathematical arguments using precise mathematical language in written or oral communication	Appears throughout, including: 3.4, p. 119 Example 5 6.6, p. 319 Exploration 1	Appears throughout, including: 1.3, p. 26 Exercise 40 2.5, p. 78 Exercise 33 6.6, p. 326 Exercise 54
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication	A.1.G.iii explain mathematical ideas using precise mathematical language in written or oral communication	Appears throughout, including: 3.2, p. 102 Example 6 5.3, p. 232 Teaching text 6.4, p. 302 Example 3	Appears throughout, including: 3.2, pp. 105–106 Exercises 39–42, 53, 54 7.4, p. 362 Exercise 39
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication	A.1.G.iv explain mathematical arguments using precise mathematical language in written or oral communication	Appears throughout, including: 2.3, p. 59 Explorations 1 and 2	Appears throughout, including: 3.1, p. 96 Exercise 34 3.5, p. 132 Exercise 48 4.3, p. 178 Exercise 31 7.4, p. 362 Exercise 45
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication	A.1.G.v justify mathematical ideas using precise mathematical language in written or oral communication	Appears throughout, including: 1.2, p. 11 Exploration 1 4.2, p. 167 Exploration 2 7.1, p. 337 Explorations 1 and 2	Appears throughout, including: 1.2, p. 17 Exercises 38, 39 6.4, p. 308 Exercise 64
A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	A.1.G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication	A.1.G.vi justify mathematical arguments using precise mathematical language in written or oral communication	Appears throughout, including: 3.2, p. 97 Exploration 1 7.3, p. 354 Example 4(b)	Appears throughout, including: 4.5, p. 192 Exercise 21 7.8, p. 388 Exercise 45 9.1, p. 474 Exercises 99, 100
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities	A.2.A.i determine the domain of a linear function in mathematical problems	3.2, pp. 100-101 Examples 4(a) and 5(b) 3.5, p. 128 Example 6(a)	3.1, pp. 95-96 Exercises 22(b), 45(c) 3.2, pp. 104–105 Exercises 27–34, 35(a), 36(a), 37(b), 38(b) 3.5, p. 130 Exercises 37(a), 38(a)
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities	A.2.A.ii determine the range of a linear function in mathematical problems	3.1, p. 93 Example 5(b) 3.5, p. 128 Example 6(a)	3.1, pp. 95–96 Exercises 21(b), 22(b), 45(c) 3.5, p. 130 Exercises 37(a), 38(a)
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities	A.2.A.iii determine reasonable domain values for real-world situations, both continuous and discrete	3.2, p. 102 Example 6	3.2, pp. 105–106 Exercises 39–42, 53, 54

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities	A.2.A.iv determine reasonable range values for real-world situations, both continuous and discrete	3.2, p. 102 Example 6	3.2, pp. 105–106 Exercises 39–42, 53, 54
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities	A.2.A.v represent domain using inequalities	3.1, p. 92 Example 4(b) 3.5, p. 128 Example 6(a)	3.1, p. 94 Exercises 19, 20, 22(b) 3.5, p. 130 Exercises 37(a), 38(a)
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities	A.2.A.vi represent range using inequalities	3.1, p. 92 Example 4(b) 3.5, p. 128 Example 6(a)	3.1, p. 94 Exercises 19, 20, 22(b) 3.5, p. 130 Exercises 37(a), 38(a)
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points	A.2.B.i write linear equations in two variables in various forms, including $y = mx + b$ given one point and the slope	3.5, p. 126 Core Concept 4.1, p. 161 Exploration 2 4.1, p. 162 Example 1	4.1, p. 165 Exercises 2–8
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points	A.2.B.ii write linear equations in two variables in various forms, including $Ax + By = C$, given two points	4.3, p. 174 Example 2	4.3, pp. 177–178 Exercises 1, 15–18
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points	A.2.B.iii write linear equations in two variables in various forms, including $y - y_1 = m(x - x_1)$, given one point and the slope	4.2, p. 167 Explorations 2 and 3 4.2, pp. 168–170 Teaching text, Core Concept, Example 2, Teaching text, Examples 3–5	4.2, p. 171 Exercises 7–38
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	A.2.C write linear equations in two variables given a table of values, a graph, and a verbal description	A.2.C.i write linear equations in two variables given a table of values	4.2, p. 170 Example 5	4.1, p. 165 Exercise 26 4.2, pp. 171–172 Exercises 31-34, 38, 44

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.C** write linear equations in two variables given a table of values, a graph, and a verbal description	**A.2.C.ii** write linear equations in two variables given a graph	4.1, p. 162 Example 2 4.2, p. 169 Example 3	4.1, pp. 165–166 Exercises 9–12, 28, 35 4.2, pp. 171–172 Exercises 15–18, 44
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.C** write linear equations in two variables given a table of values, a graph, and a verbal description	**A.2.C.iii** write linear equations in two variables given a verbal description	4.1, pp. 162–164 Examples 1, 3, 4, and 5 4.2, p. 167 Exploration 3 4.2, pp. 168–169 Examples 2 and 4	4.1, pp. 165–166 Exercises 3–8, 13–24, 27, 29(a), 30(a), 31, 34, 37 4.2, pp. 171–172 Exercises 7–14, 19–30, 37(a)
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.D** write and solve equations involving direct variation	**A.2.D.i** write equations involving direct variation	3.6, pp. 135–136 Teaching text, Examples 3 and 4	3.6, pp. 137–138 Exercises 17–22, 25(b), 26(b), 27, 28
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.D** write and solve equations involving direct variation	**A.2.D.ii** solve equations involving direct variation	3.6, pp. 134, 136 Examples 1 and 4	3.6, pp. 137–138 Exercises 3–10, 25(c), 26(c), 27, 28
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.E** write the equation of a line that contains a given point and is parallel to a given line	**A.2.E.i** write the equation of a line that contains a given point and is parallel to a given line	4.4, p. 180 Example 2	4.4, pp. 183–184 Exercises 9–12, 27, 30
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.F** write the equation of a line that contains a given point and is perpendicular to a given line	**A.2.F.i** write the equation of a line that contains a given point and is perpendicular to a given line	4.4, pp. 181–182 Examples 4 and 6	4.4, pp. 183–184 Exercises 19-22, 28, 29, 33
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.G** write an equation of a line that is parallel or perpendicular to the X or Y axis and determine whether the slope of the line is zero or undefined	**A.2.G.i** write an equation of a line that is parallel or perpendicular to the X or Y axis	4.4, p. 182 Example 5	4.4, p. 183 Exercises 23-26

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.G** write an equation of a line that is parallel or perpendicular to the X or Y axis and determine whether the slope of the line is zero or undefined	**A.2.G.ii** determine whether the slope of the line is zero or undefined	4.4, p. 182 Example 5	4.4, p. 182 Monitoring Progress Question 5
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.H** write linear inequalities in two variables given a table of values, a graph, and a verbal description	**A.2.H.i** write linear inequalities in two variables given a table of values	5.6, p. 254 Example 5	5.6, p. 257 Exercises 39–41, 43
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.H** write linear inequalities in two variables given a table of values, a graph, and a verbal description	**A.2.H.ii** write linear inequalities in two variables given a graph	5.6, p. 251 Exploration 1(c) 5.6, p. 254 Core Concept, Example 4 5.7, p. 262 Examples 4 and 5	5.6, pp. 257–258 Exercises 33–38, 42, 50 5.7, p. 264 Exercises 21–26
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.H** write linear inequalities in two variables given a table of values, a graph, and a verbal description	**A.2.H.iii** write linear inequalities in two variables given a verbal description	5.6, p. 255 Example 6	5.6, pp. 257-258 Exercises 44-47, 48(a), 52–54
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.I** write systems of two linear equations given a table of values, a graph, and a verbal description	**A.2.I.i** write systems of two linear equations given a table of values	5.4, p. 239 Example 3 5.5, p. 247 Example 2	5.4, p. 242 Exercises 25, 27, 32
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.I** write systems of two linear equations given a table of values, a graph, and a verbal description	**A.2.I.ii** write systems of two linear equations given a graph	5.3, p. 234 Example 3	5.3, pp. 235–236 Exercises 22, 28(b)
A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:	**A.2.I** write systems of two linear equations given a table of values, a graph, and a verbal description	**A.2.I.iii** write systems of two linear equations given a verbal description	5.1, p. 222 Example 3 5.2, p. 228 Example 3	5.1, p. 224 Exercises 27–29, 33 5.2, pp. 229–230 Exercises 19, 20, 21–24, 25, 30, 31, 33, 34, 35

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.A** determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$	**A.3.A.i** determine the slope of a line given a table of values	3.5, p. 125 Example 2 4.2, p. 170 Example 5	3.5, pp. 129–130 Exercises 9–12, 14, 39 4.2, pp. 171–172 Exercises 31–34, 38(a), 44
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.A** determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$	**A.3.A.ii** determine the slope of a line given a graph	3.5, p. 124 Core Concept, Example 1 4.1, p. 162 Example 2	3.5, p. 129 Exercises 5–8, 13 4.1, pp. 165–166 Exercises 9-12, 28, 36(a)
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.A** determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$	**A.3.A.iii** determine the slope of a line given two points on the line	4.1, p. 163 Example 3 4.2, p. 169 Example 3	4.1, p. 165 Exercises 13–18, 28 4.2, p. 171 Exercises 15–24
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.A** determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$	**A.3.A.iv** determine the slope of a line given an equation written in various forms, including $y = mx + b$	3.5, pp. 126–127 Core Concept, Examples 3 and 4 4.4, pp. 180–181 Examples 2, 3, and 4	3.5, p. 130 Exercises 15–18, 23-32 4.4, p. 183 Exercises 7–12, 17, 18, 19–22
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.A** determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$	**A.3.A.v** determine the slope of a line given an equation written in various forms, including $Ax + By = C$	3.5, pp. 126–127 Examples 3(c) and 4 4.3, p. 173 Exploration 2	3.5, pp. 130–131 Exercises 19–22, 29–32, 41 4.1, p. 166 Exercise 33
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.A** determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$	**A.3.A.vi** determine the slope of a line given an equation written in various forms, including $y - y_1 = m(x - x_1)$	4.2, p. 168 Example 1	4.2, p. 171 Exercises 3–6

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.B calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems	A.3.B.i calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical problems	3.5, pp. 124–126 Core Concept, Examples 1–3 4.1, p. 163 Example 4	3.5, pp. 129–130 Exercises 5–12, 15–24 4.1, p. 165 Exercises 19–24
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.B calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems	A.3.B.ii calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of real-world problems	3.5, p. 128 Example 6 4.1, p. 164 Example 5	3.5, pp. 129–130 Exercises 13, 14, 35-39 4.1, p. 166 Exercises 29, 30
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	A.3.C.i graph linear functions on the coordinate plane in mathematical problems	3.3, p. 109 Example 4 3.4, p. 117 Core Concept, Example 2	3.3, p. 111 Exercises 23–28 3.4, p. 120 Exercises 19–28
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	A.3.C.ii graph linear functions on the coordinate plane in real-world problems	3.3, p. 110 Example 5 3.4, p. 119 Example 5	3.3, p. 112 Exercise 29 3.4, p. 121 Exercises 49(a), 50(a)
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	A.3.C.iii identify key features, including *x*-intercept in mathematical problems	3.4, p. 117 Core Concept, Example 2 3.5, p. 127 Examples 4 and 5	3.4, p. 120 Exercises 9–28 3.5, p. 130 Exercises 25–34
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	A.3.C.iv identify key features, including *x*-intercept in real-world problems	3.4, p. 119 Example 5(a) 3.5, p. 128 Example 6	3.4, pp. 121–122 Exercises 49(a), 50(a), 52, 58(a) 3.5, p. 130 Exercise 36

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.C** graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	**A.3.C.v** identify key features, including *y*-intercept in mathematical problems	3.4, p. 117 Core Concept, Example 2 3.5, pp. 126–127 Core Concept, Examples 4 and 5	3.4, p. 120 Exercises 9–28 3.5, p. 130 Exercises 15–22, 33, 34
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.C** graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	**A.3.C.vi** identify key features, including *y*-intercept in real-world problems	3.4, p. 119 Example 5(a) 3.5, p. 128 Example 6	3.4, pp. 121–122 Exercises 49(a), 50(a), 52, 58(a) 3.5, p. 130 Exercises 35–39
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.C** graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	**A.3.C.vii** identify key features, including zeros in mathematical problems	3.4, p. 118 Example 3	3.4, p. 121 Exercises 31–44, 48
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.C** graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	**A.3.C.viii** identify key features, including zeros in real-world problems	3.4, p. 118 Example 4	3.4, p. 121 Exercises 45, 46, 47
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.C** graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	**A.3.C.ix** identify key features, including slope in mathematical problems	3.5, p. 123 Teaching text, Explorations 1 and 2 3.5, p. 127 Example 4	3.5, p. 130 Exercises 25–34
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.C** graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems	**A.3.C.x** identify key features, including slope in real-world problems	3.5, p. 128 Example 6(b)	3.5, pp. 129–130 Exercises 13, 35, 36, 37(b), 38(b), 39

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.D graph the solution set of linear inequalities in two variables on the coordinate plane	A.3.D.i graph the solution set of linear inequalities in two variables on the coordinate plane	5.6, pp. 253, 255 Core Concept, Examples 3 and 6 5.7, p. 261 Examples 2 and 3	5.6, pp. 256–258 Exercises 25–32, 44-47, 48(a), 50, 53, 54 5.7, p. 264 Exercises 11–20
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d	A.3.E.i determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$ for specific values of a	3.7, pp. 142–143 Core Concept, Examples 3(b) and 4(b)	3.7, pp. 145–146 Exercises 2, 21, 22, 24, 25, 28, 29, 30, 33, 37
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d	A.3.E.ii determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(x) + d$ for specific values of d	3.7, p. 139 Explorations 1, 3 3.7, p. 140 Core Concept, Example 1(a)	3.7, pp. 145–146 Exercises 2, 5, 7, 8, 12, 34
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d	A.3.E.iii determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(x - c)$ for specific values of c	3.7, p. 140 Core Concept, Example 1(b)	3.7, pp. 145–146 Exercises 2, 6, 9, 10, 11, 31, 35, 39
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d	A.3.E.iv determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(bx)$ for specific values of b	3.7, pp. 142, 143 Core Concept, Examples 3(a) and 4(a)	3.7, pp. 145–146 Exercises 2, 18, 19, 20, 23, 26, 27, 32, 36, 38, 40
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	A.3.F graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist	A.3.F.i graph systems of two linear equations in two variables on the coordinate plane	5.1, pp. 221–222 Core Concept, Examples 2 and 3 5.4, p. 238 Core Concept, Example 1 5.5, pp. 246–248 Examples 1, 2, and 3	5.1, pp. 223–224 Exercises 13–20, 23–29, 31, 33(a) 5.4, p. 241 Exercises 3–16, 23

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.F** graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist	**A.3.F.ii** determine the solutions if they exist	5.4, pp. 238, 240 Core Concept, Examples 1 and 4	5.4, p. 241 Exercises 3–16, 23
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.G** estimate graphically the solutions to systems of two linear equations with two variables in real-world problems	**A.3.G.i** estimate graphically the solutions to systems of two linear equations with two variables in real-world problems	5.1, p. 222 Example 3 5.5, p. 247 Example 2	5.1, p. 224 Exercises 27, 28, 32(a), 33(b) 5.4, p. 242 Exercise 30 5.5, p. 250 Exercises 31–33, 36
A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:	**A.3.H** graph the solution set of systems of two linear inequalities in two variables on the coordinate plane	**A.3.H.i** graph the solution set of systems of two linear inequalities in two variables on the coordinate plane	5.7, pp. 261, 263 Core Concept, Examples 2, 3 and 6	5.7, pp. 264–265 Exercises 11–20, 27–31
A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:	**A.4.A** calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association	**A.4.A.i** calculate, using technology, the correlation coefficient between two quantitative variables	4.6, p. 196 Example 3	4.6, pp. 198–199 Exercises 11–14, 17(b), 18(b), 19(b), 20(b)
A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:	**A.4.A** calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association	**A.4.A.ii** interpret this quantity as a measure of the strength of the linear association	4.6, p. 196 Example 3(b)	4.6, pp. 198–199 Exercises 11–14, 16, 17(b), 18(b), 19(b), 20(b)
A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:	**A.4.B** compare and contrast association and causation in real-world problems	**A.4.B.i** compare and contrast association and causation in real-world problems	4.6, p. 197 Teaching text, Example 5	4.6, pp. 199–200 Exercises 21-24, 25, 27(d), 30, 31(b)
A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:	**A.4.C** write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems	**A.4.C.i** write, with technology, linear functions that provide a reasonable fit to data to estimate solutions	4.6, p. 196 Example 3	4.6, pp. 198–199 Exercises 11-14, 17, 18, 19(a), 20(a)

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:	A.4.C write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems	A.4.C.ii write, without technology, linear functions that provide a reasonable fit to data to estimate solutions	4.5, p. 190 Core Concept, Example 3	4.5, p. 192 Exercises 15, 16, 24
A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:	A.4.C write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems	A.4.C.iii make predictions for real-world problems	4.6, p. 197 Teaching text, Example 4	4.6, p. 199 Exercises 19(d), 19(e), 20(d), 20(e)
A.5 Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:	A.5.A solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides	A.5.A.i solve linear equations in one variable, including those for which the application of the distributive property is necessary	1.2, p. 13 Example 3	1.2, pp. 16–17 Exercises 17–24, 38, 39
A.5 Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:	A.5.A solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides	A.5.A.ii solve linear equations in one variable, including those for which variables are included on both sides	1.3, pp. 22–24 Core Concept, Examples 1–4	1.3, pp. 25–26 Exercises 2–40
A.5 Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:	A.5.B solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides	A.5.B.i solve linear inequalities in one variable, including those for which the application of the distributive property is necessary	2.4, p. 69 Example 3	2.4, pp. 71–72 Exercises 3–16, 23–33, 35–38
A.5 Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:	A.5.B solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides	A.5.B.ii solve linear inequalities in one variable, including those for which variables are included on both sides	2.4, pp. 68–69 Examples 1–3	2.4, pp. 71–72 Exercises 3–5, 7–14, 17–32, 34–36, 38–40
A.5 Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:	A.5.C solve systems of two linear equations with two variables for mathematical and real-world problems	A.5.C.i solve systems of two linear equations with two variables for mathematical problems	5.1, p. 221 Core Concept, Example 2	5.1, pp. 223–224 Exercises 9-26, 29, 31
A.5 Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:	A.5.C solve systems of two linear equations with two variables for mathematical and real-world problems	A.5.C.ii solve systems of two linear equations with two variables for real-world problems	5.1, p. 222 Example 3	5.1, p. 224 Exercises 27, 28, 32, 33

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.A** determine the domain and range of quadratic functions and represent the domain and range using inequalities	**A.6.A.i** determine the domain of quadratic functions	8.1, pp. 406, 408 Examples 1 and 5 8.3, p. 419 Example 2	8.1, p. 409 Exercises 3, 4, 18, 19(a) 8.3, pp. 422–423 Exercises 13-18, 28(c)
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.A** determine the domain and range of quadratic functions and represent the domain and range using inequalities	**A.6.A.ii** determine the range of quadratic functions	8.1, pp. 406, 408 Examples 1 and 5 8.3, p. 419 Example 2	8.1, p. 409 Exercises 3, 4, 18, 19(a) 8.3, pp. 422–423 Exercises 13-18, 28(c)
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.A** determine the domain and range of quadratic functions and represent the domain and range using inequalities	**A.6.A.iii** represent the domain using inequalities	8.1, p. 408 Example 5	8.1, p. 409 Exercise 18, 19(a) 8.3, p. 423 Exercise 28(c) 8.4, p. 433 Exercise 56(ab)
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.A** determine the domain and range of quadratic functions and represent the domain and range using inequalities	**A.6.A.iv** represent the range using inequalities	8.1, p. 408 Example 5 8.3, p. 419 Example 2	8.1, p. 409 Exercises 3, 4, 18, 19(a) 8.3, pp. 422–423 Exercise 13-18, 28(c)
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.B** write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$)	**A.6.B.i** write equations of quadratic functions given the vertex and another point on the graph	8.4, p. 431 Example 5	8.4, pp. 433–434 Exercises 57–64
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.B** write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$)	**A.6.B.ii** write the equation [of quadratic functions] in vertex form ($f(x) = a(x - h)^2 + k$)	8.4, p. 431 Example 5	8.4, pp. 433–434 Exercises 57–68, 71–75
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.B** write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$)	**A.6.B.iii** rewrite the equation [of quadratic functions] from vertex form to standard form ($f(x) = ax^2 + bx + c$)	8.5, p. 439 Example 6(b)	8.4, p. 434 Exercise 75 8.5, p. 442 Exercises 45, 46

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.C** write quadratic functions when given real solutions and graphs of their related equations	**A.6.C.i** write quadratic functions when given real solutions	8.5, p. 439 Examples 6(a) and 7 8.6, p. 448 Example 3	8.5, p. 442 Exercises 47–52 8.6, p. 452 Exercises 21, 25, 28
A.6 Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:	**A.6.C** write quadratic functions when given real solutions and graphs of their related equations	**A.6.C.ii** write quadratic functions when given graphs of their related equations	8.5, p. 439 Example 7	8.5, p. 442 Exercises 57–60
A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:	**A.7.A** graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry	**A.7.A.i** graph quadratic functions on the coordinate plane	8.3, p. 419 Example 2 8.4, p. 430 Core Concept, Examples 3 and 4	8.3, p. 422 Exercises 13–18, 20 8.4, pp. 432–433 Exercises 23–28, 39–44, 49–54, 55(a), 55(b), 56(a), 56(b)
A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:	**A.7.A** graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry	**A.7.A.ii** use the graph to identify key attributes, if possible, including *x*-intercept	8.5, pp. 436–437 Core Concept, Examples 1 and 2	8.2, p. 415 Exercise 28 8.5, p. 441 Exercises 3–20
A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:	**A.7.A** graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry	**A.7.A.iii** use the graph to identify key attributes, if possible, including *y*-intercept	8.3, p. 420 Example 4	8.2, p. 415 Exercise 28 8.3, pp. 422–424 Exercises 3–6, 20, 31–34, 42(a)
A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:	**A.7.A** graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry	**A.7.A.iv** use the graph to identify key attributes, if possible, including zeros	9.2, pp. 478–479 Examples 5, 6(c)	9.2, pp. 482–483 Exercises 43-52, 53(c), 54(c)

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:	**A.7.C** determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d	**A.7.C.ii** determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(x) + d$ for specific values of d	8.2, pp. 412–413 Core Concept, Examples 1-3	8.2, p. 415 Exercises 2–18
A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:	**A.7.C** determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d	**A.7.C.iii** determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(x - c)$ for specific values of c	8.4, pp. 429–430 Core Concept, Example 2, Core Concept, Examples 3 and 4	8.4, pp. 432-433 Exercises 23–28, 39–44, 49–54, 55, 56
A.7 Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:	**A.7.C** determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d	**A.7.C.iv** determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(bx)$ for specific values of b	8.1, p. 408 Core Concept, Example 4	8.1, p. 409 Exercises 13–16
A.8 Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.8.A** solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula	**A.8.A.i** solve quadratic equations having real solutions by factoring	7.7, p. 380 Example 5	7.7, pp. 381–382 Exercises 25–28, 35, 36, 40, 43, 44
A.8 Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.8.A** solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula	**A.8.A.ii** solve quadratic equations having real solutions by taking square roots	9.3, pp. 486–487 Teaching text, Core Concept, Examples 1–3	9.3, p. 489 Exercises 3–31
A.8 Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.8.A** solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula	**A.8.A.iii** solve quadratic equations having real solutions by completing the square	9.4, p. 495 Examples 2 and 3	9.4, p. 499 Exercises 17–22, 23(b), 24(b), 25–32

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.8 Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.8.A** solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula	**A.8.A.iv** solve quadratic equations having real solutions by applying the quadratic formula	9.5, pp. 504–505 Core Concept, Examples 1 and 2	9.5, p. 509 Exercises 9–24
A.8 Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.8.B** write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems	**A.8.B.i** write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions	9.2, p. 480 Example 7	8.6, p. 454 Exercise 42 9.2, p. 483 Exercises 55(a), 55(b), 56(a), 56(b), 57(b), 57(d), 58(a), 58(b)
A.8 Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.8.B** write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems	**A.8.B.ii** write, using technology, quadratic functions that provide a reasonable fit to data to make predictions for real-world problems	9.2, p. 480 Example 7	8.6, p. 454 Exercise 42 9.2, p. 483 Exercises 55(c), 56(c), 57(c), 58(c)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.A** determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities	**A.9.A.i** determine the domain of exponential functions of the form $f(x) = ab^x$	6.3, pp. 293–294 Examples 3–5, and 6(b)	6.3, pp. 296–298 Exercises 25–36, 41(b), 42(b), 44, 49
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.A** determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities	**A.9.A.ii** determine the range of exponential functions of the form $f(x) = ab^x$	6.3, pp. 293–294 Examples 3–5, and 6(b)	6.3, pp. 296–298 Exercises 25–36, 41(b), 42(b), 44, 49

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.A** determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities	**A.9.A.iii** represent the domain [of exponential functions of the form $f(x) = ab^x$] using inequalities	6.3, p. 294 Example 6(b)	6.3, p. 297 Exercise 41(b), 42(b)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.A** determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities	**A.9.A.iv** represent the range [of exponential functions of the form $f(x) = ab^x$] using inequalities	6.3, pp. 293–294 Examples 3–5, 6(b)	6.3, p. 296–298 Exercises 25–36, 41(b), 42(b), 44, 49
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.B** interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems	**A.9.B.i** interpret the meaning of the values of a in exponential functions of the form $f(x) = ab^x$ in real-world problems	6.3, p. 294 Example 6(a) 6.4, pp. 302–303 Examples 3, 6	6.3, pp. 297–298 Exercises 41(a), 42(a), 56 6.4, pp. 306–307 Exercises 41–44, 57, 58
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.B** interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems	**A.9.B.ii** interpret the meaning of the values of b in exponential functions of the form $f(x) = ab^x$ in real-world problems	6.3, p. 294 Example 6(a) 6.4, pp. 302–303 Examples 3, 6	6.3, p. 298 Exercise 56 6.4, pp. 306–308 Exercises 41–44, 57, 58, 63

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.C** write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay	**A.9.C.i** write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical situations	6.3, p. 294 Example 6	6.3, p. 298 Exercises 52, 53, 58, 59 8.6, pp. 452, 454 Exercises 22, 24, 40
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.C** write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay	**A.9.C.ii** write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from real-world situations, including growth	6.4, pp. 300, 303 Core Concept, Example 1(a), Core Concept, Examples 5 and 6(a)	6.4, pp. 305, 307 Exercises 13–16, 17(a), 18(a), 53–56, 57(a), 58(a), 59(a)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.C** write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay	**A.9.C.iii** write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from real-world situations, including decay	6.4, pp. 301, 304 Core Concept, Example 7(a)	6.4, pp. 305–307 Exercises 27–30, 32, 60(a)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.D** graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems	**A.9.D.i** graph exponential functions that model growth	6.4, p. 303 Example 6(b)	6.4, pp. 307–308 Exercises 57(b), 58(b), 59(c), 64(a), 64(c)

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	A.9.D graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems	A.9.D.ii graph exponential functions that model decay	6.4, p. 304 Example 7(c)	6.4, pp. 307–308 Exercises 60(c), 64(b), 64(d)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	A.9.D graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems	A.9.D.iii identify key features, including *y*-intercept, in mathematical problems	6.3, p. 293 Teaching text, Core Concept, Examples 3 and 4	6.3, pp. 296–298 Exercises 25-30, 49
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	A.9.D graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems	A.9.D.iv identify key features, including *y*-intercept, in real-world problems	6.3, p. 294 Example 6(a)	6.3, p. 297 Exercises 41(a), 42(a)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	A.9.D graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems	A.9.D.v identify key features, including asymptote, in mathematical problems	6.3, p. 293 Core Concept, Examples 3–5	6.3, pp. 296–297 Exercises 25–36

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.D** graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems	**A.9.D.vi** identify key features, including asymptote, in real-world problems	6.4, p. 304 Example 7(c)	6.4, p. 307 Exercise 60(c)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.E** write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems	**A.9.E.i** write, using technology, exponential functions that provide a reasonable fit to data	6.3, p. 295 Example 7(a)	6.3, p. 297 Exercises 45(a), 46(a)
A.9 Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	**A.9.E** write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems	**A.9.E.ii** make predictions for real-world problems	6.3, p. 295 Example 7(b)	6.3, p. 297 Exercises 45(b), 46(b)
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	**A.10.A** add and subtract polynomials of degree one and degree two	**A.10.A.i** add polynomials of degree one	7.1, p. 340 Example 4(a)	7.1, pp. 342–344 Exercises 23, 24, 42, 55
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	**A.10.A** add and subtract polynomials of degree one and degree two	**A.10.A.ii** add polynomials of degree two	7.1, pp. 340–341 Examples 4(b) and 6(a)	7.1, pp. 342, 344 Exercises 25–28, 53(a), 60(a)

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	A.10.A add and subtract polynomials of degree one and degree two	A.10.A.iii subtract polynomials of degree one	7.1, p. 340 Example 5(a)	7.1, pp. 342–343 Exercises 31, 32, 41
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	A.10.A add and subtract polynomials of degree one and degree two	A.10.A.iv subtract polynomials of degree two	7.1, pp. 340–341 Examples 5(b) and 6(a)	7.1, pp. 342–344 Exercises 33, 34, 39, 53(a), 61(a)
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	A.10.B multiply polynomials of degree one and degree two	A.10.B.i multiply polynomials of degree one	7.2, pp. 346–347 Examples 1, 2, and 3(a)	7.2, p. 349 Exercises 3–20, 21–26, 31–34
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	A.10.B multiply polynomials of degree one and degree two	A.10.B.ii multiply polynomials of degree two	7.2, pp. 347–348 Core Concept, Examples 3(b) and 4	7.2, pp. 349–350 Exercises 27–30, 35–42, 49, 50
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	A.10.C determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend	A.10.C.i determine the quotient of a polynomial of degree one when divided by a polynomial of degree one	7.4, pp. 358–359 Teaching text, Core Concept, Example 2	7.4, pp. 361–362 Exercises 7–12, 41
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	A.10.C determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend	A.10.C.ii determine the quotient of a polynomial of degree two when divided by a polynomial of degree one	7.4, pp. 358–360 Teaching text, Core Concept, Examples 1, 3, and 5	7.4, pp. 361–362 Exercises 5, 6, 13–20, 29–36, 42, 45
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	A.10.C determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend	A.10.C.iii determine the quotient of a polynomial of degree two when divided by a polynomial of degree two	7.4, pp. 358–359 Teaching text, Core Concept, Example 4	7.4, p. 361 Exercises 21–28

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	**A.10.D** rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property	**A.10.D.i** rewrite polynomial expressions of degree one in equivalent forms using the distributive property	1.3, pp. 22–24 Examples 2, 3, and 4	1.3, p. 25 Exercises 2, 11–16, 21–24, 26
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	**A.10.D** rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property	**A.10.D.ii** rewrite polynomial expressions of degree two in equivalent forms using the distributive property	7.2, pp. 347–348 Examples 3(b) and 4	7.2, pp. 349–350 Exercises 27–30, 35–42, 49, 50
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	**A.10.E** factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two	**A.10.E.i** factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two	7.7, pp. 378–379 Examples 1–4 7.8, p. 385 Core Concept, Example 3	7.7, pp. 381–382 Exercises 2–34, 39–48 7.8, pp. 387–388 Exercises 15–22, 24–26, 37–40, 43
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	**A.10.F** decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial	**A.10.F.i** decide if a binomial can be written as the difference of two squares	7.8, p. 384 Examples 1 and 2	7.8, p. 387 Exercises 3–14
A.10 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:	**A.10.F** decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial	**A.10.F.ii** if [a binomial can be written as the difference of two squares], use the structure of a difference of two squares to rewrite the binomial	7.8, p. 384 Examples 1 and 2	7.8, p. 387 Exercises 3–14
A.11 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:	**A.11.A** simplify numerical radical expressions involving square roots	**A.11.A.i** simplify numerical radical expressions involving square roots	9.1, pp. 466–470 Core Concepts, Examples 1(a), 2(a), 5–7, 8(a), 8(b), and 9	9.1, pp. 471–473 Exercises 13–16, 21–24, 37, 45–48, 55–60, 61, 63-80, 83, 84, 87, 88, 91
A.11 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:	**A.11.B** simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents	**A.11.B.i** simplify numeric expressions using the laws of exponents, including integral exponents	6.1, pp. 278–279 All Core Concepts, Examples 1, 3(a), 3(b), and 6	6.1, pp. 282–283 Exercises 5–12, 23–26, 29, 30, 33, 35, 46, 51–56, 58
A.11 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:	**A.11.B** simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents	**A.11.B.ii** simplify numeric expressions using the laws of exponents, including rational exponents	6.2, pp. 287–288 Teaching text, Core Concept, Examples 3, 4(a), and 5	6.2, pp. 289–290 Exercises 23–28, 30–35, 37–40, 45, 46

Correlation from TEKS to Algebra 1 *(continued)*

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.11 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:	**A.11.B** simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents	**A.11.B.iii** simplify algebraic expressions using the laws of exponents, including integral exponents	6.1, pp. 278-281 All Core Concepts, Examples 2, 3(c), 4, and 5	6.1, pp. 282–284 Exercises 13–22, 27, 28, 31, 32, 34, 36–45, 47–50, 57, 59–62
A.11 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:	**A.11.B** simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents	**A.11.B.iv** simplify algebraic expressions using the laws of exponents, including rational exponents	6.2, pp. 287–288 Core Concept, Example 4(b)	6.2, p. 290 Exercises 41–44, 51-54
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.A** decide whether relations represented verbally, tabularly, graphically, and symbolically define a function	**A.12.A.i** decide whether relations represented verbally define a function	3.1, p. 90 Example 2	3.1, p. 94 Exercises 9–12
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.A** decide whether relations represented verbally, tabularly, graphically, and symbolically define a function	**A.12.A.ii** decide whether relations represented tabularly define a function	3.1, p. 90 Example 1(c)	3.1, pp. 94–95 Exercises 7, 8, 23, 24
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.A** decide whether relations represented verbally, tabularly, graphically, and symbolically define a function	**A.12.A.iii** decide whether relations represented graphically define a function	3.1, p. 91 Core Concept, Example 3	3.1, pp. 94–96 Exercises 13–16, 31, 34(a), 35
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.A** decide whether relations represented verbally, tabularly, graphically, and symbolically define a function	**A.12.A.iv** decide whether relations represented symbolically define a function	3.1, p. 90 Example 1	3.1, pp. 94–95 Exercises 3–6, 32, 33(a)
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.B** evaluate functions, expressed in function notation, given one or more elements in their domains	**A.12.B.i** evaluate functions, expressed in function notation, given one or more elements in their domains	3.3, pp. 108-110 Teaching text, Examples 1, 2, 4, and 5	3.3, pp. 111–112 Exercises 2–12, 21–28, 30–33, 34(b), 35, 36
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.C** identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes	**A.12.C.i** identify terms of arithmetic sequences when the sequences are given in function form using recursive processes	6.6, p. 320 Teaching text, Core Concept, Example 1(a)	6.6, pp. 324–325 Exercises 4, 5, 7, 8, 47
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.C** identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes	**A.12.C.ii** identify terms of geometric sequences when the sequences are given in function form using recursive processes	6.6, p. 320 Teaching text, Core Concept, Example 1(b)	6.6, pp. 324–325 Exercises 3, 6, 9–12, 48, 49

Knowledge and Skills Statement	Student Expectation	Breakout	Instruction Citations	Review Citations
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.D** write a formula for the *n*th term of arithmetic and geometric sequences, given the value of several of their terms	**A.12.D.i** write a formula for the *n*th term of arithmetic sequences, given the value of several of their terms	4.7, pp. 204–205 Teaching text, Core Concept, Examples 4 and 5(a)	4.7, p. 207 Exercises 33–38, 40, 45(a), 46(a)
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.D** write a formula for the *n*th term of arithmetic and geometric sequences, given the value of several of their terms	**A.12.D.ii** write a formula for the *n*th term of geometric sequences, given the value of several of their terms	6.5, pp. 314-315 Teaching text, Core Concept, Examples 4 and 5	6.5, pp. 316–318 Exercises 25–32, 36, 37(a), 38(a), 39, 40, 47, 50(a), 51(b), 54
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.E** solve mathematic and scientific formulas, and other literal equations, for a specified variable	**A.12.E.i** solve mathematic formulas for a specified variable	1.4, p. 29 Examples 3 and 4	1.4, pp. 32–34 Exercises 28, 29, 37, 39, 40, 42–44
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.E** solve mathematic and scientific formulas, and other literal equations, for a specified variable	**A.12.E.ii** solve scientific formulas for a specified variable	1.4, pp. 30–31 Examples 5 and 8	1.4, pp. 32–34 Exercises 30–32, 34, 36, 38, 41
A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:	**A.12.E** solve mathematic and scientific formulas, and other literal equations, for a specified variable	**A.12.E.iii** solve other literal equations for a specified variable	1.4, pp. 28, 31 Examples 1, 2, and 7	1.4, pp. 32–34 Exercises 2–27, 33, 35, 45, 46

Pacing Guide

Chapters 1–9 157 Days

Chapter 1 (12 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Quiz	1 Day
Section 3	2 Days
Section 4	2 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**12 Days**

Chapter 2 (14 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Quiz	1 Day
Section 4	2 Days
Section 5	2 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**26 Days**

Chapter 3 (19 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Quiz	1 Day
Section 4	2 Days
Section 5	2 Days
Section 6	2 Days
Section 7	3 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**45 Days**

Chapter 4 (18 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	2 Days
Quiz	1 Day
Section 5	2 Days
Section 6	2 Days
Section 7	2 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**63 Days**

Chapter 5 (19 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	2 Days
Quiz	1 Day
Section 5	2 Days
Section 6	2 Days
Section 7	3 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**82 Days**

Chapter 6 (17 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	3 Days
Quiz	1 Day
Section 5	2 Days
Section 6	2 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**99 Days**

Chapter 7 (22 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	2 Days
Section 5	2 Days
Quiz	1 Day
Section 6	2 Days
Section 7	2 Days
Section 8	2 Days
Section 9	2 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**121 Days**

Chapter 8 (18 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Quiz	1 Day
Section 4	2 Days
Section 5	3 Days
Section 6	3 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**139 Days**

Chapter 9 (18 Days)

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	3 Days
Section 2	3 Days
Section 3	2 Days
Quiz	1 Day
Section 4	3 Days
Section 5	3 Days
Chapter Review/Chapter Tests	2 Days
Year-to-Date	**157 Days**

Chapter 1 Pacing Guide

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Quiz	1 Day
Section 3	2 Days
Section 4	2 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 1	12 Days
Year-to-Date	12 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
1.1	A.5.A
1.2	A.5.A, A.10.D
1.3	A.5.A, A.10.D
1.4	A.12.E

1 Solving Linear Equations

1.1 Solving Simple Equations
1.2 Solving Multi-Step Equations
1.3 Solving Equations with Variables on Both Sides
1.4 Rewriting Equations and Formulas



Density of Pyrite *(p. 33)*



Ski Club *(p. 32)*



Boat *(p. 24)*



Biking *(p. 14)*



Average Speed *(p. 6)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

Maintaining Mathematical Proficiency

Adding and Subtracting Integers (6.3.D)

Example 1 Evaluate $4 + (-12)$.

$4 + (-12) = -8$

$|-12| > |4|$. So, subtract $|4|$ from $|-12|$.

Use the sign of -12.

Example 2 Evaluate $-7 - (-16)$.

$-7 - (-16) = -7 + 16$ Add the opposite of -16.

$= 9$ Add.

Add or subtract.

1. $-5 + (-2)$
2. $0 + (-13)$
3. $-6 + 14$
4. $19 - (-13)$
5. $-1 - 6$
6. $-5 - (-7)$
7. $17 + 5$
8. $8 + (-3)$
9. $11 - 15$

Multiplying and Dividing Integers (6.3.D)

Example 3 Evaluate $-3 \cdot (-5)$.

The integers have the same sign.

$-3 \cdot (-5) = 15$

The product is positive.

Example 4 Evaluate $15 \div (-3)$.

The integers have different signs.

$15 \div (-3) = -5$

The quotient is negative.

Multiply or divide.

10. $-3(8)$
11. $-7 \cdot (-9)$
12. $4 \cdot (-7)$
13. $-24 \div (-6)$
14. $-16 \div 2$
15. $12 \div (-3)$
16. $6 \cdot 8$
17. $36 \div 6$
18. $-3(-4)$

19. **ABSTRACT REASONING** Summarize the rules for (a) adding integers, (b) subtracting integers, (c) multiplying integers, and (d) dividing integers. Give an example of each.

Dynamic Solutions available at *BigIdeasMath.com*

1

For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make Information Frames for the following words.

- Integer
- Opposite
- Absolute Value

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

Texas Essential Knowledge and Skills

6.3.D The student is expected to add, subtract, multiply, and divide integers fluently.

ANSWERS

1. -7
2. -13
3. 8
4. 32
5. -7
6. 2
7. 22
8. 5
9. -4
10. -24
11. 63
12. -28
13. 4
14. -8
15. -4
16. 48
17. 6
18. 12
19. a. If the signs are the same, add the absolute values and attach the sign. If the signs are different, subtract the absolute values and attach the sign of the number with the greatest absolute value; *Sample answer:* $-6 + 2 = -4$
 b. Add the opposite; *Sample answer:* $5 - (-3) = 5 + 3 = 8$
 c. Multiply the absolute values. If the signs are the same, then the product is positive. If the signs are different, then the product is negative; *Sample answer:* $(-6)(-4) = 24$
 d. Divide the absolute values. If the signs are the same, then the quotient is positive. If the signs are different, then the quotient is negative; *Sample answer:* $-15 \div 3 = -5$

MONITORING PROGRESS ANSWERS

1. about 3 million per year
2. 30 mi/gal
3. about 17 min

Mathematical Thinking

Mathematically proficient students display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication. **(A.1.G)**

Specifying Units of Measure

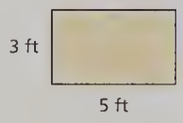
Core Concept

Operations and Unit Analysis

Addition and Subtraction

When you add or subtract quantities, they must have the same units of measure. The sum or difference will have the *same* unit of measure.

Example



Perimeter of rectangle
$= (3 \text{ ft}) + (5 \text{ ft}) + (3 \text{ ft}) + (5 \text{ ft})$
$= 16$ feet ← When you add **feet**, you get **feet**.

Multiplication and Division

When you multiply or divide quantities, the product or quotient will have a *different* unit of measure.

Example Area of rectangle $= (3 \text{ ft}) \times (5 \text{ ft})$
$= 15$ square feet ← When you multiply **feet**, you get feet squared, or **square feet**.

EXAMPLE 1 Specifying Units of Measure

You work 8 hours and earn \$72. What is your hourly wage?

SOLUTION

$$\underbrace{\text{Hourly wage (\$ per h)}}_{\text{dollars per hour}} = \underbrace{\$72 \div 8 \text{ h}}_{\text{dollars per hour}}$$
$= \$9$ per hour

← The units on each side of the equation balance. Both are specified in dollars per hour.

▶ Your hourly wage is \$9 per hour.

Monitoring Progress

Solve the problem and specify the units of measure.

1. The population of the United States was about 280 million in 2000 and about 310 million in 2010. What was the annual rate of change in population from 2000 to 2010?
2. You drive 240 miles and use 8 gallons of gasoline. What was your car's gas mileage (in miles per gallon)?
3. A bathtub is in the shape of a rectangular prism. Its dimensions are 5 feet by 3 feet by 18 inches. The bathtub is three-fourths full of water and drains at a rate of 1 cubic foot per minute. About how long does it take for all the water to drain?

For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	*If students got it...*
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

1.1 Solving Simple Equations

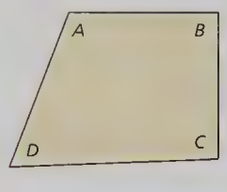
Texas Essential Knowledge and Skills
A.5.A

Essential Question How can you use simple equations to solve real-life problems?

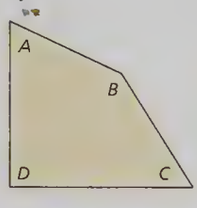
EXPLORATION 1 Measuring Angles

Work with a partner. Use a protractor to measure the angles of each quadrilateral. Copy and complete the table to organize your results. (The notation $m\angle A$ denotes the measure of angle A.) How precise are your measurements?

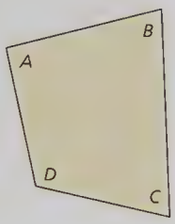
a.



b.



c.



Quadrilateral	$m\angle A$ (degrees)	$m\angle B$ (degrees)	$m\angle C$ (degrees)	$m\angle D$ (degrees)	$m\angle A + m\angle B + m\angle C + m\angle D$
a.					
b.					
c.					

UNDERSTANDING MATHEMATICAL TERMS

A **conjecture** is an unproven statement about a general mathematical concept. After the statement is proven, it is called a **rule** or a **theorem**.

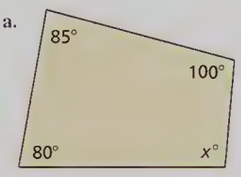
EXPLORATION 2 Making a Conjecture

Work with a partner. Use the completed table in Exploration 1 to write a conjecture about the sum of the angle measures of a quadrilateral. Draw three quadrilaterals that are different from those in Exploration 1 and use them to justify your conjecture.

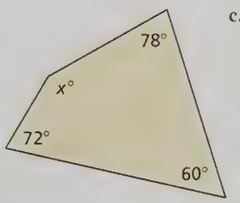
EXPLORATION 3 Applying Your Conjecture

Work with a partner. Use the conjecture you wrote in Exploration 2 to write an equation for each quadrilateral. Then solve the equation to find the value of x. Use a protractor to check the reasonableness of your answer.

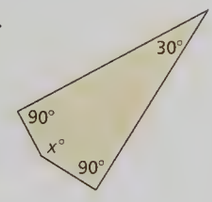
a.



b.



c.



Communicate Your Answer

4. How can you use simple equations to solve real-life problems?
5. Draw your own quadrilateral and cut it out. Tear off the four corners of the quadrilateral and rearrange them to affirm the conjecture you wrote in Exploration 2. Explain how this affirms the conjecture.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

3. a. $85 + 100 + x + 80 = 360$; $x = 95$
 b. $x + 78 + 60 + 72 = 360$; $x = 150$
 c. $90 + 30 + 90 + x = 360$; $x = 150$
4. Simple equations can relate parts of geometric shapes and can be used to find missing parts.
5. The corners can be arranged so the angles complete a full circle, which is 360°.

Dynamic Teaching Tools

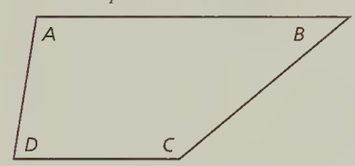
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

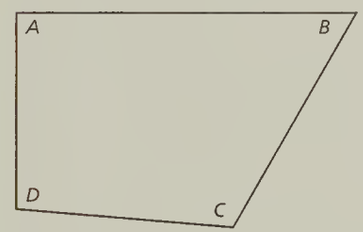
A.5.A The student is expected to solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

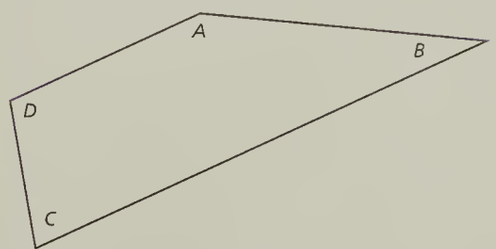
1. a. 110; 90; 92; 68; 360°
 b. 65; 147; 58; 90; 360°
 c. 91; 79; 75; 115; 360°
 Answers will vary.
2. equals 360°
 Sample answer:



$100 + 40 + 140 + 80 = 360$



$90 + 60 + 115 + 95 = 360$



$150 + 30 + 75 + 105 = 360$

Divide the quadrilateral into two triangles.

The sum of the angle measures of a triangle is 180°, so the sum of the angle measures of a quadrilateral is $2(180°) = 360°$.

Extra Example 1

Solve each equation. Justify each step. Check your answer.

a. $d - \frac{1}{4} = -\frac{1}{2}$ $d = -\frac{1}{4}$

b. $m + 4.8 = 9.2$ $m = 4.4$

MONITORING PROGRESS ANSWERS

1. $n = -10$; Subtract 3 from each side.
2. $g = -\frac{1}{3}$; Add $\frac{1}{3}$ to each side.
3. $p = -10.4$; Subtract 3.9 from each side.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

1.1 Lesson

What You Will Learn

- Solve linear equations using addition and subtraction.
- Solve linear equations using multiplication and division.
- Use linear equations to solve real-life problems.

Core Vocabulary

conjecture, *p. 3*
rule, *p. 3*
theorem, *p. 3*
equation, *p. 4*
linear equation in one variable, *p. 4*
solution, *p. 4*
inverse operations, *p. 4*
equivalent equations, *p. 4*

Previous
expression

Solving Linear Equations by Adding or Subtracting

An **equation** is a statement that two expressions are equal. A **linear equation in one variable** is an equation that can be written in the form $ax + b = 0$, where a and b are constants and $a \neq 0$. A **solution** of an equation is a value that makes the equation true.

Inverse operations are two operations that undo each other, such as addition and subtraction. When you perform the same inverse operation on each side of an equation, you produce an equivalent equation. **Equivalent equations** are equations that have the same solution(s).

Core Concept

Addition Property of Equality

Words Adding the same number to each side of an equation produces an equivalent equation.

Algebra If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality

Words Subtracting the same number from each side of an equation produces an equivalent equation.

Algebra If $a = b$, then $a - c = b - c$.

EXAMPLE 1 Solving Equations by Addition or Subtraction

Solve each equation. Justify each step. Check your answer.

a. $x - 3 = -5$ b. $0.9 = y + 2.8$

SOLUTION

a. $x - 3 = -5$ Write the equation.

Addition Property of Equality → $+3 \quad +3$ Add 3 to each side.

$x = -2$ Simplify.

▶ The solution is $x = -2$.

Check

$x - 3 = -5$

$-2 - 3 \stackrel{?}{=} -5$

$-5 = -5$ ✓

b. $0.9 = y + 2.8$ Write the equation.

Subtraction Property of Equality → $-2.8 \quad -2.8$ Subtract 2.8 from each side.

$-1.9 = y$ Simplify.

▶ The solution is $y = -1.9$.

Check

$0.9 = y + 2.8$

$0.9 \stackrel{?}{=} -1.9 + 2.8$

$0.9 = 0.9$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Justify each step. Check your solution.

1. $n + 3 = -7$
2. $g - \frac{1}{3} = -\frac{2}{3}$
3. $-6.5 = p + 3.9$

SUPPORTING English Language Learners

Have students use the following Core Vocabulary as they read the topic Solving Linear Equations by Adding or Subtracting: *equation, linear equation in one variable, solution, inverse operations, equivalent operations.*

Beginning Repeat the Core Vocabulary to improve pronunciation.
Intermediate Read aloud one sentence with Core Vocabulary.
Advanced Read aloud a paragraph.
Advanced High Explain a sentence with Core Vocabulary using their own words.

ELPS 3.D.1 Speak using grade-level content area vocabulary in context to internalize new English words.

Solving Linear Equations by Multiplying or Dividing

Core Concept

Multiplication Property of Equality

Words Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality

Words Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $a = b$, then $a \div c = b \div c$, $c \neq 0$.

REMEMBER

Multiplication and division are inverse operations.

EXAMPLE 2 Solving Equations by Multiplication or Division

Solve each equation. Justify each step. Check your answer.

a. $-\frac{n}{5} = -3$ b. $\pi x = -2\pi$ c. $1.3z = 5.2$

SOLUTION

a. $-\frac{n}{5} = -3$ Write the equation.

Multiplication Property of Equality → $-5 \cdot \left(-\frac{n}{5}\right) = -5 \cdot (-3)$ Multiply each side by -5.

$n = 15$ Simplify.

▶ The solution is $n = 15$.

Check

$-\frac{n}{5} = -3$

$-\frac{15}{5} \stackrel{?}{=} -3$

$-3 = -3$ ✓

b. $\pi x = -2\pi$ Write the equation.

Division Property of Equality → $\frac{\pi x}{\pi} = \frac{-2\pi}{\pi}$ Divide each side by π.

$x = -2$ Simplify.

▶ The solution is $x = -2$.

Check

$\pi x = -2\pi$

$\pi(-2) \stackrel{?}{=} -2\pi$

$-2\pi = -2\pi$ ✓

c. $1.3z = 5.2$ Write the equation.

Division Property of Equality → $\frac{1.3z}{1.3} = \frac{5.2}{1.3}$ Divide each side by 1.3.

$z = 4$ Simplify.

▶ The solution is $z = 4$.

Check

$1.3z = 5.2$

$1.3(4) \stackrel{?}{=} 5.2$

$5.2 = 5.2$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Justify each step. Check your solution.

4. $\frac{y}{3} = -6$ 5. $9\pi = \pi x$ 6. $0.05w = 1.4$

Extra Example 2

Solve each equation. Justify each step. Check your answer.

a. $\frac{m}{5} = -4$ $m = -20$

b. $3\pi = \pi c$ $c = 3$

c. $2.5r = 75$ $r = 30$

MONITORING PROGRESS ANSWERS

4. $y = -18$; Multiply each side by 3.
5. $x = 9$; Divide each side by π.
6. $w = 28$; Divide each side by 0.05.

Differentiated Instruction

Kinesthetic

Ask two students to assist you at the board or overhead when solving equations. Assign one student to the left side of the equation and the other student to the right side. Each student is responsible for performing the operations on his or her side. Emphasize that to keep the equality, both students must perform the same operation to solve the equation.

Extra Example 3

You clean a community park for 6.5 hours. You earn $42.25. Write and solve an equation to find how much you earn per hour. $6.5r = 42.25$; $r = 6.5$; You earn $6.50 per hour.

MONITORING PROGRESS ANSWER

7. 38.65 sec

Solving Real-Life Problems

APPLYING MATHEMATICS

Mathematically proficient students routinely check that their solutions make sense in the context of a real-life problem.

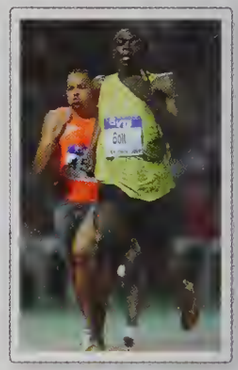
Core Concept

Four-Step Approach to Problem Solving

1. **Understand the Problem** What is the unknown? What information is being given? What is being asked?
2. **Make a Plan** This plan might involve one or more of the problem-solving strategies shown on the next page.
3. **Solve the Problem** Carry out your plan. Check that each step is correct.
4. **Look Back** Examine your solution. Check that your solution makes sense in the original statement of the problem.

EXAMPLE 3 **Modeling with Mathematics**

In the 2012 Olympics, Usain Bolt won the 200-meter dash with a time of 19.32 seconds. Write and solve an equation to find his average speed to the nearest hundredth of a meter per second.



REMEMBER

The formula that relates distance d, rate or speed r, and time t is

$$d = rt.$$

SOLUTION

1. **Understand the Problem** You know the winning time and the distance of the race. You are asked to find the average speed to the nearest hundredth of a meter per second.
2. **Make a Plan** Use the Distance Formula to write an equation that represents the problem. Then solve the equation.
3. **Solve the Problem**

$d = r \cdot t$ Write the Distance Formula.

$200 = r \cdot 19.32$ Substitute 200 for d and 19.32 for t.

$\frac{200}{19.32} = \frac{19.32r}{19.32}$ Divide each side by 19.32.

$10.35 \approx r$ Simplify.

▶ Bolt's average speed was about 10.35 meters per second.

REMEMBER

The symbol $\approx$ means "approximately equal to."

4. **Look Back** Round Bolt's average speed to 10 meters per second. At this speed, it would take

$$\frac{200 \text{ m}}{10 \text{ m/sec}} = 20 \text{ seconds}$$

to run 200 meters. Because 20 is close to 19.32, your solution is reasonable.

Monitoring Progress Help in English and Spanish at *BigIdeasMath.com*

7. Suppose Usain Bolt ran 400 meters at the same average speed that he ran the 200 meters. How long would it take him to run 400 meters? Round your answer to the nearest hundredth of a second.

6 Chapter 1 Solving Linear Equations

SUPPORTING English Language Learners

To provide listening practice, read aloud the Four-Step Approach to Problem Solving as students follow along. Provide synonyms when needed to help understanding.

Beginning Repeat the title of each step.
Intermediate State the actions used in each step.
Advanced/Advanced High Explain each step using their own words.

2.C.4 Learn academic vocabulary heard during classroom instruction and interactions.

Core Concept

Common Problem-Solving Strategies

Use a verbal model.	Guess, check, and revise.
Draw a diagram.	Sketch a graph or number line.
Write an equation.	Make a table.
Look for a pattern.	Make a list.
Work backward.	Break the problem into parts.

EXAMPLE 4 **Modeling with Mathematics**

On January 22, 1943, the temperature in Spearfish, South Dakota, fell from 54°F at 9:00 A.M. to −4°F at 9:27 A.M. How many degrees did the temperature fall?

SOLUTION

1. **Understand the Problem** You know the temperature before and after the temperature fell. You are asked to find how many degrees the temperature fell.
2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation.
3. **Solve the Problem**

Words Temperature at 9:27 A.M. = Temperature at 9:00 A.M. − Number of degrees the temperature fell

Variable Let T be the number of degrees the temperature fell.

Equation $-4 = 54 - T$

$$-4 = 54 - T$$ Write the equation.

$$-4 - 54 = 54 - 54 - T$$ Subtract 54 from each side.

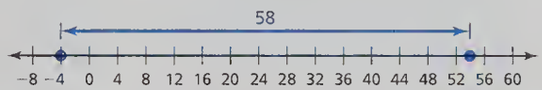
$$-58 = -T$$ Simplify.

$$58 = T$$ Divide each side by −1.

▶ The temperature fell 58°F.

REMEMBER
The distance between two points on a number line is always positive.

4. **Look Back** The temperature fell from 54 degrees *above* 0 to 4 degrees *below* 0. You can use a number line to check that your solution is reasonable.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

8. You thought the balance in your checking account was $68. When your bank statement arrives, you realize that you forgot to record a check. The bank statement lists your balance as $26. Write and solve an equation to find the amount of the check that you forgot to record.

Extra Example 4

A discounted concert ticket is $14.50 less than the original price p. You pay $53 for a discounted ticket. What is the original price of the ticket? The original ticket price is $67.50.

MONITORING PROGRESS ANSWER

8. $26 = 68 - c$; $42

Closure

- Describe in words how to solve a one-step equation. *Sample answer:* Use the same inverse operation on each side of the equation to produce an equivalent equation.
- Solve for x: $-13.8 = x - 4.3$ $x = -9.5$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–39 odd, 43, 47, 50, 57–65

Average: 1–4, 6–38 even, 39–45, 50, 57–65

Advanced: 1–4, 12–20 even, 29, 30–50 even, 51–65

HOMEWORK CHECK

Basic: 9, 11, 15, 27, 29

Average: 10, 12, 28, 34, 43

Advanced: 12, 14, 29, 30, 44

ANSWERS

1. $+$ and $-$; $\times$ and $\div$
2. yes; $x = -5$
3. Division Property of Equality; Divide each side by 14.
4. $x - 6 = 5$; It is the only one that involves addition and subtraction.
5. $x = 3$; Subtract 5 from each side.
6. $m = -7$; Subtract 9 from each side.
7. $y = 7$; Add 4 to each side.
8. $s = 3$; Add 2 to each side.
9. $w = -7$; Subtract 3 from each side.
10. $n = -1$; Add 6 to each side.
11. $p = -3$; Add 11 to each side.
12. $q = -4$; Subtract 4 from each side.
13. $r = 18$; Add 8 to each side.
14. $t = 4$; Subtract 5 from each side.
15. $p - 12.95 = 44$; \$56.95
16. $x + 12 = 195$; 183 points
17. $x + 100 + 120 + 100 = 360$; $x = 40$
18. $x + 150 + 77 + 48 = 360$; $x = 85$
19. $x + 76 + 92 + 122 = 360$; $x = 70$
20. $x + 60 + 115 + 85 = 360$; $x = 100$
21. $g = 4$; Divide each side by 5.
22. $q = 13$; Divide each side by 4.
23. $p = 15$; Multiply each side by 5.
24. $y = 7$; Multiply each side by 7.
25. $r = -8$; Divide each side by -8.
26. $x = -16$; Multiply each side by -2.
27. $x = 48$; Multiply each side by 6.
28. $w = -18$; Multiply each side by -3.
29. $s = -6$; Divide each side by 9.
30. $t = -49$; Multiply each side by 7.

1.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Which of the operations $+$, $-$, $\times$, and $\div$ are inverses of each other?
2. **VOCABULARY** Are the equations $-2x = 10$ and $-5x = 25$ equivalent? Explain.
3. **WRITING** Which property of equality would you use to solve the equation $14x = 56$? Explain.
4. **WHICH ONE DOESN'T BELONG?** Which expression does not belong with the other three? Explain your reasoning.

$8 = \frac{x}{2}$ $\quad 3 = x \div 4$ $\quad x - 6 = 5$ $\quad \frac{x}{3} = 9$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–14, solve the equation. Justify each step. Check your solution. *(See Example 1.)*

5. $x + 5 = 8$
6. $m + 9 = 2$
7. $y - 4 = 3$
8. $s - 2 = 1$
9. $w + 3 = -4$
10. $n - 6 = -7$
11. $-14 = p - 11$
12. $0 = 4 + q$
13. $r + (-8) = 10$
14. $t - (-5) = 9$

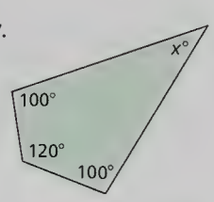
15. **MODELING WITH MATHEMATICS** A discounted amusement park ticket costs \$12.95 less than the original price p. Write and solve an equation to find the original price.

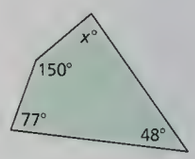


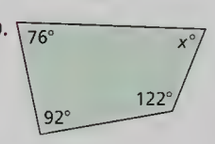
16. **MODELING WITH MATHEMATICS** You and a friend are playing a board game. Your final score x is 12 points less than your friend's final score. Write and solve an equation to find your final score.

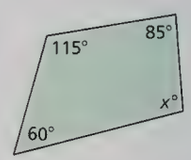
	ROUND 9	ROUND 10	FINAL SCORE
Your Friend	22	12	195
You	9	25	?

USING TOOLS **The sum of the angle measures of a quadrilateral is 360°. In Exercises 17–20, write and solve an equation to find the value of x. Use a protractor to check the reasonableness of your answer.**

17. 

18. 

19. 

20. 

In Exercises 21–30, solve the equation. Justify each step. Check your solution. *(See Example 2.)*

21. $5g = 20$
22. $4q = 52$
23. $p \div 5 = 3$
24. $y \div 7 = 1$
25. $-8r = 64$
26. $x \div (-2) = 8$
27. $\frac{x}{6} = 8$
28. $\frac{w}{-3} = 6$
29. $-54 = 9s$
30. $-7 = \frac{t}{7}$

In Exercises 31–38, solve the equation. Check your solution.

31. $\frac{3}{2} + t = \frac{1}{2}$

32. $b - \frac{3}{16} = \frac{5}{16}$

33. $\frac{3}{7}m = 6$

34. $-\frac{2}{5}y = 4$

35. $5.2 = a - 0.4$

36. $f + 3\pi = 7\pi$

37. $-108\pi = 6\pi j$

38. $x \div (-2) = 1.4$

ERROR ANALYSIS **In Exercises 39 and 40, describe and correct the error in solving the equation.**

39.

$$-0.8 + r = 12.6$$
$$r = 12.6 + (-0.8)$$
$$r = 11.8$$

40.

$$-\frac{m}{3} = -4$$
$$3 \cdot \left(-\frac{m}{3}\right) = 3 \cdot (-4)$$
$$m = -12$$

41. **ANALYZING RELATIONSHIPS** A baker orders 162 eggs. Each carton contains 18 eggs. Which equation can you use to find the number x of cartons? Explain your reasoning and solve the equation.

Ⓐ $162x = 18$ Ⓑ $\frac{x}{18} = 162$

Ⓒ $18x = 162$ Ⓓ $x + 18 = 162$

MODELING WITH MATHEMATICS **In Exercises 42–44, write and solve an equation to answer the question.** *(See Examples 3 and 4.)*

42. The temperature at 5 P.M. is 20°F. The temperature at 10 P.M. is −5°F. How many degrees did the temperature fall?

43. The length of an American flag is 1.9 times its width. What is the width of the flag?



9.5 ft

44. The balance of an investment account is $308 more than the balance 4 years ago. The current balance of the account is $4708. What was the balance 4 years ago?

45. **REASONING** Identify the property of equality that makes Equation 1 and Equation 2 equivalent.

Equation 1 $x - \frac{1}{2} = \frac{x}{4} + 3$

Equation 2 $4x - 2 = x + 12$

46. **PROBLEM SOLVING** Tatami mats are used as a floor covering in Japan. One possible layout uses four identical rectangular mats and one square mat, as shown. The area of the square mat is half the area of one of the rectangular mats.



Total area = 81 ft²

a. Write and solve an equation to find the area of one rectangular mat.

b. The length of a rectangular mat is twice the width. Use Guess, Check, and Revise to find the dimensions of one rectangular mat.

47. **PROBLEM SOLVING** You spend $30.40 on 4 CDs. Each CD costs the same amount and is on sale for 80% of the original price.



a. Write and solve an equation to find how much you spend on each CD.

b. The next day, the CDs are no longer on sale. You have $25. Will you be able to buy 3 more CDs? Explain your reasoning.

48. **ANALYZING RELATIONSHIPS** As c increases, does the value of x *increase*, *decrease*, or *stay the same* for each equation? Assume c is positive.

Equation	Value of x
$x - c = 0$	
$cx = 1$	
$cx = c$	
$\frac{x}{c} = 1$	

Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

ANSWERS

31. $t = -1$

32. $b = \frac{1}{2}$

33. $m = 14$

34. $y = -10$

35. $a = 5.6$

36. $f = 4\pi$

37. $j = -18$

38. $x = -2.8$

39. Subtract −0.8 from each side, not add; $r = 12.6 - (-0.8)$; $r = 13.4$

40. Multiply each side by −3, not 3; $-3 \cdot \left(-\frac{m}{3}\right) = -3 \cdot (-4)$; $m = 12$

41. C; Multiplying the number of eggs in each carton by the number of cartons will give the total number of eggs; 9 cartons

42. $-5 = 20 - T$; 25°F

43. $9.5 = 1.9w$; 5 ft

44. $4708 = b + 308$; $4400

45. Multiplication Property of Equality

46. **a.** $4.5A = 81$; 18 ft²

b. 6 ft by 3 ft

47. **a.** $4p = 30.40$; $7.60

b. no; Each CD costs $9.50 at the regular price, so 3 CDs would cost $28.50 which is greater than $25.

48. increase; decrease; stay the same; increase

ANSWERS

49. a. 5; 10
b. −2; 9

50. a. 100%
b. It shows the sum of all the parts is 100%; Solve it for x; $x = 31$

51. 71; Because $\frac{1}{6}$ of the girls is 6, there are 36 girls. Because $\frac{2}{7}$ of the boys is 10, there are 35 boys; $36 + 35 = 71$

52. *Sample answer:* $5x = 25 + 5$; You and 4 friends go to a movie. The total cost of the 5 tickets is \$25 and the snacks you and your friends split cost an additional \$5. How much does each person pay?; $x = 6$; Each person pays \$6.

53. $B = 12\pi$ in.^{2}

54. $h = 9$ cm

55. $B = 9\pi$ m^2

56. $h = 3.5$ ft

57. a. 132 hits
b. no; Dividing by a larger number of at-bats decreases the value of the average.

58. $8y + 24$

59. $\frac{5}{6}x + \frac{15}{4}$

60. $5m + 15 + 5n$

61. $8p + 16q + 24$

62. 300

63. 0.02

64. 0.47

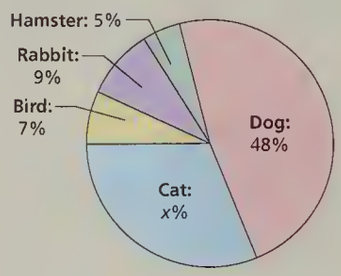
65. 298.26

49. USING STRUCTURE Use the values −2, 5, 9, and 10 to complete each statement about the equation $ax = b - 5$.

a. When $a =$ ___ and $b =$ ___, x is a positive integer.

b. When $a =$ ___ and $b =$ ___, x is a negative integer.

50. HOW DO YOU SEE IT? The circle graph shows the percents of different animals sold at a local pet store in 1 year.



a. What percent is represented by the entire circle?

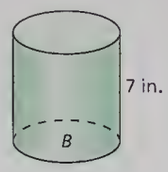
b. How does the equation $7 + 9 + 5 + 48 + x = 100$ relate to the circle graph? How can you use this equation to find the percent of cats sold?

51. REASONING One-sixth of the girls and two-sevenths of the boys in a school marching band are in the percussion section. The percussion section has 6 girls and 10 boys. How many students are in the marching band? Explain.

52. THOUGHT PROVOKING Write a real-life problem that can be modeled by an equation equivalent to the equation $5x = 30$. Then solve the equation and write the answer in the context of your real-life problem.

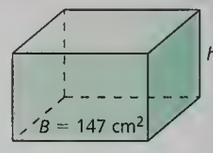
MATHEMATICAL CONNECTIONS In Exercises 53–56, find the height h or the area of the base B of the solid.

53.



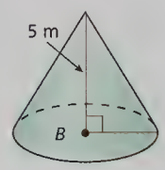
Volume = 84π in.^{3}

54.



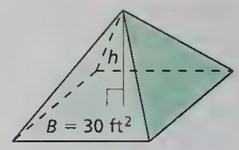
Volume = 1323 cm^3

55.



Volume = 15π m^3

56.



Volume = 35 ft^3

57. MAKING AN ARGUMENT In baseball, a player's batting average is calculated by dividing the number of hits by the number of at-bats. The table shows Player A's batting average and number of at-bats for three regular seasons.

Season	Batting average	At-bats
2010	.312	596
2011	.296	446
2012	.295	599

a. How many hits did Player A have in the 2011 regular season? Round your answer to the nearest whole number.

b. Player B had 33 fewer hits in the 2011 season than Player A but had a greater batting average. Your friend concludes that Player B had more at-bats in the 2011 season than Player A. Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the Distributive Property to simplify the expression. *(Skills Review Handbook)*

58. $8(y + 3)$ **59.** $\frac{5}{6}\left(x + \frac{1}{2} + 4\right)$ **60.** $5(m + 3 + n)$ **61.** $4(2p + 4q + 6)$

Copy and complete the statement. Round to the nearest hundredth, if necessary. *(Skills Review Handbook)*

62. $\frac{5\text{ L}}{\text{min}} = \frac{\quad\text{L}}{\text{h}}$ **63.** $\frac{68\text{ mi}}{\text{h}} \approx \frac{\quad\text{mi}}{\text{sec}}$

64. $\frac{7\text{ gal}}{\text{min}} \approx \frac{\quad\text{qt}}{\text{sec}}$ **65.** $\frac{8\text{ km}}{\text{min}} \approx \frac{\quad\text{mi}}{\text{h}}$

Mini-Assessment

Solve the equation.

1. $t + 17 = 3$ $t = -14$
2. $g - 2 = -5$ $g = -3$
3. $-13.5 = 2.7s$ $s = -5$
4. $\frac{j}{3} = 8$ $j = 24$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

1.2 Solving Multi-Step Equations

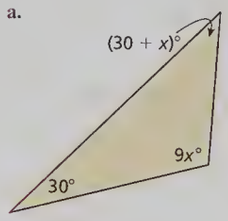
TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.5.A
A.10.D

Essential Question How can you use multi-step equations to solve real-life problems?

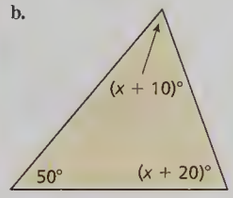
EXPLORATION 1 Solving for the Angle Measures of a Polygon

Work with a partner. The sum S of the angle measures of a polygon with n sides can be found using the formula $S = 180(n - 2)$. Write and solve an equation to find each value of x. Justify the steps in your solution. Then find the angle measures of each polygon. How can you check the reasonableness of your answers?

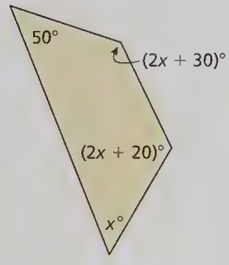
a.



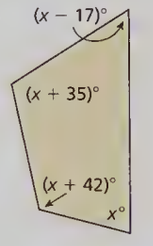
b.



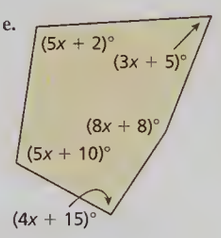
c.



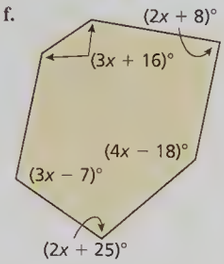
d.



e.



f.



JUSTIFYING THE SOLUTION
To be proficient in math, you need to be sure your answers make sense in the context of the problem. For instance, if you find the angle measures of a triangle, and they have a sum that is not equal to 180°, then you should check your work for mistakes.

EXPLORATION 2 Writing a Multi-Step Equation

Work with a partner.

a. Draw an irregular polygon.

b. Measure the angles of the polygon. Record the measurements on a separate sheet of paper.

c. Choose a value for x. Then, using this value, work backward to assign a variable expression to each angle measure, as in Exploration 1.

d. Trade polygons with your partner.

e. Solve an equation to find the angle measures of the polygon your partner drew. Do your answers seem reasonable? Explain.

Communicate Your Answer

3. How can you use multi-step equations to solve real-life problems?

4. In Exploration 1, you were given the formula for the sum S of the angle measures of a polygon with n sides. Explain why this formula works.

5. The sum of the angle measures of a polygon is 1080°. How many sides does the polygon have? Explain how you found your answer.

SUPPORTING English Language Learners

Students work in groups of mixed language abilities to complete Exploration 2. Beginners consult with Intermediate and Advanced students to clarify the meaning of the text. Advanced High students consult with native speakers, the teacher, or a classroom aide as needed.

Beginning Complete part (a).
Intermediate Complete part (b).
Advanced Complete part (c).
Advanced High Complete part (e) using the polygon and variable expression created by members of the group.

4.F.6 Use support from peers and teachers to read grade-appropriate content area text.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.5.A The student is expected to solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

A.10.D The student is expected to rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

ANSWERS

1. a. $(30 + x) + 9x + 30 = 180$; Write the equation.
$10x + 60 = 180$; Combine like terms.
$10x = 120$; Subtract 60 from each side.
$x = 12$; Divide each side by 10.
42°, 108°, 30°

b. $(x + 10) + (x + 20) + 50 = 180$; Write the equation.
$2x + 80 = 180$; Combine like terms.
$2x = 100$; Subtract 80 from each side.
$x = 50$; Divide each side by 2.
60°, 70°, 50°

c. $50 + (2x + 30) + (2x + 20) + x = 360$; Write the equation.
$5x + 100 = 360$; Combine like terms.
$5x = 260$; Subtract 100 from each side.
$x = 52$; Divide each side by 5.
50°, 134°, 124°, 52°

d. $(x - 17) + x + (x + 42) + (x + 35) = 360$; Write the equation.
$4x + 60 = 360$; Combine like terms.
$4x = 300$; Subtract 60 from each side.
$x = 75$; Divide each side by 4.
58°, 75°, 117°, 110°

1e–5. See Additional Answers.

Extra Example 1

Solve $-9 = 1.5r + 3$. Check your solution.

$r = -8$

Extra Example 2

Solve $5f + 4f - 8 = 19$. Check your solution.

$f = 3$

MONITORING PROGRESS ANSWERS

1. $n = -3$
2. $c = -20$
3. $x = -\frac{1}{2}$

1.2 Lesson

What You Will Learn

- Solve multi-step linear equations using inverse operations.
- Use multi-step linear equations to solve real-life problems.
- Use unit analysis to model real-life problems.

Core Vocabulary

Previous

inverse operations

mean

Solving Multi-Step Linear Equations

Core Concept

Solving Multi-Step Equations

To solve a multi-step equation, simplify each side of the equation, if necessary. Then use inverse operations to isolate the variable.

EXAMPLE 1 Solving a Two-Step Equation

Solve $2.5x - 13 = 2$. Check your solution.

SOLUTION

$2.5x - 13 = 2$		Write the equation.
Undo the subtraction. →	$+13 \quad +13$	Add 13 to each side.
$2.5x = 15$		Simplify.
Undo the multiplication. →	$\frac{2.5x}{2.5} = \frac{15}{2.5}$	Divide each side by 2.5.
$x = 6$		Simplify.

The solution is $x = 6$.

Check

$2.5x - 13 = 2$

$2.5(6) - 13 \stackrel{?}{=} 2$

$2 = 2$ ✓

EXAMPLE 2 Combining Like Terms to Solve an Equation

Solve $-12 = 9x - 6x + 15$. Check your solution.

SOLUTION

$-12 = 9x - 6x + 15$		Write the equation.
$-12 = 3x + 15$		Combine like terms.
Undo the addition. →	$-15 \quad -15$	Subtract 15 from each side.
$-27 = 3x$		Simplify.
Undo the multiplication. →	$\frac{-27}{3} = \frac{3x}{3}$	Divide each side by 3.
$-9 = x$		Simplify.

The solution is $x = -9$.

Check

$-12 = 9x - 6x + 15$

$-12 \stackrel{?}{=} 9(-9) - 6(-9) + 15$

$-12 = -12$ ✓

Monitoring Progress

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Solve the equation. Check your solution.

1. $-2n + 3 = 9$
2. $-21 = \frac{1}{2}c - 11$
3. $-2x - 10x + 12 = 18$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Visual Aids

To solve an equation, students must perform the same operation on each side of an equation, maintaining equivalency from one step to the next until they reach the solution. Use a balance scale to model this concept. Explain that the equal sign (=) separates the two sides of the equation, which must be kept in balance. Aligning the equal signs vertically on their papers makes it easy for students to identify the result of each operation on both sides of an equation as they check their work.

EXAMPLE 3 **Using Structure to Solve a Multi-Step Equation**

Solve $2(1 - x) + 3 = -8$. Check your solution.

SOLUTION

Method 1 One way to solve the equation is by using the Distributive Property.

$2(1 - x) + 3 = -8$	Write the equation.
$2(1) - 2(x) + 3 = -8$	Distributive Property
$2 - 2x + 3 = -8$	Multiply.
$-2x + 5 = -8$	Combine like terms.
$-5 \quad -5$	Subtract 5 from each side.
$-2x = -13$	Simplify.
$\frac{-2x}{-2} = \frac{-13}{-2}$	Divide each side by -2.
$x = 6.5$	Simplify.

▶ The solution is $x = 6.5$.

Check

$2(1 - x) + 3 = -8$

$2(1 - 6.5) + 3 \stackrel{?}{=} -8$

$-8 = -8$ ✓

Method 2 Another way to solve the equation is by interpreting the expression $1 - x$ as a single quantity.

ANALYZING MATHEMATICAL RELATIONSHIPS

First solve for the expression $1 - x$, and then solve for x.

$2(1 - x) + 3 = -8$	Write the equation.
$-3 \quad -3$	Subtract 3 from each side.
$2(1 - x) = -11$	Simplify.
$\frac{2(1 - x)}{2} = \frac{-11}{2}$	Divide each side by 2.
$1 - x = -5.5$	Simplify.
$-1 \quad -1$	Subtract 1 from each side.
$-x = -6.5$	Simplify.
$\frac{-x}{-1} = \frac{-6.5}{-1}$	Divide each side by -1.
$x = 6.5$	Simplify.

▶ The solution is $x = 6.5$, which is the same solution obtained in Method 1.

Monitoring Progress

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Solve the equation. Check your solution.

4. $3(x + 1) + 6 = -9$
5. $15 = 5 + 4(2d - 3)$
6. $13 = -2(y - 4) + 3y$
7. $2x(5 - 3) - 3x = 5$
8. $-4(2m + 5) - 3m = 35$
9. $5(3 - x) + 2(3 - x) = 14$

Extra Example 3

Solve $4(n - 6) - 1 = 31$. Check your solution. $n = 14$

MONITORING PROGRESS ANSWERS

4. $x = -6$
5. $d = 2.75$
6. $y = 5$
7. $x = 5$
8. $m = -5$
9. $x = 1$

Extra Example 4

Use the table to find the high temperature x on Wednesday so that the mean high temperature for the 5 days is 65°F.

Day	High Temperature (°F)
Monday	76
Tuesday	70
Wednesday	x
Thursday	68
Friday	55

The high temperature on Wednesday needs to be 56°F.

MONITORING PROGRESS ANSWER

10. 48 lb/in.^{2}

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**



Use the table to find the number of miles x you need to bike on Friday so that the mean number of miles biked per day is 5.

Day	Miles
Monday	3.5
Tuesday	5.5
Wednesday	0
Thursday	5
Friday	x

SOLUTION

1. **Understand the Problem** You know how many miles you biked Monday through Thursday. You are asked to find the number of miles you need to bike on Friday so that the mean number of miles biked per day is 5.
2. **Make a Plan** Use the definition of mean to write an equation that represents the problem. Then solve the equation.
3. **Solve the Problem** The mean of a data set is the sum of the data divided by the number of data values.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5 \qquad \text{Write the equation.}$$

$$\frac{14 + x}{5} = 5 \qquad \text{Combine like terms.}$$

$$5 \cdot \frac{14 + x}{5} = 5 \cdot 5 \qquad \text{Multiply each side by 5.}$$

$$14 + x = 25 \qquad \text{Simplify.}$$

$$-14 \qquad -14 \qquad \text{Subtract 14 from each side.}$$

$$x = 11 \qquad \text{Simplify.}$$

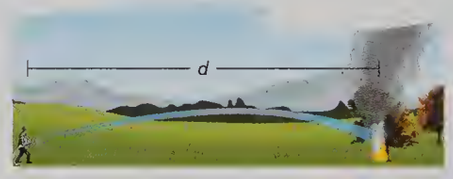
▶ You need to bike 11 miles on Friday.

4. **Look Back** Notice that on the days that you did bike, the values are close to the mean. Because you did not bike on Wednesday, you need to bike about twice the mean on Friday. Eleven miles is about twice the mean. So, your solution is reasonable.

Monitoring Progress

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10. The formula $d = \frac{1}{2}n + 26$ relates the nozzle pressure n (in pounds per square inch) of a fire hose and the maximum horizontal distance the water reaches d (in feet). How much pressure is needed to reach a fire 50 feet away?



REMEMBER

When you add **miles** to **miles**, you get **miles**. But, when you divide **miles** by **days**, you get **miles per day**.

Using Unit Analysis to Model Real-Life Problems

When you write an equation to model a real-life problem, you should check that the units on each side of the equation balance. For instance, in Example 4, notice how the units balance.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5 \qquad \frac{\text{mi}}{\text{day}} = \frac{\text{mi}}{\text{day}} \checkmark$$

miles, miles per day, per, days

EXAMPLE 5 **Solving a Real-Life Problem**

Your school's drama club charges $4 per person for admission to a play. The club borrowed $400 to pay for costumes and props. After paying back the loan, the club has a profit of $100. How many people attended the play?

SOLUTION

1. **Understand the Problem** You know how much the club charges for admission. You also know how much the club borrowed and its profit. You are asked to find how many people attended the play.
2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation.
3. **Solve the Problem**

REMEMBER

When you multiply **dollars per person** by **people**, you get **dollars**.

Words Ticket price • Number of people who attended − Amount of loan = Profit

Variable Let x be the number of people who attended.

Equation $\frac{\$4}{\text{person}} \cdot x \text{ people} - \$400 = \$100$ $\quad \$ = \$$ ✓

$$4x - 400 = 100 \qquad \text{Write the equation.}$$
$$4x - 400 + 400 = 100 + 400 \qquad \text{Add 400 to each side.}$$
$$4x = 500 \qquad \text{Simplify.}$$
$$\frac{4x}{4} = \frac{500}{4} \qquad \text{Divide each side by 4.}$$
$$x = 125 \qquad \text{Simplify.}$$

▶ So, 125 people attended the play.

4. **Look Back** To check that your solution is reasonable, multiply $4 per person by 125 people. The result is $500. After paying back the $400 loan, the club has $100, which is the profit.



Monitoring Progress

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11. You have 96 feet of fencing to enclose a rectangular pen for your dog. To provide sufficient running space for your dog to exercise, the pen should be three times as long as it is wide. Find the dimensions of the pen.

Extra Example 5

You spent $50 on supplies for the bake sale. Each item sold for $2. After the sale, you had a profit of $180. How many items did you sell? 115 items

MONITORING PROGRESS ANSWER

11. 36 ft by 12 ft

Closure

- **Exit Ticket:** Solve $8x + 9 - 4x = 25$. Check your solution. $x = 4$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–3, 5–25 odd, 35, 37, 41, 47, 50, 57–65

Average: 1, 2–34 even, 35–38, 40, 42, 47, 50, 57–65

Advanced: 1, 2, 6, 10, 14–16, 22–34 even, 35–37, 38–44 even, 46–52, 57–65

HOMEWORK CHECK

Basic: 3, 11, 15, 17, 35

Average: 10, 14, 16, 18, 35

Advanced: 6, 14, 16, 22, 36

ANSWERS

1. like terms
2. Use the Distributive Property to simplify, then solve for x; Solve for the expression $4x - 11$, then solve for x.
3. $w = 4$
4. $g = 8$
5. $q = 1$
6. $m = -3$
7. $z = -32$
8. $a = 6$
9. $h = 4$
10. $d = -16$
11. $y = 4$
12. $n = 4$
13. $v = 3$
14. $c = -2$
15. 6 min
16. $553 = 265 + 48t$; 6 h
17. $z = 3$
18. $g = -3$
19. $m = 3$
20. $h = -9$
21. $c = 5$
22. $y = -19$
23. $x = 29$
24. $r = -6$
25. $k = 45$; 45°, 90°, 45°
26. $a = 60$; 120°, 60°, 120°, 60°
27. $b = 90$; 90°, 135°, 90°, 90°, 135°
28. $x = 125$; 120°, 100°, 120°, 135°, 120°, 125°
29. $2n + 13 = 75$; $n = 31$
30. $3n - 4 = -19$; $n = -5$
31. $8 + \frac{n}{3} = -2$; $n = -30$
32. $2n + \frac{1}{2}n = 10$; $n = 4$
33. $6(n + 15) = -42$; $n = -22$
34. $4(n - 7) = 12$; $n = 10$

1.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** To solve the equation $2x + 3x = 20$, first combine $2x$ and $3x$ because they are _____.
2. **WRITING** Describe two ways to solve the equation $2(4x - 11) = 10$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–14, solve the equation. Check your solution. *(See Examples 1 and 2.)*

3. $3w + 7 = 19$
4. $2g - 13 = 3$
5. $11 = 12 - q$
6. $10 = 7 - m$
7. $5 = \frac{z}{-4} - 3$
8. $\frac{a}{3} + 4 = 6$
9. $\frac{h + 6}{5} = 2$
10. $\frac{d - 8}{-2} = 12$
11. $8y + 3y = 44$
12. $36 = 13n - 4n$
13. $12v + 10v + 14 = 80$
14. $6c - 8 - 2c = -16$
15. **MODELING WITH MATHEMATICS** The altitude a (in feet) of a plane t minutes after liftoff is given by $a = 3400t + 600$. How many minutes after liftoff is the plane at an altitude of 21,000 feet?



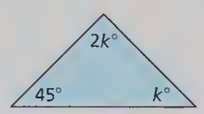
16. **MODELING WITH MATHEMATICS** A repair bill for your car is \$553. The parts cost \$265. The labor cost is \$48 per hour. Write and solve an equation to find the number of hours of labor spent repairing the car.

In Exercises 17–24, solve the equation. Check your solution. *(See Example 3.)*

17. $4(z + 5) = 32$
18. $-2(4g - 3) = 30$
19. $6 + 5(m + 1) = 26$
20. $5h + 2(11 - h) = -5$
21. $27 = 3c - 3(6 - 2c)$
22. $-3 = 12y - 5(2y - 7)$
23. $-3(3 + x) + 4(x - 6) = -4$
24. $5(r + 9) - 2(1 - r) = 1$

USING TOOLS **In Exercises 25–28, find the value of the variable. Then find the angle measures of the polygon. Use a protractor to check the reasonableness of your answer.**

25.



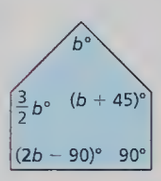
Sum of angle measures: 180°

26.



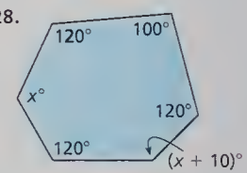
Sum of angle measures: 360°

27.



Sum of angle measures: 540°

28.



Sum of angle measures: 720°

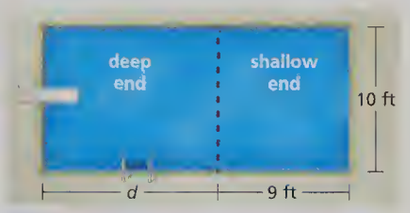
In Exercises 29–34, write and solve an equation to find the number.

29. The sum of twice a number and 13 is 75.
30. The difference of three times a number and 4 is −19.
31. Eight plus the quotient of a number and 3 is −2.
32. The sum of twice a number and half the number is 10.
33. Six times the sum of a number and 15 is −42.
34. Four times the difference of a number and 7 is 12.

USING EQUATIONS **In Exercises 35–37, write and solve an equation to answer the question. Check that the units on each side of the equation balance.** *(See Examples 4 and 5.)*

35. During the summer, you work 30 hours per week at a gas station and earn $8.75 per hour. You also work as a landscaper for $11 per hour and can work as many hours as you want. You want to earn a total of $400 per week. How many hours must you work as a landscaper?

36. The area of the surface of the swimming pool is 210 square feet. What is the length d of the deep end (in feet)?



37. You order two tacos and a salad. The salad costs $2.50. You pay 8% sales tax and leave a $3 tip. You pay a total of $13.80. How much does one taco cost?

JUSTIFYING STEPS **In Exercises 38 and 39, justify each step of the solution.**

38. $-\frac{1}{2}(5x - 8) - 1 = 6$ Write the equation.

$-\frac{1}{2}(5x - 8) = 7$

$5x - 8 = -14$

$5x = -6$

$x = -\frac{6}{5}$

39. $2(x + 3) + x = -9$ Write the equation.

$2(x) + 2(3) + x = -9$

$2x + 6 + x = -9$

$3x + 6 = -9$

$3x = -15$

$x = -5$

ERROR ANALYSIS **In Exercises 40 and 41, describe and correct the error in solving the equation.**

40.

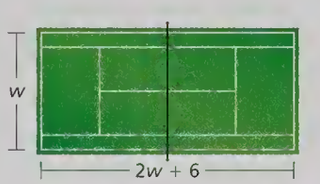
$$-2(7 - y) + 4 = -4$$
$$-14 - 2y + 4 = -4$$
$$-10 - 2y = -4$$
$$-2y = 6$$
$$y = -3$$

41.

$$\frac{1}{4}(x - 2) + 4 = 12$$
$$\frac{1}{4}(x - 2) = 8$$
$$x - 2 = 2$$
$$x = 4$$

MATHEMATICAL CONNECTIONS **In Exercises 42–44, write and solve an equation to answer the question.**

42. The perimeter of the tennis court is 228 feet. What are the dimensions of the court?



43. The perimeter of the Norwegian flag is 190 inches. What are the dimensions of the flag?



44. The perimeter of the school crossing sign is 102 inches. What is the length of each side?



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ANSWERS

35. $30(8.75) + 11t = 400$; 12.5 h

36. $10d + 10(9) = 210$; 12 ft

37. $1.08(2t + 2.50) + 3 = 13.80$; $3.75

38. Add 1 to each side; Multiply each side by -2; Add 8 to each side; Divide each side by 5.

39. Distributive Property; Simplify; Combine like terms; Subtract 6 from each side; Divide each side by 3.

40. When using the Distributive Property in the second step, the second term should be positive; $-14 + 2y + 4 = -4$; $-10 + 2y = -4$; $2y = 6$; $y = 3$

41. In the third step, the right side should be 8×4, not $8 \div 4$; $x - 2 = 32$; $x = 34$

42. $2(2w + 6) + 2w = 228$, $w = 36$; 78 ft by 36 ft

43. $2y + 2 \cdot \frac{11}{8}y = 190$, $y = 40$; 55 in. by 40 in.

44. $(s + 6) + (s + 6) + s + 2s + s = 102$; $s = 15$; 21 in., 21 in., 15 in., 30 in., 15 in.

ANSWERS

45. $x = \frac{15}{16}$; *Sample answer:* method 1; There are no fractions until the last step.

46. 6 tickets

47. no; Solving the equation $0.25(d + 8) + 0.10d = 2.80$ results in the number of dimes not being a whole number.

48. See Additional Answers.

49. 16, 18, 20; The next consecutive even integers after $2n$ are $2n + 2$ and $2n + 4$. Solve the equation $2n + (2n + 2) + (2n + 4) = 54$. Then substitute the solution into the expressions for the integers.

50. **a.** greater than 20; The attendance at the second meeting is only 1 above 20, and the attendance at the other two is more than 1 below 20, so the attendance at the fourth meeting must be greater than 20 to have a mean of 20.

b. *Sample answer:* 24

c. Find the mean of the four numbers.

51–65. See Additional Answers.

45. COMPARING METHODS Solve the equation $2(4 - 8x) + 6 = -1$ using (a) Method 1 from Example 3 and (b) Method 2 from Example 3. Which method do you prefer? Explain.

46. PROBLEM SOLVING An online ticket agency charges the amounts shown for basketball tickets. The total cost for an order is \$220.70. How many tickets are purchased?

Charge	Amount
Ticket price	\$32.50 per ticket
Convenience charge	\$3.30 per ticket
Processing charge	\$5.90 per order

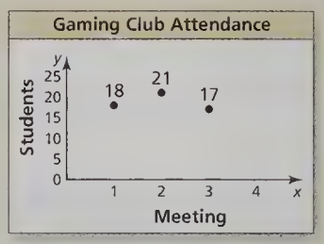
47. MAKING AN ARGUMENT You have quarters and dimes that total \$2.80. Your friend says it is possible that the number of quarters is 8 more than the number of dimes. Is your friend correct? Explain.

48. THOUGHT PROVOKING You teach a math class and assign a weight to each component of the class. You determine final grades by totaling the products of the weights and the component scores. Choose values for the remaining weights and find the necessary score on the final exam for a student to earn an A (90%) in the class, if possible. Explain your reasoning.

Component	Student's score	Weight	Score × Weight
Class Participation	92%	0.20	92% × 0.20 = 18.4%
Homework	95%		
Midterm Exam	88%		
Final Exam			
Total		1	

49. REASONING An even integer can be represented by the expression $2n$, where n is any integer. Find three consecutive even integers that have a sum of 54. Explain your reasoning.

50. HOW DO YOU SEE IT? The scatter plot shows the attendance for each meeting of a gaming club.



a. The mean attendance for the first four meetings is 20. Is the number of students who attended the fourth meeting greater than or less than 20? Explain.

b. Estimate the number of students who attended the fourth meeting.

c. Describe a way you can check your estimate in part (b).

REASONING **In Exercises 51–56, the letters a, b, and c represent nonzero constants. Solve the equation for x.**

51. $bx = -7$

52. $x + a = \frac{3}{4}$

53. $ax - b = 12.5$

54. $ax + b = c$

55. $2bx - bx = -8$

56. $cx - 4b = 5b$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

57. $4m + 5 - 3m$ **58.** $9 - 8b + 6b$ **59.** $6t + 3(1 - 2t) - 5$

Determine whether (a) $x = -1$ or (b) $x = 2$ is a solution of the equation. *(Skills Review Handbook)*

60. $x - 8 = -9$ **61.** $x + 1.5 = 3.5$ **62.** $2x - 1 = 3$

63. $3x + 4 = 1$ **64.** $x + 4 = 3x$ **65.** $-2(x - 1) = 1 - 3x$

Mini-Assessment

Solve each equation.

1. $-5d + 4 = 12$ $d = -1.6$
2. $-2n + 9n - 8 = 27$ $n = 5$
3. $-18 = 5(2 - x) + 2$ $x = 6$
4. Use the table to find the grade g you need on Quiz 4 so that your quiz grade average is 90.

Quiz	Grade
1	92
2	89
3	85
4	g

You need a grade of 94.

5. A team pays \$50 to reserve a banquet table. The players on the team pay \$9 per person. The team spends a total of \$104. How many players sit at the team table? Six players sit at the team table.

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

1.1–1.2 What Did You Learn?

Core Vocabulary

conjecture, *p. 3*
rule, *p. 3*
theorem, *p. 3*
equation, *p. 4*
linear equation in one variable, *p. 4*
solution, *p. 4*
inverse operations, *p. 4*
equivalent equations, *p. 4*

Core Concepts

Section 1.1

Addition Property of Equality, *p. 4*
Subtraction Property of Equality, *p. 4*
Multiplication Property of Equality, *p. 5*
Division Property of Equality, *p. 5*
Four-Step Approach to Problem Solving, *p. 6*
Common Problem-Solving Strategies, *p. 7*

Section 1.2

Solving Multi-Step Equations, *p. 12*
Unit Analysis, *p. 15*

Mathematical Thinking

1. How did you make sense of the relationships between the quantities in Exercise 46 on page 9?
2. What is the limitation of the tool you used in Exercises 25–28 on page 16?

Study Skills

Completing Homework Efficiently

Before doing homework, review the core concepts and examples. Use the tutorials at *BigIdeasMath.com* for additional help.

Complete homework as though you are also preparing for a quiz. Memorize different types of problems, vocabulary, rules, and so on.



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ANSWERS

1. *Sample answer:* Let A represent the area of a single rectangle. Because each rectangle has the same area, and the area of the square is half the area of a rectangle, use the expression $4A + \frac{1}{2}A$ to represent the total area.
2. *Sample answer:* A limitation of a protractor is centering it exactly on each vertex point.

ANSWERS

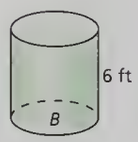
1. $x = -2$; Subtract 9 from each side.
2. $z = 12.4$; Add 3.8 to each side.
3. $r = -5$; Divide each side by -12.
4. $p = 24$; Multiply each side by $\frac{4}{3}$.
5. $B = 4\pi$ ft^2
6. $h = 3$ m
7. $m = 8$
8. $v = 5$
9. $w = \frac{1}{5}$
10. $a = -0.6$
11. $k = -9$
12. $x = 14$
13. $2 = \frac{s}{5}$; 10 sec
14. 55; *Sample answer:* There are 100 marbles and $\frac{11}{20}$ of them are blue.
15. 13, 15, 17; Let $2x + 1$ represent the first odd integer. The next consecutive odd integers are $2x + 3$ and $2x + 5$. Solve the equation $(2x + 1) + (2x + 3) + (2x + 5) = 45$. Then substitute the solution into the expressions for the integers.
16. $0.10s = 250$; \$2500
17. $12 + 2x = 15$; $1\frac{1}{2}$ ft

1.1–1.2 Quiz

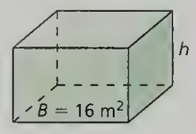
Solve the equation. Justify each step. Check your solution. *(Section 1.1)*

1. $x + 9 = 7$
2. $8.6 = z - 3.8$
3. $60 = -12r$
4. $\frac{3}{4}p = 18$

Find the height h or the area of the base B of the solid. *(Section 1.1)*

5. 

Volume = 24π ft^3

6. 

Volume = 48 m^3

Solve the equation. Check your solution. *(Section 1.2)*

7. $2m - 3 = 13$
8. $5 = 10 - v$
9. $5 = 7w + 8w + 2$
10. $-21a + 28a - 6 = -10.2$
11. $2k - 3(2k - 3) = 45$
12. $68 = \frac{1}{5}(20x + 50) + 2$

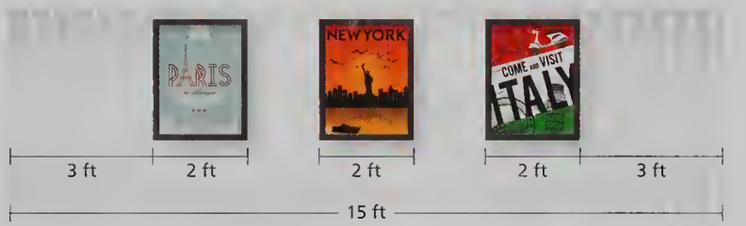
13. To estimate how many miles you are from a thunderstorm, count the seconds between when you see lightning and when you hear thunder. Then divide by 5. Write and solve an equation to determine how many seconds you would count for a thunderstorm that is 2 miles away. *(Section 1.1)*

14. A jar contains red, white, and blue marbles. One-fifth of the marbles are red and one-fourth of the marbles are white. There are 20 red marbles in the jar. How many blue marbles are in the jar? Explain. *(Section 1.1)*

15. Find three consecutive odd integers that have a sum of 45. Explain your reasoning. *(Section 1.2)*

16. You work as a salesperson for a marketing company. The position pays \$300 per week plus 10% of your total weekly sales. You normally earn \$550 each week. Write and solve an equation to determine how much your total weekly sales are normally. *(Section 1.2)*

17. You want to hang three equally-sized travel posters on a wall so that the posters on the ends are each 3 feet from the end of the wall. You want the spacing between posters to be equal. Write and solve an equation to determine how much space you should leave between the posters. *(Section 1.2)*



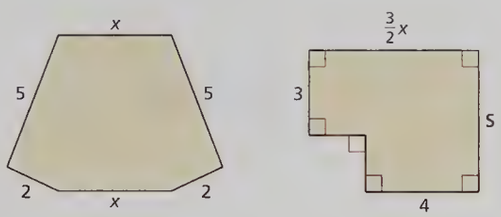
1.3 Solving Equations with Variables on Both Sides

Texas Essential Knowledge and Skills
A.5.A
A.10.D

Essential Question How can you solve an equation that has variables on both sides?

EXPLORATION 1 Perimeter

Work with a partner. The two polygons have the same perimeter. Use this information to write and solve an equation involving x. Explain the process you used to find the solution. Then find the perimeter of each polygon.



EXPLORATION 2 Perimeter and Area

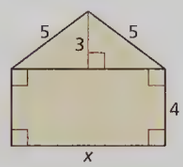
Work with a partner.

- Each figure has the unusual property that the value of its perimeter (in feet) is equal to the value of its area (in square feet). Use this information to write an equation for each figure.
- Solve each equation for x. Explain the process you used to find the solution.
- Find the perimeter and area of each figure.

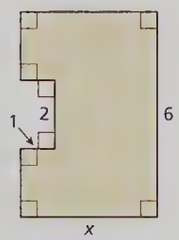
ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to visualize complex things, such as composite figures, as being made up of simpler, more manageable parts.

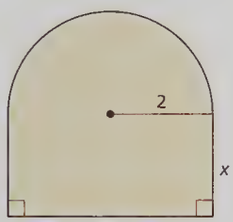
a.



b.



c.



Communicate Your Answer

3. How can you solve an equation that has variables on both sides?
4. Write three equations that have the variable x on both sides. The equations should be different from those you wrote in Explorations 1 and 2. Have your partner solve the equations.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

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Texas Essential Knowledge and Skills

A.5.A The student is expected to solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

A.10.D The student is expected to rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

ANSWERS

1. $2x + 14 = 3x + 10$; $x = 4$; Add the side lengths of each figure to get the perimeters and set them equal to each other; $P = 22$
2. a. $x + 18 = 4x + \frac{3}{2}x$; $x = 4$; Add the side lengths to get the perimeter. Add the area of the triangle to the area of the rectangle to get the total area. Then set the perimeter equal to the area; $P = 22$ ft, $A = 22$ ft^2
 b. $2x + 14 = 6x - 2$; $x = 4$; Add the side lengths to get the perimeter. Subtract the area of the small rectangle from the area of the large rectangle to get the total area. Then set the perimeter equal to the area; $P = 22$ ft, $A = 22$ ft^2
 c. $2\pi + 2x + 4 = 2\pi + 4x$; $x = 2$; Add the circumference of the semicircle to the remaining 3 side lengths to find the perimeter. Add the area of the semicircle to the area of the rectangle to find the total area. Then set the perimeter equal to the area; $P = 2\pi + 8$ ft, $A = 2\pi + 8$ ft^2
3. Collect the variable terms on one side of the equation and the constant terms on the other side of the equation, then solve.
4. *Sample answer:* $5x - 7 = 2x + 5$, $x = 4$; $-x + 3 = 4x + 13$, $x = -2$; $7x = 6x + 4$, $x = 4$

Extra Example 1

Solve $5p = 24 - 3p$. Check your solution.

$p = 3$

Extra Example 2

Solve $2(3n - 4) = \frac{1}{2}(12 - 2n)$. $n = 2$

MONITORING PROGRESS ANSWERS

1. $x = -2$
2. $h = \frac{3}{8}$
3. $n = 0$

1.3 Lesson

What You Will Learn

- Solve linear equations that have variables on both sides.
- Identify special solutions of linear equations.
- Use linear equations to solve real-life problems.

Core Vocabulary

identity, *p. 23*

Previous

inverse operations

Solving Equations with Variables on Both Sides

Core Concept

Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, simplify one or both sides of the equation, if necessary. Then use inverse operations to collect the variable terms on one side, collect the constant terms on the other side, and isolate the variable.

EXAMPLE 1 Solving an Equation with Variables on Both Sides

Solve $10 - 4x = -9x$. Check your solution.

SOLUTION

$10 - 4x = -9x$	Write the equation.
$+ 4x \quad + 4x$	Add $4x$ to each side.
$10 = -5x$	Simplify.
$\frac{10}{-5} = \frac{-5x}{-5}$	Divide each side by -5.
$-2 = x$	Simplify.

Check

$10 - 4x = -9x$

$10 - 4(-2) \stackrel{?}{=} -9(-2)$

$18 = 18$ ✓

▶ The solution is $x = -2$.

EXAMPLE 2 Solving an Equation with Grouping Symbols

Solve $3(3x - 4) = \frac{1}{4}(32x + 56)$.

SOLUTION

$3(3x - 4) = \frac{1}{4}(32x + 56)$	Write the equation.
$9x - 12 = 8x + 14$	Distributive Property
$+ 12 \quad + 12$	Add 12 to each side.
$9x = 8x + 26$	Simplify.
$- 8x \quad - 8x$	Subtract $8x$ from each side.
$x = 26$	Simplify.

▶ The solution is $x = 26$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Check your solution.

1. $-2x = 3x + 10$
2. $\frac{1}{2}(6h - 4) = -5h + 1$
3. $-\frac{3}{4}(8n + 12) = 3(n - 3)$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Visual

Give students an equation with variables on both sides. Have students highlight any term that includes a variable. Have students draw a vertical line through the equal sign. Explain how they can use inverse operations to collect all the highlighted terms on one side of the line. Then, show how they can combine those terms to simplify the equation and find the solution.

Identifying Special Solutions of Linear Equations

Core Concept

Special Solutions of Linear Equations

Equations do not always have one solution. An equation that is true for all values of the variable is an **identity** and has *infinitely many solutions*. An equation that is not true for any value of the variable has *no solution*.

EXAMPLE 3 Identifying the Number of Solutions

Solve each equation.

a. $3(5x + 2) = 15x$ b. $-2(4y + 1) = -8y - 2$

SOLUTION

a.
$$\begin{aligned} 3(5x + 2) &= 15x && \text{Write the equation.} \\ 15x + 6 &= 15x && \text{Distributive Property} \\ -15x \quad & \quad -15x && \text{Subtract } 15x \text{ from each side.} \\ 6 &= 0 \quad \times && \text{Simplify.} \end{aligned}$$

▶ The statement $6 = 0$ is never true. So, the equation has no solution.

b.
$$\begin{aligned} -2(4y + 1) &= -8y - 2 && \text{Write the equation.} \\ -8y - 2 &= -8y - 2 && \text{Distributive Property} \\ +8y \quad & \quad +8y && \text{Add } 8y \text{ to each side.} \\ -2 &= -2 && \text{Simplify.} \end{aligned}$$

▶ The statement $-2 = -2$ is always true. So, the equation is an identity and has infinitely many solutions.

REASONING

The equation $15x + 6 = 15x$ is not true because the number $15x$ cannot be equal to 6 more than itself.

READING

All real numbers are solutions of an identity.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation.

4. $4(1 - p) = -4p + 4$
5. $6m - m = \frac{5}{6}(6m - 10)$
6. $10k + 7 = -3 - 10k$
7. $3(2a - 2) = 2(3a - 3)$

Concept Summary

Steps for Solving Linear Equations

Here are several steps you can use to solve a linear equation. Depending on the equation, you may not need to use some steps.

Step 1 Use the Distributive Property to remove any grouping symbols.

Step 2 Simplify the expression on each side of the equation.

Step 3 Collect the variable terms on one side of the equation and the constant terms on the other side.

Step 4 Isolate the variable.

Step 5 Check your solution.

STUDY TIP

To check an identity, you can choose several different values of the variable.

Extra Example 3

Solve each equation.

a. $4(3d - 1) = 12d$

The statement $-4 = 0$ is never true. So, the equation has no solution.

b. $21 - 7w = 7(3 - w)$

The statement $21 = 21$ is always true. So, the equation is an identity and has infinitely many solutions.

MONITORING PROGRESS ANSWERS

4. infinitely many solutions
5. no solution
6. $k = -\frac{1}{2}$
7. infinitely many solutions

SUPPORTING English Language Learners

Have students use the steps in the Concept Summary to solve a linear equation. Explain that they have already learned the Distributive Property, which is used in Step 1. Review the property if needed.

Beginning/Advanced Work as partners to write out the solution to the problem, showing each stage of work. Once complete, Beginning students label the problem with each step number to show their understanding of the process.

Intermediate/Advanced High Work as partners. Intermediate students write out the problem and number its steps. Advanced High students explain the process by summarizing.

1.A.1 Use prior knowledge to understand meanings in English.

Extra Example 4

Paul bikes for 3 hours to reach his favorite beach. He bikes 2 miles per hour faster on the way back. So the trip back takes only 2.5 hours. How fast did he bike going to the beach?

10 miles per hour

MONITORING PROGRESS ANSWER

8. 17.5 mi

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**



A boat leaves New Orleans and travels upstream on the Mississippi River for 4 hours. The return trip takes only 2.8 hours because the boat travels 3 miles per hour faster downstream due to the current. How far does the boat travel upstream?

SOLUTION

1. **Understand the Problem** You are given the amounts of time the boat travels and the difference in speeds for each direction. You are asked to find the distance the boat travels upstream.
2. **Make a Plan** Use the Distance Formula to write expressions that represent the problem. Because the distance the boat travels in both directions is the same, you can use the expressions to write an equation.
3. **Solve the Problem** Use the formula (distance) = (rate)(time).

Words Distance upstream = Distance downstream

Variable Let x be the speed (in miles per hour) of the boat traveling upstream.

Equation $\frac{x \text{ mi}}{1 \cancel{\text{h}}} \cdot 4 \cancel{\text{h}} = \frac{(x + 3) \text{ mi}}{1 \cancel{\text{h}}} \cdot 2.8 \cancel{\text{h}}$ (mi = mi) ✓

$4x = 2.8(x + 3)$ Write the equation.

$4x = 2.8x + 8.4$ Distributive Property

$-2.8x \quad -2.8x$ Subtract 2.8x from each side.

$1.2x = 8.4$ Simplify.

$\frac{1.2x}{1.2} = \frac{8.4}{1.2}$ Divide each side by 1.2.

$x = 7$ Simplify.

▶ So, the boat travels 7 miles per hour upstream. To determine how far the boat travels upstream, multiply 7 miles per hour by 4 hours to obtain 28 miles.

4. **Look Back** To check that your solution is reasonable, use the formula for distance. Because the speed upstream is 7 miles per hour, the speed downstream is 7 + 3 = 10 miles per hour. When you substitute each speed into the Distance Formula, you get the same distance for upstream and downstream.

Upstream

$\text{Distance} = \frac{7 \text{ mi}}{1 \cancel{\text{h}}} \cdot 4 \cancel{\text{h}} = 28 \text{ mi}$

Downstream

$\text{Distance} = \frac{10 \text{ mi}}{1 \cancel{\text{h}}} \cdot 2.8 \cancel{\text{h}} = 28 \text{ mi}$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

8. A boat travels upstream on the Mississippi River for 3.5 hours. The return trip only takes 2.5 hours because the boat travels 2 miles per hour faster downstream due to the current. How far does the boat travel upstream?

Closure

- **I Used to Think … But Now I Know:** Take time for students to reflect on their current understanding of solving equations.
- **Exit Ticket:** Solve $6 - 2x = 4x - 9$. $x = 2.5$

1.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Is the equation $-2(4 - x) = 2x + 8$ an identity? Explain your reasoning.
2. **WRITING** Describe the steps in solving the linear equation $3(3x - 8) = 4x + 6$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–16, solve the equation. Check your solution. *(See Examples 1 and 2.)*

3. $15 - 2x = 3x$
4. $26 - 4s = 9s$
5. $5p - 9 = 2p + 12$
6. $8g + 10 = 35 + 3g$
7. $5t + 16 = 6 - 5t$
8. $-3r + 10 = 15r - 8$
9. $7 + 3x - 12x = 3x + 1$
10. $w - 2 + 2w = 6 + 5w$
11. $10(g + 5) = 2(g + 9)$
12. $-9(t - 2) = 4(t - 15)$
13. $\frac{2}{3}(3x + 9) = -2(2x + 6)$
14. $2(2t + 4) = \frac{3}{4}(24 - 8t)$
15. $10(2y + 2) - y = 2(8y - 8)$
16. $2(4x + 2) = 4x - 12(x - 1)$
17. **MODELING WITH MATHEMATICS** You and your friend drive toward each other. The equation $50h = 190 - 45h$ represents the number h of hours until you and your friend meet. When will you meet?
18. **MODELING WITH MATHEMATICS** The equation $1.5r + 15 = 2.25r$ represents the number r of movies you must rent to spend the same amount at each movie store. How many movies must you rent to spend the same amount at each movie store?



Membership Fee: $15



Membership Fee: Free

In Exercises 19–24, solve the equation. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*. *(See Example 3.)*

19. $3t + 4 = 12 + 3t$
20. $6d + 8 = 14 + 3d$
21. $2(h + 1) = 5h - 7$
22. $12y + 6 = 6(2y + 1)$
23. $3(4g + 6) = 2(6g + 9)$
24. $5(1 + 2m) = \frac{1}{2}(8 + 20m)$

ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in solving the equation.**

25. ✗
$$5c - 6 = 4 - 3c$$
$$2c - 6 = 4$$
$$2c = 10$$
$$c = 5$$

26. ✗
$$6(2y + 6) = 4(9 + 3y)$$
$$12y + 36 = 36 + 12y$$
$$12y = 12y$$
$$0 = 0$$
The equation has no solution.

27. **MODELING WITH MATHEMATICS** Write and solve an equation to find the month when you would pay the same total amount for each Internet service.

	Installation fee	Price per month
Company A	$60.00	$42.95
Company B	$25.00	$49.95

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–2, 3–13 odd, 17–25 odd, 34, 38, 41–44

Average: 1, 2–32 even, 33, 34, 38, 41–44

Advanced: 1–2, 10, 14–32 even, 33–44

HOMEWORK CHECK

Basic: 5, 11, 19, 23, 34

Average: 8, 12, 18, 22, 33

Advanced: 10, 16, 18, 24, 33

ANSWERS

1. no; Solving the equation gives a statement that is never true, not one that is always true.
2. Use the Distributive Property to get $9x - 24 = 4x + 6$. Subtract $4x$ from each side to get $5x - 24 = 6$. Add 24 to each side to get $5x = 30$. Divide each side by 5 to get $x = 6$.
3. $x = 3$
4. $s = 2$
5. $p = 7$
6. $g = 5$
7. $t = -1$
8. $r = 1$
9. $x = \frac{1}{2}$
10. $w = -4$
11. $g = -4$
12. $t = 6$
13. $x = -3$
14. $t = 1$
15. $y = -12$
16. $x = \frac{1}{2}$
17. 2 h
18. 20 movies
19. no solution
20. $d = 2$; one solution
21. $h = 3$; one solution
22. infinitely many solutions
23. infinitely many solutions
24. no solution
25. In the second step, you should add $3c$ to each side; $8c - 6 = 4$, $8c = 10$, $c = \frac{5}{4}$
26. $0 = 0$ is always true, so it is an identity; The equation has infinitely many solutions.
27. $60 + 42.95x = 25 + 49.95x$; 5th month

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

28. 50 g
29. $r = -2$
30. $r = 23$
31. $x = 10$; $S = 62.5\pi$ cm^2 or about 196.35 cm^2; $V = 62.5\pi$ cm^3 or about 196.35 cm^3
32. $x = 4.3$; $S = 60.38\pi$ ft^2 or about 189.9 ft^2; $V = 60.47\pi$ ft^3 or about 189.97 ft^3
33. 4 sec
34. no; The cheetah would have to maintain top speed for about 21.7 seconds, which is greater than 20 seconds.
35. $a = 5$; Both sides simplify to $10x + 15$.
36. $a = 4$; Both sides simplify to $20x - 8$.
37. 11, 12
38. a. after 6 years
 b. The left side shows the Spanish enrollment after x years, and the right side shows the French enrollment after x years; Solving the equation will indicate after how many years the enrollments will be equal.
39. a. *Sample answer:* $3x + 12 = 2x + x$; simplifies to a statement that is never true
 b. *Sample answer:* $5x + 3 = 2x + 3 + 3x$; simplifies to a statement that is always true

40–44. See Additional Answers.

Mini-Assessment

Solve each equation.

1. $8a + 96 = -24a$ $a = -3$
2. $\frac{1}{5}(10b + 60) = -4(b - 12)$ $b = 6$
3. $3(2m + 4) = 2(3m + 6)$ infinitely many solutions
4. $4y = -2(6 - 2y)$ no solution
5. Mary drove 4 hours to reach her camping spot. Driving home, she increased her speed by 15 miles per hour, reducing her travel time by 1 hour. How fast did she drive going to the camping spot? 45 miles per hour

28. PROBLEM SOLVING One serving of granola provides 4% of the protein you need daily. You must get the remaining 48 grams of protein from other sources. How many grams of protein do you need daily?

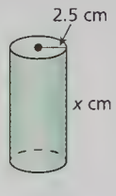
USING STRUCTURE In Exercises 29 and 30, find the value of r.

29. $8(x + 6) - 10 + r = 3(x + 12) + 5x$

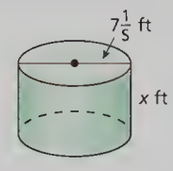
30. $4(x - 3) - r + 2x = 5(3x - 7) - 9x$

MATHEMATICAL CONNECTIONS In Exercises 31 and 32, the value of the surface area of the cylinder is equal to the value of the volume of the cylinder. Find the value of x. Then find the surface area and volume of the cylinder.

31.



32.



33. MODELING WITH MATHEMATICS A cheetah that is running 90 feet per second is 120 feet behind an antelope that is running 60 feet per second. How long will it take the cheetah to catch up to the antelope? *(See Example 4.)*

34. MAKING AN ARGUMENT A cheetah can run at top speed for only about 20 seconds. If an antelope is too far away for a cheetah to catch it in 20 seconds, the antelope is probably safe. Your friend claims the antelope in Exercise 33 will not be safe if the cheetah starts running 650 feet behind it. Is your friend correct? Explain.

REASONING In Exercises 35 and 36, for what value of a is the equation an identity? Explain your reasoning.

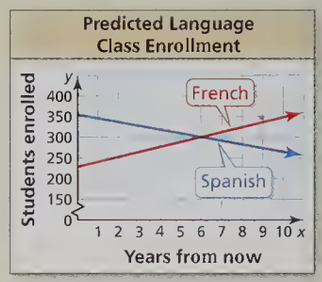
35. $a(2x + 3) = 9x + 15 + x$

36. $8x - 8 + 3ax = 5ax - 2a$

37. REASONING Two times the greater of two consecutive integers is 9 less than three times the lesser integer. What are the integers?

38. HOW DO YOU SEE IT? The table and the graph show information about students enrolled in Spanish and French classes at a high school.

	Students enrolled this year	Average rate of change
Spanish	355	9 fewer students each year
French	229	12 more students each year

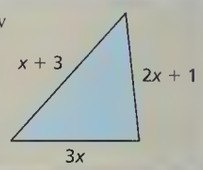


a. Use the graph to determine after how many years there will be equal enrollment in Spanish and French classes.

b. How does the equation $355 - 9x = 229 + 12x$ relate to the table and the graph? How can you use this equation to determine whether your answer in part (a) is reasonable?

39. WRITING EQUATIONS Give an example of a linear equation that has (a) no solution and (b) infinitely many solutions. Justify your answers.

40. THOUGHT PROVOKING Draw a different figure that has the same perimeter as the triangle shown. Explain why your figure has the same perimeter.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Order the values from least to greatest. *(Skills Review Handbook)*

41. $9, |-4|, -4, 5, |2|$

42. $|-32|, 22, -16, -|21|, |-10|$

43. $-18, |-24|, -19, |-18|, |22|$

44. $-|-3|, |0|, -1, |2|, -2$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

1.4 Rewriting Equations and Formulas

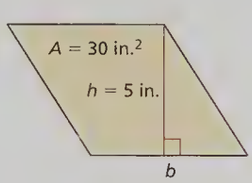
Texas Essential Knowledge and Skills
A.12.E

Essential Question How can you use a formula for one measurement to write a formula for a different measurement?

EXPLORATION 1 Using an Area Formula

Work with a partner.

a. Write a formula for the area A of a parallelogram.



b. Substitute the given values into the formula. Then solve the equation for b. Justify each step.

c. Solve the formula in part (a) for b without first substituting values into the formula. Justify each step.

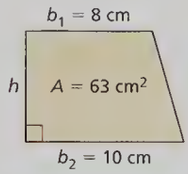
d. Compare how you solved the equations in parts (b) and (c). How are the processes similar? How are they different?

REASONING
To be proficient in math, you need to consider the given units. For instance, in Exploration 1, the area A is given in square inches and the height h is given in inches. A unit analysis shows that the units for the base b are also inches, which makes sense.

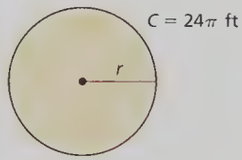
EXPLORATION 2 Using Area, Circumference, and Volume Formulas

Work with a partner. Write the indicated formula for each figure. Then write a new formula by solving for the variable whose value is not given. Use the new formula to find the value of the variable.

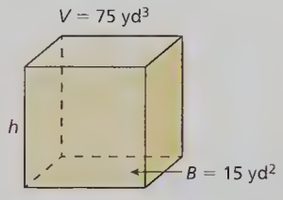
a. Area A of a trapezoid



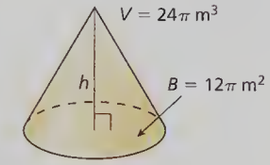
b. Circumference C of a circle



c. Volume V of a rectangular prism



d. Volume V of a cone



Communicate Your Answer

3. How can you use a formula for one measurement to write a formula for a different measurement? Give an example that is different from those given in Explorations 1 and 2.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.12.E The student is expected to solve mathematic and scientific formulas, and other literal equations, for a specified variable.

ANSWERS

1. a. $A = bh$

b. $30 = b \cdot 5$; Write the equation.
$6 = b$; Divide each side by 5.

c. $A = bh$; Write the formula.
$\frac{A}{h} = b$; Divide each side by h.

d. applied the same steps to solve; used variables instead of constants

2. a. $A = \frac{1}{2}h(b_1 + b_2)$;
$h = \frac{2A}{b_1 + b_2}$; $h = 7$ cm

b. $C = 2\pi r$; $r = \frac{C}{2\pi}$; $r = 12$ ft

c. $V = Bh$; $h = \frac{V}{B}$; $h = 5$ yd

d. $V = \frac{1}{3}Bh$; $h = \frac{3V}{B}$; $h = 6$ m

3. Solve the formula for a different variable; *Sample answer:*
(volume of a cylinder) $V = \pi r^2 h$;
$h = \frac{V}{\pi r^2}$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students of mixed language abilities work in groups to complete Exploration 2. Point out that it provides visuals to help guide the work.

Beginning Draw each figure and write its name: *trapezoid, circle, rectangular prism, cone.*
Intermediate Write the indicated formula for each figure, using the labels provided as a guide.
Advanced Work with the group to write a new formula for each figure by solving for the variable whose value is not given.
Advanced High Work with the group to find the value of each variable. Explain how the new formula is used to do this.

4.F.1 Use visual and contextual support to read grade-appropriate content area text.

Extra Example 1

Solve the literal equation $4x - 5y = 10$ for y. $y = -2 + \frac{4}{5}x$

Extra Example 2

Solve the literal equation $3w + 4wp = a$ for w.

$w = \frac{a}{3 + 4p}$

MONITORING PROGRESS ANSWERS

1. $y = 3 + \frac{1}{3}x$
2. $y = x - \frac{5}{2}$
3. $y = 5 - 2x$
4. $x = y$
5. $x = \frac{m}{2 + k}$
6. $x = \frac{y - 3}{5 - k}$

1.4 Lesson

Core Vocabulary

literal equation, *p. 28*
formula, *p. 29*

Previous
surface area

What You Will Learn

- Rewrite literal equations.
- Rewrite and use formulas for area.
- Rewrite and use other common formulas.

Rewriting Literal Equations

An equation that has two or more variables is called a **literal equation**. To rewrite a literal equation, solve for one variable in terms of the other variable(s).

EXAMPLE 1 Rewriting a Literal Equation

Solve the literal equation $3y + 4x = 9$ for y.

SOLUTION

$3y + 4x = 9$	Write the equation.
$3y + 4x - 4x = 9 - 4x$	Subtract $4x$ from each side.
$3y = 9 - 4x$	Simplify.
$\frac{3y}{3} = \frac{9 - 4x}{3}$	Divide each side by 3.
$y = 3 - \frac{4}{3}x$	Simplify.

▶ The rewritten literal equation is $y = 3 - \frac{4}{3}x$.

EXAMPLE 2 Rewriting a Literal Equation

Solve the literal equation $y = 3x + 5xz$ for x.

SOLUTION

$y = 3x + 5xz$	Write the equation.
$y = x(3 + 5z)$	Distributive Property
$\frac{y}{3 + 5z} = \frac{x(3 + 5z)}{3 + 5z}$	Divide each side by $3 + 5z$.
$\frac{y}{3 + 5z} = x$	Simplify.

▶ The rewritten literal equation is $x = \frac{y}{3 + 5z}$.

REMEMBER
Division by 0 is undefined.

In Example 2, you must assume that $z \neq -\frac{3}{5}$ in order to divide by $3 + 5z$. In general, if you have to divide by a variable or variable expression when solving a literal equation, you should assume that the variable or variable expression does not equal 0.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the literal equation for y.

1. $3y - x = 9$
2. $2x - 2y = 5$
3. $20 = 8x + 4y$

Solve the literal equation for x.

4. $y = 5x - 4x$
5. $2x + kx = m$
6. $3 + 5x - kx = y$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Inclusion

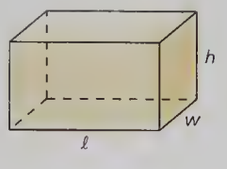
Some students may have difficulty keeping track of the designated variable as they rewrite literal equations. Have students circle each term with the variable for which they are solving. Remind them that they have solved the literal equation when that variable is alone on one side of the equation.

Rewriting and Using Formulas for Area

A **formula** shows how one variable is related to one or more other variables. A formula is a type of literal equation.

EXAMPLE 3 Rewriting a Formula for Surface Area

The formula for the surface area S of a rectangular prism is $S = 2\ell w + 2\ell h + 2wh$. Solve the formula for the length ℓ.



SOLUTION

$$S = 2\ell w + 2\ell h + 2wh$$ Write the equation.

$$S - 2wh = 2\ell w + 2\ell h + 2wh - 2wh$$ Subtract $2wh$ from each side.

$$S - 2wh = 2\ell w + 2\ell h$$ Simplify.

$$S - 2wh = \ell(2w + 2h)$$ Distributive Property

$$\frac{S - 2wh}{2w + 2h} = \frac{\ell(2w + 2h)}{2w + 2h}$$ Divide each side by $2w + 2h$.

$$\frac{S - 2wh}{2w + 2h} = \ell$$ Simplify.

▶ When you solve the formula for ℓ, you obtain $\ell = \frac{S - 2wh}{2w + 2h}$.

EXAMPLE 4 Using a Formula for Area

You own a rectangular lot that is 500 feet deep. It has an area of 100,000 square feet. To pay for a new water system, you are assessed \$5.50 per foot of lot frontage.

a. Find the frontage of your lot.

b. How much are you assessed for the new water system?

SOLUTION



a. In the formula for the area of a rectangle, let the width w represent the lot frontage.

$$A = \ell w$$ Write the formula for area of a rectangle.

$$\frac{A}{\ell} = w$$ Divide each side by ℓ to solve for w.

$$\frac{100{,}000}{500} = w$$ Substitute 100,000 for A and 500 for ℓ.

$$200 = w$$ Simplify.

▶ The frontage of your lot is 200 feet.

b. Each foot of frontage costs \$5.50, and $\frac{\$5.50}{1\ \text{ft}} \cdot 200\ \text{ft} = \$1100$.

▶ So, your total assessment is \$1100.

Monitoring Progress

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Solve the formula for the indicated variable.

7. Area of a triangle: $A = \frac{1}{2}bh$; Solve for h.

8. Surface area of a cone: $S = \pi r^2 + \pi r\ell$; Solve for ℓ.

Extra Example 3

The formula for the surface area S of a rectangular prism is $S = 2\ell w + 2\ell h + 2wh$. Solve the formula for the height h.

$h = \frac{S - 2\ell w}{2\ell + 2w}$

Extra Example 4

You have a rectangular backyard that is 90 feet wide. It has an area of 10,800 square feet. You are putting a fence along one length of the yard. The fence costs \$12.50 per linear foot.

a. Find the length of your backyard. 120 feet

b. What is the total cost of the fence for one length of the yard? \$1500

MONITORING PROGRESS ANSWERS

7. $h = \frac{2A}{b}$

8. $\ell = \frac{S - \pi r^2}{\pi r}$

Extra Example 5

Solve the distance formula $d = rt$ for r.

$r = \frac{d}{t}$

Extra Example 6

On a July day, the temperature at the peak of Mt. Everest is $-16°C$. Is that temperature warmer or colder than 0°F?

warmer

MONITORING PROGRESS ANSWER

9. no

Rewriting and Using Other Common Formulas

Core Concept

Common Formulas

Temperature	F = degrees Fahrenheit, C = degrees Celsius $C = \frac{5}{9}(F - 32)$
Simple Interest	I = interest, P = principal, r = annual interest rate (decimal form), t = time (years) $I = Prt$
Distance	d = distance traveled, r = rate, t = time $d = rt$

EXAMPLE 5 Rewriting the Formula for Temperature

Solve the temperature formula for F.

SOLUTION

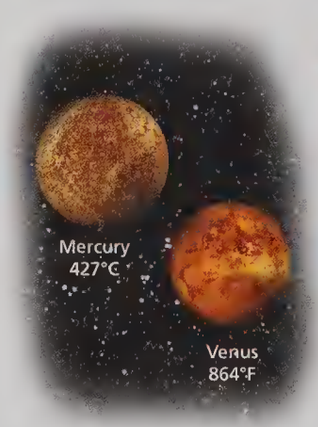
$C = \frac{5}{9}(F - 32)$ Write the temperature formula.

$\frac{9}{5}C = F - 32$ Multiply each side by $\frac{9}{5}$.

$\frac{9}{5}C + 32 = F - 32 + 32$ Add 32 to each side.

$\frac{9}{5}C + 32 = F$ Simplify.

▶ The rewritten formula is $F = \frac{9}{5}C + 32$.



EXAMPLE 6 Using the Formula for Temperature

Which has the greater surface temperature: Mercury or Venus?

SOLUTION

Convert the Celsius temperature of Mercury to degrees Fahrenheit.

$F = \frac{9}{5}C + 32$ Write the rewritten formula from Example 5.

$= \frac{9}{5}(427) + 32$ Substitute 427 for C.

$= 800.6$ Simplify.

▶ Because 864°F is greater than 800.6°F, Venus has the greater surface temperature.

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9. A fever is generally considered to be a body temperature greater than 100°F. Your friend has a temperature of 37°C. Does your friend have a fever?

EXAMPLE 7 **Using the Formula for Simple Interest**

You deposit $5000 in an account that earns simple interest. After 6 months, the account earns $162.50 in interest. What is the annual interest rate?

COMMON ERROR

The unit of t is years. Be sure to convert months to years.

$\frac{1 \text{ yr}}{12 \cancel{\text{mo}}} \cdot 6 \cancel{\text{mo}} = 0.5 \text{ yr}$

SOLUTION

To find the annual interest rate, solve the simple interest formula for r.

$I = Prt$ Write the simple interest formula.

$\frac{I}{Pt} = r$ Divide each side by Pt to solve for r.

$\frac{162.50}{(5000)(0.5)} = r$ Substitute 162.50 for I, 5000 for P, and 0.5 for t.

$0.065 = r$ Simplify.

▶ The annual interest rate is 0.065, or 6.5%.

EXAMPLE 8 **Solving a Real-Life Problem**

A truck driver averages 60 miles per hour while delivering freight to a customer. On the return trip, the driver averages 50 miles per hour due to construction. The total driving time is 6.6 hours. How long does each trip take?

SOLUTION

Step 1 Rewrite the Distance Formula to write expressions that represent the two trip times. Solving the formula $d = rt$ for t, you obtain $t = \frac{d}{r}$. So, $\frac{d}{60}$ represents the delivery time, and $\frac{d}{50}$ represents the return trip time.

Step 2 Use these expressions and the total driving time to write and solve an equation to find the distance one way.

$\frac{d}{60} + \frac{d}{50} = 6.6$ The sum of the two trip times is 6.6 hours.

$\frac{11d}{300} = 6.6$ Add the left side using the LCD.

$11d = 1980$ Multiply each side by 300 and simplify.

$d = 180$ Divide each side by 11 and simplify.

The distance one way is 180 miles.

Step 3 Use the expressions from Step 1 to find the two trip times.

▶ So, the delivery takes $180 \text{ mi} \div \frac{60 \text{ mi}}{1 \text{ h}} = 3$ hours, and the return trip takes $180 \text{ mi} \div \frac{50 \text{ mi}}{1 \text{ h}} = 3.6$ hours.

Monitoring Progress

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10. How much money must you deposit in a simple interest account to earn $500 in interest in 5 years at 4% annual interest?

11. A truck driver averages 60 miles per hour while delivering freight and 45 miles per hour on the return trip. The total driving time is 7 hours. How long does each trip take?

Extra Example 7

You deposit $3500 in an account that earns 4% simple interest. How long will you have to leave the money in the account to earn $350 interest?

2.5 years

Extra Example 8

Dan drove 330 miles at 60 miles per hour. Ryan drove 275 miles at 55 miles per hour. How much more time did Dan drive than Ryan?

Dan drove 0.5 hour longer than Ryan.

MONITORING PROGRESS ANSWERS

10. $2500

11. delivery: 3 h, return: 4 h

Closure

- **Exit Ticket:** Solve $2x + 4y = 11$ for y. $y = \frac{11 - 2x}{4}$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–2, 3–35 odd, 40, 41, 47–52

Average: 1, 2–30 even, 33–36, 39–41, 47–52

Advanced: 1–2, 10, 12, 20–38 even, 39–52

HOMEWORK CHECK

Basic: 7, 17, 29, 33, 35

Average: 10, 18, 28, 34, 36

Advanced: 12, 22, 28, 34, 36

ANSWERS

1. no; It only has one variable.
2. Solve $6y = 24 - 3x$ for y in terms of x; $y = 4 - \frac{1}{2}x$; $x = 8 - 2y$
3. $y = 13 + 3x$
4. $y = 7 - 2x$
5. $y = -13 + 9x$
6. $y = 3 - 4x$
7. $y = 9x - 45$
8. $y = 4$
9. $y = x - 3$
10. $y = 2x + 1$
11. $y = 18x + 12$
12. $y = 16 - 12x$
13. $x = \frac{1}{12}y$
14. $x = \frac{1}{9}m$
15. $x = \frac{a}{2 + 6z}$
16. $x = \frac{y}{3b - 7}$
17. $x = \frac{y - 6}{4 + r}$
18. $x = \frac{z - 8}{6 - p}$
19. $x = \frac{r}{s + t}$
20. $x = \frac{a - d}{b + c}$
21. $x = \frac{y - 12}{-5 - 4k}$
22. $x = \frac{y + 9}{1 + 2w}$
23. a. $x = \frac{C - 60}{85}$
 b. 3 trips; 5 trips
24. a. $n = \frac{d + 2}{4}$
 b. $1\frac{1}{4}$ in.; 2 in.; 3 in.

1.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Is $9r + 16 = \frac{\pi}{5}$ a literal equation? Explain.
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Solve $3x + 6y = 24$ for x.

Solve $24 - 3x = 6y$ for x.

Solve $6y = 24 - 3x$ for y in terms of x.

Solve $24 - 6y = 3x$ for x in terms of y.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the literal equation for y. *(See Example 1.)*

3. $y - 3x = 13$
4. $2x + y = 7$
5. $2y - 18x = -26$
6. $20x + 5y = 15$
7. $9x - y = 45$
8. $6 - 3y = -6$
9. $4x - 5 = 7 + 4y$
10. $16x + 9 = 9y - 2x$
11. $2 + \frac{1}{6}y = 3x + 4$
12. $11 - \frac{1}{2}y = 3 + 6x$

In Exercises 13–22, solve the literal equation for x. *(See Example 2.)*

13. $y = 4x + 8x$
14. $m = 10x - x$
15. $a = 2x + 6xz$
16. $y = 3bx - 7x$
17. $y = 4x + rx + 6$
18. $z = 8 + 6x - px$
19. $sx + tx = r$
20. $a = bx + cx + d$
21. $12 - 5x - 4kx = y$
22. $x - 9 + 2wx = y$

23. **MODELING WITH MATHEMATICS** The total cost C (in dollars) to participate in a ski club is given by the literal equation $C = 85x + 60$, where x is the number of ski trips you take.
 a. Solve the equation for x.
 b. How many ski trips do you take if you spend a total of \$315? \$485?



24. **MODELING WITH MATHEMATICS** The penny size of a nail indicates the length of the nail. The penny size d is given by the literal equation $d = 4n - 2$, where n is the length (in inches) of the nail.
 a. Solve the equation for n.
 b. Use the equation from part (a) to find the lengths of nails with the following penny sizes: 3, 6, and 10.



ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in solving the equation for x.**

25. ✗
$$12 - 2x = -2(y - x)$$
$$-2x = -2(y - x) - 12$$
$$x = (y - x) + 6$$

26. ✗
$$10 = ax - 3b$$
$$10 = x(a - 3b)$$
$$\frac{10}{a - 3b} = x$$

In Exercises 27–30, solve the formula for the indicated variable. *(See Examples 3 and 5.)*

27. Profit: $P = R - C$; Solve for C.
28. Surface area of a cylinder: $S = 2\pi r^2 + 2\pi rh$; Solve for h.
29. Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$; Solve for b_2.
30. Average acceleration of an object: $a = \frac{v_1 - v_0}{t}$; Solve for v_1.

25. The equation is not solved for x because there is still a term with x on both sides; $x = y - x + 6$; $2x = y + 6$; $x = \frac{y + 6}{2}$
26. There is no x in $3b$ to factor out; $10 = ax - 3b$; $10 + 3b = ax$; $x = \frac{10 + 3b}{a}$
27. $C = R - P$
28. $h = \frac{S - 2\pi r^2}{2\pi r}$
29. $b_2 = \frac{2A}{h} - b_1$
30. $v_1 = at + v_0$

31. REWRITING A FORMULA A common statistic used in professional football is the quarterback rating. This rating is made up of four major factors. One factor is the completion rating given by the formula

$$R = 5\left(\frac{C}{A} - 0.3\right)$$

where C is the number of completed passes and A is the number of attempted passes. Solve the formula for C.

32. REWRITING A FORMULA Newton's law of gravitation is given by the formula

$$F = G\left(\frac{m_1 m_2}{d^2}\right)$$

where F is the force between two objects of masses m_1 and m_2, G is the gravitational constant, and d is the distance between the two objects. Solve the formula for m_1.

33. MODELING WITH MATHEMATICS The sale price S (in dollars) of an item is given by the formula $S = L - rL$, where L is the list price (in dollars) and r is the discount rate (in decimal form). *(See Examples 4 and 6.)*



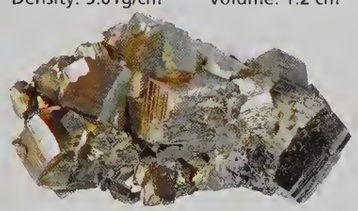
a. Solve the formula for r.

b. The list price of the shirt is \$30. What is the discount rate?

34. MODELING WITH MATHEMATICS The density d of a substance is given by the formula $d = \frac{m}{V}$, where m is its mass and V is its volume.

Pyrite

Density: 5.01g/cm^3 Volume: 1.2 cm^3



a. Solve the formula for m.

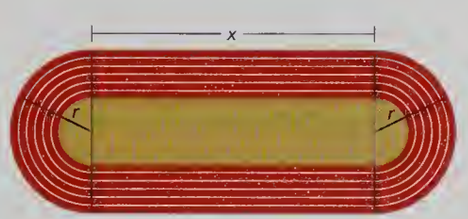
b. Find the mass of the pyrite sample.

35. PROBLEM SOLVING You deposit \$2000 in an account that earns simple interest at an annual rate of 4%. How long must you leave the money in the account to earn \$500 in interest? *(See Example 7.)*

36. PROBLEM SOLVING A flight averages 460 miles per hour. The return flight averages 500 miles per hour due to a tailwind. The total flying time is 4.8 hours. How long is each flight? Explain. *(See Example 8.)*



37. USING STRUCTURE An athletic facility is building an indoor track. The track is composed of a rectangle and two semicircles, as shown.

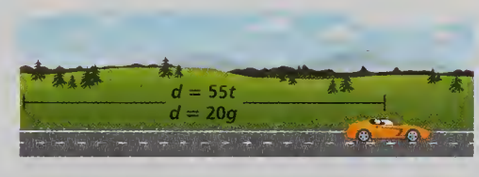


a. Write a formula for the perimeter of the indoor track.

b. Solve the formula for x.

c. The perimeter of the track is 660 feet, and r is 50 feet. Find x. Round your answer to the nearest foot.

38. MODELING WITH MATHEMATICS The distance d (in miles) you travel in a car is given by the two equations shown, where t is the time (in hours) and g is the number of gallons of gasoline the car uses.



a. Write an equation that relates g and t.

b. Solve the equation for g.

c. You travel for 6 hours. How many gallons of gasoline does the car use? How far do you travel? Explain.

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ANSWERS

31. $C = A\left(\frac{R}{5} + 0.3\right)$

32. $m_1 = \frac{Fd^2}{m_2 G}$

33. a. $r = \frac{L - S}{L}$

b. 0.4

34. a. $m = dV$

b. 6.012 g

35. 6.25 yr

36. 2.5 h; 2.3 h; $\frac{d}{460}$ represents the original trip time, and $\frac{d}{500}$ represents the return trip time. Add these expressions and solve for the one-way distance. Substituting the distance into each of the expressions gives the time for each flight.

37. a. $P = 2x + 2\pi r$

b. $x = \frac{1}{2}P - \pi r$

c. 173 ft

38. a. $55t = 20g$

b. $g = \frac{11t}{4}$

c. 16.5 gal; 330 mi; The amount of gasoline used can be found using the formula from part (b). Either of the original formulas can be used to find the distance.

ANSWERS

39. **a.** $r = \frac{C}{2\pi}$

b. 1.1 ft; 1.3 ft; 1.4 ft

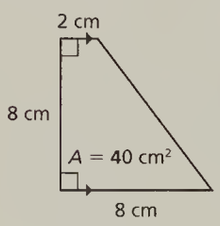
c. First find the radius using the formula from part (a), then substitute this into the formula for the area of a circle.

40. **a.** $S = 4\ell b + 2b^2$

b. *Sample answer:* ℓ; The formula contains terms with both b and b^2, but only one term with ℓ.

41. no; 70°F is about 21.1°C, which is greater than 20°C.

42. *Sample answer:* $h = 8$; missing base = 2; $A = \frac{1}{2}h(b_1 + b_2)$, $40 = \frac{1}{2} \cdot 8 \cdot (2 + 8)$



43. $A = \frac{5}{2}bh$; $h = \frac{2A}{5b}$

44. $A = 4bh$; $h = \frac{A}{4b}$

45. $a = \frac{b + c}{bx - 1}$

46. $a = \frac{yb}{y - xb}$

47–52. See Additional Answers.

Mini-Assessment

1. Solve the literal equation $9 - c = 17d$ for c. $c = 9 - 17d$
2. Solve the literal equation $nt + 6 - xt = s$ for t. $t = \frac{s - 6}{n - x}$
3. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Solve the formula for the base b. $b = \frac{2A}{h}$
4. A tour bus driver averages 60 miles per hour to the destination. The driver averages 55 miles per hour on the return trip. The total driving time is 5.75 hours. How long does each trip take? The trip to the destination takes 2.75 hours, and the return trip takes 3 hours.

39. MODELING WITH MATHEMATICS One type of stone formation found in Carlsbad Caverns in New Mexico is called a column. This cylindrical stone formation connects to the ceiling and the floor of a cave.

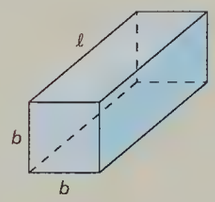


a. Rewrite the formula for the circumference of a circle, so that you can easily calculate the radius of a column given its circumference.

b. What is the radius (to the nearest tenth of a foult) of a column that has a circumference of 7 feet? 8 feet? 9 feet?

c. Explain how you can find the area of a cross section of a column when you know its circumference.

40. HOW DO YOU SEE IT? The rectangular prism shown has bases with equal side lengths.



a. Use the figure to write a formula for the surface area S of the rectangular prism.

b. Your teacher asks you to rewrite the formula by solving for one of the side lengths, b or ℓ. Which side length would you choose? Explain your reasoning.

41. MAKING AN ARGUMENT Your friend claims that Thermometer A displays a greater temperature than Thermometer B. Is your friend correct? Explain your reasoning.

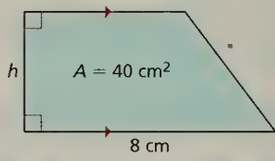


Thermometer A



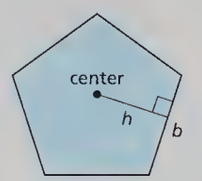
Thermometer B

42. THOUGHT PROVOKING Give a possible value for h. Justify your answer. Draw and label the figure using your chosen value of h.

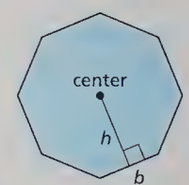


MATHEMATICAL CONNECTIONS **In Exercises 43 and 44, write a formula for the area of the regular polygon. Solve the formula for the height h.**

43.



44.



REASONING **In Exercises 45 and 46, solve the literal equation for a.**

45. $x = \frac{a + b + c}{ab}$

46. $y = x\left(\frac{ab}{a - b}\right)$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression. *(Skills Review Handbook)*

47. $15 - 5 + 5^2$ **48.** $18 \cdot 2 - 4^2 \div 8$ **49.** $3^3 + 12 \div 3 \cdot 5$ **50.** $2^5(5 - 6) + 9 \div 3$

Solve the equation. *(Section 1.2)*

51. $-15 = 6x + 3(4 - 3x)$ **52.** $-2(4 + m) + 6(m - 3) = 14$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

1.3–1.4 What Did You Learn?

Core Vocabulary

identity, *p. 23*
literal equation, *p. 28*
formula, *p. 29*

Core Concepts

Section 1.3

Solving Equations with Variables on Both Sides, *p. 22*
Special Solutions of Linear Equations, *p. 23*

Section 1.4

Rewriting Literal Equations, *p. 28*
Common Formulas, *p. 30*

Mathematical Thinking

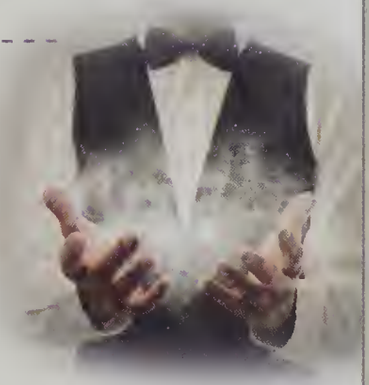
1. What definition did you use in your reasoning in Exercises 35 and 36 on page 26?
2. What entry points did you use to answer Exercises 43 and 44 on page 34?

Performance Task

Magic of Mathematics

Have you ever watched a magician perform a number trick? You can use algebra to explain how these types of tricks work.

To explore the answer to this question and more, go to *BigIdeasMath.com*.



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ANSWERS

1. the definition of "identity"
2. *Sample answer:* the center and two adjacent vertices to form congruent triangles

ANSWERS

1. $z = -9$; Subtract 3 from each side.
2. $t = -13$; Divide each side by -0.2.
3. $n = 10$; Multiply each side by -5.

1 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

1.1 Solving Simple Equations *(pp. 3–10)*

a. Solve $x - 5 = -9$. Justify each step.

$x - 5 = -9$ Write the equation.

Addition Property of Equality → $+5 \quad +5$ Add 5 to each side.

$x = -4$ Simplify.

▶ The solution is $x = -4$.

b. Solve $4x = 12$. Justify each step.

$4x = 12$ Write the equation.

Division Property of Equality → $\frac{4x}{4} = \frac{12}{4}$ Divide each side by 4.

$x = 3$ Simplify.

▶ The solution is $x = 3$.

c. The *boiling point* of a liquid is the temperature at which the liquid becomes a gas. The boiling point of mercury, 357°C, is about $\frac{41}{200}$ of the boiling point of lead. Write and solve an equation to find the boiling point of lead.

Let x be the boiling point of lead.

$\frac{41}{200}x = 357$ Write the equation.

Multiplication Property of Equality → $\frac{200}{41} \cdot \left(\frac{41}{200}x\right) = \frac{200}{41} \cdot 357$ Multiply each side by $\frac{200}{41}$.

$x \approx 1741$ Simplify.

▶ The boiling point of lead is about 1741°C.

Solve the equation. Justify each step. Check your solution.

1. $z + 3 = -6$
2. $2.6 = -0.2t$
3. $-\frac{n}{5} = -2$

1.2 Solving Multi-Step Equations *(pp. 11–18)*

a. Solve $-4p - 9 = 3$.

$-4p - 9 = 3$ Write the equation.

$-4p = 12$ Add 9 to each side.

$p = -3$ Divide each side by -4.

▶ The solution is $p = -3$.

b. Solve $-6x + 23 + 2x = 15$.

$-6x + 23 + 2x = 15$	Write the equation.
$-4x + 23 = 15$	Combine like terms.
$-4x = -8$	Subtract 23 from each side.
$x = 2$	Divide each side by -4.

▶ The solution is $x = 2$.

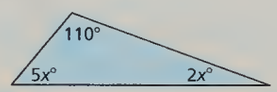
Solve the equation. Check your solution.

4. $3y + 11 = -16$ **5.** $6 = 1 - b$ **6.** $n + 5n + 7 = 43$

7. $-4(2z + 6) - 12 = 4$ **8.** $\frac{3}{2}(x - 2) - 5 = 19$ **9.** $6 = \frac{1}{5}w + \frac{7}{5}w - 4$

Find the value of x. Then find the angle measures of the polygon.

10.



Sum of angle measures: 180°

11.



Sum of angle measures: 540°

12. Use the table to write and solve an equation to find the number of points p you need to score in the fourth game so that the mean number of points scored per game is 20.

Game	1	2	3	4
Points	25	15	18	p

1.3 Solving Equations with Variables on Both Sides *(pp. 21–26)*

Solve $2(y - 4) = -4(y + 8)$.

$2(y - 4) = -4(y + 8)$	Write the equation.
$2y - 8 = -4y - 32$	Distributive Property
$6y - 8 = -32$	Add $4y$ to each side.
$6y = -24$	Add 8 to each side.
$y = -4$	Divide each side by 6.

▶ The solution is $y = -4$.

Solve the equation.

13. $3n - 3 = 4n + 1$ **14.** $5(1 + x) = 5x + 5$ **15.** $3(n + 4) = \frac{1}{2}(6n + 4)$

16. You are biking at a speed of 18 miles per hour. You are 3 miles behind your friend who is biking at a speed of 12 miles per hour. Write and solve an equation to find the amount of time it takes for you to catch up to your friend.

ANSWERS

4. $y = -9$
5. $b = -5$
6. $n = 6$
7. $z = -5$
8. $x = 18$
9. $w = \frac{25}{4}$
10. $x = 10$; 110°, 50°, 20°
11. $x = 126$; 126°, 96°, 126°, 96°, 96°
12. $\frac{25 + 15 + 18 + p}{4} = 20$; 22 points
13. $n = -4$
14. all real numbers
15. no solution
16. $18x - 3 = 12x$; 0.5 h

ANSWERS

17. $y = \frac{1}{2}x - 5$
18. $y = 2x - 2$
19. $y = \dfrac{a}{9 + 3x}$
20. **a.** $h = \dfrac{3V}{B}$
 b. 18 cm
21. **a.** $K = \frac{5}{9}(F - 32) + 273.15$
 b. about 355.37 K

1.4 Rewriting Equations and Formulas *(pp. 27–34)*

a. The slope-intercept form of a linear equation is $y = mx + b$. Solve the equation for m.

$y = mx + b$	Write the equation.
$y - b = mx + b - b$	Subtract b from each side.
$y - b = mx$	Simplify.
$\frac{y - b}{x} = \frac{mx}{x}$	Divide each side by x.
$\frac{y - b}{x} = m$	Simplify.

▶ When you solve the equation for m, you obtain $m = \dfrac{y - b}{x}$.

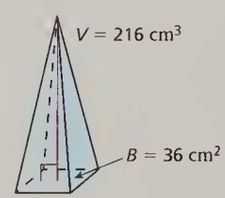
b. The formula for the surface area S of a cylinder is $S = 2\pi r^2 + 2\pi rh$. Solve the formula for the height h.

$S = 2\pi r^2 + 2\pi rh$	Write the equation.
$-2\pi r^2 \quad -2\pi r^2$	Subtract $2\pi r^2$ from each side.
$S - 2\pi r^2 = 2\pi rh$	Simplify.
$\frac{S - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$	Divide each side by $2\pi r$.
$\frac{S - 2\pi r^2}{2\pi r} = h$	Simplify.

▶ When you solve the formula for h, you obtain $h = \dfrac{S - 2\pi r^2}{2\pi r}$.

Solve the literal equation for y.

17. $2x - 4y = 20$
18. $8x - 3 = 5 + 4y$
19. $a = 9y + 3yx$
20. The volume V of a pyramid is given by the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.
 a. Solve the formula for h.
 b. Find the height h of the pyramid.



21. The formula $F = \frac{9}{5}(K - 273.15) + 32$ converts a temperature from kelvin K to degrees Fahrenheit F.
 a. Solve the formula for K.
 b. Convert 180°F to kelvin K. Round your answer to the nearest hundredth.

1 Chapter Test

Solve the equation. Justify each step. Check your solution.

1. $x - 7 = 15$
2. $\frac{2}{3}x + 5 = 3$
3. $11x + 1 = -1 + x$

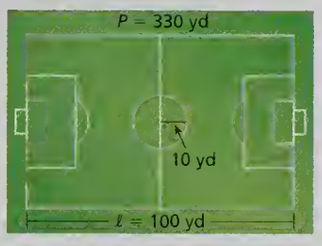
Solve the equation.

4. $-2 + 5x - 7 = 3x - 9 + 2x$
5. $3(x + 4) - 1 = -7$
6. $\frac{1}{3}(6x + 12) - 2(x - 7) = 19$

Describe the values of c for which the equation has no solution. Explain your reasoning.

7. $3x - 5 = 3x - c$
8. $4x + 1 = 2x + c$

9. The perimeter P (in yards) of a soccer field is represented by the formula $P = 2\ell + 2w$, where ℓ is the length (in yards) and w is the width (in yards).



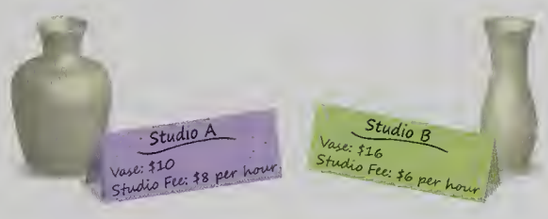
 a. Solve the formula for w.
 b. Find the width of the field.
 c. About what percent of the field is inside the circle?

10. Your car needs new brakes. You call a dealership and a local mechanic for prices.

	Cost of parts	Labor cost per hour
Dealership	$24	$99
Local mechanic	$45	$89

 a. After how many hours are the total costs the same at both places? Justify your answer.
 b. When do the repairs cost less at the dealership? at the local mechanic? Explain.

11. You want to paint a piece of pottery at an art studio. The total cost is the cost of the piece plus an hourly studio fee. There are two studios to choose from. *(Section 1.3)*



 a. After how many hours of painting are the total costs the same at both studios? Justify your answer.
 b. Studio B increases the hourly studio fee by $2. How does this affect your answer in part (a)? Explain.

12. Your friend was solving the equation shown and was confused by the result "$-8 = -8$." Explain what this result means.

$$\begin{aligned} 4(y - 2) - 2y &= 6y - 8 - 4y \\ 4y - 8 - 2y &= 6y - 8 - 4y \\ 2y - 8 &= 2y - 8 \\ -8 &= -8 \end{aligned}$$

If students need help...	If students got it...
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

ANSWERS

1. $x = 22$; Add 7 to each side.
2. $x = -3$; Subtract 5 from each side; Multiply each side by $\frac{3}{2}$.
3. $x = -\frac{1}{5}$; Subtract x and 1 from each side; Divide each side by 10.
4. all real numbers
5. $x = -6$
6. no solution
7. $c \neq 5$; If c is 5, then the equation is an identity. For all other values of c, subtracting $3x$ from each side will give a statement that is always false.
8. none; For all values of c, a value of x can always be substituted so that the statement is always true.
9. a. $w = P - \frac{2\ell}{2}$
 b. 65 yd
 c. about 4.8%
10. a. 2.1 h; The cost at the dealership is $24 + 99t$ and the cost at the local mechanic is $45 + 89t$. Set these two expressions equal and solve for the time.
 b. time is less than 2.1 h; time is greater than 2.1 h; Because the expressions are equal for 2.1 hours, that is the cutoff point from the dealership being less expensive to the local mechanic being less expensive.
11. a. 3 h; The cost at studio A is $10 + 8h$ and the cost at Studio B is $16 + 6h$. To find when the costs are the same, set these two expressions equal and solve for the time.
 b. The costs will never be the same; *Sample answer:* The cost for studio B changes to $16 + 8h$, and the new equation has no solution.
12. It is an identity, meaning that all real numbers are solutions of the equation.

ANSWERS

1. C
2. H
3. C
4. G
5. B
6. G

1 Standards Assessment

1. You are ordering concert tickets. You pay a processing fee of \$5, plus \$18 per ticket. How many tickets can you buy for \$100? *(TEKS A.5.A)*

 Ⓐ 3 Ⓑ 4

 Ⓒ 5 Ⓓ 6

2. Which equation is equivalent to $2(x + 1) + 1 = y$? *(TEKS A.10.D)*

 Ⓕ $x + 7 = y + 4$ Ⓖ $2x - 3 = y - 3$

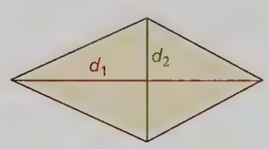
 Ⓗ $3x + 3 = y + x$ Ⓙ $4x + 3 = 2y$

3. You are renting a canoe. It costs \$35 to rent the canoe, plus \$6 per hour. You have \$60. How long can you rent the canoe? Round to the nearest hour. *(TEKS A.5.A)*

 Ⓐ 1 h Ⓑ 3 h

 Ⓒ 4 h Ⓓ 5 h

4. The area A of a rhombus is given by the formula $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals. Which equation could you use to find the value of d_1 for different values of d_2 and A? *(TEKS A.12.E)*



 Ⓕ $d_1 = A\left(\frac{d_2}{2}\right)$ Ⓖ $d_1 = \frac{2A}{d_2}$

 Ⓗ $d_1 = \frac{A}{2d_2}$ Ⓙ $d_1 = A - d_2 - \frac{1}{2}$

5. Your friend's age is 5 years less than half her mother's age. Your friend is 15 years old. How old is her mother? *(TEKS A.5.A)*

 Ⓐ 35 yr Ⓑ 40 yr

 Ⓒ 45 yr Ⓓ 50 yr

6. In biology, the surface-area-to-volume quotient Q of a single spherical cell is given by the formal $Q = \frac{3}{r}$, where r is the radius of the cell. A biologist needs to calculate the diameter d of a cell in terms of Q. Because $d = 2r$, which formula could the biologist use to find d? *(TEKS A.12.E)*

 Ⓕ $d = \frac{6}{r}$ Ⓖ $d = 2\left(\frac{3}{Q}\right)$

 Ⓗ $d = 2Q$ Ⓙ $d = \frac{3}{2r}$

7. *Flyball* is a relay race for dogs. In each of the four legs of the relay, a dog jumps over hurdles, retrieves a ball from a flybox, and runs back over the hurdles. The collie starts the course 0.3 second before the German shepherd. How many seconds does it take for the German shepherd to catch up with the collie? *(TEKS A.5.A)*



Ⓐ 0.3 sec
Ⓑ 1.8 sec
Ⓒ 11.7 sec
Ⓓ 13.3 sec

8. A ski resort offers a super-saver pass for $90 that allows you to buy lift tickets at half price. If you buy the pass, the cost (in dollars) of buying t tickets is $90 + 22.5t$. Otherwise, the cost (in dollars) is $45t$. How many lift tickets would a skier have to buy for the two costs to be equal? *(TEKS A.5.A)*

Ⓕ 2
Ⓖ 4
Ⓗ 6
Ⓙ 8

9. **GRIDDED ANSWER** What is the value of x in the equation $-7x = -\frac{3}{4}x - 5$? *(TEKS A.5.A)*

10. A music store offers a finance plan where you make a $50 down payment on a guitar and pay the remaining balance in 6 equal monthly payments. You have $50 and can afford to pay between $60 and $90 per month for a guitar. What is a reasonable price that you can afford to pay for a guitar? *(TEKS A.5.A)*

Ⓐ $542
Ⓑ $591
Ⓒ $645
Ⓓ $718

11. What is the solution of $7v - (6 - 2v) = 12$? *(TEKS A.5.A)*

Ⓕ $v = -3.6$
Ⓖ $v = -2$
Ⓗ $v = 2$
Ⓙ $v = 3.6$

12. An investor puts x dollars in Fund A and $2000 in Fund B. The total amount invested in the two funds is $4x - 1000$ dollars. How much money did the investor put in Fund A? *(TEKS A.5.A)*

Ⓐ $333
Ⓑ $1000
Ⓒ $3000
Ⓓ $3333

ANSWERS

7. C
8. G
9. $\frac{4}{5}$
10. A
11. H
12. B

Chapter 2 Pacing Guide

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Quiz	1 Day
Section 4	2 Days
Section 5	2 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 2	14 Days
Year-to-Date	26 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
2.1	Preparing for A.5.B
2.2	A.5.B
2.3	A.5.B
2.4	A.5.B
2.5	A.5.B

2 Solving Linear Inequalities

2.1 Writing and Graphing Inequalities
2.2 Solving Inequalities Using Addition or Subtraction
2.3 Solving Inequalities Using Multiplication or Division
2.4 Solving Multi-Step Inequalities
2.5 Solving Compound Inequalities



Withdraw Money *(p. 71)*



Mountain Plant Life *(p. 77)*



Digital Camera *(p. 62)*



Microwave Electricity *(p. 56)*



Natural Arch *(p. 51)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

For an overview of this chapter, formative assessment tips, and teaching strategies, see **Laurie's Notes** at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

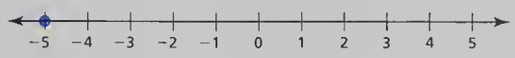
Graphing Numbers on a Number Line (6.2.C)

Example 1 Graph each number.

a. 3

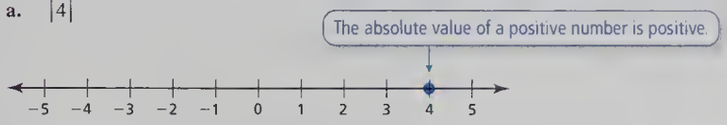


b. −5

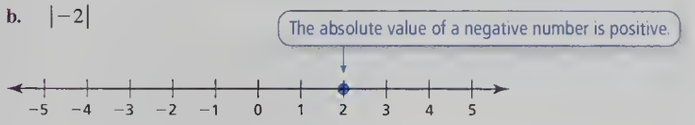


Example 2 Graph each number.

a. $|4|$



b. $|-2|$

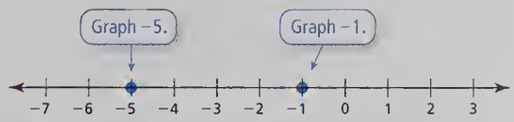


Graph the number.

1. 6
2. $|2|$
3. $|-1|$
4. $2 + |-2|$
5. $1 - |-4|$
6. $-5 + |3|$

Comparing Real Numbers (6.2.C)

Example 3 Complete the statement -1 ▭ -5 with <, >, or =.



▶ -1 is to the right of -5. So, $-1 > -5$.

Complete the statement with <, >, or =.

7. 2 ▭ 9
8. -6 ▭ 5
9. -12 ▭ -4
10. -7 ▭ -13
11. $|-8|$ ▭ $|8|$
12. -10 ▭ $|-18|$

13. **ABSTRACT REASONING** A number a is to the left of a number b on the number line. How do the numbers $-a$ and $-b$ compare?

For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make an Idea and Examples Chart for the following words.

- Absolute value
- Negative number
- Positive number

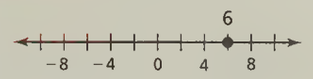
Dynamic Teaching Tools

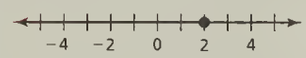
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

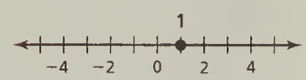
Texas Essential Knowledge and Skills

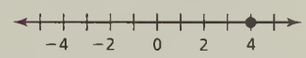
6.2.C The student is expected to locate, compare, and order integers and rational numbers using a number line.

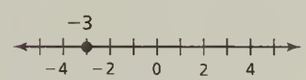
ANSWERS

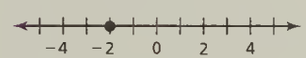
1. 

2. 

3. 

4. 

5. 

6. 

7. $<$
8. $<$
9. $<$
10. $>$
11. $=$
12. $<$
13. $-b < -a$

MONITORING PROGRESS ANSWERS

1. $x < -4$
2. $x > 3$
3. $x \geq -2$

Mathematical Thinking

Mathematically proficient students select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems. **(A.1.C)**

Using a Graphing Calculator

Core Concept

Solving an Inequality in One Variable

You can use a graphing calculator to solve an inequality.

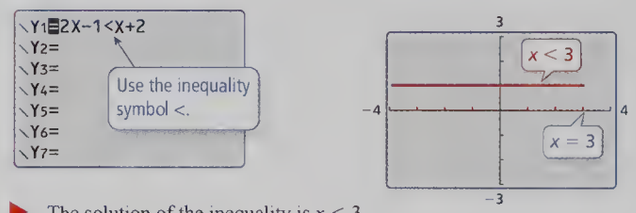
1. Enter the inequality into a graphing calculator.
2. Graph the inequality.
3. Use the graph to write the solution.

EXAMPLE 1 Using a Graphing Calculator

Use a graphing calculator to solve (a) $2x - 1 < x + 2$ and (b) $2x - 1 \leq x + 2$.

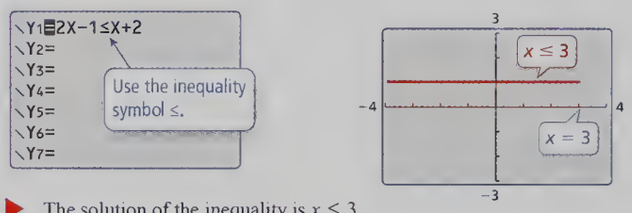
SOLUTION

a. Enter the inequality $2x - 1 < x + 2$ into a graphing calculator. Press *graph*.



▶ The solution of the inequality is $x < 3$.

b. Enter the inequality $2x - 1 \leq x + 2$ into a graphing calculator. Press *graph*.



▶ The solution of the inequality is $x \leq 3$.

Notice that the graphing calculator does not distinguish between the solutions $x < 3$ and $x \leq 3$. You must distinguish between these yourself, based on the inequality symbol used in the original inequality.

Monitoring Progress

Use a graphing calculator to solve the inequality.

1. $2x + 3 < x - 1$
2. $-x - 1 > -2x + 2$
3. $\frac{1}{2}x + 1 \leq \frac{3}{2}x + 3$

For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

2.1 Writing and Graphing Inequalities

Texas Essential Knowledge and Skills
Preparing for A.5.B

Essential Question How can you use an inequality to describe a real-life statement?

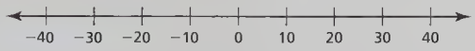
EXPLORATION 1 Writing and Graphing Inequalities

Work with a partner. Write an inequality for each statement. Then sketch the graph of the numbers that make each inequality true.

a. Statement The temperature t in Sweden is at least $-10°C$.

Inequality

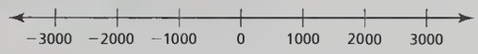
Graph



b. Statement The elevation e of Alabama is at most 2407 feet.

Inequality

Graph



USING PRECISE MATHEMATICAL LANGUAGE
To be proficient in math, you need to communicate precisely. You also need to state the meanings of the symbols you use.

EXPLORATION 2 Writing Inequalities

Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of x that make each inequality true.

a. 

b. 

c. 

d. 

Communicate Your Answer

3. How can you use an inequality to describe a real-life statement?
4. Write a real-life statement that involves each inequality.
 a. $x < 3.5$
 b. $x \leq 6$
 c. $x > -2$
 d. $x \geq 10$

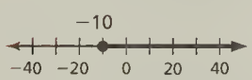
Dynamic Teaching Tools
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- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

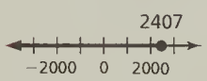
A.5.B The student is expected to solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

1. a. $t \geq 10°C$



 b. $e \leq 2407$ ft



2. a. $x \geq 1$; one and all numbers greater than one
 b. $x > 1$; all numbers greater than one
 c. $x \leq 1$; one and all numbers less than one
 d. $x < 1$; all numbers less than one
3. An inequality can indicate the largest or smallest value for a quantity.
4. a. *Sample answer:* The judge's score is less than 3.5.
 b. *Sample answer:* The time in the pool was at most 6 hours.
 c. *Sample answer:* The temperature is greater than $-2°C$.
 d. *Sample answer:* The profit was at least \$10.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Extra Example 1

Write each sentence as an inequality.

a. A number c plus 9.4 is greater than or equal to -6. $c + 9.4 \geq -6$

b. Six is greater than a number p minus 8. $6 > p - 8$

c. Ten is no more than the sum of 3 times a number y and 4. $10 \leq 3y + 4$

MONITORING PROGRESS ANSWERS

1. $b < 30.4$
2. $-\frac{7}{10} \geq 2k - 4$

2.1 Lesson

Core Vocabulary

inequality, *p. 46*
solution of an inequality, *p. 47*
solution set, *p. 47*
graph of an inequality, *p. 48*

Previous
expression

What You Will Learn

- Write linear inequalities.
- Sketch the graphs of linear inequalities.
- Write linear inequalities from graphs.

Writing Linear Inequalities

An **inequality** is a mathematical sentence that compares expressions. An inequality contains the symbol $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine what inequality symbol to use.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 **Writing Inequalities**

Write each sentence as an inequality.

a. A number w minus 3.5 is less than or equal to -2.

b. Three is less than a number n plus 5.

c. Zero is greater than or equal to twice a number x plus 1.

SOLUTION

a. A number w minus 3.5 is less than or equal to -2.

$w - 3.5$ $\leq$ -2

▶ An inequality is $w - 3.5 \leq -2$.

READING

The inequality $3 < n + 5$ is the same as $n + 5 > 3$.

b. Three is less than a number n plus 5.

3 $<$ $n + 5$

▶ An inequality is $3 < n + 5$.

c. Zero is greater than or equal to twice a number x plus 1.

0 $\geq$ $2x + 1$

▶ An inequality is $0 \geq 2x + 1$.

Monitoring Progress

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Write the sentence as an inequality.

1. A number b is fewer than 30.4.
2. $-\frac{7}{10}$ is at least twice a number k minus 4.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

To provide listening practice, read aloud the Key Phrases describing the Inequality Symbols. Check understanding.

Beginning As a phrase is read, write the symbol that represents it.
Intermediate As a phrase is read, provide the synonymous phrase.
Advanced/Advanced High Provide the meaning of the phrase using their own words.

ELPS 2.C.1 Learn new language structures heard during classroom instruction and interactions.

Sketching the Graphs of Linear Inequalities

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 5 \geq -2$	Is the inequality true?
-6	$-6 + 5 \stackrel{?}{\geq} -2$ $-1 \geq -2$ ✓	yes
-7	$-7 + 5 \stackrel{?}{\geq} -2$ $-2 \geq -2$ ✓	yes
-8	$-8 + 5 \stackrel{?}{\geq} -2$ $-3 \not\geq -2$ ✗	no

Recall that a diagonal line through an inequality symbol means the inequality is *not* true. For instance, the symbol $\not\geq$ means "is not greater than or equal to."

EXAMPLE 2 Checking Solutions

Tell whether -4 is a solution of each inequality.

a. $x + 8 < -3$

b. $-4.5x > -21$

SOLUTION

a. $x + 8 < -3$ Write the inequality.

$-4 + 8 \stackrel{?}{<} -3$ Substitute -4 for x.

$4 \not< -3$ ✗ Simplify.

4 is *not* less than -3.

▶ So, -4 is *not* a solution of the inequality.

b. $-4.5x > -21$ Write the inequality.

$-4.5(-4) \stackrel{?}{>} -21$ Substitute -4 for x.

$18 > -21$ ✓ Simplify.

18 is greater than -21.

▶ So, -4 is a solution of the inequality.

Monitoring Progress

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Tell whether -6 is a solution of the inequality.

3. $c + 4 < -1$

4. $10 \leq 3 - m$

5. $21 \div x \geq -3.5$

6. $4x - 25 > -2$

Extra Example 2

Tell whether 3 is a solution of each inequality.

a. $x - 7 > -5$ 3 is a solution.

b. $-3x \leq -12.5$ 3 is *not* a solution.

MONITORING PROGRESS ANSWERS

3. yes

4. no

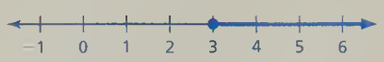
5. yes

6. no

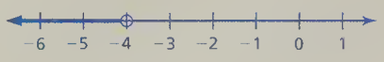
Extra Example 3

Graph each inequality.

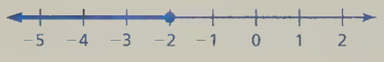
a. $x \geq 3$



b. $-4 > y$



c. $x \leq -2$

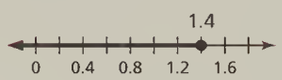


MONITORING PROGRESS ANSWERS

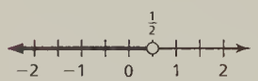
7.



8.



9.



10.



The **graph of an inequality** shows the solution set of the inequality on a number line. An open circle, ○, is used when a number is *not* a solution. A closed circle, ●, is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 Graphing Inequalities

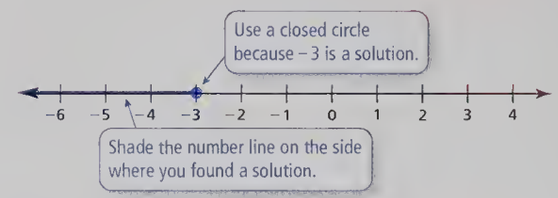
Graph each inequality.

a. $y \leq -3$ **b.** $2 < x$ **c.** $x > 0$

SOLUTION

a. Test a number to the left of −3. $y = -4$ is a solution.

Test a number to the right of −3. $y = 0$ is *not* a solution.

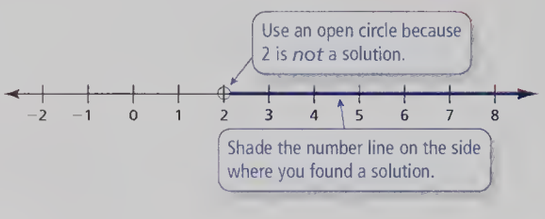


ANOTHER WAY

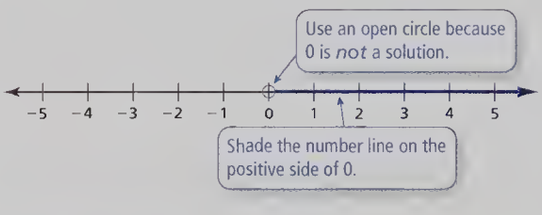
Another way to represent the solutions of an inequality is to use *set-builder notation*. In Example 3b, the solutions can be written as $\{x \mid x > 2\}$, which is read as "the set of all numbers x such that x is greater than 2."

b. Test a number to the left of 2. $x = 0$ is *not* a solution.

Test a number to the right of 2. $x = 4$ is a solution.



c. Just by looking at the inequality, you can see that it represents the set of all positive numbers.



Monitoring Progress

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Graph the inequality.

7. $b > -8$ **8.** $1.4 \geq g$

9. $r < \frac{1}{2}$ **10.** $v \geq \sqrt{36}$



Writing Linear Inequalities from Graphs

EXAMPLE 4 **Writing Inequalities from Graphs**

The graphs show the height restrictions h (in inches) for two rides at an amusement park. Write an inequality that represents the height restriction of each ride.

Ride A

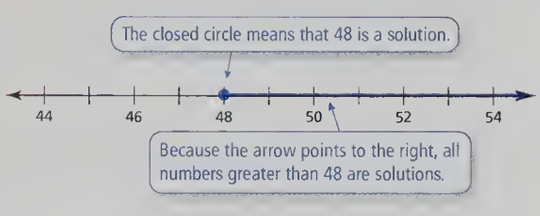


Ride B

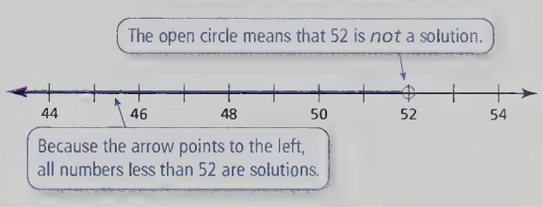


SOLUTION

Ride A



Ride B



▶ So, $h \geq 48$ represents the height restriction for Ride A, and $h < 52$ represents the height restriction for Ride B.

Monitoring Progress

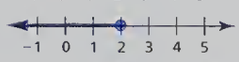
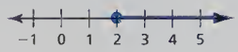
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11. Write an inequality that represents the graph.



Concept Summary

Representing Linear Inequalities

Words	Algebra	Graph
x is less than 2	$x < 2$	
x is greater than 2	$x > 2$	
x is less than or equal to 2	$x \leq 2$	
x is greater than or equal to 2	$x \geq 2$	

Extra Example 4

The graphs show the age restrictions a (in years) for two types of movie tickets. Write an inequality that represents the age restriction of each ticket.

Ticket A



Ticket B



Ticket A: $a < 12$; Ticket B: $a \geq 62$

MONITORING PROGRESS ANSWERS

11. $x \geq -6$

Closure

- Explain how to graph the inequality $x > -2$ to someone who has called you on the phone.
 Sample answer: On a number line, draw an open circle at -2 and shade the line to the right of -2.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–23 odd, 27–35 odd, 41, 45, 50, 52, 60–65

Average: 1–4, 6–46 even, 50–54 even, 60–65

Advanced: 1–4, 10–14, 22–26, 28, 34–58 even, 59–65

HOMEWORK CHECK

Basic: 9, 17, 33, 41

Average: 10, 16, 32, 44

Advanced: 12, 24, 36, 44

ANSWERS

1. inequality
2. no; If x is 5, then the value on the left simplifies to 8, which is not greater than 8.
3. Draw an open circle when a number is not part of the solution. Draw a closed circle when a number is part of the solution. Draw an arrow to the left or right to show that the graph continues in that direction.
4. w is no more than -7; $w \leq -7$; $w \geq -7$
5. $x > 3$
6. $n + 7 \leq 9$
7. $15 \leq \frac{t}{5}$
8. $3w < 18$
9. $\frac{1}{2}y > 22$
10. $3 < s + 4$
11. $13 \geq v - 1$
12. $4 \geq \frac{x}{2.1}$
13. $w \geq 1.7$
14. $x \leq 170$
15. no
16. no
17. yes
18. yes
19. yes
20. no
21. yes
22. no
23. yes
24. no
25. a. $h < 107$ in.
 b. no; A height of 9 feet is equal to 108 inches, which is not less than 107 inches.

2.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A mathematical sentence using the symbols $<$, $>$, $\leq$, or $\geq$ is called a(n)_____.
2. **VOCABULARY** Is 5 in the solution set of $x + 3 > 8$? Explain.
3. **ATTENDING TO PRECISION** Describe how to graph an inequality.
4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

w is greater than or equal to -7.

w is no less than -7.

w is no more than -7.

w is at least -7.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, write the sentence as an inequality. *(See Example 1.)*

5. A number x is greater than 3.
6. A number n plus 7 is less than or equal to 9.
7. Fifteen is no more than a number t divided by 5.
8. Three times a number w is less than 18.
9. One-half of a number y is more than 22.
10. Three is less than the sum of a number s and 4.
11. Thirteen is at least the difference of a number v and 1.
12. Four is no less than the quotient of a number x and 2.1.
13. **MODELING WITH MATHEMATICS** On a fishing trip, you catch two fish. The weight of the first fish is shown. The second fish weighs at least 0.5 pound more than the first fish. Write an inequality that represents the possible weights of the second fish.



14. **MODELING WITH MATHEMATICS** There are 430 people in a wave pool. Write an inequality that represents how many more people can enter the pool.



In Exercises 15–24, tell whether the value is a solution of the inequality. *(See Example 2.)*

15. $r + 4 > 8$; $r = 2$
16. $5 - x < 8$; $x = -3$
17. $3s \leq 19$; $s = -6$
18. $17 \geq 2y$; $y = 7$
19. $-1 > -\frac{x}{2}$; $x = 3$
20. $\frac{4}{z} \geq 3$; $z = 2$
21. $14 \geq -2n + 4$; $n = -5$
22. $-5 \div (2s) < -1$; $s = 10$
23. $20 \leq \frac{10}{2z} + 20$; $z = 5$
24. $\frac{3m}{6} - 2 > 3$; $m = 8$
25. **MODELING WITH MATHEMATICS** The tallest person who ever lived was approximately 8 feet 11 inches tall.
 a. Write an inequality that represents the heights of every other person who has ever lived.
 b. Is 9 feet a solution of the inequality? Explain.

26. DRAWING CONCLUSIONS The winner of a weight-lifting competition bench-pressed 400 pounds. The other competitors all bench-pressed at least 23 pounds less.

a. Write an inequality that represents the weights that the other competitors bench-pressed.

b. Was one of the other competitors able to bench-press 379 pounds? Explain.

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in determining whether 8 is in the solution set of the inequality.

27.

$$-y + 7 < -4$$
$$-8 + 7 \stackrel{?}{<} -4$$
$$-1 < -4$$

8 is in the solution set.

28.

$$\tfrac{1}{2}x + 2 \leq 6$$
$$\tfrac{1}{2}(8) + 2 \stackrel{?}{\leq} 6$$
$$4 + 2 \stackrel{?}{\leq} 6$$
$$6 \leq 6$$

8 is *not* in the solution set.

In Exercises 29–36, graph the inequality. *(See Example 3.)*

29. $x \geq 2$

30. $z \leq 5$

31. $-1 > t$

32. $-2 < w$

33. $v \leq -4$

34. $s < 1$

35. $\frac{1}{4} < p$

36. $r \geq -|5|$

In Exercises 37–40, write and graph an inequality for the given solution set.

37. $\{x \mid x < 7\}$

38. $\{n \mid n \geq -2\}$

39. $\{z \mid 1.3 \leq z\}$

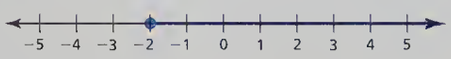
40. $\{w \mid 5.2 > w\}$

In Exercises 41–44, write an inequality that represents the graph. *(See Example 4.)*

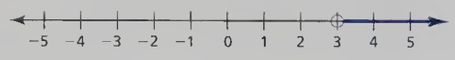
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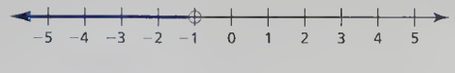
42.



43.



44.



45. ANALYZING RELATIONSHIPS The water temperature of a swimming pool must be no less than 76°F. The temperature is currently 74°F. Which graph correctly shows how much the temperature needs to increase? Explain your reasoning.

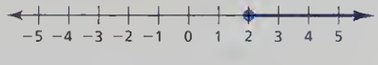
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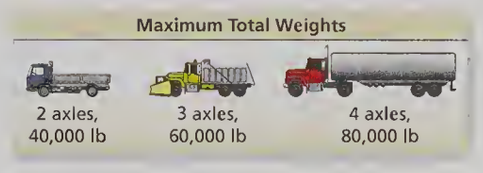
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46. MODELING WITH MATHEMATICS According to a state law for vehicles traveling on state roads, the maximum total weight of a vehicle and its contents depends on the number of axles on the vehicle. For each type of vehicle, write and graph an inequality that represents the possible total weights w (in pounds) of the vehicle and its contents.



47. PROBLEM SOLVING The Xianren Bridge is located in Guangxi Province, China. This arch is the world's longest natural arch, with a length of 400 feet. Write and graph an inequality that represents the lengths ℓ (in *inches*) of all other natural arches.



Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

ANSWERS

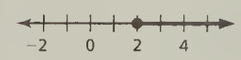
26. a. $x \leq 377$

b. no; A weight of 379 pounds is not a solution of the inequality in part (a).

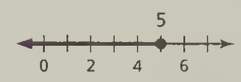
27. Because -1 is not less than -4, the final result is not true; $-1 \not< -4$; 8 is not in the solution set.

28. Because 6 equals 6, the final inequality is true; 8 is in the solution set.

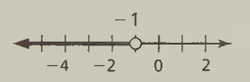
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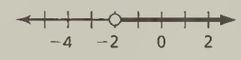
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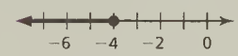
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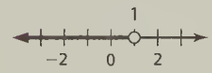
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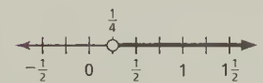
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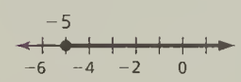
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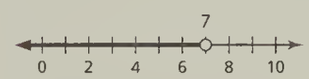
35.



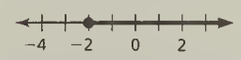
36.



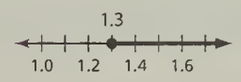
37. $x < 7$



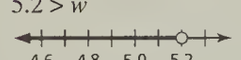
38. $n \geq -2$



39. $1.3 \leq z$



40. $5.2 > w$



41. $x \leq 4$

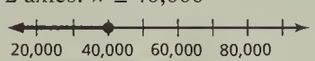
42. $x \geq -2$

43. $x > 3$

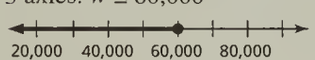
44. $x < -1$

45. C; The temperature must be at least 2°F warmer, so the increase is represented by $x \geq 2$.

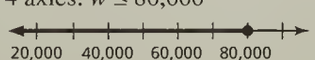
46. 2 axles: $w \leq 40{,}000$



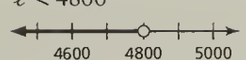
3 axles: $w \leq 60{,}000$



4 axles: $w \leq 80{,}000$

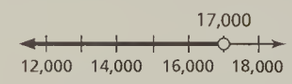


47. $\ell < 4800$

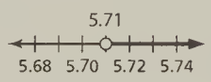


ANSWERS

48. *Sample answer:* $h \leq \frac{25}{7}$
49. *Sample answer:* You spend \$23 on admission and x dollars on snacks, and you can spend no more than \$31 total.
50. a. $T \geq -38.87$
 b. yes; The graph shows $-38.87°C$ as the lowest possible melting point.
51. $0.90x \leq 24$; yes; Because $0.9(25) = \$22.50$, which is less than \$24, the inequality is true.
52. cousin; Because the inequality is "less than or equal to," a weight equal to the given amount is possible.
53. *Sample answer:* A temperature above the freezing point of water can be represented by $T > 0$ if the temperature is in degrees Celsius, or by $T > 32$ if the temperature is in degrees Fahrenheit.
54. $b < 17{,}000$



55. $x < 14$
56. $x \geq 1.6$
57. $x < 3$
58. $x > 6$
59. a. $r > \frac{40}{7}$ (about 5.71)

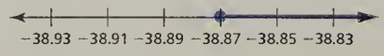


 b. no; The graph includes speeds beyond the maximum speed a human can run.
60. $x = 1$
61. $y = 14$
62. $y = 2$
63. $y = -1$
64. Addition Property of Equality
65. Division Property of Equality

48. **THOUGHT PROVOKING** A student works no more than 25 hours each week at a part-time job. Write an inequality that represents how many hours the student can work each day.

49. **WRITING** Describe a real-life situation modeled by the inequality $23 + x \leq 31$.

50. **HOW DO YOU SEE IT?** The graph represents the known melting points of all metallic elements (in degrees Celsius).



 a. Write an inequality represented by the graph.
 b. Is it possible for a metallic element to have a melting point of $-38.87°C$? Explain.

51. **DRAWING CONCLUSIONS** A one-way ride on a subway costs \$0.90. A monthly pass costs \$24. Write an inequality that represents how many one-way rides you can buy before it is cheaper to buy the monthly pass. Is it cheaper to pay the one-way fare for 25 rides? Explain.

Subway Prices	
One-way ride	\$0.90
Monthly pass	\$24.00

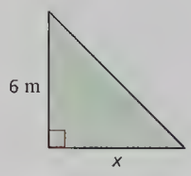
52. **MAKING AN ARGUMENT** The inequality $x \leq 1324$ represents the weights (in pounds) of all mako sharks ever caught using a rod and reel. Your friend says this means no one using a rod and reel has ever caught a mako shark that weighs 1324 pounds. Your cousin says this means someone using a rod and reel *has* caught a mako shark that weighs 1324 pounds. Who is correct? Explain your reasoning.

53. **CRITICAL THINKING** Describe a real-life situation that can be modeled by more than one inequality.

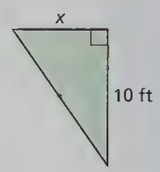
54. **MODELING WITH MATHEMATICS** In 1997, Superman's cape from the 1978 movie *Superman* was sold at an auction. The winning bid was \$17,000. Write and graph an inequality that represents the amounts all the losing bids.

MATHEMATICAL CONNECTIONS In Exercises 55–58, write an inequality that represents the missing dimension x.

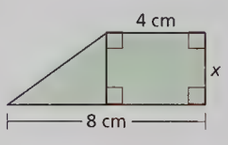
55. The area is less than 42 square meters.



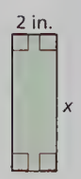
56. The area is greater than or equal to 8 square feet.



57. The area is less than 18 square centimeters.



58. The area is greater than 12 square inches.



59. **WRITING** A runner finishes a 200-meter dash in 35 seconds. Let r represent any speed (in meters per second) faster than the runner's speed.
 a. Write an inequality that represents r. Then graph the inequality.
 b. Every point on the graph represents a speed faster than the runner's speed. Do you think every point could represent the speed of a runner? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.1)*

60. $x + 2 = 3$
61. $y - 9 = 5$
62. $6 = 4 + y$
63. $-12 = y - 11$

Identify the property of equality that makes Equation 1 and 2 equivalent. *(Section 1.1)*

64.

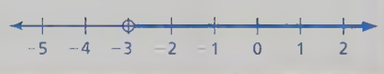
Equation 1	$3x + 8 = x - 1$
Equation 2	$3x + 9 = x$

65.

Equation 1	$4y = 28$
Equation 2	$y = 7$

Mini-Assessment

1. Write the sentence as an inequality.
 One-half is at least 3 times a number p. $\frac{1}{2} \geq 3p$
2. Tell whether -2 is a solution of the inequality $n - 5 < 8$. -2 is a solution.
3. Graph the inequality $s > -3$.



If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

2.2 Solving Inequalities Using Addition or Subtraction

Texas Essential Knowledge and Skills
A.5.B

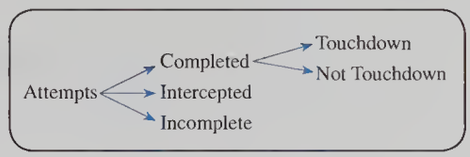
Essential Question
How can you use addition or subtraction to solve an inequality?

EXPLORATION 1 Quarterback Passing Efficiency

Work with a partner. The National Collegiate Athletic Association (NCAA) uses the following formula to rank the passing efficiencies P of quarterbacks.

$$P = \frac{8.4Y + 100C + 330T - 200N}{A}$$

Y = total length of all completed passes (in Yards)
C = Completed passes
T = passes resulting in a Touchdown
N = iNtercepted passes
A = Attempted passes
M = incoMplete passes



Determine whether each inequality must be true. Explain your reasoning.

a. $T < C$ b. $C + N \leq A$ c. $N < A$ d. $A - C \geq M$

APPLYING MATHEMATICS
To be proficient in math, you need to identify and analyze important relationships and then draw conclusions, using tools such as diagrams, flowcharts, and formulas.

EXPLORATION 2 Finding Solutions of Inequalities

Work with a partner. Use the passing efficiency formula to create a passing record that makes each inequality true. Record your results in the table. Then describe the values of P that make each inequality true.

Attempts	Completions	Yards	Touchdowns	Interceptions

a. $P < 0$

b. $P + 100 \geq 250$

c. $P - 250 > -80$

Communicate Your Answer

3. How can you use addition or subtraction to solve an inequality?
4. Solve each inequality.
 a. $x + 3 < 4$ b. $x - 3 \geq 5$
 c. $4 > x - 2$ d. $-2 \leq x + 1$

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.5.B The student is expected to solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

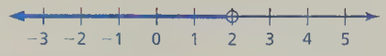
1. a. no; It is also possible for the number of touchdowns to equal the number of completed passes.
 b. yes; Completed and intercepted passes are both included in attempts.
 c. no; It is also possible for the number of intercepted passes to equal the number of attempts.
 d. yes; After subtracting completed passes from attempts, all remaining passes could be incomplete.
2. a. *Sample answer:* 25; 5; 100; 0; 10; negative numbers
 b. *Sample answer:* 10; 9; 600; 4; 0; numbers greater than or equal to 150
 c. *Sample answer:* 10; 7; 300; 2; 1; numbers greater than 170
3. Add or subtract the same number to or from each side of the inequality to create an equivalent inequality.
4. a. $x < 1$
 b. $x \geq 8$
 c. $x < 6$
 d. $x \geq -3$

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Extra Example 1

Solve $x - 5 < -3$. Graph the solution.

$x < 2$

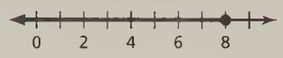


MONITORING PROGRESS ANSWERS

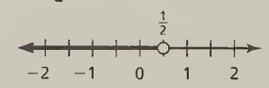
1. $b > -7$



2. $m \leq 8$



3. $y < \frac{1}{2}$



2.2 Lesson

What You Will Learn

- Solve inequalities using addition.
- Solve inequalities using subtraction.
- Use inequalities to solve real-life problems.

Core Vocabulary

equivalent inequalities, *p. 54*

Previous

inequality

Solving Inequalities Using Addition

Just as you used the properties of equality to produce equivalent equations, you can use the properties of inequality to produce equivalent inequalities. **Equivalent inequalities** are inequalities that have the same solutions.

Core Concept

Addition Property of Inequality

Words Adding the same number to each side of an inequality produces an equivalent inequality.

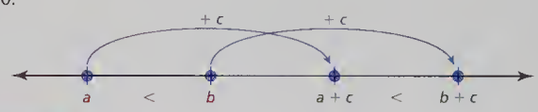
Numbers

$$\begin{array}{rrrr} -3 & < & 2 & \\ \underline{+4} & & \underline{+4} & \\ 1 & < & 6 & \end{array} \qquad \begin{array}{rrrr} -3 & \geq & -10 & \\ \underline{+3} & & \underline{+3} & \\ 0 & \geq & -7 & \end{array}$$

Algebra If $a > b$, then $a + c > b + c$. If $a \geq b$, then $a + c \geq b + c$.

If $a < b$, then $a + c < b + c$. If $a \leq b$, then $a + c \leq b + c$.

The diagram shows one way to visualize the Addition Property of Inequality when $c > 0$.



EXAMPLE 1 Solving an Inequality Using Addition

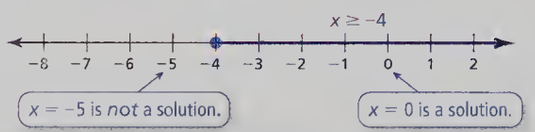
Solve $x - 6 \geq -10$. Graph the solution.

SOLUTION

$$\begin{array}{rrrr} x - 6 & \geq & -10 & \text{Write the inequality.} \\ \underline{+6} & & \underline{+6} & \text{Add 6 to each side.} \\ x & \geq & -4 & \text{Simplify.} \end{array}$$

Addition Property of Inequality

▶ The solution is $x \geq -4$.



REMEMBER

To check this solution, substitute a few numbers to the left and right of −4 into the original inequality.

Monitoring Progress

Help In English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution.

1. $b - 2 > -9$
2. $m - 3 \leq 5$
3. $\frac{1}{4} > y - \frac{1}{4}$

For classroom suggestions on teaching this lesson, see Laurie's Notes at *BigIdeasMath.com*.

Differentiated Instruction

Kinesthetic

To help students understand the difference between the solutions of equations and inequalities, write the equation $x + 5 = 10$ and the inequality $x + 5 \geq 10$ on the board. Ask two students to assist you at the board. Assign the equation to one student to solve and graph on a number line. Assign the inequality to the second student to solve and graph on a number line. Have students discuss how and why the solutions are different.

Solving Inequalities Using Subtraction

Core Concept

Subtraction Property of Inequality

Words Subtracting the same number from each side of an inequality produces an equivalent inequality.

Numbers

$-3 \leq 1$ $\quad$ $7 > -20$

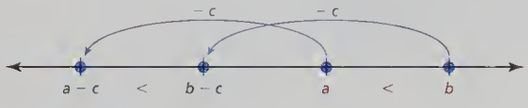
$-5 \quad -5$ $\quad$ $-7 \quad -7$

$-8 \leq -4$ $\quad$ $0 > -27$

Algebra If $a > b$, then $a - c > b - c$. $\quad$ If $a \geq b$, then $a - c \geq b - c$.

If $a < b$, then $a - c < b - c$. $\quad$ If $a \leq b$, then $a - c \leq b - c$.

The diagram shows one way to visualize the Subtraction Property of Inequality when $c > 0$.



EXAMPLE 2 Solving an Inequality Using Subtraction

Solve each inequality. Graph the solution.

a. $y + 8 \leq 5$ $\quad$ **b.** $-8 < 1.4 + m$

SOLUTION

a. $y + 8 \leq 5$ Write the inequality.

Subtraction Property of Inequality → $-8 \quad -8$ Subtract 8 from each side.

$y \leq -3$ Simplify.

▶ The solution is $y \leq -3$.

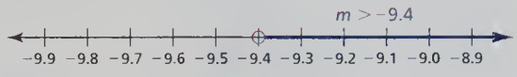


b. $-8 < 1.4 + m$ Write the inequality.

Subtraction Property of Inequality → $-1.4 \quad -1.4$ Subtract 1.4 from each side.

$-9.4 < m$ Simplify.

▶ The solution is $m > -9.4$.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

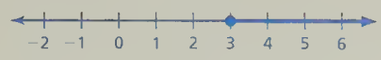
Solve the inequality. Graph the solution.

4. $k + 5 \leq -3$ $\quad$ **5.** $\frac{5}{6} \leq z + \frac{1}{6}$ $\quad$ **6.** $p + 0.7 > -2.3$

Extra Example 2

Solve each inequality. Graph the solution.

a. $s + 18 \geq 21$ $s \geq 3$



b. $-5.2 > x + 2$ $x < -7.2$

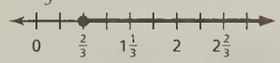


MONITORING PROGRESS ANSWERS

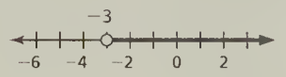
4. $k \leq -8$



5. $z \geq \frac{2}{3}$



6. $p > -3$



Extra Example 3

An electric grill that uses 1600 watts of electricity is plugged into the circuit described in Example 3.

a. Write and solve an inequality that represents how many watts you can add to the circuit without overloading the circuit. $1600 + w < 1800$; $w < 200$; You can add up to 200 watts.

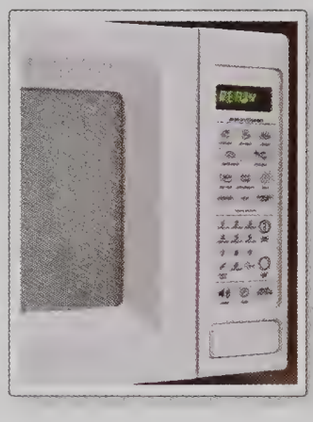
b. In addition to the electric grill, which of the appliances listed in the table in Example 3 can you plug into the circuit at the same time without overloading the circuit? clock radio

MONITORING PROGRESS ANSWERS

7. no; The additional electricity must be less than 800 watts, and the toaster uses 800 watts.

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**



A circuit overloads at 1800 watts of electricity. You plug a microwave oven that uses 1100 watts of electricity into the circuit.

a. Write and solve an inequality that represents how many watts you can add to the circuit without overloading the circuit.

b. In addition to the microwave oven, which of the following appliances can you plug into the circuit at the same time without overloading the circuit?

Appliance	Watts
Clock radio	50
Blender	300
Hot plate	1200
Toaster	800

SOLUTION

1. **Understand the Problem** You know that the microwave oven uses 1100 watts out of a possible 1800 watts. You are asked to write and solve an inequality that represents how many watts you can add without overloading the circuit. You also know the numbers of watts used by four other appliances. You are asked to identify the appliances you can plug in at the same time without overloading the circuit.

2. **Make a Plan** Use a verbal model to write an inequality. Then solve the inequality and identify other appliances that you can plug into the circuit at the same time without overloading the circuit.

3. **Solve the Problem**

Words Watts used by microwave oven + Additional watts < Overload wattage

Variable Let w be the additional watts you can add to the circuit.

Inequality $1100 + w < 1800$

$1100 + w < 1800$ Write the inequality.

Subtraction Property of Inequality → $-1100 \quad -1100$ Subtract 1100 from each side.

$w < 700$ Simplify.

▶ You can add up to 700 watts to the circuit, which means that you can also plug in the clock radio and the blender.

4. **Look Back** You can check that your answer is correct by adding the numbers of watts used by the microwave oven, clock radio, and blender.

$1100 + 50 + 300 = 1450$

The circuit will not overload because the total wattage is less than 1800 watts.

Monitoring Progress

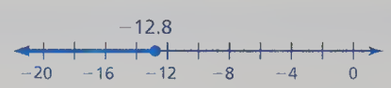
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7. The microwave oven uses only 1000 watts of electricity. Does this allow you to have both the microwave oven and the toaster plugged into the circuit at the same time? Explain your reasoning.

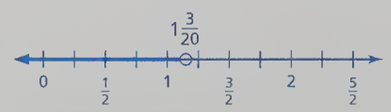
Closure

- **Think-Pair-Share:** Solve and graph.

$x + 3.8 \leq -9$ $\quad x \leq -12.8$



$\frac{2}{5} > x - \frac{3}{4}$ $\quad x < 1\frac{3}{20}$



2.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Why is the inequality $x \leq 6$ equivalent to the inequality $x - 5 \leq 6 - 5$?
2. **WRITING** Compare solving equations using addition with solving inequalities using addition.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, tell which number you would add to or subtract from each side of the inequality to solve it.

3. $k + 11 < -3$
4. $v - 2 > 14$
5. $-1 \geq b - 9$
6. $-6 \leq 17 + p$

In Exercises 7–20, solve the inequality. Graph the solution. *(See Examples 1 and 2.)*

7. $x - 4 < -5$
8. $1 \leq s - 8$
9. $6 \geq m - 1$
10. $c - 12 > -4$
11. $r + 4 < 5$
12. $-8 \leq 8 + y$
13. $9 + w > 7$
14. $15 \geq q + 3$
15. $h - (-2) \geq 10$
16. $-6 > t - (-13)$
17. $j + 9 - 3 < 8$
18. $1 - 12 + y \geq -5$
19. $10 \geq 3p - 2p - 7$
20. $18 - 5z + 6z > 3 + 6$

In Exercises 21–24, write the sentence as an inequality. Then solve the inequality.

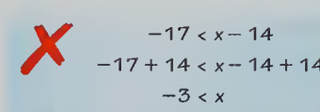
21. A number plus 8 is greater than 11.
22. A number minus 3 is at least -5.
23. The difference of a number and 9 is fewer than 4.
24. Six is less than or equal to the sum of a number and 15.
25. **MODELING WITH MATHEMATICS** You are riding a train. Your carry-on bag can weigh no more than 50 pounds. Your bag weighs 38 pounds. *(See Example 3.)*
 a. Write and solve an inequality that represents how much weight you can add to your bag.
 b. Can you add both a 9-pound laptop and a 5-pound pair of boots to your bag without going over the weight limit? Explain.
26. **MODELING WITH MATHEMATICS** You order the hardcover book shown from a website that offers free shipping on orders of $25 or more. Write and solve an inequality that represents how much more you must spend to get free shipping.



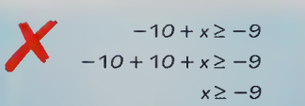
Price: $19.76

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in solving the equation or graphing the solution.**

27.
$$-17 < x - 14$$
$$-17 + 14 < x - 14 + 14$$
$$-3 < x$$



28.
$$-10 + x \geq -9$$
$$-10 + 10 + x \geq -9$$
$$x \geq -9$$



29. **PROBLEM SOLVING** An NHL hockey player has 59 goals so far in a season. What are the possible numbers of additional goals the player can score to match or break the NHL record of 92 goals in a season?

SUPPORTING English Language Learners

Exercise 25 describes a trip. Have students describe the process of planning for a trip. Ask what is done with bags before the trip. (They are weighed.)

Beginning Tell what the carry-on bag weighs. (38 pounds)
Intermediate Use a complete sentence to give the weight of the bag.
Advanced Describe the difference between the weight of the bag and the weight limit.
Advanced High Provide the explanation for part (b).

ELPS 1.A.2 Use prior experiences to understand meanings in English.

Assignment Guide and Homework Check

ASSIGNMENT

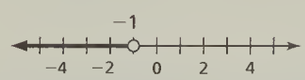
Basic: 1–6, 7–29 odd, 30, 36, 39–46
Average: 1, 2–32 even, 36–46
Advanced: 1, 2, 16–19, 20–24 even, 26–38 even, 39–46

HOMEWORK CHECK

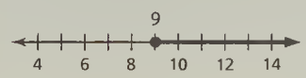
Basic: 5, 7, 13, 25
Average: 8, 18, 22, 26
Advanced: 16, 17, 24, 26

ANSWERS

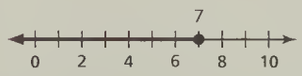
1. Subtraction Property of Inequality
2. *Sample answer:* In both cases the same number is added to each side to obtain an equivalent mathematical sentence.
3. subtract 11
4. add 2
5. add 9
6. subtract 17
7. $x < -1$



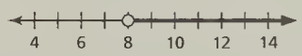
8. $s \geq 9$



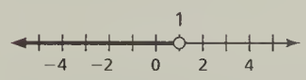
9. $m \leq 7$



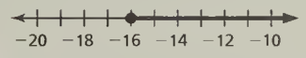
10. $c > 8$



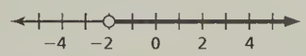
11. $r < 1$



12. $y \geq -16$



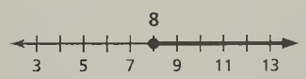
13. $w > -2$



14. $q \leq 12$



15. $h \geq 8$



16–29. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

30. **a.** score greater than 117.4 points

b. they both are; Both scores are greater than 117.4 points.

31. A; D; Subtract 3 from each side; order of inequality reverses for opposites

32. $14.2 + 15.5 + x < 51.3$; $x < 21.6$

33. $6.4 + 4.9 + 4.1 + x \leq 18.7$; $x \leq 3.3$

34. *Sample answer:* $x + 34 \leq 50$; A shelf can safely support 50 pounds. The combined weight of items on the shelf is 34 pounds. x represents the possible weights of items that can be put on the shelf safely.

35. no; *Sample answer:* 3, 7, 8, 9, and 12; There are infinitely many solutions. Check 8 and a few numbers greater than and less than 8.

36–46. See Additional Answers.

Mini-Assessment

Solve the inequality. Graph the solution.

1. $x - 2 > -3$ $x > -1$



2. $4 \geq m - 3$ $m \leq 7$



3. $r + 5 < 3$ $r < -2$



4. $-8 + y \geq -4$ $y \geq 4$



5. A shirt that you want to buy costs \$15.50. You can use a \$5 discount coupon if you spend at least \$20. Write and solve an inequality that represents how much more you must spend to use the \$5 coupon. $x + 15.5 \geq 20$; $x \geq 4.5$; You must spend at least \$4.50 more.

30. MAKING AN ARGUMENT In an aerial ski competition, you perform two acrobatic ski jumps. The scores on the two jumps are then added together.

Ski jump	Competitor's score	Your score
1	117.1	119.5
2	119.8	

a. Describe the score that you must earn on your second jump to beat your competitor.

b. Your coach says that you will beat your competitor if you score 118.4 points. A teammate says that you only need 117.5 points. Who is correct? Explain.

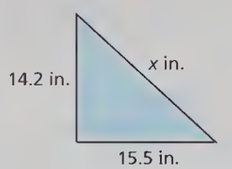
31. REASONING Which of the following inequalities are equivalent to the inequality $x - b < 3$, where b is a constant? Justify your answer.

Ⓐ $x - b - 3 < 0$ Ⓑ $0 > b - x + 3$

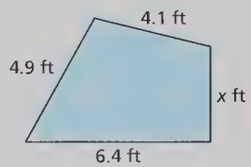
Ⓒ $x < 3 - b$ Ⓓ $-3 < b - x$

MATHEMATICAL CONNECTIONS **In Exercises 32 and 33, write and solve an inequality to find the possible values of x.**

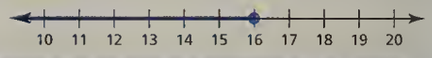
32. Perimeter < 51.3 inches



33. Perimeter ≤ 18.7 feet

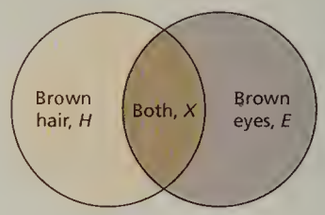


34. THOUGHT PROVOKING Write an inequality that has the solution shown in the graph. Describe a real-life situation that can be modeled by the inequality.



35. WRITING Is it possible to check all the numbers in the solution set of an inequality? When you solve the inequality $x - 11 \geq -3$, which numbers can you check to verify your solution? Explain your reasoning.

36. HOW DO YOU SEE IT? The diagram represents the numbers of students in a school with brown eyes, brown hair, or both.



Determine whether each inequality must be true. Explain your reasoning.

a. $H \geq E$ **b.** $H + 10 \geq E$

c. $H \geq X$ **d.** $H + 10 \geq X$

e. $H > X$ **f.** $H + 10 > X$

37. REASONING Write and graph an inequality that represents the numbers that are *not* solutions of each inequality.

a. $x + 8 < 14$

b. $x - 12 \geq 5.7$

38. PROBLEM SOLVING Use the inequalities $c - 3 \geq d$, $b + 4 < a + 1$, and $a - 2 \leq d - 7$ to order a, b, c, and d from least to greatest.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the product or quotient. *(Skills Review Handbook)*

39. $7 \cdot (-9)$ **40.** $-11 \cdot (-12)$ **41.** $-27 \div (-3)$ **42.** $20 \div (-5)$

Solve the equation. Check your solution. *(Section 1.1)*

43. $6x = 24$ **44.** $-3y = -18$ **45.** $\frac{s}{-8} = 13$ **46.** $\frac{n}{4} = -7.3$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

2.3 Solving Inequalities Using Multiplication or Division

Texas Essential Knowledge and Skills
A.5.B

Essential Question
How can you use division to solve an inequality?

EXPLORATION 1 Writing a Rule

Work with a partner.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < 3x$. Write the solution of the inequality.

x	-1	0	1	2	3	4	5
$3x$	-3						
$6 \stackrel{?}{<} 3x$	No						



b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

i. $2x < 4$ **ii.** $3 \geq 3x$ **iii.** $2x < 8$ **iv.** $6 \geq 3x$

EXPLORATION 2 Writing a Rule

Work with a partner.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < -3x$. Write the solution of the inequality.

x	-5	-4	-3	-2	-1	0	1
$-3x$							
$6 \stackrel{?}{<} -3x$							



b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

i. $-2x < 4$ **ii.** $3 \geq -3x$ **iii.** $-2x < 8$ **iv.** $6 \geq -3x$

Communicate Your Answer

3. How can you use division to solve an inequality?

4. Use the rules you wrote in Explorations 1(b) and 2(b) to solve each inequality.

a. $7x < -21$ **b.** $12 \leq 4x$ **c.** $10 < -5x$ **d.** $-3x \leq 0$

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to investigate relationships, observe patterns, and use your observations to write general rules.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.5.B The student is expected to solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

1. a. 0; 3; 6; 9; 12; 15; no; no; no; yes; yes; yes; right graph; $x > 2$

b. i. $x < 2$
ii. $x \leq 1$
iii. $x < 4$
iv. $x \leq 2$
Dividing each side of an inequality by the same positive number produces an equivalent inequality.

2. a. 15; 12; 9; 6; 3; 0; -3; yes; yes; yes; no; no; no; no; left graph; $x < -2$

b. i. $x > -2$
ii. $x \geq -1$
iii. $x > -4$
iv. $x \geq -2$
When dividing each side of an inequality by the same negative number, the direction of the inequality must be reversed to produce an equivalent inequality.

3. Divide each side of the inequality by the same number. If the number is positive, this produces an equivalent inequality. If the number is negative, the inequality must be reversed to be equivalent.

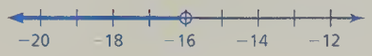
4. a. $x < -3$
b. $x \geq 3$
c. $x < -2$
d. $x \geq 0$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Extra Example 1

a. Solve $-4 > \frac{x}{4}$. Graph the solution.

$x < -16$



b. Solve $2n \geq -14$. Graph the solution.

$n \geq -7$

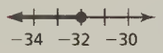


MONITORING PROGRESS ANSWERS

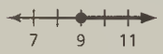
1. $n \geq -7$



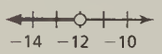
2. $w \leq -32$



3. $b \geq 9$



4. $q < -12$



2.3 Lesson

What You Will Learn

- Solve inequalities by multiplying or dividing by *positive* numbers.
- Solve inequalities by multiplying or dividing by *negative* numbers.
- Use inequalities to solve real-life problems.

Multiplying or Dividing by Positive Numbers

Core Concept

Multiplication and Division Properties of Inequality ($c > 0$)

Words Multiplying or dividing each side of an inequality by the same *positive* number produces an equivalent inequality.

Numbers

$-6 < 8$ $\qquad$ $6 > -8$

$2 \cdot (-6) < 2 \cdot 8$ $\qquad$ $\frac{6}{2} > \frac{-8}{2}$

$-12 < 16$ $\qquad$ $3 > -4$

Algebra If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 Multiplying or Dividing by Positive Numbers

Solve (a) $\frac{x}{8} > -5$ and (b) $-24 \geq 3x$. Graph each solution.

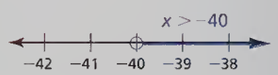
SOLUTION

a. $\frac{x}{8} > -5$ Write the inequality.

Multiplication Property of Inequality → $8 \cdot \frac{x}{8} > 8 \cdot (-5)$ Multiply each side by 8.

$x > -40$ Simplify.

▶ The solution is $x > -40$.



b. $-24 \geq 3x$ Write the inequality.

Division Property of Inequality → $\frac{-24}{3} \geq \frac{3x}{3}$ Divide each side by 3.

$-8 \geq x$ Simplify.

▶ The solution is $x \leq -8$.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution.

1. $\frac{n}{7} \geq -1$ 2. $-6.4 \geq \frac{1}{5}w$ 3. $4b \geq 36$ 4. $-18 > 1.5q$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Pair Activity

Have students work in pairs to solve inequalities. Each student solves a different inequality. Students share their work and explain their reasoning while their partners follow along.

Multiplying or Dividing by Negative Numbers

Core Concept

Multiplication and Division Properties of Inequality ($c < 0$)

Words When multiplying or dividing each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed to produce an equivalent inequality.

Numbers $-6 < 8$ $\qquad 6 > -8$

$-2 \cdot (-6) > -2 \cdot 8$ $\qquad \frac{6}{-2} < \frac{-8}{-2}$

$12 > -16$ $\qquad -3 < 4$

Algebra If $a > b$ and $c < 0$, then $ac < bc$. If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

If $a < b$ and $c < 0$, then $ac > bc$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

COMMON ERROR

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol, as shown in Example 1.

Only reverse the inequality symbol when you multiply or divide each side by a negative number.

EXAMPLE 2 Multiplying or Dividing by Negative Numbers

Solve each inequality. Graph each solution.

a. $2 < \frac{y}{-3}$ $\qquad$ b. $-7y \leq -35$

SOLUTION

a. $2 < \frac{y}{-3}$ Write the inequality.

Multiplication Property of Inequality → $-3 \cdot 2 > -3 \cdot \frac{y}{-3}$ Multiply each side by -3. Reverse the inequality symbol.

$-6 > y$ Simplify.

▶ The solution is $y < -6$.

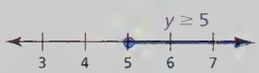


b. $-7y \leq -35$ Write the inequality.

Division Property of Inequality → $\frac{-7y}{-7} \geq \frac{-35}{-7}$ Divide each side by -7. Reverse the inequality symbol.

$y \geq 5$ Simplify.

▶ The solution is $y \geq 5$.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution.

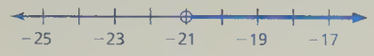
5. $\frac{p}{-4} < 7$ $\qquad$ 6. $\frac{x}{-5} \leq -5$ $\qquad$ 7. $1 \geq -\frac{1}{10}z$

8. $-9m > 63$ $\qquad$ 9. $-2r \geq -22$ $\qquad$ 10. $-0.4y \geq -12$

Extra Example 2

Solve each inequality. Graph each solution.

a. $3 > \frac{x}{-7}$ $x > -21$



b. $-5m \geq -45$ $m \leq 9$



MONITORING PROGRESS ANSWERS

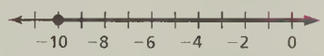
5. $p > -28$



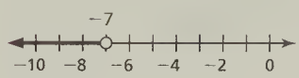
6. $x \geq 25$



7. $z \geq -10$



8. $m < -7$



9. $r \leq 11$



10. $y \leq 30$



Extra Example 3

You earn $8.50 per hour at the sub shop. Write and solve an inequality that represents the numbers of hours you need to work to earn $187 to buy a tablet computer. $8.5h \geq 187$; $h \geq 22$; at least 22 hours

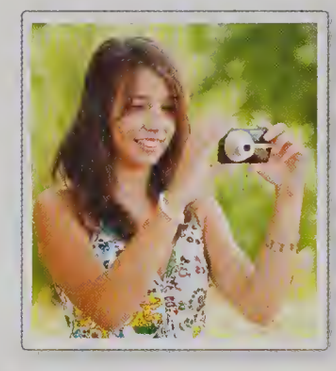
MONITORING PROGRESS ANSWERS

11. $0.25c \leq 3.65, c \leq 14.6$

12. $\frac{165}{t} \leq 55, t \geq 3$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**



You earn $9.50 per hour at your summer job. Write and solve an inequality that represents the numbers of hours you need to work to buy a digital camera that costs $247.

SOLUTION

1. **Understand the Problem** You know your hourly wage and the cost of the digital camera. You are asked to write and solve an inequality that represents the numbers of hours you need to work to buy the digital camera.
2. **Make a Plan** Use a verbal model to write an inequality. Then solve the inequality.
3. **Solve the Problem**

Words Hourly wage $\cdot$ Hours worked $\geq$ Cost of camera

Variable Let n be the number of hours worked.

Inequality $9.5 \cdot n \geq 247$

$9.5n \geq 247$ Write the inequality.

Division Property of Inequality → $\frac{9.5n}{9.5} \geq \frac{247}{9.5}$ Divide each side by 9.5.

$n \geq 26$ Simplify.

▶ You need to work at least 26 hours for your gross pay to be at least $247. If you have payroll deductions, such as Social Security taxes, you need to work more than 26 hours.

REMEMBER

Compatible numbers are numbers that are easy to compute mentally.

4. **Look Back** You can use estimation to check that your answer is reasonable.

$\$247 \div \$9.50/\text{h}$

$\downarrow \qquad \downarrow$

$\$250 \div \$10/\text{h} = 25 \text{ h}$ Use compatible numbers.

Your hourly wage is about $10 per hour. So, to earn about $250, you need to work about 25 hours.

Unit Analysis Each time you set up an equation or inequality to represent a real-life problem, be sure to check that the units balance.

$\frac{\$9.50}{\text{h}} \times 26\text{h} = \$247$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

11. You have at most $3.65 to make copies. Each copy costs $0.25. Write and solve an inequality that represents the numbers of copies you can make.

12. The maximum speed limit for a school bus is 55 miles per hour. Write and solve an inequality that represents the numbers of hours it takes to travel 165 miles in a school bus.

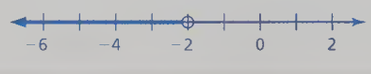
Closure

- **I Used to Think ... But Now I Know:** Take time for students to reflect on their current understanding of solving inequalities involving multiplication and division.
- **Exit Ticket:** Solve and graph.

$\frac{x}{-3} \leq -9$ $x \geq 27$



$-8 > 4x$ $x < -2$



2.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how solving $2x < -8$ is different from solving $-2x < 8$.
2. **OPEN-ENDED** Write an inequality that is solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the inequality. Graph the solution. *(See Example 1.)*

3. $4x < 8$
4. $3y \leq -9$
5. $-20 \leq 10n$
6. $35 < 7t$
7. $\frac{x}{2} > -2$
8. $\frac{a}{4} < 10.2$
9. $20 \geq \frac{4}{5}w$
10. $-16 \leq \frac{8}{3}t$

In Exercises 11–18, solve the inequality. Graph the solution. *(See Example 2.)*

11. $-6t < 12$
12. $-9y > 9$
13. $-10 \geq -2z$
14. $-15 \leq -3c$
15. $\frac{n}{-3} \geq 1$
16. $\frac{w}{-5} \leq 16$
17. $-8 < -\frac{1}{4}m$
18. $-6 > -\frac{2}{3}y$

19. **MODELING WITH MATHEMATICS** You have \$12 to buy five goldfish for your new fish tank. Write and solve an inequality that represents the prices you can pay per fish. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** A weather forecaster predicts that the temperature in Antarctica will decrease 8°F each hour for the next 6 hours. Write and solve an inequality to determine how many hours it will take for the temperature to drop at least 36°F.

USING TOOLS In Exercises 21–26, solve the inequality. Use a graphing calculator to verify your answer.

21. $36 < 3y$
22. $17v \geq 51$
23. $2 \leq -\frac{2}{9}x$
24. $4 > \frac{n}{-4}$
25. $2x > \frac{3}{4}$
26. $1.1y < 4.4$

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in solving the inequality.

27.

$$-6 > \frac{2}{3}x$$
$$\frac{3}{2} \cdot (-6) < \frac{3}{2} \cdot \frac{2}{3}x$$
$$-\frac{18}{2} < x$$
$$-9 < x$$

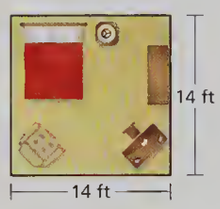
The solution is $x > -9$.

28.

$$-4y \leq -32$$
$$\frac{-4y}{-4} \leq \frac{-32}{-4}$$
$$y \leq 8$$

The solution is $y \leq 8$.

29. **ATTENDING TO PRECISION** You have \$700 to buy new carpet for your bedroom. Write and solve an inequality that represents the costs per square foot that you can pay for the new carpet. Specify the units of measure in each step.



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–19 odd, 27, 30–31, 33, 40–47

Average: 1, 2–30 even, 31, 35, 40–47

Advanced: 1, 2, 6–28 even, 29–31, 32–38 even, 40–47

HOMEWORK CHECK

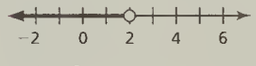
Basic: 3, 7, 11, 15, 19

Average: 4, 8, 12, 16, 20

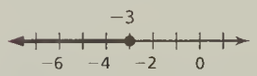
Advanced: 6, 8, 16, 18, 29

ANSWERS

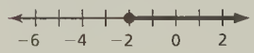
1. When solving $2x < -8$, the inequality symbol is not reversed when dividing each side by 2. When solving $-2x < 8$, the inequality is reversed when dividing each side by -2.
2. *Sample answer:* $-5x \geq 25$
3. $x < 2$



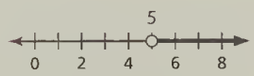
4. $y \leq -3$



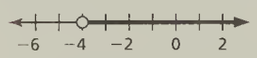
5. $n \geq -2$



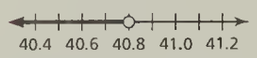
6. $t > 5$



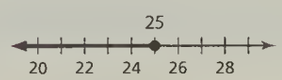
7. $x > -4$



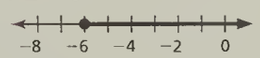
8. $a < 40.8$



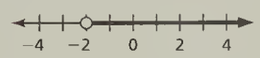
9. $w \leq 25$



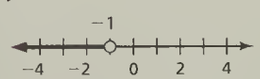
10. $t \geq -6$



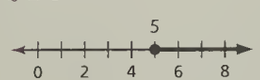
11. $t > -2$



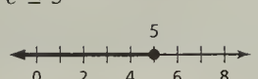
12. $y < -1$



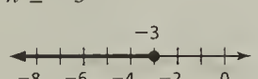
13. $z \geq 5$



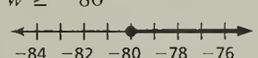
14. $c \leq 5$



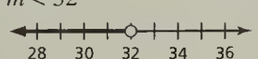
15. $n \leq -3$



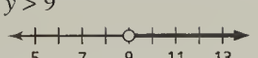
16. $w \geq -80$



17. $m < 32$



18. $y > 9$



19. $5p \leq 12, p \leq 2.4$
20. $8t \geq 36, t \geq 4.5$
21. $y > 12$
22. $v \geq 3$
23. $x \leq -9$
24. $n > -16$
25. $x > \frac{3}{8}$
26. $y < 4$
27–29. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

30. **a.** C; Multiplying both sides by m gives $x < -m$.

b. A; Multiplying both sides by m gives $x > m$.

c. B; Multiplying both sides by m gives $x < m$.

d. D; Multiplying both sides by $-m$ and reversing the inequality symbol gives $x > -m$.

31. **a.** $d \leq 6.3(2)$, $d \leq 12.6$

b. yes; The distance traveled in 4 hours would be no more than 25.2 miles, which is less than the distance required for a marathon.

32. *Sample answer:* $x < 21$

33. more than 300 million pennies

34. no; To get the second inequality, each side of the first is multiplied by -3. To be equivalent, the inequality symbol also needs to be reversed.

35. **a.** $A > B$ or $B < A$

b. $-A < -B$ or $-B > -A$

c. As numbers move farther away from zero, their absolute value becomes larger. $A > B$ and $|A| > |B|$. $-A < -B$ and $|A| > |B|$.

36. Because x may represent a positive or negative number, the inequality may need to be reversed when multiplying by x.

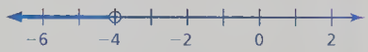
37. $\frac{C}{2\pi} > 5$, $C > 10\pi$

38–47. See Additional Answers.

Mini-Assessment

Solve the inequality. Graph the solution.

1. $6x < -24$ $x < -4$



2. $\frac{s}{-4} \leq -2$ $s \geq 8$



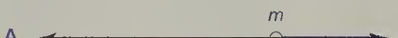
3. $18 < -\frac{2}{3}y$ $y < -27$

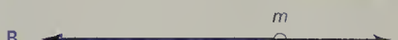


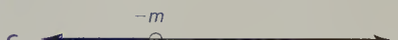
30. **HOW DO YOU SEE IT?** Let $m > 0$. Match each inequality with its graph. Explain your reasoning.

a. $\frac{x}{m} < -1$ **b.** $\frac{x}{m} > 1$

c. $\frac{x}{m} < 1$ **d.** $-\frac{x}{m} < 1$

A.


B.


C.


D.


31. **MAKING AN ARGUMENT** You run for 2 hours at a speed no faster than 6.3 miles per hour.

a. Write and solve an inequality that represents the possible numbers of miles you run.

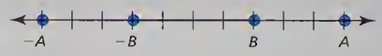
b. A marathon is approximately 26.2 miles. Your friend says that if you continue to run at this speed, you will not be able to complete a marathon in less than 4 hours. Is your friend correct? Explain.

32. **THOUGHT PROVOKING** The inequality $\frac{x}{4} \leq 5$ has a solution of $x = p$. Write a second inequality that also has a solution of $x = p$.

33. **PROBLEM SOLVING** The U.S. Mint pays \$0.02 to produce every penny. How many pennies are produced when the U.S. Mint pays more than \$6 million in production costs?

34. **REASONING** Are $x \leq \frac{2}{3}$ and $-3x \leq -2$ equivalent? Explain your reasoning.

35. **ANALYZING RELATIONSHIPS** Consider the number line shown.



a. Write an inequality relating A and B.

b. Write an inequality relating $-A$ and $-B$.

c. Use the results from parts (a) and (b) to explain why the direction of the inequality symbol must be reversed when multiplying or dividing each side of an inequality by the same negative number.

36. **REASONING** Why might solving the inequality $\frac{4}{x} \geq 2$ by multiplying each side by x lead to an error? (*Hint:* Consider $x > 0$ and $x < 0$.)

37. **MATHEMATICAL CONNECTIONS** The radius of a circle is represented by the formula $r = \frac{C}{2\pi}$. Write and solve an inequality that represents the possible circumferences C of the circle.



38. **CRITICAL THINKING** A water-skiing instructor recommends that a boat pulling a beginning skier has a speed less than 18 miles per hour. Write and solve an inequality that represents the possible distances d (in miles) that a beginner can travel in 45 minutes of practice time.

39. **CRITICAL THINKING** A local zoo employs 36 people to take care of the animals each day. At most, 24 of the employees work full time. Write and solve an inequality that represents the fraction of employees who work part time. Graph the solution.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.2 and Section 1.3)*

40. $5x + 3 = 13$

41. $\frac{1}{2}y - 8 = -10$

42. $-3n + 2 = 2n - 3$

43. $\frac{1}{2}z + 4 = \frac{5}{2}z - 8$

Tell which number is greater. *(Skills Review Handbook)*

44. 0.8, 85%

45. $\frac{16}{30}$, 50%

46. 120%, 0.12

47. 60%, $\frac{2}{3}$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

2.1–2.3 What Did You Learn?

Core Vocabulary

inequality, *p. 46*
solution of an inequality, *p. 47*
solution set, *p. 47*
graph of an inequality, *p. 48*
equivalent inequalities, *p. 54*

Core Concepts

Section 2.1

Representing Linear Inequalities, *p. 49*

Section 2.2

Addition Property of Inequality, *p. 54*
Subtraction Property of Inequality, *p. 55*

Section 2.3

Multiplication and Division Properties of Inequality ($c > 0$), *p. 60*
Multiplication and Division Properties of Inequality ($c < 0$), *p. 61*

Mathematical Thinking

1. Explain the meaning of the inequality symbol in your answer to Exercise 47 on page 51. How did you know which symbol to use?
2. In Exercise 30 on page 58, why is it important to check the reasonableness of your answer in part (a) before answering part (b)?
3. Explain how considering the units involved in Exercise 29 on page 63 helped you answer the question.

Study Skills

Analyzing Your Errors

Application Errors

What Happens: You can do numerical problems, but you struggle with problems that have context.

How to Avoid This Error: Do not just mimic the steps of solving an application problem. Explain out loud what the question is asking and why you are doing each step. After solving the problem, ask yourself, "Does my solution make sense?"



65

Dynamic Teaching Tools

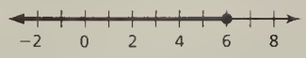
- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
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ANSWERS

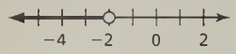
1. The inequality symbol means that all the other bridges are shorter; Because the Xianren Bridge is the longest, all other bridges must have a shorter length.
2. The value from part (a) is used to answer part (b), so checking its reasonableness is important to be sure that the correct result is used.
3. Showing the units in each step helps to compare the units in the final answer with the expected units.

ANSWERS

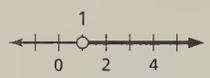
1. $z - 6 \geq 11$
2. $12 \leq -1.5w + 4$
3. $x < 0$
4. $x \geq 8$
5. $q \leq 6$



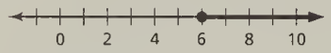
6. $z < -2$



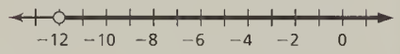
7. $y > 1$



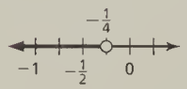
8. $p \geq 6$



9. $w > -12$



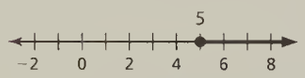
10. $x < -\frac{1}{4}$



11. a. $s \geq 100$



$t \geq 5$



$u \geq 10$



b. no; One of the requirements is to swim 100 yards, which is equal to 300 feet. If you only swim 250 feet, you do not meet this requirement.

12. $V + 100 \leq 700$; $V \leq 600$
13. $a = 7$
14. $5b < 35$; $b < 7$ ft
15. $57 \geq 0.3x$; $x \leq 190$

2.1–2.3 Quiz

Write the sentence as an inequality. *(Section 2.1)*

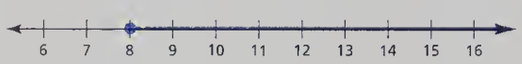
1. A number z minus 6 is greater than or equal to 11.
2. Twelve is no more than the sum of -1.5 times a number w and 4.

Write an inequality that represents the graph. *(Section 2.1)*

3.



4.



Solve the inequality. Graph the solution. *(Section 2.2 and Section 2.3)*

5. $9 + q \leq 15$
6. $z - (-7) < 5$
7. $-3 < y - 4$
8. $3p \geq 18$
9. $6 > \frac{w}{-2}$
10. $-20x > 5$

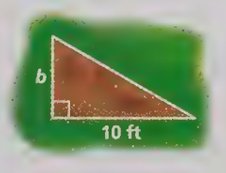
11. Three requirements for a lifeguard training course are shown. *(Section 2.1)*



 a. Write and graph three inequalities that represent the requirements.
 b. You can swim 250 feet, tread water for 6 minutes, and swim 35 feet underwater without taking a breath. Do you satisfy the requirements of the course? Explain.

12. The maximum volume of an American white pelican's bill is about 700 cubic inches. A pelican scoops up 100 cubic inches of water. Write and solve an inequality that represents the additional volumes the pelican's bill can contain. *(Section 2.2)*

13. The solution of $x - a > 4$ is $x > 11$. What is the value of a? *(Section 2.2)*



14. The area of the triangular garden must be less than 35 square feet. Write and solve an inequality that represents the value of b. *(Section 2.3)*

15. A candidate for class president receives 57 votes, which is at least 30% of the total number of votes. Write and solve an inequality that represents the numbers of students who cast votes. *(Section 2.3)*

SUPPORTING English Language Learners

Point out the ad for lifeguards in Exercise 11. Explain that the meaning of *lifeguard* can be guessed by knowing the meanings of the words *life* and *guard*. The picture of a lifesaver also helps convey the meaning of the ad.

Beginning State two actions a lifeguard must perform. (swim, tread water)
Intermediate State one training requirement.
Advanced Summarize the training requirements.
Advanced High Summarize the ad.

ELPS 4.C.2 Derive meaning of environmental print.

2.4 Solving Multi-Step Inequalities

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.5.B

Essential Question
How can you solve a multi-step inequality?

EXPLORATION 1 Solving a Multi-Step Inequality

Work with a partner.

- Use what you already know about solving equations and inequalities to solve each multi-step inequality. Justify each step.
- Match each inequality with its graph. Use a graphing calculator to check your answer.

JUSTIFYING STEPS
To be proficient in math, you need to justify each step in a solution and communicate your justification to others.

a. $2x + 3 \leq x + 5$

b. $-2x + 3 > x + 9$

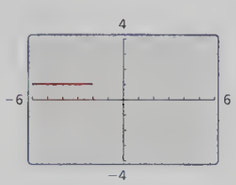
c. $27 \geq 5x + 4x$

d. $-8x + 2x - 16 < -5x + 7x$

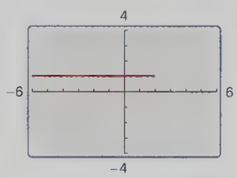
e. $3(x - 3) - 5x > -3x - 6$

f. $-5x - 6x \leq 8 - 8x - x$

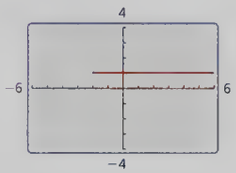
A.



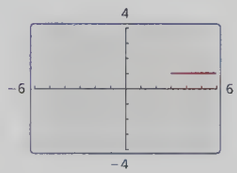
B.



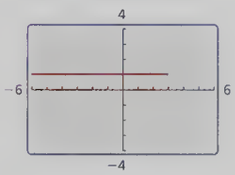
C.



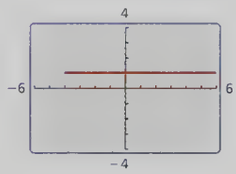
D.



E.



F.



Communicate Your Answer

2. How can you solve a multi-step inequality?
3. Write two different multi-step inequalities whose solutions are represented by the graph.



Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.5.B The student is expected to solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

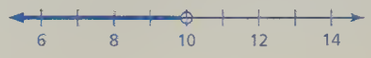
1. **a.** $x \leq 2$; Subtract x and 3 from each side; B
 b. $x < -2$; Subtract x and 3 from each side; Divide each side by -3; A
 c. $3 \geq x$; Divide each side by 9; E
 d. $-2 < x$; Add $6x$ to each side; Divide each side by 8; C
 e. $x > 3$; Add $3x$ and 9 to each side; D
 f. $x \geq -4$; Add $9x$ to each side; Divide each side by -2; F
2. Simplify each side, if possible, then use inverse operations to isolate the variable. Reverse the inequality symbol if multiplying or dividing by a negative number.
3. *Sample answer:* $3x + 4 < 1$, $-2x - 10 > -8$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Extra Example 1

Solve each inequality. Graph each solution.

a. $\frac{x}{-2} + 4 > -1$ $x < 10$



b. $3c - 2 \leq -11$ $c \leq -3$



MONITORING PROGRESS ANSWERS

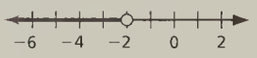
1. $b < 2$



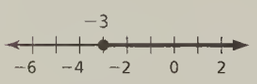
2. $c \leq 4$



3. $n < -2$



4. $v \geq -3$



2.4 Lesson

What You Will Learn

- Solve multi-step inequalities.
- Use multi-step inequalities to solve real-life problems.

Solving Multi-Step Inequalities

To solve a multi-step inequality, simplify each side of the inequality, if necessary. Then use inverse operations to isolate the variable. Be sure to reverse the inequality symbol when multiplying or dividing by a negative number.

EXAMPLE 1 Solving Multi-Step Inequalities

Solve each inequality. Graph each solution.

a. $\frac{y}{-6} + 7 < 9$ b. $2v - 4 \geq 8$

SOLUTION

a. $\frac{y}{-6} + 7 < 9$ Write the inequality.

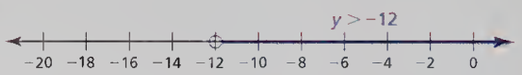
$-7 \quad -7$ Subtract 7 from each side.

$\frac{y}{-6} < 2$ Simplify.

$-6 \cdot \frac{y}{-6} > -6 \cdot 2$ Multiply each side by −6. Reverse the inequality symbol.

$y > -12$ Simplify.

▶ The solution is $y > -12$.



b. $2v - 4 \geq 8$ Write the inequality.

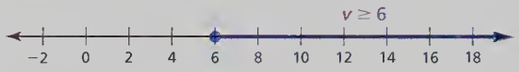
$+4 \quad +4$ Add 4 to each side.

$2v \geq 12$ Simplify.

$\frac{2v}{2} \geq \frac{12}{2}$ Divide each side by 2.

$v \geq 6$ Simplify.

▶ The solution is $v \geq 6$.



Monitoring Progress

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Solve the inequality. Graph the solution.

1. $4b - 1 < 7$
2. $8 - 9c \geq -28$
3. $\frac{n}{-2} + 11 > 12$
4. $6 \geq 5 - \frac{v}{3}$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Build on Past Knowledge

Explain to students that when they solve multi-step inequalities, they will use a process similar to the one they used to solve multi-step equations. They will be using inverse operations and performing one step at a time in order to isolate the variable.

EXAMPLE 2 Solving an Inequality with Variables on Both Sides

Solve $6x - 5 < 2x + 11$.

SOLUTION

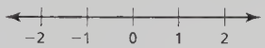
$6x - 5 < 2x + 11$	Write the inequality.
$+5 \quad +5$	Add 5 to each side.
$6x < 2x + 16$	Simplify.
$-2x \quad -2x$	Subtract $2x$ from each side.
$4x < 16$	Simplify.
$\frac{4x}{4} < \frac{16}{4}$	Divide each by 4.
$x < 4$	Simplify.

▶ The solution is $x < 4$.

When solving an inequality, if you obtain an equivalent inequality that is true, such as $-5 < 0$, the solutions of the inequality are *all real numbers*. If you obtain an equivalent inequality that is false, such as $3 \leq -2$, the inequality has *no solution*.



Graph of an inequality whose solutions are all real numbers



Graph of an inequality that has no solution

EXAMPLE 3 Inequalities with Special Solutions

Solve (a) $8b - 3 > 4(2b + 3)$ and (b) $2(5w - 1) \leq 7 + 10w$.

SOLUTION

a.

$8b - 3 > 4(2b + 3)$	Write the inequality.
$8b - 3 > 8b + 12$	Distributive Property
$-8b \quad -8b$	Subtract $8b$ from each side.
$-3 > 12$ ✗	Simplify.

▶ The inequality $-3 > 12$ is false. So, there is no solution.

b.

$2(5w - 1) \leq 7 + 10w$	Write the inequality.
$10w - 2 \leq 7 + 10w$	Distributive Property
$-10w \quad -10w$	Subtract $10w$ from each side.
$-2 \leq 7$	Simplify.

▶ The inequality $-2 \leq 7$ is true. So, all real numbers are solutions.

ANALYZING MATHEMATICAL RELATIONSHIPS

When the variable terms on each side of an inequality are the same, the constant terms will determine whether the inequality is true or false.

Monitoring Progress

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Solve the inequality.

5. $5x - 12 \leq 3x - 4$
6. $2(k - 5) < 2k + 5$
7. $-4(3n - 1) > -12n + 5.2$
8. $3(2a - 1) \geq 10a - 11$

Extra Example 2

Solve $5x - 6 \geq 3x + 4$. $x \geq 5$

Extra Example 3

a. Solve $6t - 4 > 2(3t + 5)$. The inequality $-4 > 10$ is false. So, there is no solution.

b. Solve $2(2x - 3) \leq 9 + 4x$. The inequality $-6 \leq 9$ is true. So, all real numbers are solutions.

MONITORING PROGRESS ANSWERS

5. $x \leq 4$
6. all real numbers
7. no solution
8. $a \leq 2$

Extra Example 4

Your grades on 4 math tests are 79, 85, 86, and 88. You want your mean grade to be at least 85. What grade on your fifth test will give you a mean score of at least 85?

a grade of at least 87

MONITORING PROGRESS ANSWERS

9. a score of at least 73

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

You need a mean score of at least 90 points to advance to the next round of the touch-screen trivia game. What scores in the fifth game will allow you to advance?



SOLUTION

1. Understand the Problem You know the scores of your first four games. You are asked to find the scores in the fifth game that will allow you to advance.

2. Make a Plan Use the definition of the mean of a set of numbers to write an inequality. Then solve the inequality and answer the question.

3. Solve the Problem Let x be your score in the fifth game.

$$\frac{95 + 91 + 77 + 89 + x}{5} \geq 90$$ Write an inequality.

$$\frac{352 + x}{5} \geq 90$$ Simplify.

$$5 \cdot \frac{352 + x}{5} \geq 5 \cdot 90$$ Multiply each side by 5.

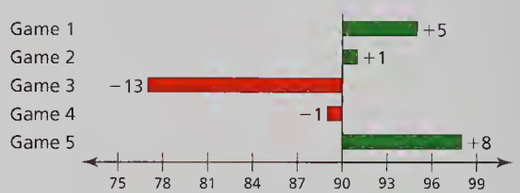
$$352 + x \geq 450$$ Simplify.

$$-352 \quad -352$$ Subtract 352 from each side.

$$x \geq 98$$ Simplify.

▶ A score of at least 98 points will allow you to advanee.

4. Look Back You can draw a diagram to check that your answer is reasonable. The horizontal bar graph shows the differences between the game scores and the desired mean of 90.



To have a mean of 90, the sum of the differences must be zero.

$5 + 1 - 13 - 1 + 8 = 0$ ✓

REMEMBER

The mean in Example 4 is equal to the sum of the game scores divided by the number of games.

Monitoring Progress

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9. WHAT IF? You need a mean score of at least 85 points to advance to the next round. What scores in the fifth game will allow you to advance?

Closure

- **Writing:** How are these inequalities alike? How are they different?

$$3n - 4 \geq -25$$
$$3n - 4 > -25$$
$$3n - 4 \leq -25$$

Sample answer: They are alike because they each use the expressions $(3n - 4)$ and (-25). They are different because of the way the expressions are related: greater than or equal to $(\geq)$, greater than $(>)$, and less than or equal to $(\leq)$.

SUPPORTING English Language Learners

Point out the graph in Step 4. Explain that graphs help confirm answers visually.

Beginning/Advanced Work as partners to explain Step 4. Beginning students recreate the graph and Advanced students explain how it illustrates the answer.

Intermediate/Advanced High Work as partners to explain Step 4. Intermediate students explain what each bar represents. Advanced High students explain how the graph illustrates the answer.

ELPS 4.F.2 Use visual and contextual support to enhance and confirm understanding.

2.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Compare solving multi-step inequalities and solving multi-step equations.
2. **WRITING** Without solving, how can you tell that the inequality $4x + 8 \le 4x - 3$ has no solution?

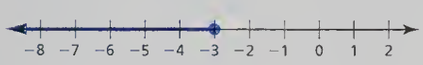
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the inequality with its graph.

3. $7b - 4 \le 10$
4. $4p + 4 \ge 12$
5. $-6g + 2 \ge 20$
6. $3(2 - f) \le 15$

A.


B.


C.


D.


In Exercises 7–16, solve the inequality. Graph the solution. *(See Example 1.)*

7. $2x - 3 > 7$
8. $5y + 9 \le 4$
9. $-9 \le 7 - 8v$
10. $2 > -3t - 10$
11. $\frac{w}{2} + 4 > 5$
12. $1 + \frac{m}{3} \le 6$
13. $\frac{p}{-8} + 9 > 13$
14. $3 + \frac{r}{-4} \le 6$
15. $6 \ge -6(a + 2)$
16. $18 \le 3(b - 4)$

In Exercises 17–28, solve the inequality. *(See Examples 2 and 3.)*

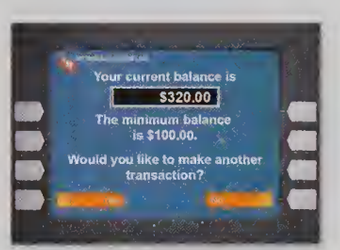
17. $4 - 2m > 7 - 3m$
18. $8n + 2 \le 8n - 9$
19. $-2d - 2 < 3d + 8$
20. $8 + 10f > 14 - 2f$
21. $8g - 5g - 4 \le -3 + 3g$
22. $3w - 5 > 2w + w - 7$
23. $6(\ell + 3) < 3(2\ell + 6)$
24. $2(5c - 7) \ge 10(c - 3)$
25. $4\left(\frac{1}{2}t - 2\right) > 2(t - 3)$
26. $15\left(\frac{1}{3}b + 3\right) \le 6(b + 9)$
27. $9j - 6 + 6j \ge 3(5j - 2)$
28. $6h - 6 + 2h < 2(4h - 3)$

ERROR ANALYSIS **In Exercises 29 and 30, describe and correct the error in solving the inequality.**

29. ✗
$$\frac{x}{4} + 6 \ge 3$$
$$x + 6 \ge 12$$
$$x \ge 6$$

30. ✗
$$-2(1 - x) \le 2x - 7$$
$$-2 + 2x \le 2x - 7$$
$$-2 \le -7$$
All real numbers are solutions.

31. **MODELING WITH MATHEMATICS** Write and solve an inequality that represents how many \$20 bills you can withdraw from the account without going below the minimum balance. *(See Example 4.)*



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–23 odd, 29, 31, 34, 36, 41–43

Average: 1, 2–30 even, 31–36, 41–43

Advanced: 1, 2, 13–16, 22–30 even, 31–43

HOMEWORK CHECK

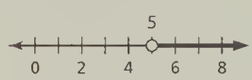
Basic: 7, 13, 17, 19, 31

Average: 8, 14, 18, 20, 32

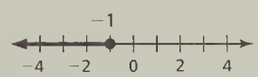
Advanced: 14, 16, 22, 28, 32

ANSWERS

1. *Sample answer:* The same steps can be applied when solving multi-step inequalities and multi-step equations, except that when each side of an inequality is divided by a negative number, the inequality must be reversed.
2. *Sample answer:* Because the terms with the variable are the same, they will cancel, and $8 \le -3$ is not true.
3. B
4. A
5. C
6. D
7. $x > 5$



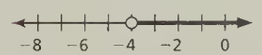
8. $y \le -1$



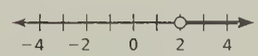
9. $v \le 2$



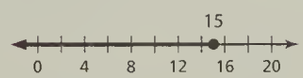
10. $t > -4$



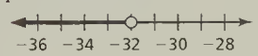
11. $w > 2$



12. $m \le 15$



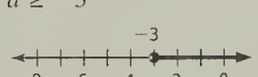
13. $p < -32$



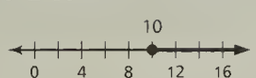
14. $r \ge -12$



15. $a \ge -3$



16. $b \ge 10$



17. $m > 3$
18. no solution
19. $d > -2$
20. $f > 0.5$
21. all real numbers
22. all real numbers
23. no solution
24. all real numbers
25. no solution
26. $b \ge -9$
27. all real numbers
28. no solution
29. In the first step, you need to use the Distributive Property on the left side; $x + 24 \ge 12$; $x \ge -12$
30. Because the final statement is false, there is no solution.
31. $20n + 100 \le 320$, $n \le 11$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

32. $25h + 125 \leq 500, h \leq 15$

33. $12(2x - 3) > 60, x > 4$

34. no; For Woodland to be cheaper, its total cost needs to be less than Forest Park's total cost. Solving the inequality $20 + 55n \leq 100 + 35n$ indicates that Woodland is cheaper for no more than 4 nights, not more than 4 nights.

35. 7 stories; Using the Pythagorean Theorem, the 74-foot ladder can reach at most 70 feet. Solving the inequality $10n - 8 \leq 70$ gives $n \leq 7.8$, so the ladder cannot quite reach the 8th story.

36. **a.** \$40

b. \$3.55; \$8

c. $3.55x + 8 \leq 40$

d. about 9 gal

37. $r \geq 3$

38. *Sample answer:*

$$\frac{25.5 + 24.3 + 24.8 + 23.5 + 3x}{7} < 24;$$

$x < 23.3$; run three more races in less than 23.3 minutes each

39–43. See Additional Answers.

Mini-Assessment

1. Solve the inequality $-2x + 6 > 16$. Graph the solution. $x < -5$



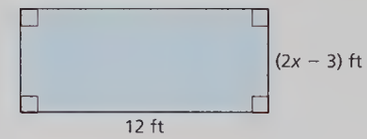
Solve the inequality.

2. $6n + 5 > 2n - 3$ $n > -2$
3. $10x - 3 < 2(5x + 1)$ The inequality $-3 < 2$ is true. So, all real numbers are solutions.
4. $3(4s - 3) \geq 5 + 12s$ The inequality $-9 \geq 5$ is false. So, there is no solution.
5. During the first 3 weeks of training, a runner runs 20 miles, 22 miles, and 21 miles. How many miles must she run during the fourth week to have a mean of at least 22 miles per week? at least 25 miles

32. MODELING WITH MATHEMATICS A woodworker wants to earn at least \$25 an hour making and selling cabinets. He pays \$125 for materials. Write and solve an inequality that represents how many hours the woodworker can spend building the cabinet.

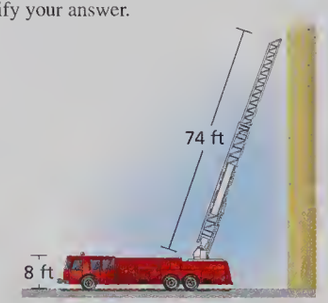


33. MATHEMATICAL CONNECTIONS The area of the rectangle is greater than 60 square feet. Write and solve an inequality to find the possible values of x.

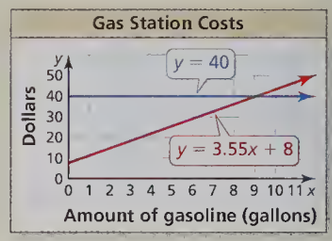


34. MAKING AN ARGUMENT Forest Park Campgrounds charges a \$100 membership fee plus \$35 per night. Woodland Campgrounds charges a \$20 membership fee plus \$55 per night. Your friend says that if you plan to camp for four or more nights, then you should choose Woodland Campgrounds. Is your friend correct? Explain.

35. PROBLEM SOLVING The height of one story of a building is about 10 feet. The bottom of the ladder on the fire truck must be at least 24 feet away from the building. How many stories can the ladder reach? Justify your answer.



36. HOW DO YOU SEE IT? The graph shows your budget, and the total cost of x gallons of gasoline and a car wash. You want to determine the possible amounts (in gallons) of gasoline you can buy within your budget.



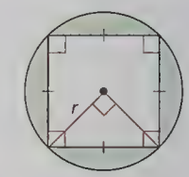
a. What is your budget?

b. How much does a gallon of gasoline cost? How much does a car wash cost?

c. Write an inequality that represents the possible amounts of gasoline you can buy.

d. Use the graph to estimate the solution of your inequality in part (c).

37. PROBLEM SOLVING For what values of r will the area of the shaded region be greater than or equal to $9(\pi - 2)$?



38. THOUGHT PROVOKING A runner's times (in minutes) in the four races he has completed are 25.5, 24.3, 24.8, and 23.5. The runner plans to run at least one more race and wants to have an average time less than 24 minutes. Write and solve an inequality to show how the runner can achieve his goal.

REASONING In Exercises 39 and 40, find the value of a for which the solution of the inequality is all real numbers.

39. $a(x + 3) < 5x + 15 - x$

40. $3x + 8 + 2ax \geq 3ax - 4a$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. *(Section 2.1)*

41. Six times a number y is less than or equal to 10.

42. A number p plus 7 is greater than 24.

43. The quotient of a number r and 7 is no more than 18.

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

2.5 Solving Compound Inequalities

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.5.B

Essential Question How can you use inequalities to describe intervals on the real number line?

EXPLORATION 1 **Describing Intervals on the Real Number Line**

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.

REASONING
To be proficient in math, you need to create a clear representation of the problem at hand.

a. Half-Open Interval



b. Half-Open Interval



c. Closed Interval



d. Open Interval

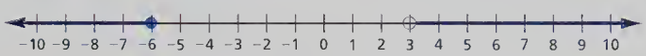


e. Do you use "and" or "or" to connect the two inequalities in parts (a)–(d)? Explain.

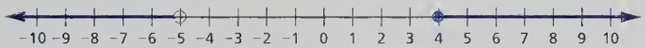
EXPLORATION 2 **Describing Two Infinite Intervals**

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.

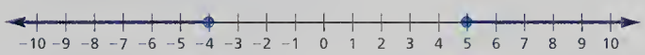
a.



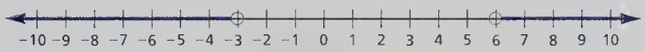
b.



c.



d.



e. Do you use "and" or "or" to connect the two inequalities in parts (a)–(d)? Explain.

Communicate Your Answer

3. How can you use inequalities to describe intervals on the real number line?

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.5.B The student is expected to solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

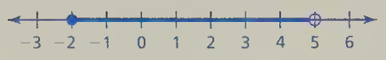
1. a. $x \geq -6$ and $x < 3$
 b. $x > -5$ and $x \leq 4$
 c. $x \geq -4$ and $x \leq 5$
 d. $x > -3$ and $x < 6$
 e. and; Both inequalities need to be true for values that are in the interval.
2. a. $x \leq -6$ or $x > 3$
 b. $x < -5$ or $x \geq 4$
 c. $x \leq -4$ or $x \geq 5$
 d. $x < -3$ or $x > 6$
 e. or; Either inequality needs to be true for values that are in the interval.
3. Write 2 inequalities joined by "and" or "or."

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

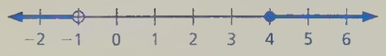
Extra Example 1

Write each sentence as an inequality. Graph each inequality.

a. A number c is less than 5 and greater than or equal to -2. $-2 \le c < 5$



b. A number t is less than -1 or greater than or equal to 4. $t < -1$ or $t \ge 4$

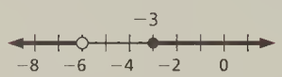


MONITORING PROGRESS ANSWERS

1. $0 < d < 10$



2. $a < -6$ or $a \ge -3$



2.5 Lesson

What You Will Learn

- Write and graph compound inequalities.
- Solve compound inequalities.
- Use compound inequalities to solve real-life problems.

Core Vocabulary

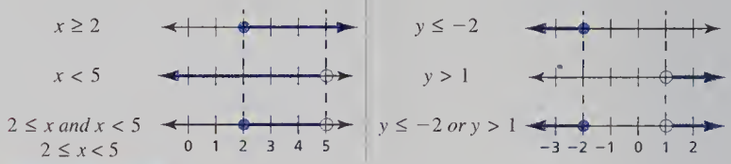
compound inequality, *p. 74*

Writing and Graphing Compound Inequalities

A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

The graph of a compound inequality with "and" is the *intersection* of the graphs of the inequalities. The graph shows numbers that are solutions of *both* inequalities.

The graph of a compound inequality with "or" is the *union* of the graphs of the inequalities. The graph shows numbers that are solutions of *either* inequality.



EXAMPLE 1 Writing and Graphing Compound Inequalities

Write each sentence as an inequality. Graph each inequality.

a. A number x is greater than -8 and less than or equal to 4.

b. A number y is at most 0 or at least 2.

REMEMBER

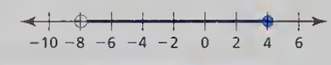
A compound inequality with "and" can be written as a single inequality. For example, you can write $x > -8$ and $x \le 4$ as $-8 < x \le 4$.

SOLUTION

a. A number x is greater than -8 and less than or equal to 4.

$x > -8$ *and* $x \le 4$

▶ An inequality is $-8 < x \le 4$.



Graph the intersection of the graphs of $x > -8$ and $x \le 4$.

b. A number y is at most 0 or at least 2.

$y \le 0$ *or* $y \ge 2$

▶ An inequality is $y \le 0$ *or* $y \ge 2$.



Graph the union of the graphs of $y \le 0$ and $y \ge 2$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write the sentence as an inequality. Graph the inequality.

1. A number d is more than 0 and less than 10.
2. A number a is fewer than -6 or no less than -3.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the topic Writing and Graphing Compound Inequalities and Example 1 that follows. Have students practice speaking using the Monitoring Progress questions.

Beginning Repeat Questions 1 and 2 after the teacher models them.
Intermediate Read aloud Questions 1 and 2.
Advanced/Advanced High Explain the answers to each question.

ELPS 3.C.4 Speak using a variety of connecting words with increasing accuracy and ease as more English is acquired.

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

Solving Compound Inequalities

You can solve a compound inequality by solving two inequalities separately. When a compound inequality with "and" is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

EXAMPLE 2 Solving Compound Inequalities with "And"

Solve each inequality. Graph each solution.

a. $-4 < x - 2 < 3$ b. $-3 < -2x + 1 \leq 9$

SOLUTION

a. Separate the compound inequality into two inequalities, then solve.

$-4 < x - 2$	*and*	$x - 2 < 3$		Write two inequalities.
$+2 \quad +2$		$+2 \quad +2$		Add 2 to each side.
$-2 < x$	*and*	$x < 5$		Simplify.

▶ The solution is $-2 < x < 5$.



b.

$-3 < -2x + 1 \leq 9$			Write the inequality.
$-1 \quad -1 \quad -1$			Subtract 1 from each expression.
$-4 < -2x$	$\leq$	8	Simplify.
$\frac{-4}{-2} > \frac{-2x}{-2}$	$\geq$	$\frac{8}{-2}$	Divide each expression by -2. Reverse each inequality symbol.
$2 > x$	$\geq$	-4	Simplify.

▶ The solution is $-4 \leq x < 2$.



EXAMPLE 3 Solving a Compound Inequality with "Or"

Solve $3y - 5 < -8$ *or* $2y - 1 > 5$. Graph the solution.

SOLUTION

$3y - 5 < -8$	*or*	$2y - 1 > 5$		Write the inequality.
$+5 \quad +5$		$+1 \quad +1$		Addition Property of Inequality
$3y < -3$		$2y > 6$		Simplify.
$\frac{3y}{3} < \frac{-3}{3}$		$\frac{2y}{2} > \frac{6}{2}$		Division Property of Inequality
$y < -1$	*or*	$y > 3$		Simplify.

▶ The solution is $y < -1$ *or* $y > 3$.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution.

3. $5 \leq m + 4 < 10$
4. $-3 < 2k - 5 < 7$
5. $4c + 3 \leq -5$ *or* $c - 8 > -1$
6. $2p + 1 < -7$ *or* $3 - 2p \leq -1$

Extra Example 2

Solve each inequality. Graph each solution.

a. $-3 < x + 1 < 5$ $-4 < x < 4$

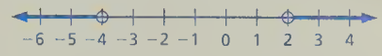


b. $-7 < -3w - 4 \leq 2$ $-2 \leq w < 1$



Extra Example 3

Solve $2x + 6 < -2$ *or* $4x - 5 > 3$. Graph the solution. $x < -4$ or $x > 2$

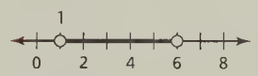


MONITORING PROGRESS ANSWERS

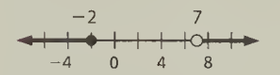
3. $1 \leq m < 6$



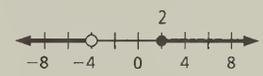
4. $1 < k < 6$



5. $c \leq -2$ or $c > 7$



6. $p < -4$ or $p \geq 2$



Differentiated Instruction

Kinesthetic/Visual

Have pairs of students work together to graph compound inequalities. Have one student use a red pencil to graph the first part of the inequality. The other student uses a different color to graph the second part on the same number line. For a compound inequality with "and," the graph is the interval where both colors overlap.

Extra Example 4

A recommended storage temperature for chocolate is from 15°C to 20°C.

a. Write and solve a compound inequality that represents the possible storage temperatures (in degrees Fahrenheit) for chocolate. $15 \le \frac{5}{9}(F - 32) \le 20$; $59 \le F \le 68$; The storage temperature is 59°F to 68°F.

b. Describe one situation in which the surrounding temperature could be below the storage temperature range and one in which it could be above. *Sample answer:* below: outdoors on a cold day; above: a car parked in the sun on a hot day

MONITORING PROGRESS ANSWERS

7. $-40 \le \frac{5}{9}(F - 32) \le 15$, $-40 \le F \le 59$

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**



Operating temperature: 0°C to 35°C

Electrical devices should operate effectively within a specified temperature range. Outside the operating temperature range, the device may fail.

a. Write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone.

b. Describe one situation in which the surrounding temperature could be below the operating range and one in which it could be above.

SOLUTION

1. **Understand the Problem** You know the operating temperature range in degrees Celsius. You are asked to write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone. Then you are asked to describe situations outside this range.

2. **Make a Plan** Write a compound inequality in degrees Celsius. Use the formula $C = \frac{5}{9}(F - 32)$ to rewrite the inequality in degrees Fahrenheit. Then solve the inequality and describe the situations.

3. **Solve the Problem** Let C be the temperature in degrees Celsius, and let F be the temperature in degrees Fahrenheit.

STUDY TIP

You can also solve the inequality by multiplying each expression by $\frac{9}{5}$.

$$
\begin{aligned}
0 \le\ & C & \le 35 && \text{Write the inequality using } C.\\
0 \le\ & \tfrac{5}{9}(F - 32) & \le 35 && \text{Substitute } \tfrac{5}{9}(F - 32) \text{ for } C.\\
9 \cdot 0 \le\ & 9 \cdot \tfrac{5}{9}(F - 32) & \le 9 \cdot 35 && \text{Multiply each expression by 9.}\\
0 \le\ & 5(F - 32) & \le 315 && \text{Simplify.}\\
0 \le\ & 5F - 160 & \le 315 && \text{Distributive Property}\\
+ 160 & + 160 & + 160 && \text{Add 160 to each expression.}\\
160 \le\ & 5F & \le 475 && \text{Simplify.}\\
\frac{160}{5} \le & \frac{5F}{5} & \le \frac{475}{5} && \text{Divide each expression by 5.}\\
32 \le\ & F & \le 95 && \text{Simplify.}
\end{aligned}
$$

▶ The solution is $32 \le F \le 95$. So, the operating temperature range of the smartphone is 32°F to 95°F. One situation when the surrounding temperature could be below this range is winter in Alaska. One situation when the surrounding temperature could be above this range is daytime in the Mojave Desert of the American Southwest.

4. **Look Back** You can use the formula $C = \frac{5}{9}(F - 32)$ to check that your answer is correct. Substitute 32 and 95 for F in the formula to verify that 0°C and 35°C are the minimum and maximum operating temperatures in degrees Celsius.



−40°C to 15°C

Monitoring Progress

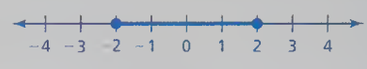
Help in English and Spanish at *BigIdeasMath.com*

7. Write and solve a compound inequality that represents the temperature rating (in degrees Fahrenheit) of the winter boots.

Closure

- **Exit Ticket:** Solve and graph.

a. $-1 \le 2x + 3 \le 7$ $\quad -2 \le x \le 2$



b. $4x + 1 \le -11$ or $3x - 4 \ge 5$ $\quad x \le -3$ or $x \ge 3$



2.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Compare the graph of $-6 \le x \le -4$ with the graph of $x \le -6$ *or* $x \ge -4$.

2. **WHICH ONE DOESN'T BELONG?** Which compound inequality does *not* belong with the other three? Explain your reasoning.

$a > 4$ *or* $a < -3$ $\quad$ $a < -2$ *or* $a > 8$ $\quad$ $a > 7$ *or* $a < -5$ $\quad$ $a < 6$ *or* $a > -9$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, write a compound inequality that is represented by the graph.

3. 

4. 

5. 

6. 

In Exercises 7–10, write the sentence as an inequality. Graph the inequality. *(See Example 1.)*

7. A number p is less than 6 and greater than 2.

8. A number n is less than or equal to -7 or greater than 12.

9. A number m is more than $-7\frac{2}{3}$ or at most -10.

10. A number r is no less than -1.5 and fewer than 9.5.

11. **MODELING WITH MATHEMATICS** Slitsnails are large mollusks that live in deep waters. They have been found in the range of elevations shown. Write and graph a compound inequality that represents this range.



12. **MODELING WITH MATHEMATICS** The life zones on Mount Rainier, a mountain in Washington, can be approximately classified by elevation, as follows.

Low-elevation forest: above 1700 feet to 2500 feet
Mid-elevation forest: above 2500 feet to 4000 feet
Subalpine: above 4000 fcet to 6500 feet
Alpine: above 6500 feet to the summit



Elevation of Mount Rainier: 14,410 ft

Write a compound inequality that represents the elevation range for each type of plant life.

a. trees in the low-elevation forest zone

b. flowers in the subalpine and alpine zones

In Exercises 13–20, solve the inequality. Graph the solution. *(See Examples 2 and 3.)*

13. $6 < x + 5 \le 11$

14. $24 > -3r \ge -9$

15. $v + 8 < 3$ *or* $-8v < -40$

16. $-14 > w + 3$ *or* $3w \ge -27$

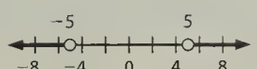
17. $2r + 3 < 7$ *or* $-r + 9 \le 2$

18. $-6 < 3n + 9 < 21$

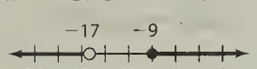
19. $-12 < \frac{1}{2}(4x + 16) < 18$

20. $35 < 7(2 - b)$ *or* $\frac{1}{3}(15b - 12) \ge 21$

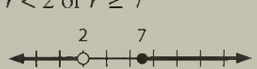
15. $v < -5$ or $v > 5$



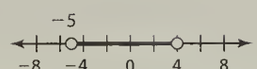
16. $w < -17$ or $w \ge -9$



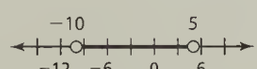
17. $r < 2$ or $r \ge 7$



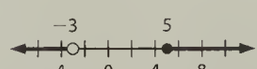
18. $-5 < n < 4$



19. $-10 < x < 5$



20. $b < -3$ or $b \ge 5$



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–2, 3–11 odd, 12, 13–17 odd, 21, 25, 33–40

Average: 1, 2–18 even, 22, 23, 24–28 even, 33–40

Advanced: 1–2, 6, 9–12, 18–20, 22–24, 28–40

HOMEWORK CHECK

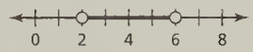
Basic: 5, 7, 13, 15, 21

Average: 8, 10, 14, 16, 23

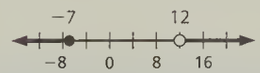
Advanced: 9, 10, 19, 20, 23

ANSWERS

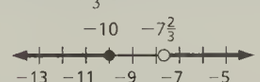
1. The graph of $-6 \le x \le -4$ shows a single segment between -6 and -4. The graph of $x \le -6$ or $x \ge -4$ shows two opposite rays with endpoints at -6 and -4.
2. $a < 6$ or $a > -9$; The graph of this inequality is the only one with overlapping rays that represent all real numbers.
3. $-3 < x \le 2$
4. $7 < x < 14$
5. $x \le -7$ or $x \ge -4$
6. $x \le 4$ or $x > 6$
7. $2 < p < 6$



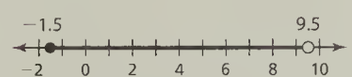
8. $n \le -7$ or $n > 12$



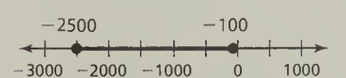
9. $m > -7\frac{2}{3}$ or $m \le -10$



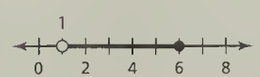
10. $-1.5 \le r < 9.5$



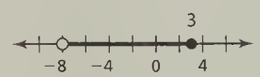
11. $-2500 \le e \le -100$



12. a. $1700 < h \le 2500$
 b. $4000 < h \le 14{,}410$
13. $1 < x \le 6$



14. $-8 < r \le 3$

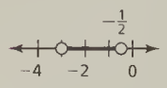


Dynamic Teaching Tools

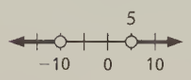
- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

21. In the second step, 3 should have been subtracted from 4 on the left side;
$1 < -2x < 6$; $-\frac{1}{2} > x > -3$



22. The graph should have 2 opposite rays, not 1 segment.

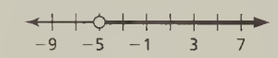


23. $-20 \le \frac{5}{9}(F - 32) \le -15$, $-4 \le F \le 5$

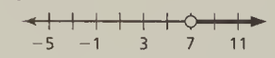
24. $1.16h < 150$ or $1.16h > 220$; height less than about 129.3 cm or greater than about 189.7 cm

25. no solution

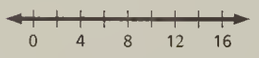
26. $m > -5$



27. $y > 7$



28. all real numbers



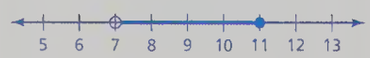
29–40. See Additional Answers.

Mini-Assessment

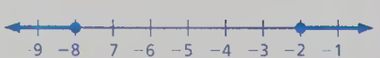
1. Write the sentence as an inequality. Graph the inequality.
A number p is greater than -3 and less than 4.
$-3 < p < 4$



2. Solve $22 < 4x - 6 \le 38$. Graph the solution. $7 < x \le 11$



3. Solve $2c + 5 \le -11$ or $3c - 1 \ge -7$. Graph the solution.
$c \le -8$ or $c \ge -2$



ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in solving the inequality or graphing the solution.

21.

$4 < -2x + 3 < 9$
$4 < -2x < 6$
$-2 > x > -3$



22.

$x - 2 > 3$ or $x + 8 < -2$
$x > 5$ or $x < -10$



23. MODELING WITH MATHEMATICS Write and solve a compound inequality that represents the possible temperatures (in degrees Fahrenheit) of the interior of the iceberg. *(See Example 4.)*



24. PROBLEM SOLVING A ski shop sells skis with lengths ranging from 150 centimeters to 220 centimeters. The shop says the length of the skis should be about 1.16 times a skier's height (in centimeters). Write and solve a compound inequality that represents the heights of skiers the shop does *not* provide skis for.

In Exercises 25–30, solve the inequality. Graph the solution, if possible.

25. $22 < -3c + 4 < 14$

26. $2m - 1 \ge 5$ *or* $5m > -25$

27. $-y + 3 \le 8$ *and* $y + 2 > 9$

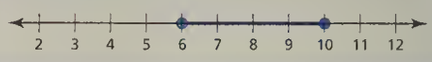
28. $x - 8 \le 4$ *or* $2x + 3 > 9$

29. $2n + 19 \le 10 + n$ *or* $-3n + 3 < -2n + 33$

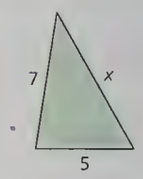
30. $3x - 18 < 4x - 23$ *and* $x - 16 < -22$

31. REASONING Fill in the compound inequality $4(x - 6)$ ☐ $2(x - 10)$ *and* $5(x + 2) \ge 2(x + 8)$ with $<$, $\le$, $>$, or $\ge$ so that the solution is only one value.

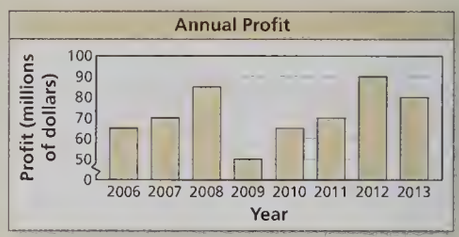
32. THOUGHT PROVOKING Write a real-life story that can be modeled by the graph.



33. MAKING AN ARGUMENT The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Use the triangle shown to write and solve three inequalities. Your friend claims the value of x can be 1. Is your friend correct? Explain.



34. HOW DO YOU SEE IT? The graph shows the annual profits of a company from 2006 to 2013.



a. Write a compound inequality that represents the annual profits from 2006 to 2013.

b. You can use the formula $P = R - C$ to find the profit P, where R is the revenue and C is the cost. From 2006 to 2013, the company's annual cost was about \$125 million. Is it possible the company had an annual revenue of \$160 million from 2006 to 2013? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the ordered pair in a coordinate plane. Describe the location of the point. *(Skills Review Handbook)*

35. $A(1, 3)$ **36.** $B(0, -3)$ **37.** $C(-4, -2)$ **38.** $D(-1, 2)$

Find and interpret the mean absolute deviation of the data. *(Skills Review Handbook)*

39. 1, 1, 2, 5, 6, 8, 10, 12, 12, 13 **40.** 24, 26, 28, 28, 30, 30, 32, 32, 34, 36

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

2.4–2.5 What Did You Learn?

Core Vocabulary

compound inequality, *p. 74*

Core Concepts

Section 2.4

Solving Multi-Step Inequalities, *p. 68*
Special Solutions of Linear Inequalities, *p. 69*

Section 2.5

Writing and Graphing Compound Inequalities, *p. 74*
Solving Compound Inequalities, *p. 75*

Mathematical Thinking

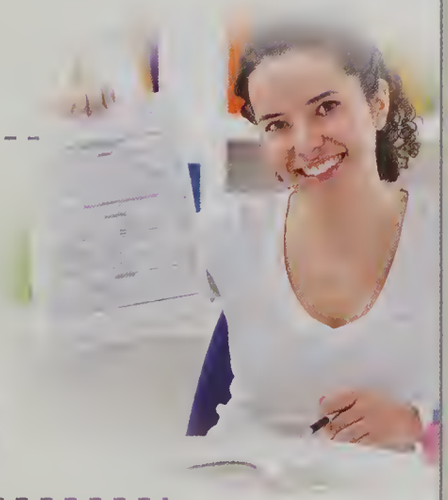
1. How can you use a diagram to help you solve Exercise 12 on page 77?
2. In Exercises 13 and 14 on page 77, how can you use structure to break down the compound inequality into two inequalities?

Performance Task

Grading Calculations

You are not doing as well as you had hoped in one of your classes. So, you want to figure out the minimum grade you need on the final exam to receive the semester grade that you want. Is it still possible to get an A? How would you explain your calculations to a classmate?

To explore the answers to these questions and more, go to ***BigIdeasMath.com***.



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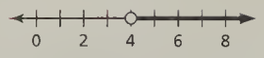
Dynamic Teaching Tools

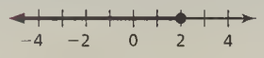
- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

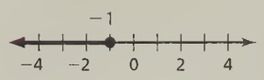
ANSWERS

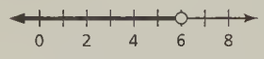
1. A diagram can be used to show the different life zones, and the elevation ranges for each.
2. Take the left expression, first inequality symbol, and middle expression to form the first inequality. Take the middle expression, second inequality symbol, and last expression to form the second inequality.

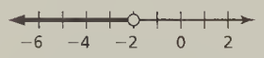
ANSWERS

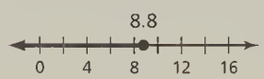
1. $d - 2 < -1$
2. $5h \leq 10$
3. 

4. 

5. 

6. $p < 6$


7. $r < -2$


8. $m \leq 8.8$


2 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

2.1 Writing and Graphing Inequalities *(pp. 45–52)*

a. A number x plus 36 is no more than 40. Write this sentence as an inequality.

A number x plus 36 | is no more than | 40.

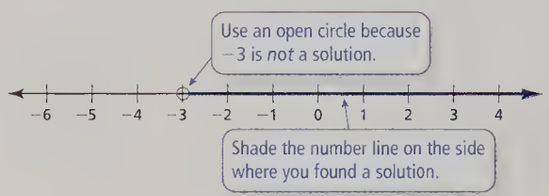
$x + 36$ | $\leq$ | 40

▶ An inequality is $x + 36 \leq 40$.

b. Graph $w > -3$.

Test a number to the left of -3. $w = -4$ is *not* a solution.

Test a number to the right of -3. $w = 0$ is a solution.



Write the sentence as an inequality.

1. A number d minus 2 is less than -1.
2. Ten is at least the product of a number h and 5.

Graph the inequality.

3. $x > 4$
4. $y \leq 2$
5. $-1 \geq z$

2.2 Solving Inequalities Using Addition or Subtraction *(pp. 53–58)*

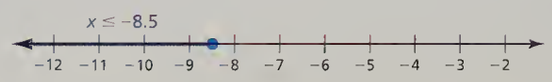
Solve $x + 2.5 \leq -6$. Graph the solution.

$x + 2.5 \leq -6$ Write the inequality.

Subtraction Property of Inequality → $-2.5 \quad -2.5$ Subtract 2.5 from each side.

$x \leq -8.5$ Simplify.

▶ The solution is $x \leq -8.5$.



Solve the inequality. Graph the solution.

6. $p + 4 < 10$
7. $r - 4 < -6$
8. $2.1 \geq m - 6.7$

2.3 Solving Inequalities Using Multiplication or Division *(pp. 59–64)*

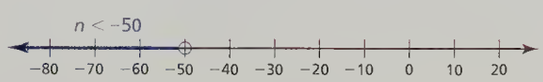
Solve $\frac{n}{-10} > 5$. Graph the solution.

$\frac{n}{-10} > 5$ — Write the inequality.

Multiplication Property of Inequality → $-10 \cdot \frac{n}{-10} < -10 \cdot 5$ — Multiply each side by -10. Reverse the inequality symbol.

$n < -50$ — Simplify.

▶ The solution is $n < -50$.



Solve the inequality. Graph the solution.

9. $3x > -21$ **10.** $-4 \le \frac{g}{5}$ **11.** $-\frac{3}{4}n \le 3$

12. $\frac{s}{-8} \ge 11$ **13.** $36 < 2q$ **14.** $-1.2k > 6$

2.4 Solving Multi-Step Inequalities *(pp. 67–72)*

Solve $22 + 3y \ge 4$. Graph the solution.

$22 + 3y \ge 4$ — Write the inequality.

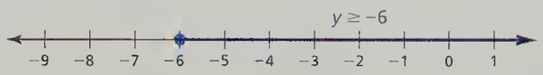
$-22 \quad -22$ — Subtract 22 from each side.

$3y \ge -18$ — Simplify.

$\frac{3y}{3} \ge \frac{-18}{3}$ — Divide each side by 3.

$y \ge -6$ — Simplify.

▶ The solution is $y \ge -6$.



Solve the inequality. Graph the solution, if possible.

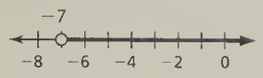
15. $3x - 4 > 11$ **16.** $-4 < \frac{b}{2} + 9$

17. $7 - 3n \le n + 3$ **18.** $2(-4s + 2) \ge -5s - 10$

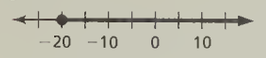
19. $6(2t + 9) \le 12t - 1$ **20.** $3r - 8 > 3(r - 6)$

ANSWERS

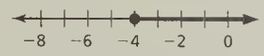
9. $x > -7$



10. $g \ge -20$



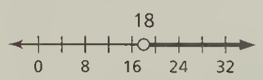
11. $n \ge -4$



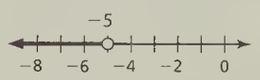
12. $s \le -88$



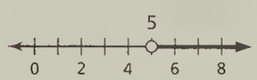
13. $q > 18$



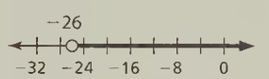
14. $k < -5$



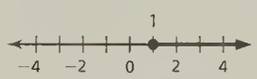
15. $x > 5$



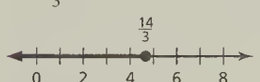
16. $b > -26$



17. $n \ge 1$



18. $s \le \frac{14}{3}$



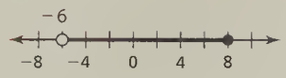
19. no solution

20. all real numbers

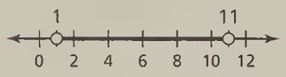


ANSWERS

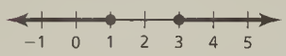
21. $-6 < x \leq 8$



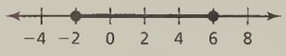
22. $1 < x < 11$



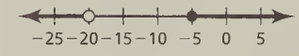
23. $x \leq 1$ or $x \geq 3$



24. $-2 \leq z \leq 6$



25. $r < -20$ or $r \geq -5$



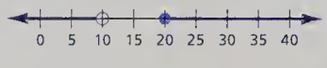
2.5 Solving Compound Inequalities *(pp. 73–78)*

a. A number m is less than 10 or at least 20. Write this sentence as an inequality. Graph the inequality.

A number m is less than 10 or at least 20.

$m < 10$ *or* $m \geq 20$

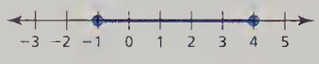
▶ An inequality is $m < 10$ *or* $m \geq 20$.



b. Solve $-1 \leq -2d + 7 \leq 9$. Graph the solution.

$-1 \leq -2d + 7 \leq 9$	Write the inequality.
$-7 \quad -7 \quad -7$	Subtract 7 from each expression.
$-8 \leq -2d \leq 2$	Simplify.
$\frac{-8}{-2} \geq \frac{-2d}{-2} \geq \frac{2}{-2}$	Divide each expression by -2. Reverse each inequality symbol.
$4 \geq d \geq -1$	Simplify.

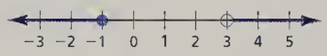
▶ The solution is $-1 \leq d \leq 4$.



c. Solve $2y - 3 \leq -5$ *or* $3y - 1 > 8$. Graph the solution.

$2y - 3 \leq -5$ *or* $3y - 1 > 8$	Write the inequality.
$+3 \quad +3 \qquad +1 \quad +1$	Addition Property of Inequality
$2y \leq -2 \qquad 3y > 9$	Simplify.
$\frac{2y}{2} \leq \frac{-2}{2} \qquad \frac{3y}{3} > \frac{9}{3}$	Division Property of Inequality
$y \leq -1$ *or* $y > 3$	Simplify.

▶ The solution is $y \leq -1$ *or* $y > 3$.



21. A number x is more than -6 and at most 8. Write this sentence as an inequality. Graph the inequality.

Solve the inequality. Graph the solution.

22. $4 > x - 7 > -6$

23. $2x + 2 \leq 4$ *or* $x + 2 \geq 5$

24. $19 \geq 3z + 1 \geq -5$

25. $\frac{r}{4} < -5$ *or* $-2r - 7 \leq 3$

2 Chapter Test

Write the sentence as an inequality.

1. The sum of a number y and 9 is at least -1.
2. A number r is more than 0 or less than or equal to -8.
3. A number k is less than 3 units from 10.

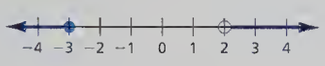
Solve the inequality. Graph the solution, if possible.

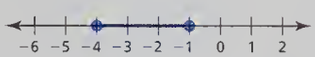
4. $\frac{x}{2} - 5 \geq -9$
5. $-4s < 6s + 1$
6. $4p + 3 \geq 2(2p + 1)$
7. $3y - 7 \geq 17$
8. $8(3g - 2) \leq 12(2g + 1)$
9. $6(2x - 1) \geq 3(4x + 1)$
10. $-7 < 2c - 1 < 10$
11. $-2 \leq 4 - 3a \leq 13$
12. $-5 < 2 - h$ *or* $6h + 5 > 71$

13. You start a small baking business, and you want to earn a profit of at least \$250 in the first month. The expenses in the first month are \$155. What are the possible revenues that you need to earn to meet the profit goal?

14. Let a, b, c, and d be constants. Describe the possible solution sets of the inequality $ax + b < cx + d$.

Write and graph a compound inequality that represents the numbers that are *not* solutions of the inequality represented by the graph shown. Explain your reasoning.

15. 

16. 

17. You save \$15 per week to purchase one of the bikes shown.
 a. Write and solve an inequality to find the number of weeks you need to save to purchase a bike.
 b. Your parents give you \$65 to help you buy the new bike. How does this affect your answer in part (a)? Use an inequality to justify your answer.



18. A state imposes a sales tax on items of clothing that cost more than \$175. The tax applies only to the difference of the price of the item and \$175.
 a. Use the receipt shown to find the tax rate (as a percent).
 b. A shopper has \$430 to spend on a winter coat. Write and solve an inequality to find the prices p of coats that the shopper can afford. Assume that $p \geq 175$.
 c. Another state imposes a 5% sales tax on the entire price of an item of clothing. For which prices would paying the 5% tax be cheaper than paying the tax described above? Write and solve an inequality to find your answer and list three prices that are solutions.


The STYLE store
PURCHASE DATE: 03/29/14
STORE#: 1006
ITEM: SUIT

PRICE:	$295.00
TAX:	$ 7.50
TOTAL:	**$302.50**

THANK YOU

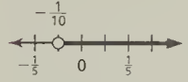
If students need help...	If students got it...
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

ANSWERS

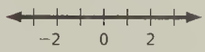
1. $y + 9 \geq -1$
2. $r > 0$ or $r \leq -8$
3. $|k - 10| < 3$
4. $x \geq -8$



5. $s > -\frac{1}{10}$



6. all real numbers



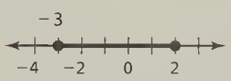
7. $y \geq 8$



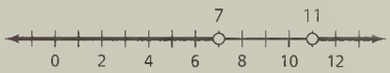
8. all real numbers



9. no solution
10. See Additional Answers.
11. $-3 \leq a \leq 2$



12. $h < 7$ or $h > 11$

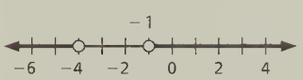


13. at least \$405
14. all real numbers; no solution; $x < \frac{d - b}{a - c}$; $x > \frac{d - b}{a - c}$
15. $-3 < x \leq 2$



The values between -3 and 2, including 2, are not solutions.

16. $x < -4$ or $x > -1$



The values greater than -1 or less than -4 are not solutions.

17. a. $15w \geq 120$; $w \geq 8$
 b. The number of weeks is reduced; $65 + 15w \geq 120$; $w \geq 3\frac{2}{3}$
18. a. 6.25%
 b. $175 + 1.0625(p - 175) \leq 430$; $p \leq 415$
 c. $1.05p < 175 + 1.0625(p - 175)$; $p > 875$; *Sample answers:* \$900, \$950, \$1000

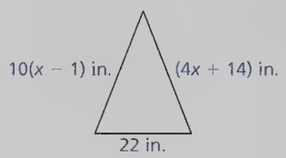
ANSWERS

1. B
2. H
3. D
4. 5
5. J
6. C

2 Standards Assessment

1. The triangle shown has a perimeter of 82 inches. What is the value of x? *(TEKS A.5.A)*

 Ⓐ 3.4
 Ⓑ 4.0
 Ⓒ 7.1
 Ⓓ 42.0



2. The sum of 5 times a number b and 8 is no less than b. Which inequality describes b? *(TEKS A.5.B)*

 Ⓕ $b > -2$
 Ⓖ $b < -2$
 Ⓗ $b \geq -2$
 Ⓙ $b \leq -2$

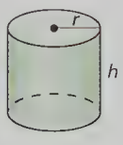
3. A skateboarding park charges \$7 per session to skate and \$4 per session to rent safety equipment. Your friend rents safety equipment every time he skates. Last year, he spent \$99 total for skating charges and equipment rentals. How much did he pay to rent safety equipment last year? *(TEKS A.5.A)*

 Ⓐ \$4
 Ⓑ \$9
 Ⓒ \$11
 Ⓓ \$36

4. **GRIDDED ANSWER** The cost C of parking in a parking garage is given by the equation $C = 2(x - 2) + 3$, where x is the amount of time in hours. You need to spend less than \$10 for parking. What is the maximum whole number of hours you can park in the garage? *(TEKS A.5.B)*

5. The volume V of a cylinder is given by the formula $V = \pi r^2 h$. Solve the formula for h. *(TEKS A.12.E)*

 Ⓕ $h = V - \pi r^2$
 Ⓖ $h = -\frac{V}{\pi r^2}$
 Ⓗ $h = \frac{V}{\pi r}$
 Ⓙ $h = \frac{V}{\pi r^2}$



6. The area of a rectangle is given by $3(x - 17) - x + 19$. Which inequality represents the possible values of x? *(TEKS A.5.B)*

 Ⓐ $x \geq 16$
 Ⓑ $x > -16$
 Ⓒ $x > 16$
 Ⓓ $-16 < x < 16$

7. What is the solution of $8x + 2x = 15x - 10$? *(TEKS A.5.A)*

Ⓕ $x = -2$ Ⓖ $x = 0.4$

Ⓗ $x = 2$ Ⓙ $x = 5$

8. In 1862, the United States imposed a tax on annual income to pay for the expenses of the Civil War. The table shows the tax rates for different incomes. Which compound inequality describes the range of incomes x (in dollars) for which the tax was between \$450 and \$600? *(TEKS A.5.B)*

Annual income	Tax rate
\$600 to \$10,000	3% of income
Greater than \$10,000	3% of first \$10,000 plus 5% of income over \$10,000

Ⓐ $3000 \le x \le 6000$ Ⓑ $9000 \le x \le 12{,}000$

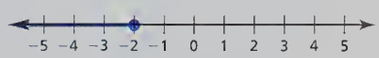
Ⓒ $13{,}000 \le x \le 16{,}000$ Ⓓ $15{,}000 \le x \le 20{,}000$

9. What is the solution of $4y + y + 1 = 7(y - 1)$? *(TEKS A.5.A)*

Ⓕ $y = 4$ Ⓖ $y = 3$

Ⓗ $y = -3$ Ⓙ $y = -4$

10. The solution set of which inequality is represented by the graph? *(TEKS A.5.B)*



Ⓐ $4x + 1 \le 2(x - 1) - 1$

Ⓑ $4x + 1 \ge 2(x - 1) - 1$

Ⓒ $4x - 1 \le 2(x + 1) + 1$

Ⓓ none of the above

11. A plumber charges \$64 per hour for labor and x dollars for replacement parts. The total bill is \$284 and includes 3 hours of labor. How much does the plumber charge for the replacement parts? *(TEKS A.5.A)*

Ⓕ \$22 Ⓖ \$24

Ⓗ \$92 Ⓙ \$118

12. When $2(a + b) + a$ is negative, which statement must be true? *(TEKS A.5.B)*

Ⓐ $a < 0, b < 0$ Ⓑ $a < 0, b > 0$

Ⓒ $a < -\frac{2}{3}b$ Ⓓ $a < b$

ANSWERS

7. H
8. C
9. F
10. A
11. H
12. C

Chapter 3 Pacing Guide

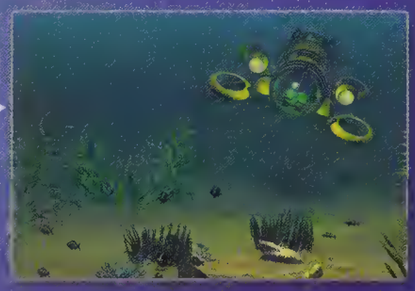
Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Quiz	1 Day
Section 4	2 Days
Section 5	2 Days
Section 6	2 Days
Section 7	3 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 3	19 Days
Year-to-Date	45 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
3.1	A.12.A
3.2	A.2.A, A.3.C
3.3	A.3.C, A.12.B
3.4	A.3.C
3.5	A.2.A, A.3.A, A.3.B, A.3.C
3.6	A.2.D
3.7	A.3.E

3 Graphing Linear Functions

3.1 Functions
3.2 Linear Functions
3.3 Function Notation
3.4 Graphing Linear Equations in Standard Form
3.5 Graphing Linear Equations in Slope-Intercept Form
3.6 Modeling Direct Variation
3.7 Transformations of Graphs of Linear Functions



Submersible *(p. 128)*



Basketball *(p. 122)*



Speed of Light *(p. 111)*



Coins *(p. 102)*



Taxi Ride *(p. 95)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

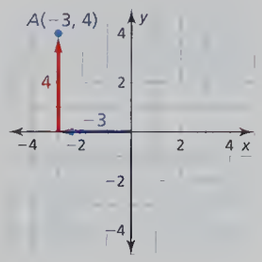
For an overview of this chapter, formative assessment tips, and teaching strategies, see Laurie's Notes at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

Plotting Points (6.11)

Example 1 Plot the point $A(-3, 4)$ in a coordinate plane. Describe the location of the point.

Start at the origin. Move 3 units left and 4 units up. Then plot the point. The point is in Quadrant II.



Plot the point in a coordinate plane. Describe the location of the point.

1. $A(3, 2)$
2. $B(-5, 1)$
3. $C(0, 3)$
4. $D(-1, -4)$
5. $E(-3, 0)$
6. $F(2, -1)$

Evaluating Expressions (6.3.D)

Example 2 Evaluate $4x - 5$ when $x = 3$.

$4x - 5 = 4(3) - 5$ Substitute 3 for x.

$= 12 - 5$ Multiply.

$= 7$ Subtract.

Example 3 Evaluate $-2x + 9$ when $x = -8$.

$-2x + 9 = -2(-8) + 9$ Substitute -8 for x.

$= 16 + 9$ Multiply.

$= 25$ Add.

Evaluate the expression for the given value of x.

7. $3x - 4; x = 7$
8. $-5x + 8; x = 3$
9. $10x + 18; x = 5$
10. $-9x - 2; x = -4$
11. $24 - 8x; x = -2$
12. $15x + 9; x = -1$
13. **ABSTRACT REASONING** Let a and b be positive real numbers. Describe how to plot (a, b), $(-a, b)$, $(a, -b)$, and $(-a, -b)$.

Dynamic Solutions available at *BigIdeasMath.com*

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For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make a Four Square for each topic.

- Plotting points
- Evaluating expressions

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

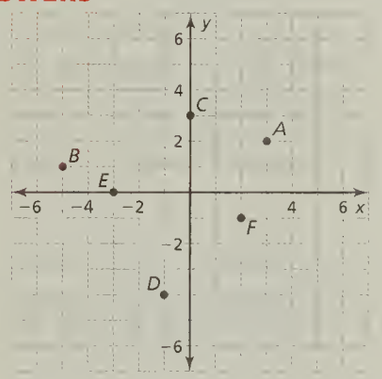
Texas Essential Knowledge and Skills

6.11 The student applies mathematical process standards to use coordinate geometry to identify locations on a plane. The student is expected to graph points in all four quadrants using ordered pairs of rational numbers.

6.3.D The student is expected to add, subtract, multiply, and divide integers fluently.

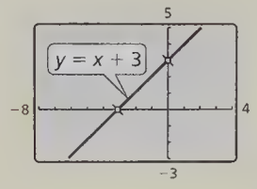
ANSWERS

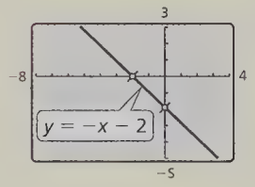
1–6.

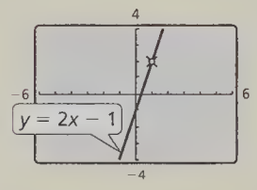


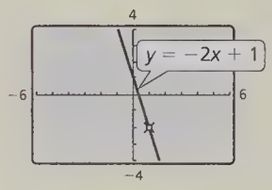
1. in Quadrant I
2. in Quadrant II
3. on positive y-axis
4. in Quadrant III
5. on negative x-axis
6. in Quadrant IV
7. 17
8. -7
9. 68
10. 34
11. 40
12. -6
13. Start at the origin. Move a units right and b units up. Plot the point; Start at the origin. Move a units left and b units up. Plot the point; Start at the origin. Move a units right and b units down. Plot the point; Start at the origin. Move a units left and b units down. Plot the point.

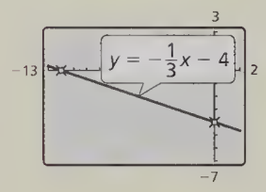
MONITORING PROGRESS ANSWERS

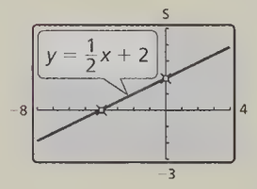
1. yes; 10 : 15 is a 2 : 3 ratio.
2. no; 18 : 12 is not a 2 : 3 ratio.
3. yes; 24 : 36 is a 2 : 3 ratio.
4. 

5. 

6. 

7. 

8. 

9. 

10. The slope of a line in a square viewing window appears to be steeper than in a standard viewing window.

Mathematical Thinking

Mathematically proficient students select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems. **(A.1.C)**

Using a Graphing Calculator

Core Concept

Standard and Square Viewing Windows

A typical graphing calculator screen has a height to width ratio of 2 to 3. This means that when you use the *standard viewing window* of −10 to 10 (on each axis), the graph will not be in its true perspective.

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
```
This is the standard viewing window.

To see a graph in its true perspective, you need to use a *square viewing window*, in which the tick marks on the x-axis are spaced the same as the tick marks on the y-axis.

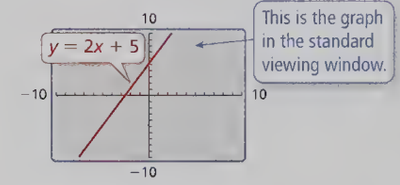
```
WINDOW
Xmin=-9
Xmax=9
Xscl=1
Ymin=-6
Ymax=6
Yscl=1
```
This is a square viewing window.

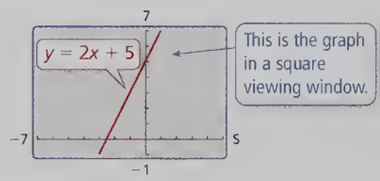
EXAMPLE 1 Using a Graphing Calculator

Use a graphing calculator to graph $y = 2x + 5$.

SOLUTION

Enter the equation $y = 2x + 5$ into your calculator. Then graph the equation. The standard viewing window does not show the graph in its true perspective. Notice that the tick marks on the y-axis are closer together than the tick marks on the x-axis. To see the graph in its true perspective, use a square viewing window.





Monitoring Progress

Determine whether the viewing window is square. Explain.

1. $-8 \le x \le 7, -3 \le y \le 7$
2. $-6 \le x \le 6, -9 \le y \le 9$
3. $-18 \le x \le 18, -12 \le y \le 12$

Use a graphing calculator to graph the equation. Use a square viewing window.

4. $y = x + 3$
5. $y = -x - 2$
6. $y = 2x - 1$
7. $y = -2x + 1$
8. $y = -\frac{1}{3}x - 4$
9. $y = \frac{1}{2}x + 2$
10. How does the appearance of the slope of a line change between a standard viewing window and a square viewing window?

For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

3.1 Functions

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.12.A

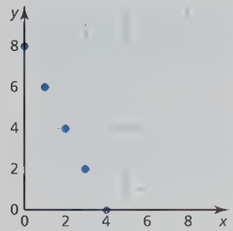
Essential Question What is a function?

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x-coordinates are inputs and the y-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

EXPLORATION 1 Describing a Function

Work with a partner. Functions can be described in many ways.

- by an equation
- by an input-output table
- using words
- by a graph
- as a set of ordered pairs



a. Explain why the graph shown represents a function.

b. Describe the function in two other ways.

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to analyze relationships mathematically to draw conclusions.

EXPLORATION 2 Identifying Functions

Work with a partner. Determine whether each relation represents a function. Explain your reasoning.

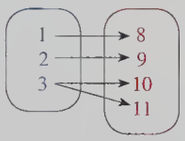
a.

Input, x	0	1	2	3	4
Output, y	8	8	8	8	8

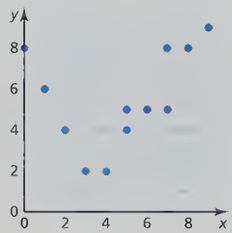
b.

Input, x	8	8	8	8	8
Output, y	0	1	2	3	4

c.



d.



e. $(-2, 5), (-1, 8), (0, 6), (1, 6), (2, 7)$

f. $(-2, 0), (-1, 0), (-1, 1), (0, 1), (1, 2), (2, 2)$

g. Each radio frequency x in a listening area has exactly one radio station y.

h. The same television station x can be found on more than one channel y.

i. $x = 2$

j. $y = 2x + 3$

Communicate Your Answer

3. What is a function? Give examples of relations, other than those in Explorations 1 and 2, that (a) are functions and (b) are not functions.

SUPPORTING English Language Learners

As students work in pairs to complete the explorations and Communicate Your Answer, circulate through the classroom and listen to their conversations. Coach students in the following self-improvement strategies.

Beginning Repeat a word when necessary to improve pronunciation. Use gestures or visuals to help convey its meaning.

Intermediate Repeat a phrase or use other language to describe its meaning if it is unclear.

Advanced/Advanced High Combine short, simple sentences to create longer, compound sentences.

ELPS 1.B.1 Monitor oral language production and employ self-corrective techniques or other resources.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.12.A The student is expected to decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

ANSWERS

1. a. Each x-coordinate is paired with exactly one y-coordinate.

b. *Sample answer:* (0, 8), (1, 6), (2, 4), (3, 2), (4, 0);

Input, x	0	1	2	3	4
Output, y	8	6	4	2	0

2. a. function; Each input value is paired with exactly one output value.

b. not a function; The input value of 8 is paired with more than one output value.

c. not a function; The input value of 3 is paired with more than one output value.

d. not a function; The input values of 5 and 7 are each paired with more than one output value.

e. function; Each input value is paired with exactly one output value.

f. not a function; The input value of -1 is paired with more than one output value.

g. function; Each input frequency is paired with exactly one output radio station.

h. not a function; Each input television station is paired with more than one output channel.

i. not a function; The input value of 2 is paired with every possible output value of y.

j. function; Each input value of x is paired with exactly one output value of y.

3. a relation that pairs each input with exactly one output

a. *Sample answer:* (0, 10), (1, 20), (2, 30); $y = -5x$

b. *Sample answer:* (0, 10), (0, 20), (0, 30); $x = -5$

Extra Example 1

Determine whether each relation is a function. Explain.

a. $(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1)$

The relation is a function. Every input has exactly one output.

b. $(1, 3), (2, 4), (3, 5), (1, 5), (4, 6)$

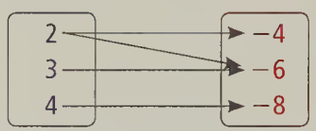
The relation is not a function. The input 1 has two outputs, 3 and 5.

c.

Input, x	0	2	4	6	8	10
Output, y	0	1	2	3	4	5

The relation is a function. Every input has exactly one output.

d. Input, x Output, y



The relation is not a function. The input 2 has two outputs, -4 and -6.

Extra Example 2

Determine whether each relation is a function. Explain.

a. $y = -2x$ with inputs $x = 0$, $x = 1$, and $x = 2$

The relation is a function. The input 0 has exactly one output, 0. The input 1 has exactly one output, -2. The input 2 has exactly one output, -4.

b. $x + 1 = y^2$ with inputs $x = 0$ and $x = 3$ The relation is not a function. The input 0 has two outputs, -1 and 1. The input 3 has two outputs, -2 and 2.

3.1 Lesson

What You Will Learn

- Determine whether relations are functions.
- Find the domain and range of a function.
- Identify the independent and dependent variables of functions.

Core Vocabulary

relation, *p. 90*
function, *p. 90*
domain, *p. 92*
range, *p. 92*
independent variable, *p. 93*
dependent variable, *p. 93*

Previous
ordered pair
mapping diagram

Determining Whether Relations Are Functions

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x-coordinates are inputs and the y-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

EXAMPLE 1 Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

a. $(-2, 2), (-1, 2), (0, 2), (1, 0), (2, 0)$

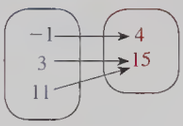
b. $(4, 0), (8, 7), (6, 4), (4, 3), (5, 2)$

c.

Input, x	−2	−1	0	0	1	2
Output, y	3	4	5	6	7	8

REMEMBER
A relation can be represented by a mapping diagram.

d. Input, x Output, y



SOLUTION

a. Every input has exactly one output. So, the relation is a function.

b. The input 4 has two outputs, 0 and 3. So, the relation is *not* a function.

c. The input 0 has two outputs, 5 and 6. So, the relation is *not* a function.

d. Every input has exactly one output. So, the relation is a function.

EXAMPLE 2 Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

a. $y = 5x$ with inputs $x = 1$, $x = 2$, and $x = 3$

b. $x = y^2$ with inputs $x = 0$ and $x = 1$

SOLUTION

a. The input 1 has exactly one output, 5. The input 2 has exactly one output, 10. The input 3 has exactly one output, 15.

▶ So, the relation is a function.

b. The input 0 has exactly one output, 0, but the input 1 has two outputs, 1 and -1.

▶ So, the relation is *not* a function.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Determine whether the relation is a function. Explain.

1. $(-5, 0), (0, 0), (5, 0), (5, 10)$
2. $(-4, 8), (-1, 2), (2, -4), (5, -10)$
3.

Input, x	2	4	6
Output, y	2.6	5.2	7.8

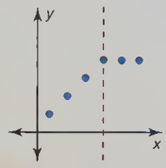
4. $y = 2x + 4$ with inputs $x = 2$ and $x = 4$

Core Concept

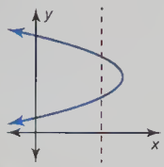
Vertical Line Test

Words A graph represents a function when no vertical line passes through more than one point on the graph.

Examples Function



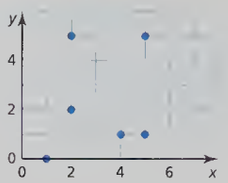
Not a function



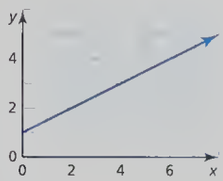
EXAMPLE 3 Using the Vertical Line Test

Determine whether each graph represents a function. Explain.

a.



b.



SOLUTION

a. You can draw a vertical line through $(2, 2)$ and $(2, 5)$.

▶ So, the graph does *not* represent a function.

b. No vertical line can be drawn through more than one point on the graph.

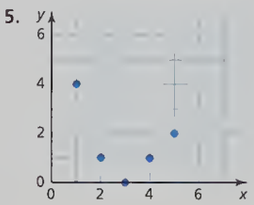
▶ So, the graph represents a function.

Monitoring Progress

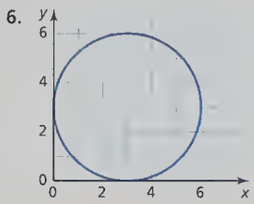
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Determine whether the graph represents a function. Explain.

5.



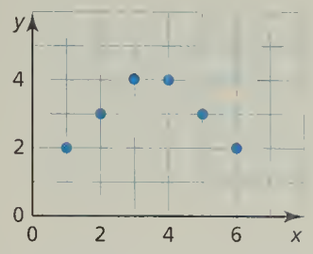
6.



Extra Example 3

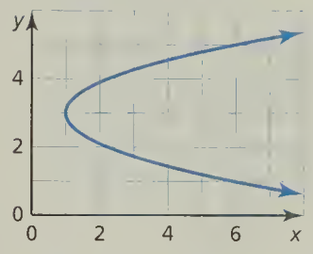
Determine whether each graph represents a function. Explain.

a.



The graph represents a function. No vertical line can be drawn through more than one point on the graph.

b.



The graph does not represent a function. *Sample answer:* A vertical line can be drawn through (6, 1) and (6, 5).

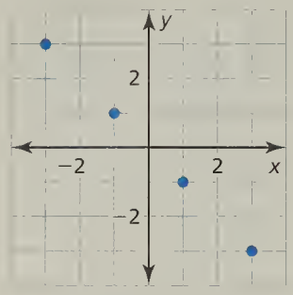
MONITORING PROGRESS ANSWERS

1. not a function; The input 5 has two outputs, 0 and 10.
2. function; Every input has exactly one output.
3. function; Every input has exactly one output.
4. function; Every input has exactly one output.
5. function; No vertical line can be drawn through more than one point on the graph.
6. not a function; A vertical line can be drawn through more than one point on the graph at several places, such as (3, 0) and (3, 6).

Extra Example 4

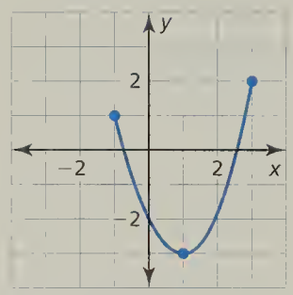
Find the domain and range of the function represented by the graph.

a.



The domain is −3, −1, 1, and 3. The range is 3, 1, −1, and −3.

b.



The domain is $-1 \le x \le 3$. The range is $-3 \le y \le 2$.

MONITORING PROGRESS ANSWERS

7. domain: −2, −1, 0, 1, 2; range: 0, 2, 4
8. domain: $1 \le x \le 5$; range: $0 \le y \le 4$

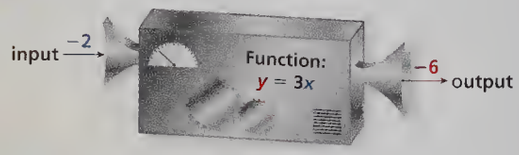
Finding the Domain and Range of a Function

Core Concept

The Domain and Range of a Function

The **domain** of a function is the set of all possible input values.

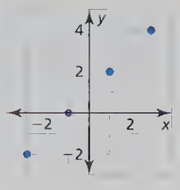
The **range** of a function is the set of all possible output values.



EXAMPLE 4 Finding the Domain and Range from a Graph

Find the domain and range of the function represented by the graph.

a.

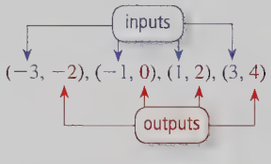


b.



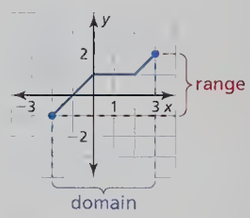
SOLUTION

a. Write the ordered pairs. Identify the inputs and outputs.



▶ The domain is −3, −1, 1, and 3. The range is −2, 0, 2, and 4.

b. Identify the x- and y-values represented by the graph.



▶ The domain is $-2 \le x \le 3$. The range is $-1 \le y \le 2$.

STUDY TIP

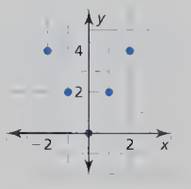
A relation also has a domain and a range.

Monitoring Progress

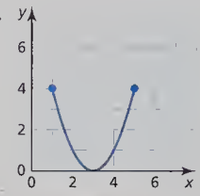
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Find the domain and range of the function represented by the graph.

7.



8.



English Language Learners

Vocabulary

Have students make a table to show how the terms used to describe a function are related.

Input	Output
x-values	y-values
Domain	Range
Independent variable	Dependent variable

Identifying Independent and Dependent Variables

The variable that represents the input values of a function is the **independent variable** because it can be *any* value in the domain. The variable that represents the output values of a function is the **dependent variable** because it *depends* on the value of the independent variable. When an equation represents a function, the dependent variable is defined in terms of the independent variable. The statement "y is a function of x" means that y varies depending on the value of x.

$$y = -x + 10$$

dependent variable, y — independent variable, x

EXAMPLE 5 **Identifying Independent and Dependent Variables**



The function $y = -3x + 12$ represents the amount y (in fluid ounces) of juice remaining in a bottle after you take x gulps.

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, 3, and 4. What is the range?

SOLUTION

a. The amount y of juice remaining depends on the number x of gulps.

▶ So, y is the dependent variable, and x is the independent variable.

b. Make an input-output table to find the range.

Input, x	$-3x + 12$	Output, y
0	$-3(0) + 12$	12
1	$-3(1) + 12$	9
2	$-3(2) + 12$	6
3	$-3(3) + 12$	3
4	$-3(4) + 12$	0

▶ The range is 12, 9, 6, 3, and 0.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

9. The function $a = -4b + 14$ represents the number a of avocados you have left after making b batches of guacamole.

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, and 3. What is the range?

10. The function $t = 19m + 65$ represents the temperature t (in degrees Fahrenheit) of an oven after preheating for m minutes.

a. Identify the independent and dependent variables.

b. A recipe calls for an oven temperature of 350°F. Describe the domain and range of the function.

Extra Example 5

The function $c = 6t + 8$ represents the cost c for t T-shirts plus shipping.

a. Identify the independent and dependent variables. independent variable: t; dependent variable: c

b. The domain is 1, 2, 3, 4, and 5. What is the range? 14, 20, 26, 32, 38

MONITORING PROGRESS ANSWERS

9. a. a is the dependent variable and b is the independent variable.

b. 14, 10, 6, 2

10. a. t is the dependent variable and m is the independent variable.

b. domain: $0 \le m \le 15$; range: $65 \le t \le 350$

Closure

- **Exit Ticket:** Make an input-output table for the function $y = -2x + 3$ using the inputs -2, 0, 2, 4.

x	-2	0	2	4
y	7	3	-1	-5

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–25 odd, 34, 35, 50–57

Average: 1, 2–30 even, 34, 35, 42, 44, 50–57

Advanced: 1, 2, 6–34 even, 35–39, 40–50 even, 51–57

HOMEWORK CHECK

Basic: 3, 9, 13, 17, 21

Average: 4, 10, 14, 18, 22

Advanced: 6, 12, 16, 20, 22

ANSWERS

1. The independent variable can be any value in the domain, but the dependent variable depends on the value of the independent variable.
2. Find the range of the function represented by the table; $-1, 5, 7$; $-1, 0, 1$
3. function; Every input has exactly one output.
4. not a function; The input 3 has two outputs, -8 and 6.
5. not a function; The input 2 has two outputs, 3 and 2.
6. function; Every input has exactly one output.
7. not a function; The input 16 has two outputs, -2 and 2, and the input 1 has two outputs, -1 and 1.
8. function; Every input has exactly one output.
9. function; Every input has exactly one output.
10. function; Every input has exactly one output.
11. not a function; The input 4 has two outputs, -2 and 2, and the input 9 has two outputs, -3 and 3.
12. not a function; The input 1 has two outputs, -1 and 1, the input 2 has two outputs, -2 and 2, and the input 3 has two outputs, -3 and 3.
13. function; No vertical line can be drawn through more than one point on the graph.
14. not a function; Three vertical lines can be drawn through more than one point on the graph, including one through (0, 2) and (0, 4).
15. not a function; A vertical line can be drawn through more than one point on the graph in many places, such as (4, 0) and (4, 6).
16. function; No vertical line can be drawn through more than one point on the graph.
17. domain: $-2, -1, 0, 1, 2$; range: $-2, 0, 2$
18. domain: $-2, 0, 2, 4$; range: 3
19. domain: $-4 \le x \le 2$; range: $2 \le y \le 6$
20. domain: $2 < x < 7$; range: $1 < y < 6$

3.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are independent variables and dependent variables different?
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the range of the function represented by the table.

Find the inputs of the function represented by the table.

x	-1	0	1
y	7	5	-1

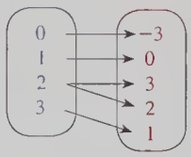
Find the x-values of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

Find the domain of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

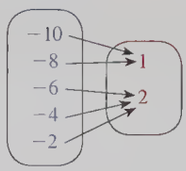
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine whether the relation is a function. Explain. *(See Example 1.)*

3. $(1, -2), (2, 1), (3, 6), (4, 13), (5, 22)$
4. $(7, 4), (5, -1), (3, -8), (1, -5), (3, 6)$
5. Input, x Output, y



6. Input, x Output, y



7.

Input, x	16	1	0	1	16
Output, y	-2	-1	0	1	2

8.

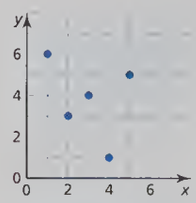
Input, x	-3	0	3	6	9
Output, y	11	5	-1	-7	-13

In Exercises 9–12, determine whether the relation is a function. Explain. *(See Example 2.)*

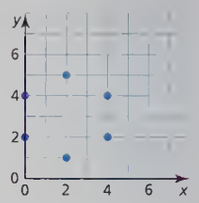
9. $y = -3x$ with inputs $x = 0$, $x = 1$, and $x = 2$
10. $y = 4x - 1$ with inputs $x = -3$, $x = -2$, and $x = -1$
11. $x = y^2$ with inputs $x = 4$ and $x = 9$
12. $x = |y|$ with inputs $x = 1$, $x = 2$, and $x = 3$

In Exercises 13–16, determine whether the graph represents a function. Explain. *(See Example 3.)*

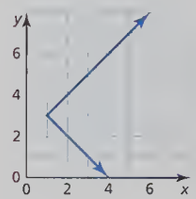
13.



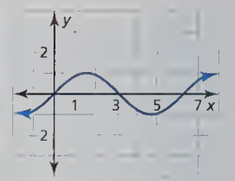
14.



15.

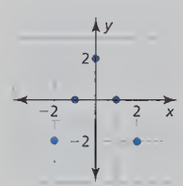


16.

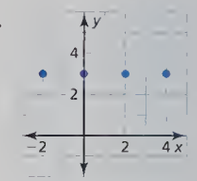


In Exercises 17–20, find the domain and range of the function represented by the graph. *(See Example 4.)*

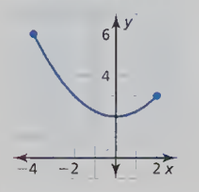
17.



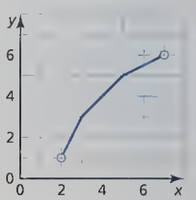
18.



19.



20.



21. MODELING WITH MATHEMATICS The function $y = 25x + 500$ represents your monthly rent y (in dollars) when you pay x days late. *(See Example 5.)*

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, 3, 4, and 5. What is the range?

22. MODELING WITH MATHEMATICS The function $y = 3.5x + 2.8$ represents the cost y (in dollars) of a taxi ride of x miles.



a. Identify the independent and dependent variables.

b. You have enough money to travel at most 20 miles in the taxi. Find the domain and range of the function.

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in the statement about the relation shown in the table.**

Input, x	1	2	3	4	5
Output, y	6	7	8	6	9

23.

✗ The relation is *not* a function. One output is paired with two inputs.

24.

✗ The relation is a function. The range is 1, 2, 3, 4, and 5.

ANALYZING RELATIONSHIPS **In Exercises 25–28, identify the independent and dependent variables.**

25. The length of your hair depends on the amount of time since your last haircut.

26. A baseball team's rank depends on the number of games the team wins.

27. The number of quarters you put into a parking meter affects the amount of time you have on the meter.

28. The battery power remaining on your MP3 player is based on the amount of time you listen to it.

29. MULTIPLE REPRESENTATIONS The balance y (in dollars) of your savings account is a function of the month x.

Month, x	0	1	2	3	4
Balance (dollars), y	100	125	150	175	200

a. Describe this situation in words.

b. Write the function as a set of ordered pairs.

c. Plot the ordered pairs in a coordinate plane.

30. MULTIPLE REPRESENTATIONS The function $1.5x + 0.5y = 12$ represents the number of hardcover books x and softcover books y you can buy at a used book sale.

a. Solve the equation for y.

b. Make an input-output table to find ordered pairs for the function.

c. Plot the ordered pairs in a coordinate plane.

31. OPEN-ENDED Graph a relation that fails the Vertical Line Test at exactly one value of x.

32. OPEN-ENDED List the inputs and outputs of a relation such that the relation is a function, but when the inputs and outputs are switched, the relation is not a function.

33. ANALYZING RELATIONSHIPS You select items in a vending machine by pressing one letter and then one number.



a. Explain why the relation that pairs letter-number combinations with food or drink items is a function.

b. Identify the independent and dependent variables.

c. Find the domain and range of the function.

Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

ANSWERS

21. a. y is the dependent variable and x is the independent variable.

b. 500, 525, 550, 575, 600, 625

22. a. y is the dependent variable and x is the independent variable.

b. domain: $0 \le x \le 20$; range: $2.8 \le y \le 72.8$

23. It is not a function if one input is paired with more than one output; The relation is a function. No input is paired with more than one output.

24. The given list is the domain of the function; The range is 6, 7, 8, 9.

25. The length of your hair is the dependent variable and the amount of time since your last haircut is the independent variable.

26. A baseball team's rank is the dependent variable and the number of games the team wins is the independent variable.

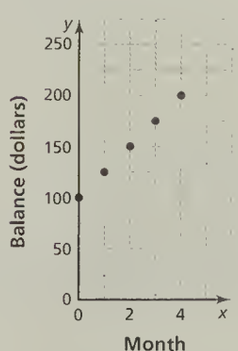
27. The amount of time is the dependent variable and the number of quarters is the independent variable.

28. The battery power remaining is the dependent variable and the amount of time you listen is the independent variable.

29. a. *Sample answer:* The balance of the savings account is \$100 in month 0 and increases by \$25 per month through month 4.

b. (0, 100), (1, 125), (2, 150), (3, 175), (4, 200)

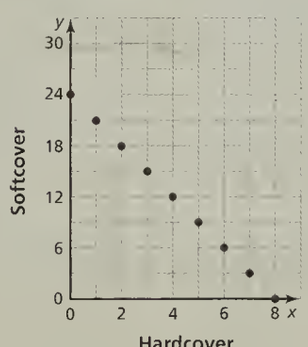
c.



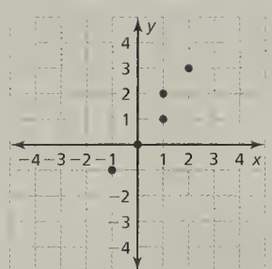
30. a. $y = 24 - 3x$

b. (0, 24), (1, 21), (2, 18), (3, 15), (4, 12), (5, 9), (6, 6), (7, 3), (8, 0)

c.



31. *Sample answer:*



32. *Sample answer:* inputs: −2, 0, 1, 3, 4; outputs: 1, 2, 2, 3, 4

33. See Additional Answers.

ANSWERS

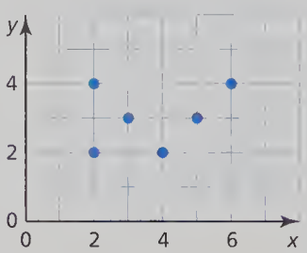
34–57. See Additional Answers.

Mini-Assessment

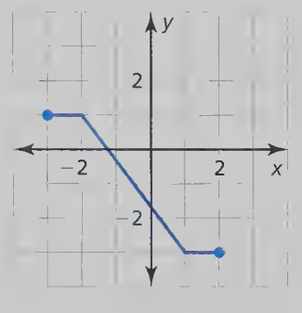
Determine whether the relation is a function. Explain.

1. $(-5, -1), (-4, 1), (-3, 3), (-1, 5), (1, 7)$ The relation is a function. Every input has exactly one output.
2. $y = x + 2$ with inputs $x = 2$, $x = 3$, and $x = 4$

 The relation is a function. The input 2 has exactly one output, 4. The input 3 has exactly one output, 5. The input 4 has exactly one output, 6.
3. Determine whether the graph represents a function. Explain.

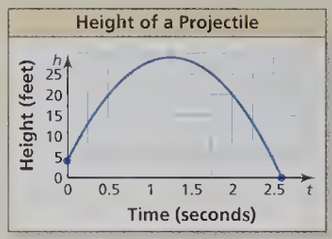


 The graph does not represent a function. A vertical line can be drawn through (2, 2) and (2, 4).
4. Find the domain and range of the function represented by the graph.



 The domain is $-3 \le x \le 2$.
 The range is $-3 \le y \le 1$.
5. The function $y = 65x + 50$ represents the cost y (in dollars) for x hours of plumbing work.
 a. Identify the independent and dependent variables.
 independent variable: x; dependent variable: y
 b. The domain is 1, 2, 3, 4, and 5. What is the range? 115, 180, 245, 310, 375

34. HOW DO YOU SEE IT? The graph represents the height h of a projectile after t seconds.



a. Explain why h is a function of t.

b. Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

c. Approximate the domain of the function.

d. Is t a function of h? Explain.

35. MAKING AN ARGUMENT Your friend says that a line always represents a function. Is your friend correct? Explain.

36. THOUGHT PROVOKING Write a function in which the inputs and/or the outputs are not numbers. Identify the independent and dependent variables. Then find the domain and range of the function.

ATTENDING TO PRECISION In Exercises 37–40, determine whether the statement uses the word *function* in a way that is mathematically correct. Explain your reasoning.

37. The selling price of an item is a function of the cost of making the item.

38. The sales tax on a purchased item in a given state is a function of the selling price.

39. A function pairs each student in your school with a homeroom teacher.

40. A function pairs each chaperone on a school trip with 10 students.

REASONING In Exercises 41–44, tell whether the statement is true or false. If it is false, explain why.

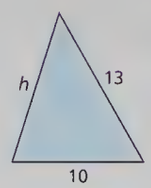
41. Every function is a relation.

42. Every relation is a function.

43. When you switch the inputs and outputs of any function, the resulting relation is a function.

44. When the domain of a function has an infinite number of values, the range always has an infinite number of values.

45. MATHEMATICAL CONNECTIONS Consider the triangle shown.



a. Write a function that represents the perimeter of the triangle.

b. Identify the independent and dependent variables.

c. Describe the domain and range of the function. (*Hint:* The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.)

REASONING In Exercises 46–49, find the domain and range of the function.

46. $y = |x|$ **47.** $y = -|x|$

48. $y = |x| - 6$ **49.** $y = 4 - |x|$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. *(Section 2.1)*

50. A number y is less than 16.

51. Three is no less than a number x.

52. Seven is at most the quotient of a number d and -5.

53. The sum of a number w and 4 is more than -12.

Evaluate the expression. *(Skills Review Handbook)*

54. 11^2 **55.** $(-3)^4$ **56.** -5^2 **57.** 2^5

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

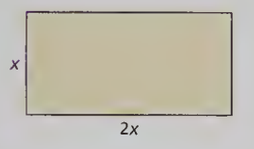
3.2 Linear Functions

Texas Essential Knowledge and Skills
A.2.A
A.3.C

Essential Question How can you determine whether a function is linear or nonlinear?

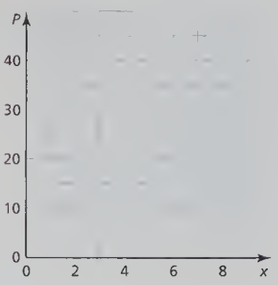
EXPLORATION 1 Finding Patterns for Similar Figures

Work with a partner. Copy and complete each table for the sequence of similar figures. (In parts (a) and (b), use the rectangle shown.) Graph the data in each table. Decide whether each pattern is linear or nonlinear. Justify your conclusion.



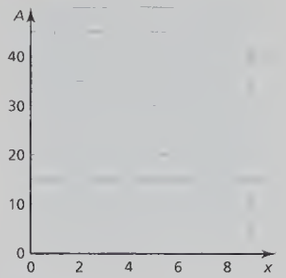
a. perimeters of similar rectangles

x	1	2	3	4	5
P					



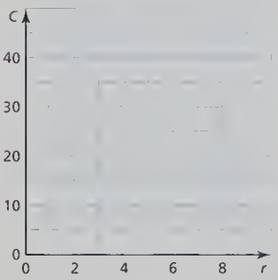
b. areas of similar rectangles

x	1	2	3	4	5
A					



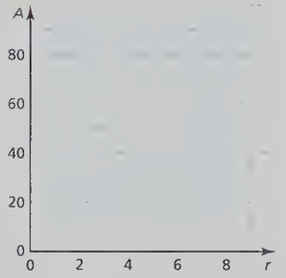
c. circumferences of circles of radius r

r	1	2	3	4	5
C					



d. areas of circles of radius r

r	1	2	3	4	5
A					



SELECTING TOOLS
To be proficient in math, you need to identify relationships using tools, such as tables and graphs.

Communicate Your Answer

2. How do you know that the patterns you found in Exploration 1 represent functions?
3. How can you determine whether a function is linear or nonlinear?
4. Describe two real-life patterns: one that is linear and one that is nonlinear. Use patterns that are different from those described in Exploration 1.

SUPPORTING English Language Learners

Read aloud the instructions for Exploration 1. Point out that students are being asked to copy, complete, and decide whether patterns are linear or nonlinear. Have them demonstrate understanding of this basic vocabulary by working in groups of mixed language abilities to do the exploration.

Beginning Copy the tables for use by the group.
Intermediate Complete the tables and graph results with the help of group members.
Advanced Consult with group members and decide which patterns are linear and which are nonlinear.
Advanced High Explain the conclusions of the group.

ELPS 2.C.3 Learn basic vocabulary heard during classroom instruction and interactions.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

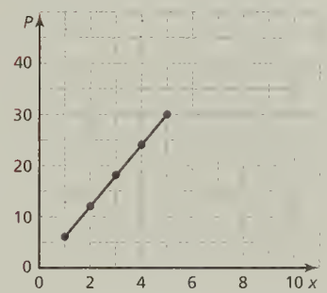
Texas Essential Knowledge and Skills

A.2.A The student is expected to determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

A.3.C The student is expected to graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

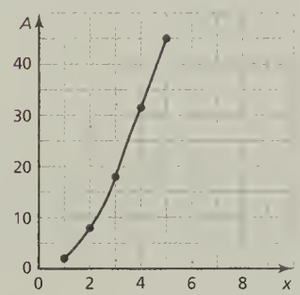
ANSWERS

1. a. 6, 12, 18, 24, 30;



linear; The graph is a line.

b. 2, 8, 18, 32, 50;



nonlinear; The graph is not a line.

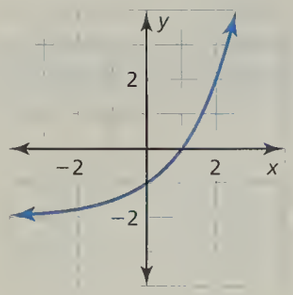
1c–4. See Additional Answers.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Extra Example 1

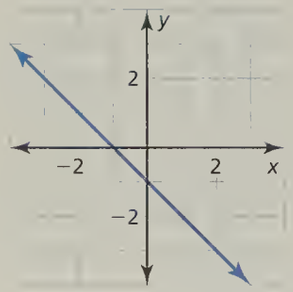
Does the graph represent a *linear* or *nonlinear* function? Explain.

a.



The function is nonlinear because the graph is not a line.

b.



The function is linear because the graph is a line.

Extra Example 2

Does the table represent a *linear* or *nonlinear* function? Explain.

a.

x	2	4	6	8
y	9	18	27	36

The function is linear because the rate of change is constant. As x increases by 2, y increases by 9.

b.

x	1	3	5	7
y	1	27	125	343

The function is nonlinear because the rate of change is not constant. As x increases by 2, y increases by different amounts.

3.2 Lesson

Core Vocabulary

linear equation in two variables, *p. 98*
linear function, *p. 98*
nonlinear function, *p. 98*
solution of a linear equation in two variables, *p. 100*
discrete domain, *p. 100*
continuous domain, *p. 100*

Previous
whole number

What You Will Learn

- Identify linear functions using graphs, tables, and equations.
- Graph linear functions using discrete and continuous data.
- Write real-life problems to fit data.

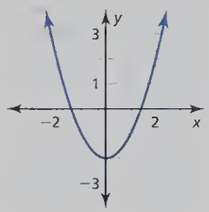
Identifying Linear Functions

A **linear equation in two variables**, x and y, is an equation that can be written in the form $y = mx + b$, where m and b are constants. The graph of a linear equation is a line. Likewise, a **linear function** is a function whose graph is a nonvertical line. A linear function has a constant rate of change and can be represented by a linear equation in two variables. A **nonlinear function** does not have a constant rate of change. So, its graph is *not* a line.

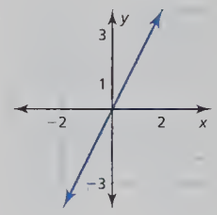
EXAMPLE 1 Identifying Linear Functions Using Graphs

Does the graph represent a *linear* or *nonlinear* function? Explain.

a.



b.



SOLUTION

a. The graph is *not* a line.
▶ So, the function is nonlinear.

b. The graph is a line.
▶ So, the function is linear.

EXAMPLE 2 Identifying Linear Functions Using Tables

Does the table represent a *linear* or *nonlinear* function? Explain.

a.

x	3	6	9	12
y	36	30	24	18

b.

x	1	3	5	7
y	2	9	20	35

SOLUTION

a.

	+3	+3	+3	
x	3	6	9	12
y	36	30	24	18
	−6	−6	−6	

As x increases by 3, y decreases by 6. The rate of change is constant.
▶ So, the function is linear.

b.

	+2	+2	+2	
x	1	3	5	7
y	2	9	20	35
	+7	+11	+15	

As x increases by 2, y increases by different amounts. The rate of change is *not* constant.
▶ So, the function is nonlinear.

REMEMBER
A constant rate of change describes a quantity that changes by equal amounts over equal intervals.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

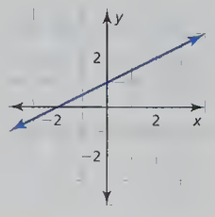
Differentiated Instruction

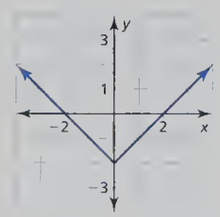
Visual
In Example 2, have students graph the functions represented by tables of ordered pairs. Students can look at the graphs to reinforce their understanding of linear and nonlinear functions.

Monitoring Progress

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Does the graph or table represent a *linear* or *nonlinear* function? Explain.

1. 

2. 

3.

x	0	1	2	3
y	3	5	7	9

4.

x	1	2	3	4
y	16	8	4	2

EXAMPLE 3 Identifying Linear Functions Using Equations

Which of the following equations represent linear functions? Explain.

$$y = 3.8,\ y = \sqrt{x},\ y = 3^x,\ y = \frac{2}{x},\ y = 6(x - 1),\ \text{and}\ x^2 - y = 0$$

SOLUTION

You cannot rewrite the equations $y = \sqrt{x}$, $y = 3^x$, $y = \frac{2}{x}$, and $x^2 - y = 0$ in the form $y = mx + b$. So, these equations cannot represent linear functions.

▶ You can rewrite the equation $y = 3.8$ as $y = 0x + 3.8$ and the equation $y = 6(x - 1)$ as $y = 6x - 6$. So, they represent linear functions.

Monitoring Progress

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Does the equation represent a *linear* or *nonlinear* function? Explain.

5. $y = x + 9$

6. $y = \frac{3x}{5}$

7. $y = 5 - 2x^2$

Concept Summary

Representations of Functions

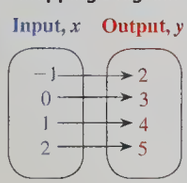
Words An output is 3 more than the input.

Equation $y = x + 3$

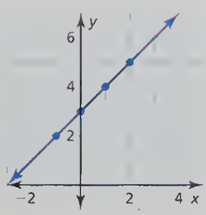
Input-Output Table

Input, x	Output, y
−1	2
0	3
1	4
2	5

Mapping Diagram



Graph



Extra Example 3

Which of the following equations represent linear functions? Explain.

$y = 2^x,\ y = 5x,\ y = 3 - 2x^2,$
$x - y = 12,\ y = \sqrt{x} + 3$

The equations $y = 5x$ and $x - y = 12$ represent linear functions because they can be written in the form $y = mx + b$. You can rewrite $y = 5x$ as $y = 5x + 0$ and you can rewrite $x - y = 12$ as $y = x - 12$. The other equations cannot be written in the form $y = mx + b$, so they cannot represent linear functions.

MONITORING PROGRESS ANSWERS

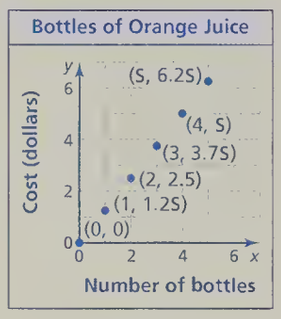
1. linear; The graph is a line.
2. nonlinear; The graph is not a line.
3. linear; As x increases by 1, y increases by 2. The rate of change is constant.
4. nonlinear; As x increases by 1, y decreases by different amounts. The rate of change is not constant.
5. linear; It can be written in the form $y = 1x + 9$.
6. linear; It can be written in the form $y = \frac{3}{5}x + 0$.
7. nonlinear; It cannot be written in the form $y = mx + b$.

Extra Example 4

The linear function $y = 1.25x$ represents the cost y (in dollars) of x bottles of orange juice. Each customer can buy a maximum of five bottles.

a. Find the domain of the function. Is the domain discrete or continuous? Explain. The maximum number of bottles a customer can buy is five. So the domain is 0, 1, 2, 3, 4, and 5. It is discrete because x represents the number of bottles, which must be a whole number.

b. Graph the function using its domain.

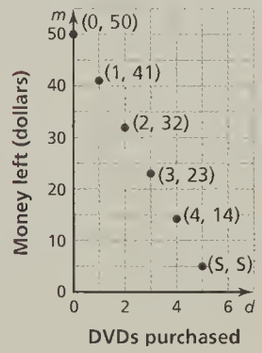


c. Find the range of the function. The range is 0, 1.25, 2.5, 3.75, 5, and 6.25.

MONITORING PROGRESS ANSWERS

8. a. 0, 1, 2, 3, 4, 5; discrete; *Sample answer:* The number of DVDs bought must be a whole number and m must be at least 0.

b.



c. 5, 14, 23, 32, 41, 50

Graphing Linear Functions

A **solution of a linear equation in two variables** is an ordered pair (x, y) that makes the equation true. The graph of a linear equation in two variables is the set of points (x, y) in a coordinate plane that represents all solutions of the equation. Sometimes the points are distinct, and other times the points are connected.

Core Concept

Discrete and Continuous Domains

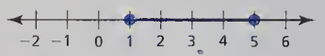
A **discrete domain** is a set of input values that consists of only certain numbers in an interval.

Example: Integers from 1 to 5



A **continuous domain** is a set of input values that consists of all numbers in an interval.

Example: All numbers from 1 to 5



EXAMPLE 4 Graphing Discrete Data

The linear function $y = 15.95x$ represents the cost y (in dollars) of x tickets for a museum. Each customer can buy a maximum of three tickets. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain. (c) Find the range of the function.

STUDY TIP

The domain of a function depends on the real-life context of the function, not just the equation that represents the function.

SOLUTION

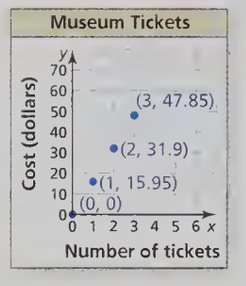
a. You cannot buy part of a ticket, only a certain number of tickets. Because x represents the number of tickets, it must be a whole number. The maximum number of tickets a customer can buy is three.

▶ So, the domain is 0, 1, 2, and 3, and it is discrete.

b. **Step 1** Make an input-output table to find the ordered pairs.

Input, x	$15.95x$	Output, y	(x, y)
0	15.95(0)	0	(0, 0)
1	15.95(1)	15.95	(1, 15.95)
2	15.95(2)	31.9	(2, 31.9)
3	15.95(3)	47.85	(3, 47.85)

Step 2 Plot the ordered pairs. The domain is discrete. So, the graph consists of individual points.



c. Use the input-output table to find the range.

▶ The range is 0, 15.95, 31.9, and 47.85.

Monitoring Progress

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8. The linear function $m = 50 - 9d$ represents the amount m (in dollars) of money you have after buying d DVDs. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain. (c) Find the range of the function.

EXAMPLE 5 Graphing Continuous Data

A cereal bar contains 130 calories. The number c of calories consumed is a function of the number b of bars eaten.

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

d. Find the range of the function.

STUDY TIP

When the domain of a linear function is not specified or cannot be obtained from a real-life context, it is understood to be all real numbers.

SOLUTION

a. As b increases by 1, c increases by 130. The rate of change is constant.

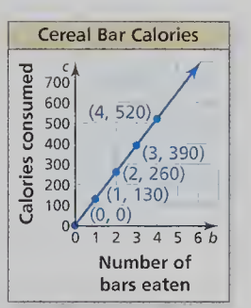
▶ So, this situation represents a linear function.

b. You can eat part of a cereal bar. The number b of bars eaten can be any value greater than or equal to 0.

▶ So, the domain is $b \geq 0$, and it is continuous.

c. **Step 1** Make an input-output table to find ordered pairs.

Input, b	Output, c	(b, c)
0	0	(0, 0)
1	130	(1, 130)
2	260	(2, 260)
3	390	(3, 390)
4	520	(4, 520)



Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points. The line should start at (0, 0) and continue to the right. Use an arrow to indicate that the line continues without end, as shown. The domain is continuous. So, the graph is a line with a domain of $b \geq 0$.

d. From the graph or table, you can see that the output values are greater than or equal to 0.

▶ So, the range is $c \geq 0$.

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9. Is the domain discrete or continuous? Explain.

Input Number of stories, x	1	2	3
Output Height of building (feet), y	12	24	36

10. A 20-gallon bathtub is draining at a rate of 2.5 gallons per minute. The number g of gallons remaining is a function of the number m of minutes. (a) Does this situation represent a linear function? Explain. (b) Find the domain of the function. Is the domain discrete or continuous? Explain. (c) Graph the function using its domain. (d) Find the range of the function.

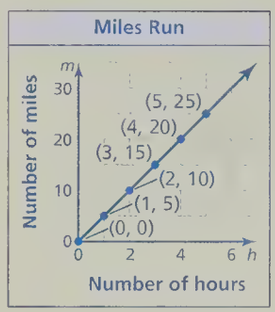
Extra Example 5

A runner's speed is 5 miles per hour. The number m of miles run is a function of the number of hours h spent running.

a. Does this situation represent a linear function? Explain. This situation represents a linear function. As h increases by 1, m increases by 5. The rate of change is constant.

b. Find the domain of the function. Is the domain discrete or continuous? Explain. The domain is $h \geq 0$, and it is continuous. The number of hours spent running can be any value greater than or equal to 0.

c. Graph the function using its domain.



d. Find the range of the function. The range is $m \geq 0$.

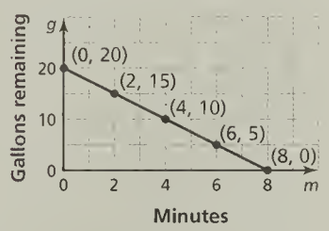
MONITORING PROGRESS ANSWERS

9. discrete; The number of stories must be a whole number.

10. **a.** yes; As m increases by 1, g decreases by 2.5. The rate of change is constant.

b. $0 \leq m \leq 8$; continuous; A tub can drain for part of a minute, so the time can be any value between 0 and 8.

c.

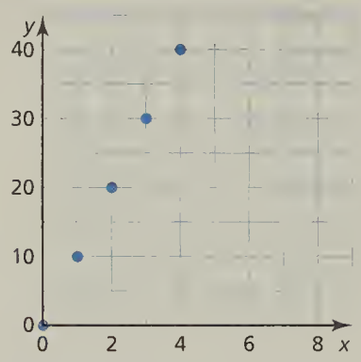


d. $0 \leq g \leq 20$

Extra Example 6

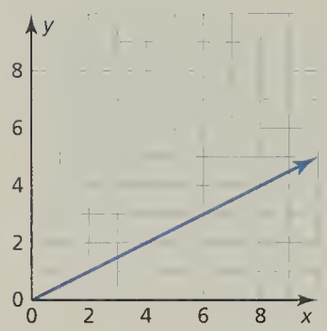
Write a real-life problem to fit the data shown in each graph. Is the domain of each function *discrete* or *continuous*? Explain.

a.



Sample answer: The books on one website each cost $10. Because it is not possible to buy part of a book, the domain is discrete.

b.



Sample answer: The frozen yogurt at one store costs $0.50 for each ounce. Because it is possible to buy any portion of yogurt, the domain is continuous.

MONITORING PROGRESS ANSWERS

11. *Sample answer:* A teacher gives every student 2 bonus points on a 6-point quiz. x is the original score and y is the recorded score; discrete; The original score must be a whole number.

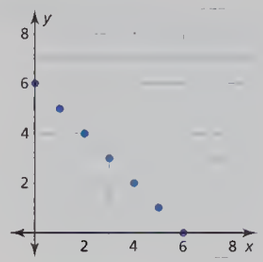
12. *Sample answer:* A marble starts rolling 2 centimeters from a wall and rolls at a constant rate of 0.5 centimeters per second for 6 seconds; continuous; The rolling time can be any value between 0 and 6.

Writing Real-Life Problems

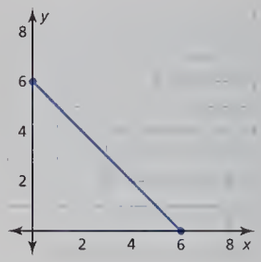
EXAMPLE 6 **Writing Real-Life Problems**

Write a real-life problem to fit the data shown in each graph. Is the domain of each function *discrete* or *continuous*? Explain.

a.



b.





SOLUTION

a. You want to think of a real-life situation in which there are two variables, x and y. Using the graph, notice that the sum of the variables is always 6, and the value of each variable must be a whole number from 0 to 6.

x	0	1	2	3	4	5	6
y	6	5	4	3	2	1	0

Discrete domain

▶ One possibility is two people bidding against each other on six coins at an auction. Each coin will be purchased by one of the two people. Because it is not possible to purchase part of a coin, the domain is discrete.

b. You want to think of a real-life situation in which there are two variables, x and y. Using the graph, notice that the sum of the variables is always 6, and the value of each variable can be any real number from 0 to 6.

$x + y = 6$ or $y = -x + 6$ Continuous domain

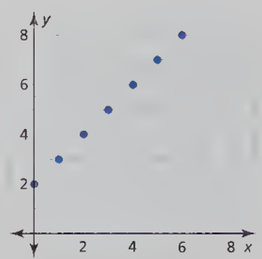
▶ One possibility is two people bidding against each other on 6 ounces of gold dust at an auction. All the dust will be purchased by the two people. Because it is possible to purchase any portion of the dust, the domain is continuous.

Monitoring Progress

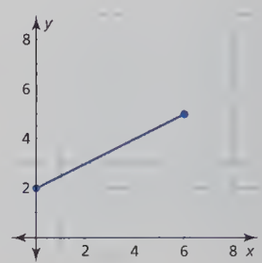
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Write a real-life problem to fit the data shown in the graph. Is the domain of the function *discrete* or *continuous*? Explain.

11.



12.



Closure

- **Exit Ticket:** Explain how to determine whether a function has a continuous or discrete domain. A continuous domain is a set of input values that consist of all numbers in an interval. A discrete domain is a set of input values that consist of only certain numbers.

3.2 Exercises

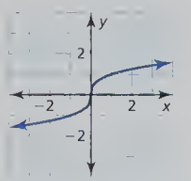
Dynamic Solutions available at *BigIdeasMath.com*

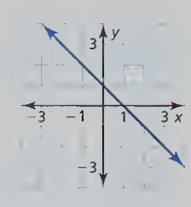
Vocabulary and Core Concept Check

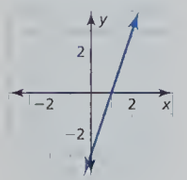
1. **COMPLETE THE SENTENCE** A linear equation in two variables is an equation that can be written in the form _____, where m and b are constants.
2. **VOCABULARY** Compare linear functions and nonlinear functions.
3. **VOCABULARY** Compare discrete domains and continuous domains.
4. **WRITING** How can you tell whether a graph shows a discrete domain or a continuous domain?

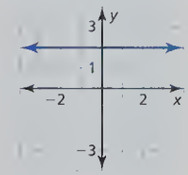
Monitoring Progress and Modeling with Mathematics

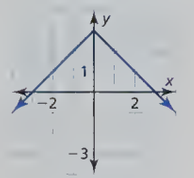
In Exercises 5–10, determine whether the graph represents a *linear* or *nonlinear* function. Explain. *(See Example 1.)*

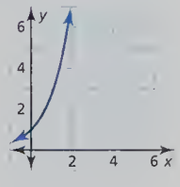
5. 

6. 

7. 

8. 

9. 

10. 

In Exercises 11–14, determine whether the table represents a *linear* or *nonlinear* function. Explain. *(See Example 2.)*

11.

x	1	2	3	4
y	5	10	15	20

12.

x	5	7	9	11
y	−9	−3	−1	3

13.

x	4	8	12	16
y	16	12	7	1

14.

x	−1	0	1	2
y	35	20	5	−10

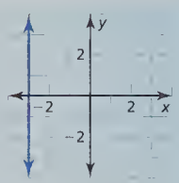
ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in determining whether the table or graph represents a linear function.**

15. ✗

	+2	+2	+2	
x	2	4	6	8
y	4	16	64	256
	×4	×4	×4	

As x increases by 2, y increases by a constant factor of 4. So, the function is linear.

16. ✗



The graph is a line. So, the graph represents a linear function.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–19 odd, 27–39 odd, 44, 52, 55–66

Average: 1–3, 4–34 even, 35–37, 38–46 even, 52, 55–66

Advanced: 1–4, 8–24 even, 25, 26–54 even, 55–66

HOMEWORK CHECK

Basic: 5, 11, 17, 35, 39

Average: 6, 12, 20, 36, 40

Advanced: 10, 14, 22, 36, 42

ANSWERS

1. $y = mx + b$
2. A linear function has a constant rate of change and a graph that is a nonvertical line. A nonlinear function does not have a constant rate of change and has a graph that is not a line.
3. Discrete domains consist of only certain numbers in an interval. Continuous domains consist of all numbers in an interval.
4. A graph with a discrete domain will consist of individual points, a graph with a continuous domain will show a line or curve.
5. nonlinear; The graph is not a line.
6. linear; The graph is a line.
7. linear; The graph is a line.
8. linear; The graph is a line.
9. nonlinear; The graph is not a line.
10. nonlinear; The graph is not a line.
11. linear; As x increases by 1, y increases by 5. The rate of change is constant.
12. nonlinear; As x increases by 2, y increases by different amounts. The rate of change is not constant.
13. nonlinear; As x increases by 4, y decreases by different amounts. The rate of change is not constant.
14. linear; As x increases by 1, y decreases by 15. The rate of change is constant.
15. The increase in y needs to be done by adding or subtracting the same amount to be linear, not multiplying; As x increases by 2, y increases by different amounts. The rate of change is not constant. So, the function is nonlinear.
16. A vertical line is not a function; The graph is a vertical line. So, the graph does not represent a function.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

17. nonlinear; It cannot be rewritten in the form $y = mx + b$.
18. linear; It can be rewritten as $y = -3x + 7$.
19. linear; It can be rewritten as $y = -1x + 2$.
20. nonlinear; It cannot be rewritten in the form $y = mx + b$.
21. linear; It can be rewritten as $y = 18x + 12$.
22. linear; It can be rewritten as $y = \frac{9}{5}x + 0$.
23. linear; It can be rewritten as $y = 9x - 13$.
24. nonlinear; It cannot be rewritten in the form $y = mx + b$.
25. A, C, F; None of these can be rewritten in the form $y = mx + b$.
26. 2, 5, 8
27. 2, 4, 6; discrete; The graph consists of individual points.
28. $0 \le x \le 7$; continuous; The graph is a line segment.
29. discrete; The number of bags must be a whole number.
30. continuous; The number of years can be any value greater than or equal to 0.
31. continuous; The time can be any value greater than or equal to 0.
32. discrete; The number of relay teams must be a whole number.
33. There is no point with an x-value of 2.5; 2.5 is not in the domain.
34. The graph shows a continuous domain; The graph ends at $x = 6$, so the domain is $0 \le x \le 6$.

In Exercises 17–24, determine whether the equation represents a *linear* or *nonlinear* function. Explain. *(See Example 3.)*

17. $y = x^2 + 13$
18. $y = 7 - 3x$
19. $y = \sqrt[3]{8} - x$
20. $y = 4x(8 - x)$
21. $2 + \frac{1}{6}y = 3x + 4$
22. $y - x = 2x - \frac{2}{3}y$
23. $18x - 2y = 26$
24. $2x + 3y = 9xy$

25. **CLASSIFYING FUNCTIONS** Which of the following equations *do not* represent linear functions? Explain.

Ⓐ $12 = 2x^2 + 4y^2$ Ⓑ $y - x + 3 = x$

Ⓒ $x = 8$ Ⓓ $x = 9 - \frac{3}{4}y$

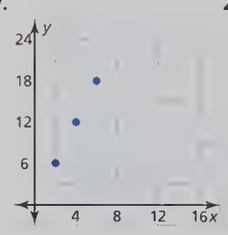
Ⓔ $y = \frac{5x}{11}$ Ⓕ $y = \sqrt{x} + 3$

26. **USING STRUCTURE** Fill in the table so it represents a linear function.

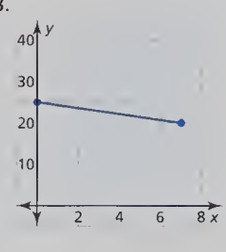
x	5	10	15	20	25
y	−1				11

In Exercises 27 and 28, find the domain of the function represented by the graph. Determine whether the domain is *discrete* or *continuous*. Explain.

27.



28.



In Exercises 29–32, determine whether the domain is *discrete* or *continuous*. Explain.

29.

Input Bags, x	2	4	6
Output Marbles, y	20	40	60

30.

Input Years, x	1	2	3
Output Height of tree (feet), y	6	9	12

31.

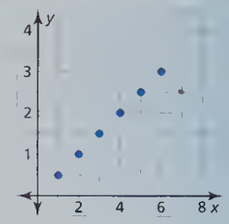
Input Time (hours), x	3	6	9
Output Distance (miles), y	150	300	450

32.

Input Relay teams, x	0	1	2
Output Athletes, y	0	4	8

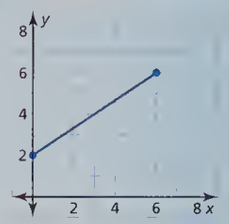
ERROR ANALYSIS In Exercises 33 and 34, describe and correct the error in the statement about the domain.

33. ✗



2.5 is in the domain.

34. ✗



The graph ends at $x = 6$, so the domain is discrete.

35. **MODELING WITH MATHEMATICS** The linear function $m = 55 - 8.5b$ represents the amount m (in dollars) of money that you have after buying b books. *(See Example 4.)*

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

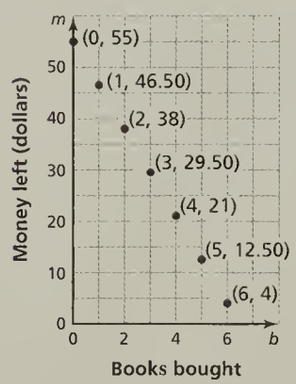
b. Graph the function using its domain.

c. Find the range of the function.



35. a. 0, 1, 2, 3, 4, 5, 6; discrete; The number of books must be a whole number.

b.



c. 4, 12.50, 21, 29.50, 38, 46.50, 55

36. MODELING WITH MATHEMATICS The number y of calories burned after x hours of rock climbing is represented by the linear function $y = 650x$.



a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

c. Find the range of the function.

37. MODELING WITH MATHEMATICS You are researching the speed of sound waves in dry air at 86°F. The table shows the distances d (in miles) sound waves travel in t seconds. *(See Example 5.)*

Time (seconds), t	Distance (miles), d
2	0.434
4	0.868
6	1.302
8	1.736
10	2.170

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

d. Find the range of the function.

38. MODELING WITH MATHEMATICS The function $y = 30 + 5x$ represents the cost y (in dollars) of having your dog groomed and buying x extra services.



a. Does this situation represent a linear function? Explain.

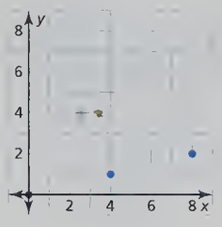
b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

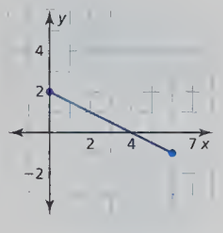
d. Find the range of the function.

WRITING In Exercises 39–42, write a real-life problem to fit the data shown in the graph. Determine whether the domain of the function is *discrete* or *continuous*. Explain. *(See Example 6.)*

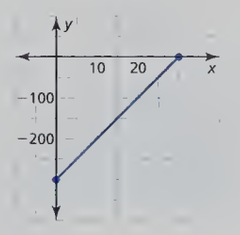
39.



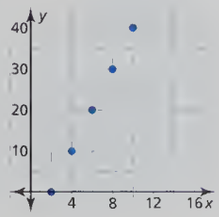
40.



41.



42.



43. USING STRUCTURE The table shows your earnings y (in dollars) for working x hours.

a. What is the missing y-value that makes the table represent a linear function?

b. What is your hourly pay rate?

Time (hours), x	Earnings (dollars), y
4	40.80
5	
6	61.20
7	71.40

44. MAKING AN ARGUMENT The linear function $d = 50t$ represents the distance d (in miles) Car A is from a car rental store after t hours. The table shows the distances Car B is from the rental store.

Time (hours), t	Distance (miles), d
1	60
3	180
5	310

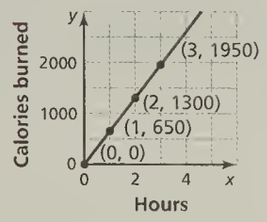
a. Does the table represent a linear or nonlinear function? Explain.

b. Your friend claims Car B is moving at a faster rate. Is your friend correct? Explain.

ANSWERS

36. a. $x \geq 0$; continuous; The time can be any number of hours greater than or equal to 0.

b.

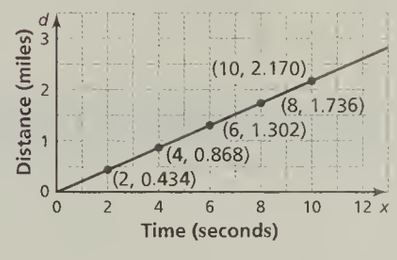


c. $y \geq 0$

37. a. yes; As t increases by 2, d increases by 0.434. The rate of change is constant.

b. $t \geq 0$; continuous; The time can be any value greater than or equal to 0.

c.

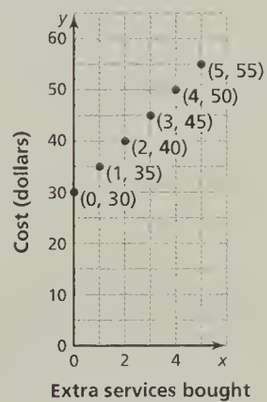


d. $d \geq 0$

38. a. yes; The equation can be rewritten as $y = 5x + 30$.

b. 0, 1, 2, 3, 4, 5; discrete; The number of extra services must be a whole number.

c.



d. 30, 35, 40, 45, 50, 55

39. *Sample answer:* The number of hours on a parking meter is a function of the number of tokens used, 4 tokens for 1 hour and a maximum time of 2 hours; discrete; The number of tokens used must be 0, 4, or 8.

40. *Sample answer:* The temperature of a substance in degrees Celsius as a function of time in minutes; continuous; The time can be any value between 0 and 6 minutes.

41. *Sample answer:* The depth (in fect) of a scuba diver returning to the surface of an ocean as a function of the time; continuous; The time can be any value from 0 to 30.

42. *Sample answer:* The total discount on gloves as a function of the number of gloves purchased on sale, where a discount of \$10 per pair is applied after paying full price for the first pair, with a maximum of 5 pairs; discrete; The number of gloves bought must be an even number no more than 10.

43. a. 51.00

b. \$10.20

44. a. nonlinear; As t increases by 2, d increases by different amounts. The rate of change is not constant.

b. yes; Using $d = 50t$, Car A has traveled a shorter distance for each of the times given in the chart for Car B.

ANSWERS

45. nonlinear; $V = 9s^2$ cannot be written in linear form.

46. linear; The formula can be written in the form $V = 6b + 0$.

47. linear; The formula can be written in the form $V = (4\pi)h + 0$.

48. nonlinear; $V = 5\pi r^2$ cannot be written in linear form.

49. **a.** the total gallons of water in one jug of each type; continuous

b. the total number of jugs of both types; discrete

c. the total gallons of water in all the jugs of the first type; continuous

d. the total gallons of water in all the jugs of both types; continuous

50–66. See Additional Answers.

Mini-Assessment

1. Does the table represent a *linear* or *nonlinear* function? Explain.

x	3	5	7	9
y	−6	0	4	2

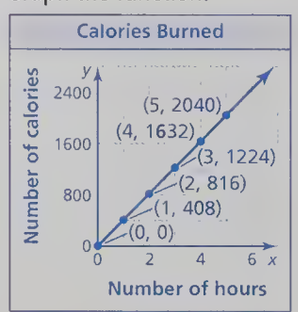
The function is nonlinear. The rate of change is not constant.

2. Does $y = x^2 + 1$ represent a *linear* or *nonlinear* function? Explain. Nonlinear; It cannot be written in the form $y = mx + b$.

3. The function $y = 408x$ represents the number of calories burned y after x hours of hiking.

 a. Find the domain of the function. Is the domain discrete or continuous? Explain. domain: $x \geq 0$; It is continuous because the number of hours can be any value ≥ 0.

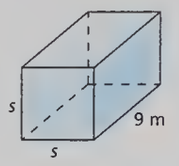
 b. Graph the function.



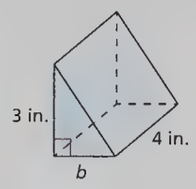
 c. Find the range of the function. The range is $y \geq 0$.

MATHEMATICAL CONNECTIONS **In Exercises 45–48, tell whether the volume of the solid is a linear or nonlinear function of the missing dimension(s). Explain.**

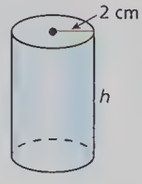
45.



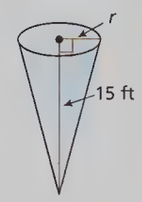
46.



47.



48.



49. REASONING A water company fills two different-sized jugs. The first jug can hold x gallons of water. The second jug can hold y gallons of water. The company fills A jugs of the first size and B jugs of the second size. What does each expression represent? Does each expression represent a set of discrete or continuous values?

a. $x + y$

b. $A + B$

c. Ax

d. $Ax + By$

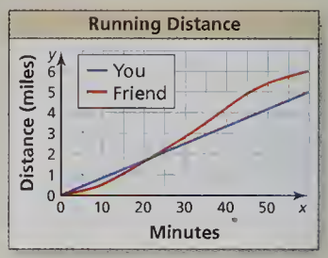


50. THOUGHT PROVOKING You go to a farmer's market to buy tomatoes. Graph a function that represents the cost of buying tomatoes. Explain your reasoning.

51. CLASSIFYING A FUNCTION Is the function represented by the ordered pairs linear or nonlinear? Explain your reasoning.

(0, 2), (3, 14), (5, 22), (9, 38), (11, 46)

52. HOW DO YOU SEE IT? You and your friend go running. The graph shows the distances you and your friend run.



a. Describe your run and your friend's run. Who runs at a constant rate? How do you know? Why might a person not run at a constant rate?

b. Find the domain of each function. Describe the domains using the context of the problem.

WRITING **In Exercises 53 and 54, describe a real-life situation for the constraints.**

53. The function has at least one negative number in the domain. The domain is continuous.

54. The function gives at least one negative number as an output. The domain is discrete.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when $x = 2$. *(Skills Review Handbook)*

55. $6x + 8$ **56.** $10 - 2x + 8$ **57.** $4(x + 2 - 5x)$ **58.** $\frac{x}{2} + 5x - 7$

Solve the equation. Check your solution. *(Section 1.2)*

59. $2x + 10 = 16$ **60.** $8 = 5 - 3p$ **61.** $\frac{5t + 2}{7} = 6$ **62.** $12 = y - 7 - 3y$

Solve the inequality. Graph the solution. *(Section 2.3)*

63. $8x \geq 4$ **64.** $-3t < 24$ **65.** $\frac{r}{16} \leq -1$ **66.** $-\frac{1}{3}a > 6$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

3.3 Function Notation

Texas Essential Knowledge and Skills
A.3.C
A.12.B

Essential Question How can you use function notation to represent a function?

The notation $f(x)$, called **function notation**, is another name for y. This notation is read as "the value of f at x" or "f of x." The parentheses do not imply multiplication. You can use letters other than f to name a function. The letters g, h, j, and k are often used to name functions.

USING PRECISE MATHEMATICAL LANGUAGE
To be proficient in math, you need to use clear definitions and state the meanings of the symbols you use.

EXPLORATION 1 Matching Functions with Their Graphs

Work with a partner. Match each function with its graph.

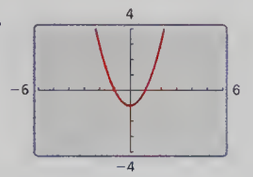
a. $f(x) = 2x - 3$

b. $g(x) = -x + 2$

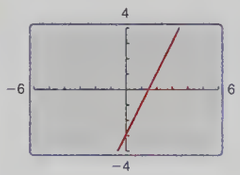
c. $h(x) = x^2 - 1$

d. $j(x) = 2x^2 - 3$

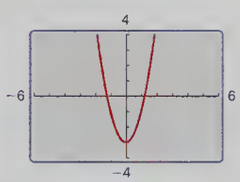
A.



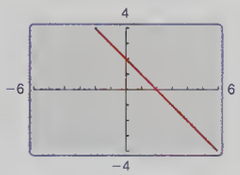
B.



C.



D.



EXPLORATION 2 Evaluating a Function

Work with a partner. Consider the function

$$f(x) = -x + 3.$$

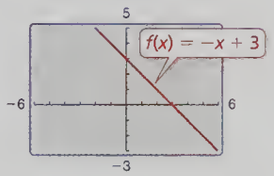
Locate the points $(x, f(x))$ on the graph. Explain how you found each point.

a. $(-1, f(-1))$

b. $(0, f(0))$

c. $(1, f(1))$

d. $(2, f(2))$



Communicate Your Answer

3. How can you use function notation to represent a function? How are standard notation and function notation similar? How are they different?

Standard Notation	*Function Notation*
$y = 2x + 5$	$f(x) = 2x + 5$

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.3.C The student is expected to graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

A.12.B The student is expected to evaluate functions, expressed in function notation, given one or more elements in their domains.

ANSWERS

1. **a.** B
 b. D
 c. A
 d. C
2. **a.** $(-1, 4)$
 b. $(0, 3)$
 c. $(1, 2)$
 d. $(2, 1)$
 Locate the x-coordinate, trace it up to the line, and then trace over to the y-axis to determine the y-coordinate.
3. Replace "y" with "$f(x)$" in the equation; They both give the rule for determining the output; Standard notation uses "y" to indicate the output, function notation uses "$f(x)$" to indicate the output.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Read aloud the instructions that accompany the Essential Question. Have students work in pairs of mixed language abilities to complete the explorations and Communicate Your Answer.

Beginning/Advanced Beginning students read aloud the examples of function notation as modeled by the teacher in the instructions (the value of f at x, f of x). Then work with Advanced students to determine answers. Advanced students state the answers using complete sentences.

Intermediate/Advanced High Intermediate students read aloud the examples of function notation and state their answers. Advanced High students provide explanations for the answers.

ELPS 3.D.2 Speak using grade-level content area vocabulary in context to build academic language proficiency.

Extra Example 1

Evaluate $f(x) = 2x - 7$ when $x = 4$ and $x = -4$.

When $x = 4$, $f(x) = 1$, and when $x = -4$, $f(x) = -15$.

Extra Example 2

Let $f(t)$ be the outside temperature (°F) t hours after 10 A.M. Explain the meaning of each statement.

a. $f(0) = 75$

The temperature at 10 A.M. is 75°F.

b. $f(3) = n$

The temperature at 1 P.M. (3 hours after 10 A.M.) is n°F.

c. $f(1) > f(9)$

The temperature at 11 A.M. (1 hour after 10 A.M.) is greater than the temperature at 7 P.M. (9 hours after 10 A.M.).

MONITORING PROGRESS ANSWERS

1. $-13; -5; 1$
2. $3; -1; -4$
3. a. The temperature is 75°F at 1 P.M.
 b. The temperature is 70°F m hours after 9 A.M.
 c. The temperature at 11 A.M. is the same as the temperature at 6 P.M.
 d. The temperature at 3 P.M. is greater than the temperature at 9 A.M.

3.3 Lesson

What You Will Learn

- Use function notation to evaluate and interpret functions.
- Use function notation to solve and graph functions.
- Solve real-life problems using function notation.

Core Vocabulary

function notation, *p. 108*

Previous

linear function
quadrant

Using Function Notation to Evaluate and Interpret

You know that a linear function can be written in the form $y = mx + b$. By naming a linear function f, you can also write the function using **function notation**.

$f(x) = mx + b$ Function notation

The notation $f(x)$ is another name for y. If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x. You can use letters other than f to name a function, such as g or h.

READING

The notation $f(x)$ is read as "the value of f at x" or "f of x." It does not mean "f times x."

EXAMPLE 1 Evaluating a Function

Evaluate $f(x) = -4x + 7$ when $x = 2$ and $x = -2$.

SOLUTION

$f(x) = -4x + 7$	Write the function.	$f(x) = -4x + 7$
$f(2) = -4(2) + 7$	Substitute for x.	$f(-2) = -4(-2) + 7$
$= -8 + 7$	Multiply.	$= 8 + 7$
$= -1$	Add.	$= 15$

▶ When $x = 2$, $f(x) = -1$, and when $x = -2$, $f(x) = 15$.

EXAMPLE 2 Interpreting Function Notation

Let $f(t)$ be the outside temperature (°F) t hours after 6 A.M. Explain the meaning of each statement.

a. $f(0) = 58$ **b.** $f(6) = n$ **c.** $f(3) < f(9)$

SOLUTION

a. The initial value of the function is 58. So, the temperature at 6 A.M. is 58°F.

b. The output of f when $t = 6$ is n. So, the temperature at noon (6 hours after 6 A.M.) is n°F.

c. The output of f when $t = 3$ is less than the output of f when $t = 9$. So, the temperature at 9 A.M. (3 hours after 6 A.M.) is less than the temperature at 3 P.M. (9 hours after 6 A.M.).

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Evaluate the function when $x = -4$, 0, and 3.

1. $f(x) = 2x - 5$
2. $g(x) = -x - 1$
3. **WHAT IF?** In Example 2, let $f(t)$ be the outside temperature (°F) t hours after 9 A.M. Explain the meaning of each statement.

 a. $f(4) = 75$ **b.** $f(m) = 70$ **c.** $f(2) = f(9)$ **d.** $f(6) > f(0)$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Comprehension

Make sure that students understand that $f(x)$ is another name for y. Some students may confuse the function notation with the use of parentheses as a multiplication symbol because they know that the expression $2(x)$ means 2 times x.

Using Function Notation to Solve and Graph

EXAMPLE 3 **Solving for the Independent Variable**

For $h(x) = \frac{2}{3}x - 5$, find the value of x for which $h(x) = -7$.

SOLUTION

$h(x) = \frac{2}{3}x - 5$	Write the function.
$-7 = \frac{2}{3}x - 5$	Substitute -7 for $h(x)$.
$+5 \quad +5$	Add 5 to each side.
$-2 = \frac{2}{3}x$	Simplify.
$\frac{3}{2} \cdot (-2) = \frac{3}{2} \cdot \frac{2}{3}x$	Multiply each side by $\frac{3}{2}$.
$-3 = x$	Simplify.

▶ When $x = -3$, $h(x) = -7$.

EXAMPLE 4 **Graphing a Linear Function in Function Notation**

Graph $f(x) = 2x + 5$.

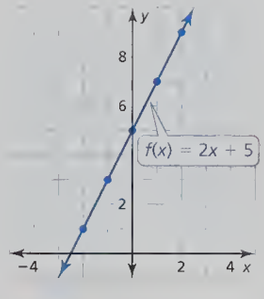
SOLUTION

Step 1 Make an input-output table to find ordered pairs.

x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points.



STUDY TIP

The graph of $y = f(x)$ consists of the points $(x, f(x))$.

Monitoring Progress

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Find the value of x so that the function has the given value.

4. $f(x) = 6x + 9$; $f(x) = 21$

5. $g(x) = -\frac{1}{2}x + 3$; $g(x) = -1$

Graph the linear function.

6. $f(x) = 3x - 2$

7. $g(x) = -x + 4$

8. $h(x) = -\frac{3}{4}x - 1$

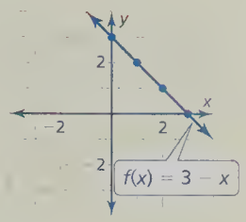
Extra Example 3

For $g(x) = \frac{1}{3}x - 2$ find the value of x for which $g(x) = -4$.

When $x = -6$, $g(x) = -4$.

Extra Example 4

Graph $f(x) = 3 - x$.

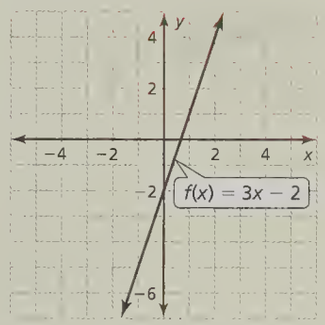


MONITORING PROGRESS ANSWERS

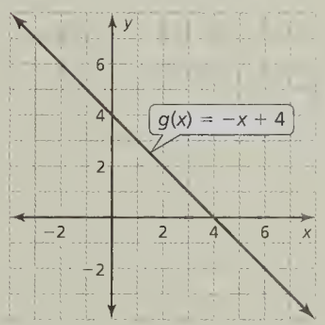
4. $x = 2$

5. $x = 8$

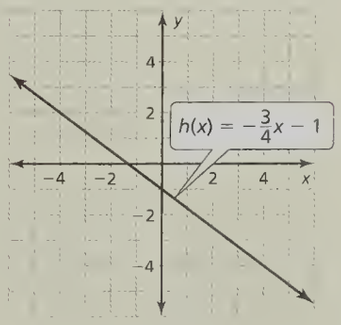
6.



7.



8.



Extra Example 5

Use the graph in Example 5 for the first flight of the helicopter. Let $f(x) = 300 - 75x$ represent the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

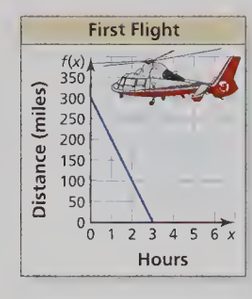
The first flight takes less time. For the first flight, when $f(x) = 0$, $x = 3$. For the second flight, when $f(x) = 0$, $x = 4$.

MONITORING PROGRESS ANSWER

9. the first flight; The graph of the second flight shows that when $f(x) = 0$, x is slightly more than 3.

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**



The graph shows the number of miles a helicopter is from its destination after x hours on its first flight. On its second flight, the helicopter travels 50 miles farther and increases its speed by 25 miles per hour. The function $f(x) = 350 - 125x$ represents the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

SOLUTION

1. Understand the Problem You are given a graph of the first flight and an equation of the second flight. You are asked to compare the flight times to determine which flight takes less time.

2. Make a Plan Graph the function that represents the second flight. Compare the graph to the graph of the first flight. The x-value that corresponds to $f(x) = 0$ represents the flight time.

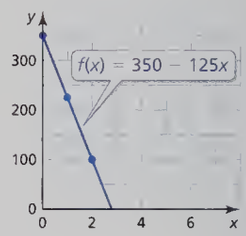
3. Solve the Problem Graph $f(x) = 350 - 125x$.

Step 1 Make an input-output table to find the ordered pairs.

x	0	1	2	3
$f(x)$	350	225	100	−25

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points. Note that the function only makes sense when x and $f(x)$ are positive. So, only draw the line in the first quadrant.



▶ From the graph of the first flight, you can see that when $f(x) = 0$, $x = 3$. From the graph of the second flight, you can see that when $f(x) = 0$, x is slightly less than 3. So, the second flight takes less time.

4. Look Back You can check that your answer is correct by finding the value of x for which $f(x) = 0$.

$f(x) = 350 - 125x$ Write the function.

$0 = 350 - 125x$ Substitute 0 for $f(x)$.

$-350 = -125x$ Subtract 350 from each side.

$2.8 = x$ Divide each side by −125.

So, the second flight takes 2.8 hours, which is less than 3.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

9. WHAT IF? Let $f(x) = 250 - 75x$ represent the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

Closure

- **Writing Prompt:** What do you understand when you see $f(x) = 4x - 3$? *Sample answer:* The equation is a function with the name f. To find the value of f at x, substitute a value of x into the function and evaluate.

3.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** When you write the function $y = 2x + 10$ as $f(x) = 2x + 10$, you are using _____.

2. **REASONING** Your height can be represented by a function h, where the input is your age. What does $h(14)$ represent?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, evaluate the function when $x = -2$, 0, and 5. *(See Example 1.)*

3. $f(x) = x + 6$
4. $g(x) = 3x$
5. $h(x) = -2x + 9$
6. $r(x) = -x - 7$
7. $p(x) = -3 + 4x$
8. $b(x) = 18 - 0.5x$
9. $v(x) = 12 - 2x - 5$
10. $n(x) = -1 - x + 4$

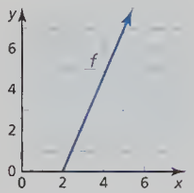
11. **INTERPRETING FUNCTION NOTATION** Let $c(t)$ be the number of customers in a restaurant t hours after 8 A.M. Explain the meaning of each statement. *(See Example 2.)*
 a. $c(0) = 0$
 b. $c(3) = c(8)$
 c. $c(n) = 29$
 d. $c(13) < c(12)$

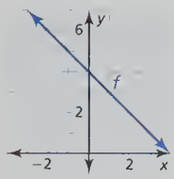
12. **INTERPRETING FUNCTION NOTATION** Let $H(x)$ be the percent of U.S. households with Internet use x years after 1980. Explain the meaning of each statement.
 a. $H(23) = 55$
 b. $H(4) = k$
 c. $H(27) \geq 61$
 d. $H(17) + H(21) \approx H(29)$

In Exercises 13–18, find the value of x so that the function has the given value. *(See Example 3.)*

13. $h(x) = -7x$; $h(x) = 63$
14. $t(x) = 3x$; $t(x) = 24$
15. $m(x) = 4x + 15$; $m(x) = 7$
16. $k(x) = 6x - 12$; $k(x) = 18$
17. $q(x) = \frac{1}{2}x - 3$; $q(x) = -4$
18. $j(x) = -\frac{4}{5}x + 7$; $j(x) = -5$

In Exercises 19 and 20, find the value of x so that $f(x) = 7$.

19.


20.


21. **MODELING WITH MATHEMATICS** The function $C(x) = 17.5x - 10$ represents the cost (in dollars) of buying x tickets to the orchestra with a $10 coupon.
 a. How much does it cost to buy five tickets?
 b. How many tickets can you buy with $130?

22. **MODELING WITH MATHEMATICS** The function $d(t) = 300{,}000t$ represents the distance (in kilometers) that light travels in t seconds.
 a. How far does light travel in 15 seconds?
 b. How long does it take light to travel 12 million kilometers?



In Exercises 23–28, graph the linear function. *(See Example 4.)*

23. $p(x) = 4x$
24. $h(x) = -5$
25. $d(x) = -\frac{1}{2}x - 3$
26. $w(x) = \frac{3}{5}x + 2$
27. $g(x) = -4 + 7x$
28. $f(x) = 3 - 6x$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–15 odd, 19–31 odd, 34, 37–42

Average: 1, 2–30 even, 31, 33, 34, 37–42

Advanced: 1, 2, 8–12 even, 17, 18–30 even, 31–42

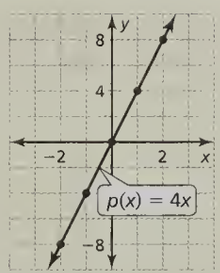
HOMEWORK CHECK

Basic: 3, 11, 13, 23, 29

Average: 6, 12, 14, 24, 30

Advanced: 8, 12, 18, 26, 30

ANSWERS

1. function notation
2. your height at 14 years old
3. 4; 6; 11
4. −6; 0; 15
5. 13; 9; −1
6. −5; −7; −12
7. −11; −3; 17
8. 19; 18; 15.5
9. 11; 7; −3
10. 5; 3; −2
11. a. There are no customers in the restaurant at 8 A.M.
 b. There are the same number of customers in the restaurant at 11 A.M. as there are at 4 P.M.
 c. There are 29 customers in the restaurant n hours after 8 A.M.
 d. There are fewer customers in the restaurant at 9 P.M. than there are at 8 P.M.
12. a. 55% of U.S. households had Internet use in 2003.
 b. k% of U.S. households had Internet use in 1984.
 c. At least 61% of U.S. households had Internet use in 2007.
 d. The sum of the percents of U.S. households with Internet use in 1997 and 2001 is about the same as the percent of U.S. households with Internet use in 2009.
13. $x = -9$
14. $x = 8$
15. $x = -2$
16. $x = 5$
17. $x = -2$
18. $x = 15$
19. $x = 5$
20. $x = -3$
21. a. $77.50
 b. 8 tickets
22. a. 4,500,000 km
 b. 40 sec
23.


24–28. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

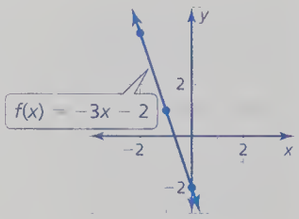
ANSWERS

29. the tablet computer; The graph of the tablet computer shows that when $p(t) = 0$, t is 6, which is greater than 5.

30–42. See Additional Answers.

Mini-Assessment

1. Evaluate $f(x) = 3x - 6$ when $x = 2$ and $x = -4$. When $x = 2$, $f(x) = 0$, and when $x = -4$, $f(x) = -18$.
2. Let $f(r)$ be the number of riders on a bus r hours after 8 A.M. Explain the meaning of the statement $f(5) = 23$. At 1 P.M. there are 23 riders.
3. For $g(x) = -5x + 1$, find the value of x for which $g(x) = -9$. When $x = 2$, $g(x) = -9$.
4. Graph $f(x) = -3x - 2$.

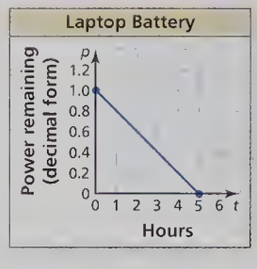


5. The table shows the number of miles a car is from its destination after x hours on its first trip.

Hours	Miles
1	150
2	100
3	50

On its second trip, the car travels 10 miles farther and increases its speed by 10 miles per hour. The function $f(x) = 210 - 60x$ represents the second trip. Which trip takes less time? Explain. The second trip takes less time. The first trip takes 4 hours, and the second trip takes 3.5 hours.

29. PROBLEM SOLVING The graph shows the percent p (in decimal form) of battery power remaining in a laptop computer after t hours of use. A tablet computer initially has 75% of its battery power remaining and loses 12.5% per hour. Which computer's battery will last longer? Explain. *(See Example 5.)*



30. PROBLEM SOLVING The function $C(x) = 25x + 50$ represents the labor cost (in dollars) for Certified Remodeling to build a deck, where x is the number of hours of labor. The table shows sample labor costs from its main competitor, Master Remodeling. The deck is estimated to take 8 hours of labor. Which company would you hire? Explain.

Hours	Cost
2	$130
4	$160
6	$190

31. MAKING AN ARGUMENT Let $P(x)$ be the number of people in the U.S. who own a cell phone x years after 1990. Your friend says that $P(x + 1) > P(x)$ for any x because $x + 1$ is always greater than x. Is your friend correct? Explain.

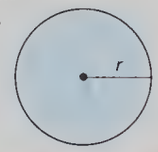
32. THOUGHT PROVOKING Let $B(t)$ be your bank account balance after t days. Describe a situation in which $B(0) < B(4) < B(2)$.

33. MATHEMATICAL CONNECTIONS Rewrite each geometry formula using function notation. Evaluate each function when $r = 5$ feet. Then explain the meaning of the result.

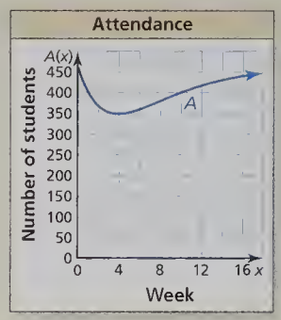
a. Diameter, $d = 2r$

b. Area, $A = \pi r^2$

c. Circumference, $C = 2\pi r$



34. HOW DO YOU SEE IT? The function $y = A(x)$ represents the attendance at a high school x weeks after a flu outbreak. The graph of the function is shown.



a. What happens to the school's attendance after the flu outbreak?

b. Estimate $A(13)$ and explain its meaning.

c. Use the graph to estimate the solution(s) of the equation $A(x) = 400$. Explain the meaning of the solution(s).

d. What was the least attendance? When did that occur?

e. How many students do you think are enrolled at this high school? Explain your reasoning.

35. INTERPRETING FUNCTION NOTATION Let f be a function. Use each statement to find the coordinates of a point on the graph of f.

a. $f(5)$ is equal to 9.

b. A solution of the equation $f(n) = -3$ is 5.

36. REASONING Given a function f, tell whether the statement

$$f(a + b) = f(a) + f(b)$$

is true or false for all inputs a and b. If it is false, explain why.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. *(Section 2.5)*

37. $-2 \leq x - 11 \leq 6$

38. $5a < -35$ *or* $a - 14 > 1$

39. $-16 < 6k + 2 < 0$

40. $2d + 7 < -9$ *or* $4d - 1 > -3$

41. $5 \leq 3y + 8 < 17$

42. $4v + 9 \leq 5$ *or* $-3v \geq -6$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

3.1–3.3 What Did You Learn?

Core Vocabulary

relation, *p. 90*
function, *p. 90*
domain, *p. 92*
range, *p. 92*
independent variable, *p. 93*
dependent variable, *p. 93*
linear equation in two variables, *p. 98*
linear function, *p. 98*
nonlinear function, *p. 98*
solution of a linear equation in two variables, *p. 100*
discrete domain, *p. 100*
continuous domain, *p. 100*
function notation, *p. 108*

Core Concepts

Section 3.1

Determining Whether Relations Are Functions, *p. 90*
Vertical Line Test, *p. 91*
The Domain and Range of a Function, *p. 92*
Independent and Dependent Variables, *p. 93*

Section 3.2

Linear and Nonlinear Functions, *p. 98*
Representations of Functions, *p. 99*
Discrete and Continuous Domains, *p. 100*

Section 3.3

Using Function Notation, *p. 108*

Mathematical Thinking

1. How can you use technology to confirm your answers in Exercises 46–49 on page 96?
2. How can you use patterns to solve Exercise 43 on page 105?
3. How can you make sense of the quantities in the function in Exercise 21 on page 111?

Study Skills

Staying Focused during Class

As soon as class starts, quickly review your notes from the previous class and start thinking about math.

Repeat what you are writing in your head.

When a particular topic is difficult, ask for another example.



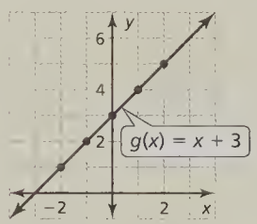
Dynamic Teaching Tools

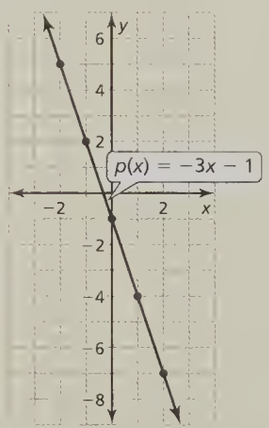
- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

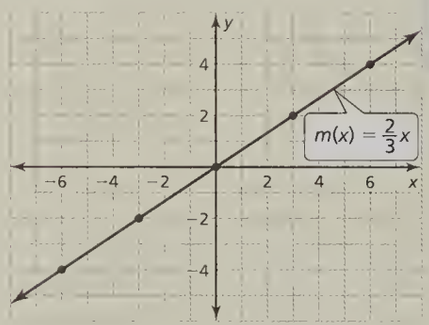
ANSWERS

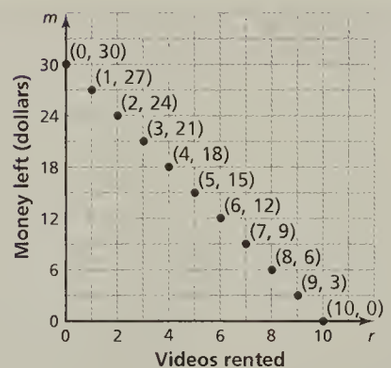
1. Use a graphing calculator to graph the functions and check the domain and range of the graphs.
2. Look for a pattern in the change in earnings.
3. The function represents the cost in dollars for the tickets at $17.50 each minus the $10 discount. All quantities are in dollars.

ANSWERS

1. function; Each input has exactly one output.
2. not a function; The input -10 has two outputs, 2 and 6, and the input -8 has two outputs, 3 and 8.
3. domain: 0, 1, 2, 3, 4; range: $-5, -3, -1, 1$
4. domain: all real numbers; range: $y \le 3$
5. domain: $-3 \le x \le 3$; range: $-1 \le y \le 3$
6. linear; The graph is a line.
7. nonlinear; As x increases by 5, y increases by different amounts. The rate of change is not constant.
8. nonlinear; It cannot be written in the form $y = mx + b$.
9. continuous; The depth can be any number of feet.
10. discrete; The number of hats must be a whole number.
11. $x = 5$
12. 

13. 

14. 

15. a. m is the dependent variable and r is the independent variable.
 b. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; discrete; The number of video games rented must be a whole number.
 c. 

 d. 0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
16. a. 495 mi
 b. 12.5 h

3.1–3.3 Quiz

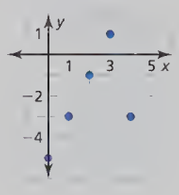
Determine whether the relation is a function. Explain. *(Section 3.1)*

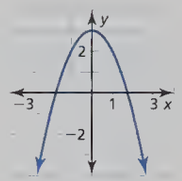
1.

Input, x	−1	0	1	2	3
Output, y	0	1	4	4	8

2. $(-10, 2), (-8, 3), (-6, 5), (-8, 8), (-10, 6)$

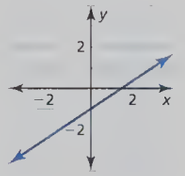
Find the domain and range of the function represented by the graph. *(Section 3.1)*

3. 

4. 

5. 

Determine whether the graph, table, or equation represents a *linear* or *nonlinear* function. Explain. *(Section 3.2)*

6. 

7.

x	y
−5	3
0	7
5	10

8. $y = x(2 - x)$

Determine whether the domain is *discrete* or *continuous*. Explain. *(Section 3.2)*

9.

Depth (feet), x	33	66	99
Pressure (ATM), y	2	3	4

10.

Hats, x	2	3	4
Cost (dollars), y	36	54	72

11. For $w(x) = -2x + 7$, find the value of x for which $w(x) = -3$. *(Section 3.3)*

Graph the linear function. *(Section 3.3)*

12. $g(x) = x + 3$
13. $p(x) = -3x - 1$
14. $m(x) = \frac{2}{3}x$

15. The function $m = 30 - 3r$ represents the amount m (in dollars) of money you have after renting r video games. *(Section 3.1 and Section 3.2)*
 a. Identify the independent and dependent variables.
 b. Find the domain of the function. Is the domain discrete or continuous? Explain.
 c. Graph the function using its domain.
 d. Find the range of the function.

16. The function $d(x) = 1375 - 110x$ represents the distance (in miles) a high-speed train is from its destination after x hours. *(Section 3.3)*
 a. How far is the train from its destination after 8 hours?
 b. How long does the train travel before reaching its destination?

3.4 Graphing Linear Equations in Standard Form

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.3.C

Essential Question How can you describe the graph of the equation $Ax + By = C$?

EXPLORATION 1 Using a Table to Plot Points

Work with a partner. You sold a total of \$16 worth of tickets to a fundraiser. You lost track of how many of each type of ticket you sold. Adult tickets are \$4 each. Child tickets are \$2 each.

$$\frac{\text{adult}}{\text{adult}} \cdot \text{Number of adult tickets} + \frac{\text{child}}{\text{child}} \cdot \text{Number of child tickets} = \text{[illegible]}$$

FORMULATING A PLAN
To be proficient in math, you need to find an entry point into the solution of a problem. Determining what information you know, and what you can do with that information, can help you find an entry point.

a. Let x represent the number of adult tickets. Let y represent the number of child tickets. Use the verbal model to write an equation that relates x and y.

b. Copy and complete the table to show the different combinations of tickets you might have sold.

x						
y						

c. Plot the points from the table. Describe the pattern formed by the points.

d. If you remember how many adult tickets you sold, can you determine how many child tickets you sold? Explain your reasoning.

EXPLORATION 2 Rewriting and Graphing an Equation

Work with a partner. You sold a total of \$48 worth of cheese. You forgot how many pounds of each type of cheese you sold. Swiss cheese costs \$8 per pound. Cheddar cheese costs \$6 per pound.

$$\frac{\text{[illegible]}}{\text{pound}} \cdot \text{Pounds of Swiss} + \frac{\text{[illegible]}}{\text{pound}} \cdot \text{Pounds of cheddar} = \text{[illegible]}$$

a. Let x represent the number of pounds of Swiss cheese. Let y represent the number of pounds of cheddar cheese. Use the verbal model to write an equation that relates x and y.

b. Solve the equation for y. Then use a graphing calculator to graph the equation. Given the real-life context of the problem, find the domain and range of the function.

c. The ***x*-intercept** of a graph is the x-coordinate of a point where the graph crosses the x-axis. The ***y*-intercept** of a graph is the y-coordinate of a point where the graph crosses the y-axis. Use the graph to determine the x- and y-intercepts.

d. How could you use the equation you found in part (a) to determine the x- and y-intercepts? Explain your reasoning.

e. Explain the meaning of the intercepts in the context of the problem.

Communicate Your Answer

3. How can you describe the graph of the equation $Ax + By = C$?

4. Write a real-life problem that is similar to those shown in Explorations 1 and 2.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students work in pairs of similar language abilities to complete the explorations. Have them read portions aloud to check their reading ability.

Beginning State examples of words found in the directions that they frequently see and automatically know. *You, a, of, how, many, each,* and *are* would all be possibilities.
Intermediate Read the direction lines aloud.
Advanced/Advanced High Read the direction lines and restate them in their own words.

ELPS 4.C.1 Develop basic sight vocabulary used routinely in written classroom materials.

Dynamic Teaching Tools

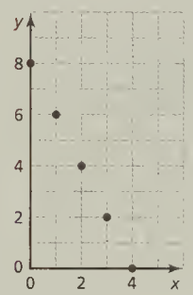
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.3.C The student is expected to graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

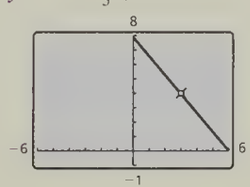
ANSWERS

1. **a.** $4x + 2y = 16$
 b. 0, 1, 2, 3, 4; 8, 6, 4, 2, 0
 c.



 The points create a line with a slope of -2. For each adult ticket sold, the number of child tickets sold decreases by 2.
 d. yes; The relationship can be expressed as an equation with 2 variables. If the value of one of the variables is known, the other can be found.

2. **a.** $8x + 6y = 48$
 b. $y = 8 - \frac{4}{3}x$;

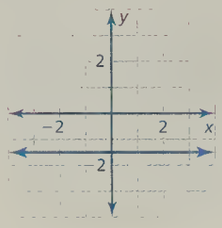


 domain: $0 \le x \le 6$;
 range: $0 \le x \le 8$
 c. x-intercept: 6; y-intercept: 8
 d. To find the x-intercept, set $y = 0$ and solve for x, to find the y-intercept, set $x = 0$ and solve for y; Because the intercepts are where the graph crosses the axes, the y-coordinate of the x-intercept must be 0, and the x-coordinate of the y-intercept must be 0.

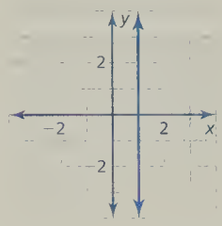
2e–4. See Additional Answers.

Extra Example 1

a. Graph $y = -1.5$.

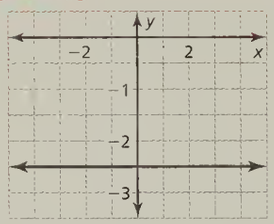


b. Graph $x = 1$.

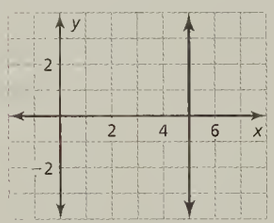


MONITORING PROGRESS ANSWERS

1.



2.



3.4 Lesson

What You Will Learn

- Graph equations of horizontal and vertical lines.
- Graph linear equations in standard form using intercepts.
- Find zeros of functions.
- Use linear equations to solve real-life problems.

Core Vocabulary

standard form, *p. 116*
x-intercept, *p. 117*
y-intercept, *p. 117*
zero of a function, *p. 118*

Previous
ordered pair
quadrant

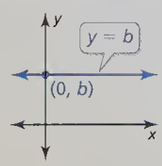
Horizontal and Vertical Lines

The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

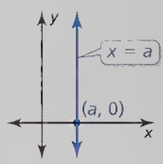
Consider what happens when $A = 0$ or when $B = 0$. When $A = 0$, the equation becomes $By = C$, or $y = \frac{C}{B}$. Because $\frac{C}{B}$ is a constant, you can write $y = b$. Similarly, when $B = 0$, the equation becomes $Ax = C$, or $x = \frac{C}{A}$, and you can write $x = a$.

Core Concept

Horizontal and Vertical Lines



The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.



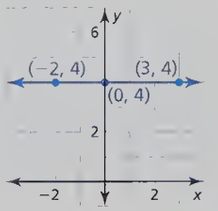
The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

EXAMPLE 1 Horizontal and Vertical Lines

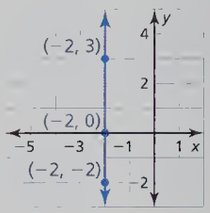
Graph (a) $y = 4$ and (b) $x = -2$.

SOLUTION

a. For every value of x, the value of y is 4. The graph of the equation $y = 4$ is a horizontal line 4 units above the x-axis.



b. For every value of y, the value of x is -2. The graph of the equation $x = -2$ is a vertical line 2 units to the left of the y-axis.



STUDY TIP

For every value of x, the ordered pair $(x, 4)$ is a solution of $y = 4$.

Monitoring Progress

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Graph the linear equation.

1. $y = -2.5$
2. $x = 5$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Using Intercepts to Graph Linear Equations

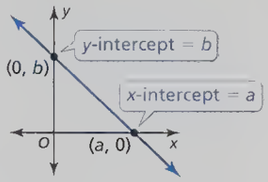
You can use the fact that two points determine a line to graph a linear equation. Two convenient points are the points where the graph crosses the axes.

Core Concept

Using Intercepts to Graph Equations

The ***x*-intercept** of a graph is the x-coordinate of a point where the graph crosses the x-axis. It occurs when $y = 0$.

The ***y*-intercept** of a graph is the y-coordinate of a point where the graph crosses the y-axis. It occurs when $x = 0$.



To graph the linear equation $Ax + By = C$, find the intercepts and draw the line that passes through the two intercepts.

- To find the x-intercept, let $y = 0$ and solve for x.
- To find the y-intercept, let $x = 0$ and solve for y.

EXAMPLE 2 Using Intercepts to Graph a Linear Equation

Use intercepts to graph the equation $3x + 4y = 12$.

SOLUTION

Step 1 Find the intercepts.

To find the x-intercept, substitute 0 for y and solve for x.

$3x + 4y = 12$	Write the original equation.
$3x + 4(0) = 12$	Substitute 0 for y.
$x = 4$	Solve for x.

To find the y-intercept, substitute 0 for x and solve for y.

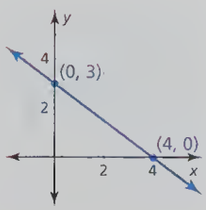
$3x + 4y = 12$	Write the original equation.
$3(0) + 4y = 12$	Substitute 0 for x.
$y = 3$	Solve for y.

STUDY TIP

As a check, you can find a third solution of the equation and verify that the corresponding point is on the graph. To find a third solution, substitute any value for one of the variables and solve for the other variable.

Step 2 Plot the points and draw the line.

The x-intercept is 4, so plot the point (4, 0).
The y-intercept is 3, so plot the point (0, 3).
Draw a line through the points.



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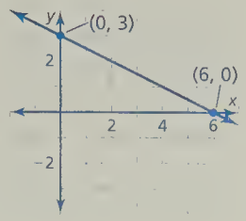
Use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

3. $2x - y = 4$

4. $x + 3y = -9$

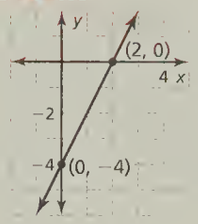
Extra Example 2

Use intercepts to graph the equation $x + 2y = 6$.

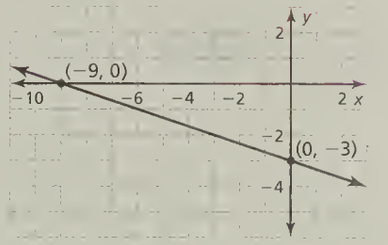


MONITORING PROGRESS ANSWERS

3.



4.



Differentiated Instruction

Auditory/Visual

Show students three different graphs—one with only an x-intercept, one with only a y-intercept, and one with both an x- and y-intercept. Ask one student to name each x-intercept and a second student to name each y-intercept. Emphasize that an intercept is not an ordered pair, but rather it is the x- or y-coordinate of a point where the line crosses an axis.

Extra Example 3

Find the zero of the function $f(x) = -7x - 21$. The zero of the function is -3.

Extra Example 4

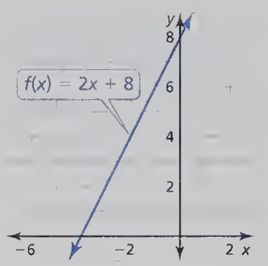
A family is driving to the beach, traveling an average of 50 miles per hour. The function $f(x) = -50x + 200$ represents the distance left to drive at that speed, where x is the number of hours driven. Find the zero of the function. Explain what the zero means in this situation. The zero of the function is 4. The zero of the function represents the number of hours the family must drive to get to the beach, traveling 50 miles per hour. So, the family must drive 4 hours at that speed to get to the beach.

MONITORING PROGRESS ANSWERS

5. 4
6. -4
7. 10
8. $-\frac{1}{3}$
9. 12.5; The tub is empty after 12.5 seconds.

Finding the Zeros of Functions

A **zero of a function** f is an x-value for which $f(x) = 0$ (or $y = 0$). A zero of a function is an x-intercept of the graph of the function.



EXAMPLE 3 Finding the Zero of a Function

Find the zero of the function $f(x) = 2x + 8$.

SOLUTION

Substitute 0 for $f(x)$ in the function and solve for x.

$f(x) = 2x + 8$ Write the function.

$0 = 2x + 8$ Substitute 0 for $f(x)$.

$x = -4$ Solve for x.

▶ The zero of the function is -4.

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Find the zero of the function.

5. $h(x) = 4x - 16$
6. $r(x) = -3x - 12$
7. $r(x) = -\frac{1}{2}x + 5$
8. $g(x) = 2x + \frac{2}{3}$

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics

An artist rents a booth at an art show for \$300. The function $f(x) = 50x - 300$ represents the artist's profit, where x is the number of paintings the artist sells. Find the zero of the function. Explain what the zero means in this situation.

SOLUTION

Substitute 0 for $f(x)$ in the function and solve for x.

$f(x) = 50x - 300$ Write the function.

$0 = 50x - 300$ Substitute 0 for $f(x)$.

$x = 6$ Solve for x.

▶ The zero of the function is 6. Because the function represents the artist's profit, the zero of the function represents the number of paintings the artist must sell to recover the cost of renting the booth. So, the artist must sell 6 paintings to recover the cost of renting the booth.

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9. The function $f(t) = -2t + 25$ represents the amount (in gallons) of water remaining in a tub after t seconds. Find the zero of the function. Explain what the zero means in this situation.

EXAMPLE 5 Modeling with Mathematics

You are planning an awards banquet for your school. You need to rent tables to seat 180 people. Tables come in two sizes. Small tables seat 6 people, and large tables seat 10 people. The equation $6x + 10y = 180$ models this situation, where x is the number of small tables and y is the number of large tables.

a. Graph the equation. Interpret the intercepts.

b. Find four possible solutions in the context of the problem.

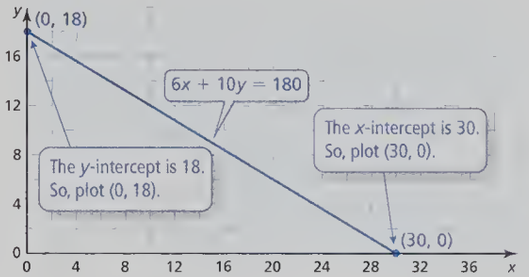
SOLUTION

1. **Understand the Problem** You know the equation that models the situation. You are asked to graph the equation, interpret the intercepts, and find four solutions.
2. **Make a Plan** Use intercepts to graph the equation. Then use the graph to interpret the intercepts and find other solutions.
3. **Solve the Problem**

a. Use intercepts to graph the equation. Neither x nor y can be negative, so only graph the equation in the first quadrant.

STUDY TIP

Although x and y represent whole numbers, it is convenient to draw a line segment that includes points whose coordinates are not whole numbers.



▶ The x-intercept shows that you can rent 30 small tables when you do not rent any large tables. The y-intercept shows that you can rent 18 large tables when you do not rent any small tables.

Check

$6x + 10y = 180$

$6(10) + 10(12) \stackrel{?}{=} 180$

$180 = 180$ ✓

$6x + 10y = 180$

$6(20) + 10(6) \stackrel{?}{=} 180$

$180 = 180$ ✓

b. Only whole-number values of x and y make sense in the context of the problem. Besides the intercepts, it appears that the line passes through the points (10, 12) and (20, 6). To verify that these points are solutions, check them in the equation, as shown.

▶ So, four possible combinations of tables that will seat 180 people are 0 small and 18 large, 10 small and 12 large, 20 small and 6 large, and 30 small and 0 large.

4. **Look Back** The graph shows that as the number x of small tables increases, the number y of large tables decreases. This makes sense in the context of the problem. So, the graph is reasonable.

Monitoring Progress

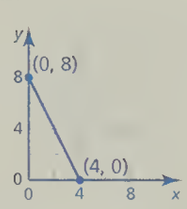
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10. WHAT IF? You decide to rent tables from a different company. The situation can be modeled by the equation $4x + 6y = 180$, where x is the number of small tables and y is the number of large tables. Graph the equation and interpret the intercepts.

Extra Example 5

You are going to the fair and have \$16 to spend on rides and games. Ride tickets cost \$4 and game tickets cost \$2. The equation $4x + 2y = 16$ models this situation, where x is the number of ride tickets and y is the number of game tickets.

a. Graph the equation. Interpret the intercepts.

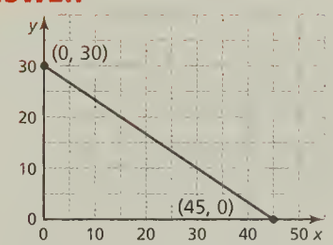


The x-intercept shows that you can buy 4 ride tickets when you do not buy any game tickets. The y-intercept shows that you can buy 8 game tickets when you do not buy any ride tickets.

b. Find four possible solutions in the context of the problem. *Sample answer:* 0 ride and 8 game tickets, 2 ride and 4 game tickets, 3 ride and 2 game tickets, 4 ride and 0 game tickets

MONITORING PROGRESS ANSWER

10.



The x-intercept shows that you can rent 45 small tables and 0 large tables. The y-intercept shows that you can rent 30 large tables and 0 small tables.

Closure

- **Writing Prompt:** To graph the equation $2x + y = 4$ … *Sample answer:* find and plot the points for the x- and y-intercepts. Then draw a line through these two points.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–23 odd, 29–45 odd, 49, 51, 58, 63–66

Average: 1, 2–46 even, 50, 51, 52–58 even, 63–66

Advanced: 1, 2, 8, 12–34 even, 40–50 even, 51, 52–62 even, 63–66

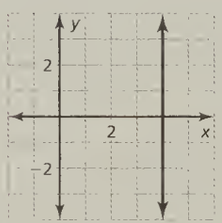
HOMEWORK CHECK

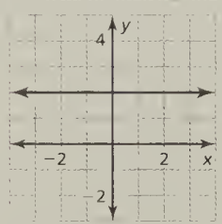
Basic: 3, 19, 35, 45, 49

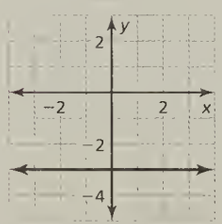
Average: 4, 20, 36, 46, 50

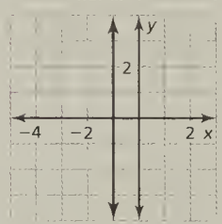
Advanced: 8, 26, 40, 46, 50

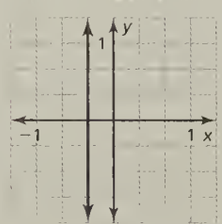
ANSWERS

1. They are both places where a graph crosses an axis; The x-intercept is where a graph crosses the x-axis. The y-intercept is where a graph crosses the y-axis.
2. $(4, -3)$; It is the only one that is not on an axis.
3. 

4. 

5. 

6. 

7. 

3.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are x-intercepts and y-intercepts alike? How are they different?
2. **WHICH ONE DOESN'T BELONG?** Which point does not belong with the other three? Explain your reasoning.

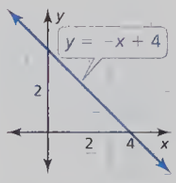
$(0, -3)$ $(0, 0)$ $(4, -3)$ $(4, 0)$

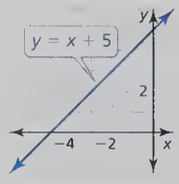
Monitoring Progress and Modeling with Mathematics

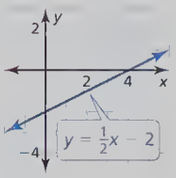
In Exercises 3–8, graph the linear equation. *(See Example 1.)*

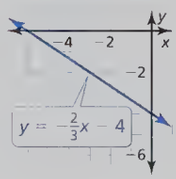
3. $x = 4$
4. $y = 2$
5. $y = -3$
6. $x = -1$
7. $x = -\frac{1}{3}$
8. $y = \frac{7}{2}$

In Exercises 9–12, find the x- and y-intercepts of the graph of the linear equation.

9. 

10. 

11. 

12. 

In Exercises 13–18, find the x- and y-intercepts of the graph of the linear equation.

13. $2x + 3y = 12$
14. $3x + 6y = 24$
15. $-4x + 8y = -16$
16. $-6x + 9y = -18$
17. $3x - 6y = 2$
18. $-x + 8y = 4$

In Exercises 19–28, use intercepts to graph the linear equation. Label the points corresponding to the intercepts. *(See Example 2.)*

19. $5x + 3y = 30$
20. $4x + 6y = 12$
21. $-12x + 3y = 24$
22. $-2x + 6y = 18$
23. $-4x + 3y = -30$
24. $-2x + 7y = -21$
25. $-x + 2y = 7$
26. $3x - y = -5$
27. $-\frac{5}{2}x + y = 10$
28. $-\frac{1}{2}x + y = -4$

ERROR ANALYSIS **In Exercises 29 and 30, describe and correct the error in finding the intercepts of the graph of the equation.**

29. ✗

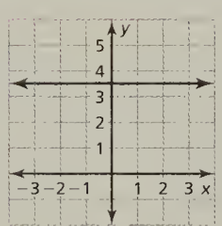
$3x + 12y = 24$	$3x + 12y = 24$
$3x + 12(0) = 24$	$3(0) + 12y = 24$
$3x = 24$	$12y = 24$
$x = 8$	$y = 2$

The intercept is at $(8, 2)$.

30. ✗

$4x + 10y = 20$	$4x + 10y = 20$
$4x + 10(0) = 20$	$4(0) + 10y = 20$
$4x = 20$	$10y = 20$
$x = 5$	$y = 2$

The x-intercept is at $(0, 5)$, and the y-intercept is at $(2, 0)$.

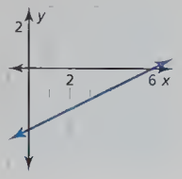
8. 

9. x-intercept: 4, y-intercept: 4
10. x-intercept: -5, y-intercept: 5
11. x-intercept: 4, y-intercept: -2
12. x-intercept: -6, y-intercept: -4
13. x-intercept: 6, y-intercept: 4
14. x-intercept: 8, y-intercept: 4
15. x-intercept: 4, y-intercept: -2
16. x-intercept: 3, y-intercept: -2
17. x-intercept: $\frac{2}{3}$, y-intercept: $-\frac{1}{3}$
18. x-intercept: -4, y-intercept: $\frac{1}{2}$

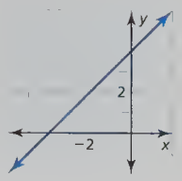
19–30. See Additional Answers.

In Exercises 31–34, use the graph to find the zero of the function.

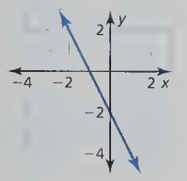
31. $f(x) = \frac{1}{2}x - 3$



32. $f(x) = x + 4$



33. $f(x) = -2x - 2$



34. $f(x) = -\frac{1}{3}x - 1$



In Exercises 35–44, find the zero of the function. *(See Example 3.)*

35. $f(x) = x + 2$

36. $r(x) = x - 5$

37. $m(x) = 3x - 18$

38. $g(x) = 2x + 4$

39. $q(x) = -5x - 20$

40. $n(x) = -6x + 18$

41. $h(x) = 4x + 2$

42. $f(x) = -3x + 1$

43. $g(x) = -\frac{1}{2}x + 7$

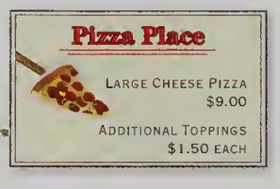
44. $p(x) = \frac{1}{4}x - 3$

45. **MODELING WITH MATHEMATICS** The function $f(x) = -200x + 1000$ represents the balance (in dollars) in a checking account after x months. Find the zero of the function. Explain what the zero means in this situation. *(See Example 4.)*

46. **MODELING WITH MATHEMATICS** The function $f(t) = -10t + 3000$ represents the height (in feet) of a skydiver t seconds after opening the parachute. Find the zero of the function. Explain what the zero means in this situation.



47. **REASONING** The function $c(x) = 9 + 1.50x$ represents the total cost (in dollars) of a large pizza, where x is the number of additional toppings. Find the zero of the function. Does the zero make sense in this situation? Explain.



48. **OPEN-ENDED** Consider the equation $8 = 4x + 16$. Write a function so that the solution of the equation is the zero of the function. Explain your reasoning.

49. **MODELING WITH MATHEMATICS** A football team has an away game, and the bus breaks down. The coaches decide to drive the players to the game in cars and vans. Four players can ride in each car. Six players can ride in each van. There are 48 players on the team. The equation $4x + 6y = 48$ models this situation, where x is the number of cars and y is the number of vans. *(See Example 5.)*

a. Graph the equation. Interpret the intercepts.

b. Find four possible solutions in the context of the problem.

50. **MODELING WITH MATHEMATICS** You are ordering shirts for the math club at your school. Short-sleeved shirts cost \$10 each. Long-sleeved shirts cost \$12 each. You have a budget of \$300 for the shirts. The equation $10x + 12y = 300$ models the total cost, where x is the number of short-sleeved shirts and y is the number of long-sleeved shirts.



a. Graph the equation. Interpret the intercepts.

b. Twelve students decide they want short-sleeved shirts. How many long-sleeved shirts can you order?

51. **MAKING AN ARGUMENT** You overhear your friend explaining how to find intercepts to a classmate. Your friend says, "When you want to find the x-intercept, just substitute 0 for x and continue to solve the equation." Is your friend's explanation correct? Explain.

Dynamic Teaching Tools

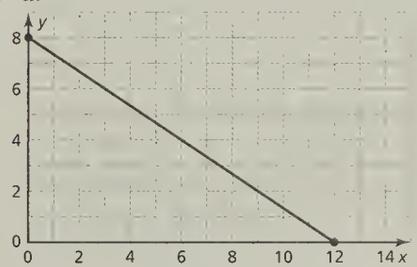
Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

ANSWERS

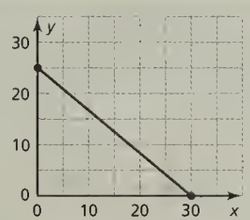
31. 6
32. −4
33. −1
34. −3
35. −2
36. 5
37. 6
38. −2
39. −4
40. 3
41. $-\frac{1}{2}$
42. $\frac{1}{3}$
43. 14
44. 12
45. 5; The balance of the account will be \$0 after 5 months.
46. 300; The skydiver will hit the ground after 300 seconds.
47. −6; no; The number of additional toppings cannot be a negative number.
48. *Sample answer:* $f(x) = 4x + 8$; Rewrite the equation as $0 = 4x + 8$. Then, substitute $f(x)$ for 0.
49. a.



The x-intercept shows they can take 12 cars and 0 vans. The y-intercept shows they can take 8 vans and 0 cars.

b. *Sample answer:* 12 cars, 0 vans; 9 cars, 2 vans; 6 cars, 4 vans; 0 cars, 8 vans

50. a.



The x-intercept shows that you can buy 30 short-sleeved shirts and 0 long-sleeved shirts. The y-intercept shows that you can buy 25 long-sleeved shirts and 0 short-sleeved shirts.

b. 15

51. no; You have to substitute 0 for y to find the x-intercept, not for x.

ANSWERS

52. See Additional Answers.

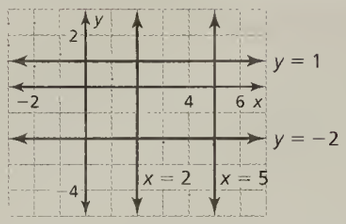
53. A

54. C

55. D

56. B

57.

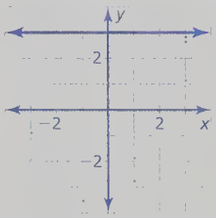


square; The graphs are 2 horizontal lines and 2 vertical lines, which will intersect at right angles. The length of each side is 3 units.

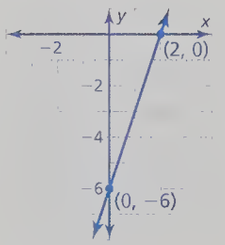
58–66. See Additional Answers.

Mini-Assessment

1. Graph $y = 3$.



2. Use intercepts to graph the equation $3x - y = 6$.



3. Find the zero of the function $f(x) = 5x - 35$. 7

4. The function $f(x) = 448 - 28x$ represents the grams of cereal left in a box after x servings are eaten. Find the zero of the function. Explain what the zero means in this situation. 16; The zero represents the number of servings in the box of cereal. So, there are 16 servings in the box of cereal.

52. ANALYZING RELATIONSHIPS You lose track of how many 2-point baskets and 3-point baskets a team makes in a basketball game. The team misses all the 1-point baskets and still scores 54 points. The equation $2x + 3y = 54$ models the total points scored, where x is the number of 2-point baskets made and y is the number of 3-point baskets made.

a. Find and interpret the intercepts.

b. Can the number of 3-point baskets made be odd? Explain your reasoning.

c. Graph the equation. Find two more possible solutions in the context of the problem.



MULTIPLE REPRESENTATIONS In Exercises 53–56, match the equation with its graph.

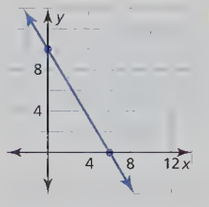
53. $5x + 3y = 30$

54. $5x + 3y = -30$

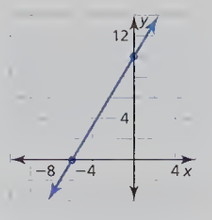
55. $5x - 3y = 30$

56. $5x - 3y = -30$

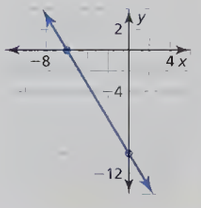
A.



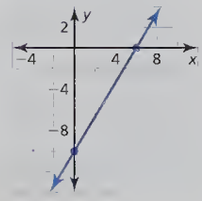
B.



C.

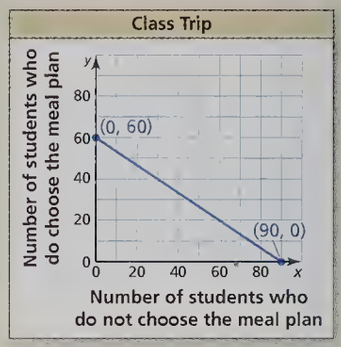


D.



57. MATHEMATICAL CONNECTIONS Graph the equations $x = 5$, $x = 2$, $y = -2$, and $y = 1$. What enclosed shape do the lines form? Explain your reasoning.

58. HOW DO YOU SEE IT? You are organizing a class trip to an amusement park. The cost to enter the park is \$30. The cost to enter with a meal plan is \$45. You have a budget of \$2700 for the trip. The equation $30x + 45y = 2700$ models the total cost for the class to go on the trip, where x is the number of students who do not choose the meal plan and y is the number of students who do choose the meal plan.



a. Interpret the intercepts of the graph.

b. Describe the domain and range in the context of the problem.

59. REASONING Use the values to fill in the equation $\square x + \square y = 30$ so that the x-intercept of the graph is -10 and the y-intercept of the graph is 5.

-10 $\quad -3 \quad$ 1 $\quad$ 5 $\quad$ 6

60. THOUGHT PROVOKING Write an equation in standard form of a line whose intercepts are integers. Explain how you know the intercepts are integers.

61. WRITING Are the equations of horizontal and vertical lines written in standard form? Explain your reasoning.

62. ABSTRACT REASONING The x- and y-intercepts of the graph of the equation $3x + 5y = k$ are integers. Describe the values of k. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

63. $\dfrac{2 - (-2)}{4 - (-4)}$

64. $\dfrac{14 - 18}{0 - 2}$

65. $\dfrac{-3 - 9}{8 - (-7)}$

66. $\dfrac{12 - 17}{-5 - (-2)}$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

3.5 Graphing Linear Equations in Slope-Intercept Form

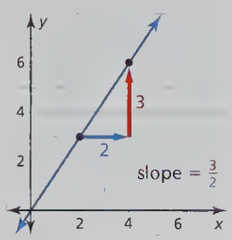
Texas Essential Knowledge and Skills
A.2.A
A.3.A
A.3.B
A.3.C

Essential Question How can you describe the graph of the equation $y = mx + b$?

Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the ratio of the change in y (vertical change) to the change in x (horizontal change).

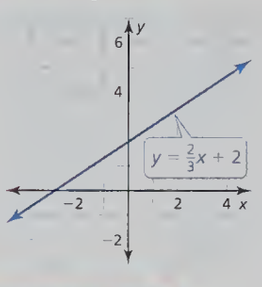
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



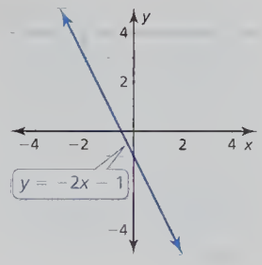
EXPLORATION 1 Finding Slopes and y-Intercepts

Work with a partner. Find the slope and y-intercept of each line.

a.



b.



ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you first need to collect and organize data. Then make conjectures about the patterns you observe in the data.

EXPLORATION 2 Writing a Conjecture

Work with a partner. Graph each equation. Then copy and complete the table. Use the completed table to write a conjecture about the relationship between the graph of $y = mx + b$ and the values of m and b.

Equation	Description of graph	Slope of graph	y-Intercept
a. $y = -\frac{2}{3}x + 3$	Line	$-\frac{2}{3}$	3
b. $y = 2x - 2$			
c. $y = -x + 1$			
d. $y = x - 4$			

Communicate Your Answer

3. How can you describe the graph of the equation $y = mx + b$?
 a. How does the value of m affect the graph of the equation?
 b. How does the value of b affect the graph of the equation?
 c. Check your answers to parts (a) and (b) by choosing one equation from Exploration 2 and (1) varying only m and (2) varying only b.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.A The student is expected to determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

A.3.A The student is expected to determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

A.3.B The student is expected to calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems.

A.3.C The student is expected to graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

ANSWERS

1. a. $\frac{2}{3}$; 2
 b. -2; -1

2–3. See Additional Answers.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the Essential Question and the introduction. Point out that the graph illustrates the concept of slope. Have students work in pairs of mixed language abilities to complete the explorations and Communicate Your Answer. Remind them that the graphs and tables should help them work through concepts. Explain that the activities on this page help them prepare for what they will learn in Lesson 3.5.

Beginning/Advanced Beginning students recreate the graphs and table for use in the explorations. They then work with Advanced students to complete the explorations. Advanced students state the answers.

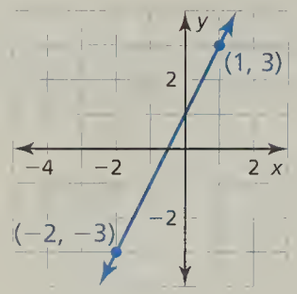
Intermediate/Advanced High Intermediate students recreate the graphs and table and complete the explorations with Advanced High students. Advanced High students explain the rationale for their answers.

ELPS 4.D Use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text.

Extra Example 1

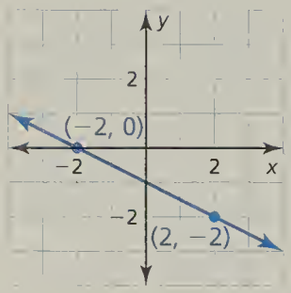
Describe the slope of each line. Then find the slope.

a.



The line rises from left to right.
So, the slope is positive; $m = 2$.

b.



The line falls from left to right.
So, the slope is negative; $m = -\frac{1}{2}$.

MONITORING PROGRESS ANSWERS

1. negative; $-\frac{2}{5}$
2. positive; $\frac{2}{3}$
3. positive; $\frac{7}{3}$

3.5 Lesson

What You Will Learn

- Find the slope of a line.
- Use the slope-intercept form of a linear equation.
- Use slopes and *y*-intercepts to solve real-life problems.

Core Vocabulary

slope, *p. 124*
rise, *p. 124*
run, *p. 124*
slope-intercept form, *p. 126*
constant function, *p. 126*

Previous
dependent variable
independent variable

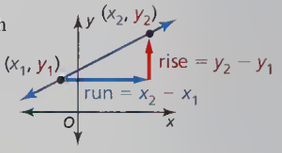
The Slope of a Line

Core Concept

Slope

The **slope** m of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2) is the ratio of the **rise** (change in y) to the **run** (change in x).

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

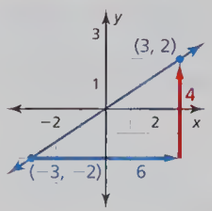


When the line rises from left to right, the slope is positive. When the line falls from left to right, the slope is negative.

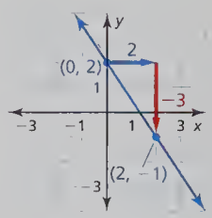
EXAMPLE 1 Finding the Slope of a Line

Describe the slope of each line. Then find the slope.

a.



b.



STUDY TIP

When finding slope, you can label either point as (x_1, y_1) and the other point as (x_2, y_2). The result is the same.

READING

In the slope formula, x_1 is read as "x sub one" and y_2 is read as "y sub two." The numbers 1 and 2 in x_1 and y_2 are called *subscripts*.

SOLUTION

a. The line rises from left to right.
So, the slope is positive.
Let $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

b. The line falls from left to right.
So, the slope is negative.
Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (2, -1)$.

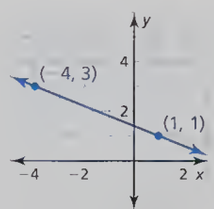
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{2 - 0} = \frac{-3}{2} = -\frac{3}{2}$$

Monitoring Progress

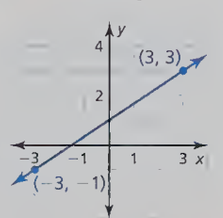
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Describe the slope of the line. Then find the slope.

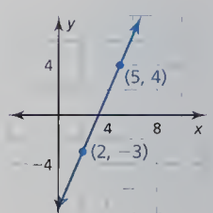
1.



2.



3.



For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Visual/Kinesthetic

Have students graph the equations $y = 2x$, $y = x$, $y = -x$, and $y = -2x$ on the same coordinate plane, using a different color for each graph. Have students describe what happens to the line as the slope decreases.

EXAMPLE 2 Finding Slope from a Table

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

a.

x	y
4	20
7	14
10	8
13	2

b.

x	y
−1	2
1	2
3	2
5	2

c.

x	y
−3	−3
−3	0
−3	6
−3	9

STUDY TIP

As a check, you can plot the points represented by the table to verify that the line through them has a slope of −2.

SOLUTION

a. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (4, 20)$ and $(x_2, y_2) = (7, 14)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 20}{7 - 4} = \frac{-6}{3}, \text{ or } -2$$

▶ The slope is −2.

b. Note that there is no change in y. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (5, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-1)} = \frac{0}{6}, \text{ or } 0$$

The change in y is 0.

▶ The slope is 0.

c. Note that there is no change in x. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (-3, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{-3 - (-3)} = \frac{6}{0}$$ ✗

The change in x is 0.

▶ Because division by zero is undefined, the slope of the line is undefined.

Monitoring Progress

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The points represented by the table lie on a line. How can you find the slope of the line from the table? What is the slope of the line?

4.

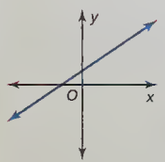
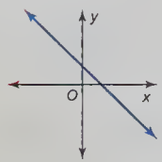
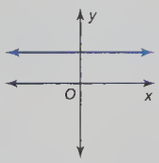
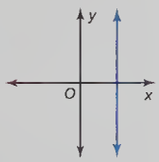
x	2	4	6	8
y	10	15	20	25

5.

x	5	5	5	5
y	−12	−9	−6	−3

Concept Summary

Slope

Positive slope	*Negative slope*	*Slope of 0*	*Undefined slope*
			
The line rises from left to right.	The line falls from left to right.	The line is horizontal.	The line is vertical.

Extra Example 2

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

a.

x	−3	−1	0	3
y	−1	3	5	11

Choose any two points from the table and use the slope formula. The slope is 2.

b.

x	−4	0	4	8
y	−2	−2	−2	−2

Note that there is no change in y. Choose any two points from the table and use the slope formula. The slope is 0.

c.

x	4	4	4	4
y	−5	−2	1	4

Note that there is no change in x. Choose any two points from the table and use the slope formula. Because division by 0 is undefined, the slope is undefined.

MONITORING PROGRESS ANSWERS

4. Choose any 2 points from the table and use the slope formula; $\frac{5}{2}$
5. Choose any 2 points from the table and use the slope formula; undefined

Extra Example 3

Find the slope and the y-intercept of the graph of each linear equation.

a. $y = 2x - 3$

The slope is 2, and the y-intercept is -3.

b. $y = -4.5$

The slope is 0, and the y-intercept is -4.5.

c. $6x - y = 8$

The slope is 6, and the y-intercept is -8.

MONITORING PROGRESS ANSWERS

6. slope: -6; y-intercept: 1
7. slope: 0; y-intercept: 8
8. slope: $-\frac{1}{4}$; y-intercept: $-\frac{5}{2}$

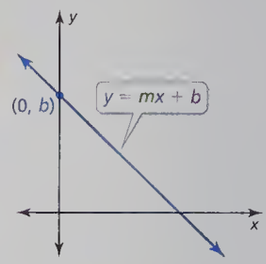
Using the Slope-Intercept Form of a Linear Equation

Core Concept

Slope-Intercept Form

Words A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.

Algebra $y = mx + b$ (slope, y-intercept)



A linear equation written in the form $y = 0x + b$, or $y = b$, is a **constant function**. The graph of a constant function is a horizontal line.

EXAMPLE 3 Identifying Slopes and y-Intercepts

Find the slope and the y-intercept of the graph of each linear equation.

a. $y = 3x - 4$ **b.** $y = 6.5$ **c.** $-5x - y = -2$

SOLUTION

a. $y = mx + b$ (slope, y-intercept) — Write the slope-intercept form.

$y = 3x + (-4)$ — Rewrite the original equation in slope-intercept form.

▶ The slope is 3, and the y-intercept is -4.

b. The equation represents a constant function. The equation can also be written as $y = 0x + 6.5$.

▶ The slope is 0, and the y-intercept is 6.5.

c. Rewrite the equation in slope-intercept form by solving for y.

$-5x - y = -2$ — Write the original equation.

$+5x \quad +5x$ — Add $5x$ to each side.

$-y = 5x - 2$ — Simplify.

$\frac{-y}{-1} = \frac{5x - 2}{-1}$ — Divide each side by -1.

$y = -5x + 2$ — Simplify.

▶ The slope is -5, and the y-intercept is 2.

STUDY TIP

For a constant function, every input has the same output. For instance, in Example 3b, every input has an output of 6.5.

STUDY TIP

When you rewrite a linear equation in slope-intercept form, you are expressing y as a function of x.

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Find the slope and the y-intercept of the graph of the linear equation.

6. $y = -6x + 1$ 7. $y = 8$ 8. $x + 4y = -10$

EXAMPLE 4 Using Slope-Intercept Form to Graph

STUDY TIP

You can use the slope to find points on a line in either direction. In Example 4, note that the slope can be written as $\frac{2}{-1}$. So, you could move 1 unit left and 2 units up from (0, 2) to find the point (−1, 4).

Graph $2x + y = 2$. Identify the x-intercept.

SOLUTION

Step 1 Rewrite the equation in slope-intercept form.

$$y = -2x + 2$$

Step 2 Find the slope and the y-intercept.

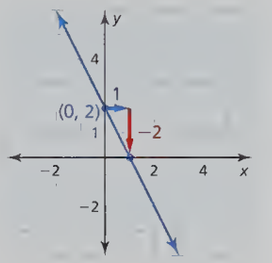
$m = -2$ and $b = 2$

Step 3 The y-intercept is 2. So, plot (0, 2).

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

Plot the point that is 1 unit right and 2 units down from (0, 2). Draw a line through the two points.



▶ The line crosses the x-axis at (1, 0). So, the x-intercept is 1.

REMEMBER

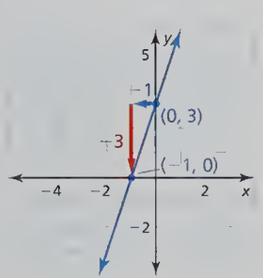
You can also find the x-intercept by substituting 0 for y in the equation $2x + y = 2$ and solving for x.

EXAMPLE 5 Graphing from a Verbal Description

A linear function g models a relationship in which the dependent variable increases 3 units for every 1 unit the independent variable increases. Graph g when $g(0) = 3$. Identify the slope, y-intercept, and x-intercept of the graph.

SOLUTION

Because the function g is linear, it has a constant rate of change. Let x represent the independent variable and y represent the dependent variable.



Step 1 **Find the slope.** When the dependent variable increases by 3, the change in y is +3. When the independent variable increases by 1, the change in x is +1.

So, the slope is $\frac{3}{1}$, or 3.

Step 2 **Find the y-intercept.** The statement $g(0) = 3$ indicates that when $x = 0$, $y = 3$. So, the y-intercept is 3. Plot (0, 3).

Step 3 **Use the slope to find another point on the line.** A slope of 3 can be written as $\frac{-3}{-1}$. Plot the point that is 1 unit left and 3 units down from (0, 3). Draw a line through the two points. The line crosses the x-axis at (−1, 0). So, the x-intercept is −1.

▶ The slope is 3, the y-intercept is 3, and the x-intercept is −1.

Monitoring Progress

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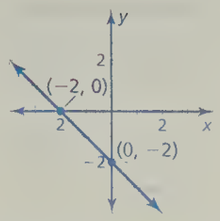
Graph the linear equation. Identify the x-intercept.

9. $y = 4x - 4$ **10.** $3x + y = -3$ **11.** $x + 2y = 6$

12. A linear function h models a relationship in which the dependent variable decreases 2 units for every 5 units the independent variable increases. Graph h when $h(0) = 4$. Identify the slope, y-intercept, and x-intercept of the graph.

Extra Example 4

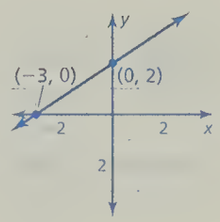
Graph $y + x = -2$. Identify the x-intercept.



The line crosses the x-axis at (−2, 0). So, the x-intercept is −2.

Extra Example 5

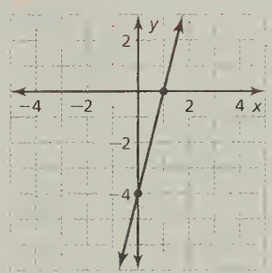
A linear function g models a relationship in which the dependent variable increases 2 units for every 3 units the independent variable increases. Graph g when $g(0) = 2$. Identify the slope, y-intercept, and x-intercept of the graph.



The slope is $\frac{2}{3}$, the y-intercept is 2, and the x-intercept is −3.

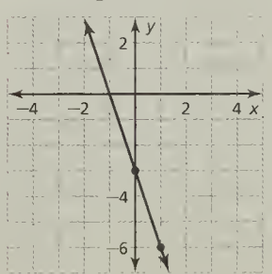
MONITORING PROGRESS ANSWERS

9.



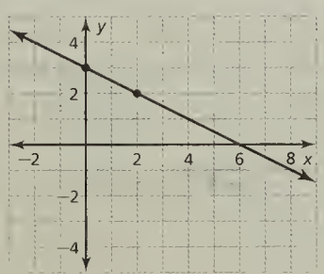
x-intercept: 1

10.



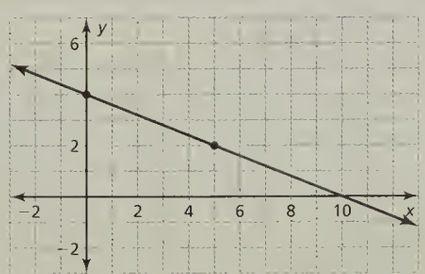
x-intercept: −1

11.



x-intercept: 6

12.

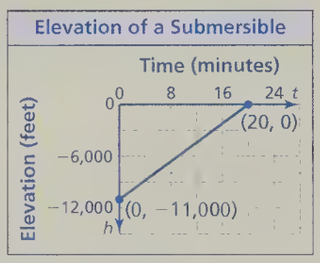


slope: $-\frac{2}{5}$; y-intercept: 4; x-intercept: 10

Extra Example 6

The elevation h (in feet) of a submersible is modeled by the function $h(t) = 550t - 11{,}000$, where t is the time (in minutes) since the submersible began to ascend.

a. Graph the function and identify its domain and range.



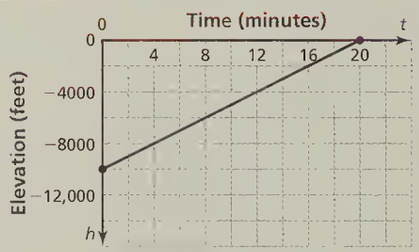
domain: $0 \le t \le 20$;
range: $-11{,}000 \le h \le 0$

b. Interpret the slope and the intercepts of the graph.

slope: The submersible ascends at a rate of 550 ft/min. h-intercept: The elevation of the submersible when the ascent begins is −11,000 ft. t-intercept: The submersible takes 20 minutes to reach sea level.

MONITORING PROGRESS ANSWERS

13. a.



domain: $0 \le t \le 20$;
range: $-10{,}000 \le h \le 0$

b. The slope is 500. So, the submersible ascends at a rate of 500 feet per minute; The h-intercept is −10,000. So, the elevation of the submersible when the ascent begins is −10,000 feet; The t-intercept is 20. So, the submersible takes 20 minutes to reach sea level.

Solving Real-Life Problems

In most real-life problems, slope is interpreted as a rate, such as miles per hour, dollars per hour, or people per year.

EXAMPLE 6 **Modeling with Mathematics**



A submersible that is exploring the ocean floor begins to ascend to the surface. The elevation h (in feet) of the submersible is modeled by the function $h(t) = 650t - 13{,}000$, where t is the time (in minutes) since the submersible began to ascend.

a. Graph the function and identify its domain and range.

b. Interpret the slope and the intercepts of the graph.

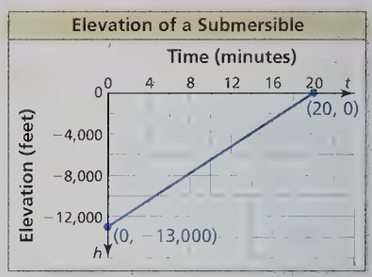
SOLUTION

1. **Understand the Problem** You know the function that models the elevation. You are asked to graph the function and identify its domain and range. Then you are asked to interpret the slope and intercepts of the graph.

2. **Make a Plan** Use the slope-intercept form of a linear equation to graph the function. Only graph values that make sense in the context of the problem. Examine the graph to interpret the slope and the intercepts.

3. **Solve the Problem**

 a. The time t must be greater than or equal to 0. The elevation h is below sea level and must be less than or equal to 0. Use the slope of 650 and the h-intercept of −13,000 to graph the function in Quadrant IV.



 ▶ The domain is $0 \le t \le 20$, and the range is $-13{,}000 \le h \le 0$.

 b. The slope is 650. So, the submersible ascends at a rate of 650 feet per minute. The h-intercept is −13,000. So, the elevation of the submersible after 0 minutes, or when the ascent begins, is −13,000 feet. The t-intercept is 20. So, the submersible takes 20 minutes to reach an elevation of 0 feet, or sea level.

4. **Look Back** You can check that your graph is correct by substituting the t-intercept for t in the function. If $h = 0$ when $t = 20$, the graph is correct.

$h = 650(20) - 13{,}000$ Substitute 20 for t in the original equation.

$h = 0$ ✓ Simplify.

STUDY TIP

Because t is the independent variable, the horizontal axis is the t-axis and the graph will have a "t-intercept." Similarly, the vertical axis is the h-axis and the graph will have an "h-intercept."

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

13. **WHAT IF?** The elevation of the submersible is modeled by $h(t) = 500t - 10{,}000$. (a) Graph the function and identify its domain and range. (b) Interpret the slope and the intercepts of the graph.

Closure

- **Point of Most Significance:** Ask students to identify, aloud or on a paper to be collected, the most significant point (or part) in the lesson that aided their learning.

3.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

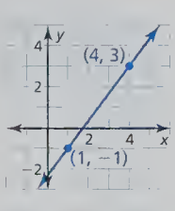
1. **COMPLETE THE SENTENCE** The _____ of a nonvertical line passing through two points is the ratio of the rise to the run.
2. **VOCABULARY** What is a constant function? What is the slope of a constant function?
3. **WRITING** What is the slope-intercept form of a linear equation? Explain why this form is called the slope-intercept form.
4. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

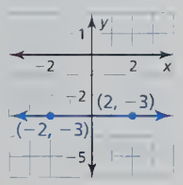
$y = -5x - 1$ $\quad 2x - y = 8$ $\quad y = x + 4$ $\quad y = -3x + 13$

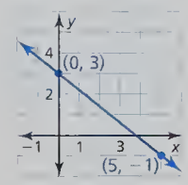
Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, describe the slope of the line. Then find the slope. *(See Example 1.)*

5. 

6. 

7. 

8. 

In Exercises 9–12, the points represented by the table lie on a line. Find the slope of the line. *(See Example 2.)*

9.

x	−9	−5	−1	3
y	−2	0	2	4

10.

x	−1	2	5	8
y	−6	−6	−6	−6

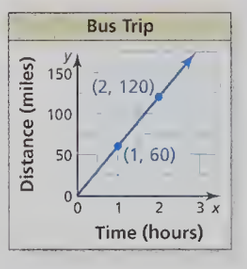
11.

x	0	0	0	0
y	−4	0	4	8

12.

x	−4	−3	−2	−1
y	2	−5	−12	−19

13. **ANALYZING A GRAPH** The graph shows the distance y (in miles) that a bus travels in x hours. Find and interpret the slope of the line.



14. **ANALYZING A TABLE** The table shows the amount x (in hours) of time you spend at a theme park and the admission fee y (in dollars) to the park. The points represented by the table lie on a line. Find and interpret the slope of the line.

Time (hours), x	Admission (dollars), y
6	54.99
7	54.99
8	54.99

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–33 odd, 37, 41, 46, 50, 54–61

Average: 1–3, 4–42 even, 46, 50, 54–61

Advanced: 1–3, 4–14 even, 20–24 even, 30–54 even, 55–61

HOMEWORK CHECK

Basic: 5, 15, 25, 33, 37

Average: 6, 16, 26, 34, 38

Advanced: 8, 20, 32, 34, 38

ANSWERS

1. slope
2. a linear function of the form $y = b$; 0
3. $y = mx + b$; The equation gives the slope m and the y-intercept b.
4. $2x - y = 8$; It is in standard form, not slope-intercept form.
5. negative; $-\frac{3}{5}$
6. positive; $\frac{4}{3}$
7. zero; 0
8. negative; $-\frac{4}{5}$
9. $\frac{1}{2}$
10. 0
11. undefined
12. -7
13. $m = 60$; The bus is traveling at a speed of 60 miles per hour.
14. $m = 0$; The admission fee does not change based on the time spent in the park.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

15. slope: -3; y-intercept: 2

16. slope: 4; y-intercept: -7

17. slope: 6; y-intercept: 0

18. slope: 0; y-intercept: -1

19. slope: 2; y-intercept: 4

20. slope: -1; y-intercept: -6

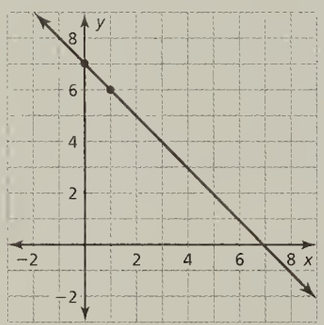
21. slope: 5; y-intercept: 8

22. slope: 7; y-intercept: $\frac{1}{2}$

23. To be in slope-intercept form the equation needs to be solved for y, not x; $y = -\frac{1}{4}x$; The slope is $-\frac{1}{4}$ and the y-intercept is 0.

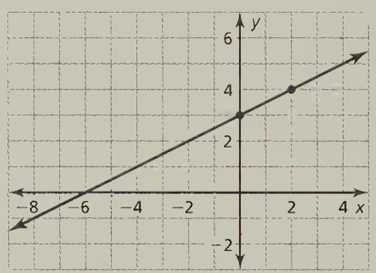
24. To be in slope-intercept form, the equation needs to be rewritten as $y = 3x + (-6)$; The slope is 3 and the y-intercept is -6.

25.



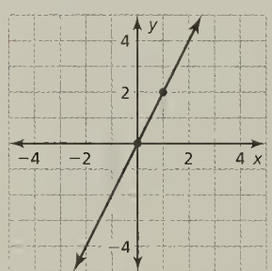
x-intercept: 7

26.



x-intercept: -6

27.



x-intercept: 0

In Exercises 15–22, find the slope and the y-intercept of the graph of the linear equation. *(See Example 3.)*

15. $y = -3x + 2$

16. $y = 4x - 7$

17. $y = 6x$

18. $y = -1$

19. $-2x + y = 4$

20. $x + y = -6$

21. $-5x = 8 - y$

22. $0 = 1 - 2y + 14x$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in finding the slope and the y-intercept of the graph of the equation.**

23.

$x = -4y$

The slope is -4, and the y-intercept is 0.

24.

$y = 3x - 6$

The slope is 3, and the y-intercept is 6.

In Exercises 25–32, graph the linear equation. Identify the x-intercept. *(See Example 4.)*

25. $y = -x + 7$

26. $y = \frac{1}{2}x + 3$

27. $y = 2x$

28. $y = -x$

29. $3x + y = -1$

30. $x + 4y = 8$

31. $-y + 5x = 0$

32. $2x - y + 6 = 0$

In Exercises 33 and 34, graph the function with the given description. Identify the slope, y-intercept, and x-intercept of the graph. *(See Example 5.)*

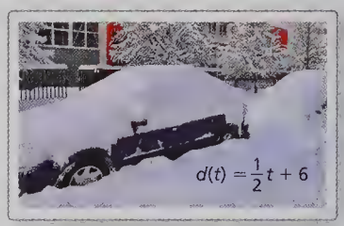
33. A linear function f models a relationship in which the dependent variable decreases 4 units for every 2 units the independent variable increases. The value of the function at 0 is -2.

34. A linear function h models a relationship in which the dependent variable increases 1 unit for every 5 units the independent variable decreases. The value of the function at 0 is 3.

35. GRAPHING FROM A VERBAL DESCRIPTION A linear function r models the growth of your right index fingernail. The length of the fingernail increases 0.7 millimeter every week. Graph r when $r(0) = 12$. Identify the slope and interpret the y-intercept of the graph.

36. GRAPHING FROM A VERBAL DESCRIPTION A linear function m models the amount of milk sold by a farm per month. The amount decreases 500 gallons for every \$1 increase in price. Graph m when $m(0) = 3000$. Identify the slope and interpret the x- and y-intercepts of the graph.

37. MODELING WITH MATHEMATICS The function shown models the depth d (in inches) of snow on the ground during the first 9 hours of a snowstorm, where t is the time (in hours) after the snowstorm begins. *(See Example 6.)*



a. Graph the function and identify its domain and range.

b. Interpret the slope and the d-intercept of the graph.

38. MODELING WITH MATHEMATICS The function $c(x) = 0.5x + 70$ represents the cost c (in dollars) of renting a truck from a moving company, where x is the number of miles you drive the truck.

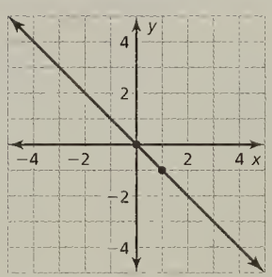
a. Graph the function and identify its domain and range.

b. Interpret the slope and the c-intercept of the graph.

39. COMPARING FUNCTIONS A linear function models the cost of renting a truck from a moving company. The table shows the cost y (in dollars) when you drive the truck x miles. Graph the function and compare the slope and the y-intercept of the graph with the slope and the c-intercept of the graph in Exercise 38.

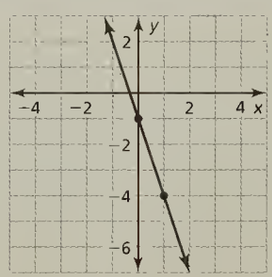
Miles, x	Cost (dollars), y
0	40
50	80
100	120

28.



x-intercept: 0

29.

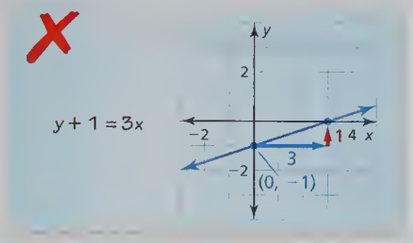


x-intercept: $-\frac{1}{3}$

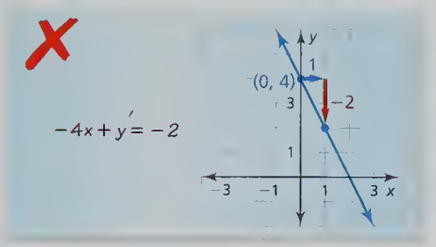
30–39. See Additional Answers.

ERROR ANALYSIS In Exercises 40 and 41, describe and correct the error in graphing the function.

40.



41.



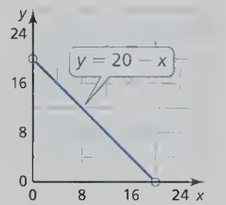
42. **MATHEMATICAL CONNECTIONS** Graph the four equations in the same coordinate plane.

$$3y = -x - 3$$
$$2y - 14 = 4x$$
$$4x - 3 - y = 0$$
$$x - 12 = -3y$$

a. What enclosed shape do you think the lines form? Explain.

b. Write a conjecture about the equations of parallel lines.

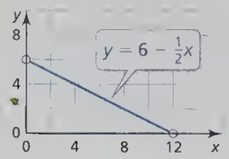
43. **MATHEMATICAL CONNECTIONS** The graph shows the relationship between the width y and the length x of a rectangle in inches. The perimeter of a second rectangle is 10 inches less than the perimeter of the first rectangle.



a. Graph the relationship between the width and length of the second rectangle.

b. How does the graph in part (a) compare to the graph shown?

44. **MATHEMATICAL CONNECTIONS** The graph shows the relationship between the base length x and the side length (of the two equal sides) y of an isosceles triangle in meters. The perimeter of a second isosceles triangle is 8 meters more than the perimeter of the first triangle.



a. Graph the relationship between the base length and the side length of the second triangle.

b. How does the graph in part (a) compare to the graph shown?

45. **ANALYZING EQUATIONS** Determine which of the equations could be represented by each graph.

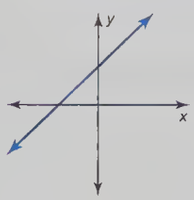
$y = -3x + 8$ $\quad y = -x - \frac{4}{3}$

$y = -7x$ $\quad y = 2x - 4$

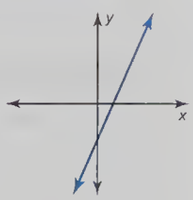
$y = \frac{7}{4}x - \frac{1}{4}$ $\quad y = \frac{1}{3}x + 5$

$y = -4x - 9$ $\quad y = 6$

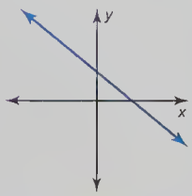
a.



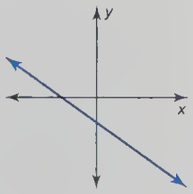
b.



c.



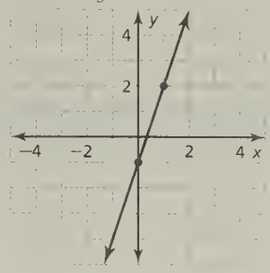
d.



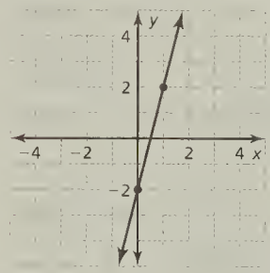
46. **MAKING AN ARGUMENT** Your friend says that you can write the equation of any line in slope-intercept form. Is your friend correct? Explain your reasoning.

ANSWERS

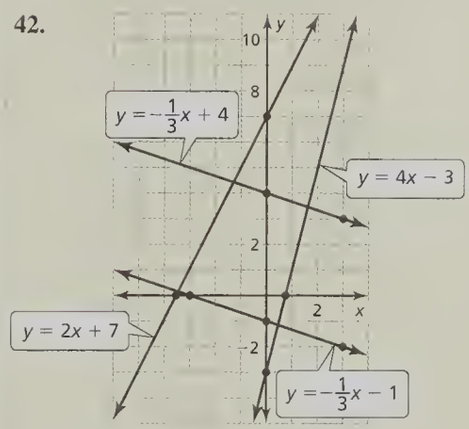
40. The slope is 3, but the graph shows a slope of $\frac{1}{3}$.



41. The slope was interpreted as the y-intercept, and the y-intercept was interpreted as the slope.



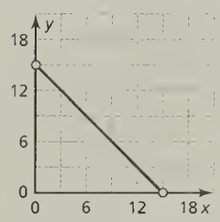
42.



a. trapezoid; The 2 lines with a slope of $-\frac{1}{3}$ are parallel, and the other 2 lines are not parallel.

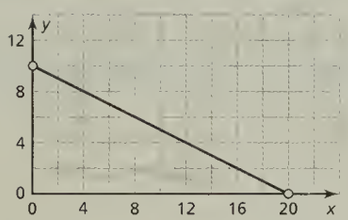
b. Parallel lines have the same slope.

43. a.



b. The slope of each graph is the same. The open circles in the graph in part (a) are closer to the origin.

44. a.



b. The slope of each graph is the same. The open circles in the graph in part (a) are farther from the origin.

45. a. $y = \frac{1}{3}x + 5$

b. $y = \frac{7}{4}x - \frac{1}{4}$; $y = 2x - 4$

c. $y = -3x + 8$

d. $y = -4x - 9$; $y = -x - \frac{4}{3}$

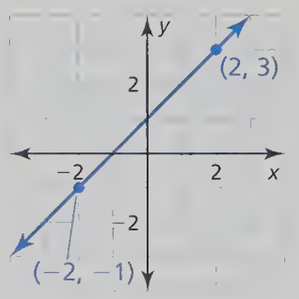
46. no; The equation of a vertical line cannot be written in slope-intercept form because its slope is undefined and it either has no y-intercept or infinite y-intercepts.

ANSWERS

47–61. See Additional Answers.

Mini-Assessment

1. Describe the slope of the line. Then find the slope.

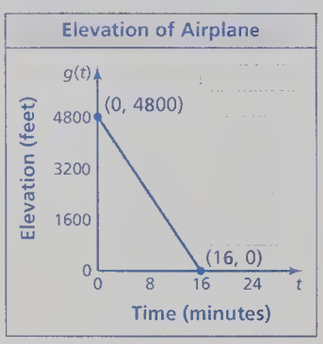


The line rises from left to right. The slope is positive; $m = 1$.

2. Find the slope and the y-intercept of the graph of the linear equation $y - 5x = -6$. slope: 5; y-intercept: -6

3. An airplane begins to descend for a landing. The elevation g (in feet) of the airplane is modeled by the function $g(t) = 4800 - 300t$, where t is the time (in minutes) since the airplane began to descend.

 a. Graph the function and identify its domain and range.



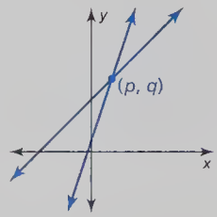
The domain is $0 \le t \le 16$ and the range is $0 \le g \le 4800$.

 b. Interpret the slope and the intercepts of the graph.
slope $= -300$: The airplane descends at a rate of 300 ft/min.
g-intercept $= 4800$: The elevation of the airplane when the descent begins is 4800 ft.
t-intercept $= 16$: The airplane takes 16 min to land.

47. WRITING Write the definition of the slope of a line in two different ways.

48. THOUGHT PROVOKING Your family goes on vacation to a beach 300 miles from your house. You reach your destination 6 hours after departing. Draw a graph that describes your trip. Explain what each part of your graph represents.

49. ANALYZING A GRAPH The graphs of the functions $g(x) = 6x + a$ and $h(x) = 2x + b$, where a and b are constants, are shown. They intersect at the point (p, q).

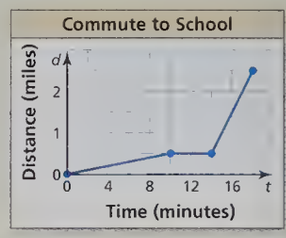


a. Label the graphs of g and h.

b. What do a and b represent?

c. Starting at the point (p, q), trace the graph of g until you get to the point with the x-coordinate $p + 2$. Mark this point C. Do the same with the graph of h. Mark this point D. How much greater is the y-coordinate of point C than the y-coordinate of point D?

50. HOW DO YOU SEE IT? You commute to school by walking and by riding a bus. The graph represents your commute.



a. Describe your commute in words.

b. Calculate and interpret the slopes of the different parts of the graph.

PROBLEM SOLVING **In Exercises 51 and 52, find the value of k so that the graph of the equation has the given slope or y-intercept.**

S1. $y = 4kx - 5$; $m = \frac{1}{2}$

S2. $y = -\frac{1}{3}x + \frac{5}{6}k$; $b = -10$

S3. ABSTRACT REASONING To show that the slope of a line is constant, let (x_1, y_1) and (x_2, y_2) be any two points on the line $y = mx + b$. Use the equation of the line to express y_1 in terms of x_1 and y_2 in terms of x_2. Then use the slope formula to show that the slope between the points is m.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. *(Section 2.4)*

54. $8a - 7 \le 2(3a - 1)$

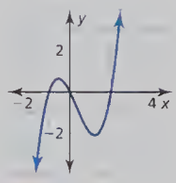
55. $-3(2p + 4) > -6p - 5$

56. $4(3h + 1.5) \ge 6(2h - 2)$

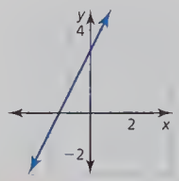
57. $-4(x + 6) < 2(2x - 9)$

Determine whether the graph or table represents a *linear* or *nonlinear* function. Explain. *(Section 3.2)*

58.



59.



60.

x	0	1	2	3	4
y	$\frac{1}{2}$	1	2	4	8

61.

x	2	4	6	8	10
y	0	−1	−2	−3	−4

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

3.6 Modeling Direct Variation

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.2.D

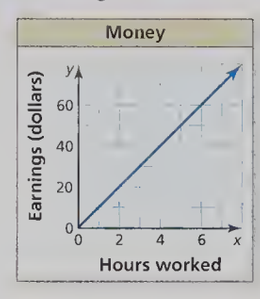
Essential Question How can you describe the relationship between two quantities that vary directly?

Two quantities x and y show **direct variation** when $y = ax$ and $a \neq 0$.

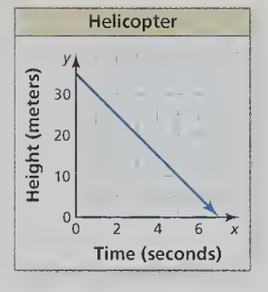
EXPLORATION 1 Identifying Direct Variation

Work with a partner. Determine whether x and y show direct variation. Explain your reasoning.

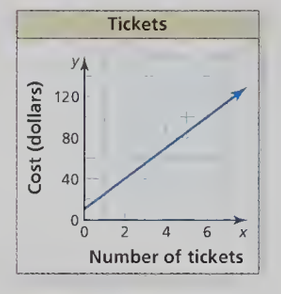
a.



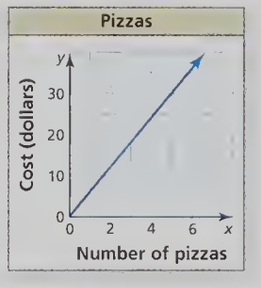
b.



c.



d.



e.

Laps, x	1	2	3	4
Time (seconds), y	90	200	325	480

f.

Cups of sugar, x	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
Cups of flour, y	1	2	3	4

REASONING
To be proficient in math, you need to reason abstractly. You also need to make sense of quantities and their relationships in problem situations.

EXPLORATION 2 Analyzing Relationships

Work with a partner. For the relationships that show direct variation in Exploration 1, do the following.

- Find the slope of the line.
- Find the value of y for the ordered pair $(1, y)$.

What do you notice? What does the value of y represent?

Communicate Your Answer

3. How can you describe the relationship between two quantities that vary directly?
4. Give a real-life example of two quantities that show direct variation. Write an equation that represents the relationship and sketch its graph.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Dynamic Teaching Tools

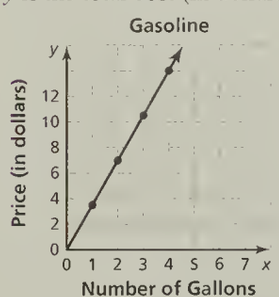
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.D The student is expected to write and solve equations involving direct variation.

ANSWERS

1. **a.** yes; The line passes through the origin.
 b. no; The line does not pass through the origin.
 c. no; The line does not pass through the origin.
 d. yes; The line passes through the origin.
 e. no; The rate of change is not constant.
 f. yes; The rate of change is constant, and the relation can be represented by the equation $y = 2x$.
2. part (a): 10, 10; part (d): 6, 6; part (f): 2, 2; They are the same; the value of a in the direct variation equation
3. *Sample answer:* The graph of the data is a line that passes through the origin.
4. *Sample answer:* gallons of gas and price; $y = 3.5x$, where x is the number of gallons of gasoline and y is the total cost (in dollars)



Extra Example 1

Determine whether x and y show direct variation. If so, identify the constant of variation.

a. $3x - y = 0$

The equation can be rewritten in the form $y = ax$. So, x and y show direct variation. The constant of variation is 3.

b. $x + 2y = 6$

The equation cannot be rewritten in the form $y = ax$. So, x and y do not show direct variation.

MONITORING PROGRESS ANSWERS

1. no
2. yes; -2
3. yes; $\frac{4}{5}$

3.6 Lesson

Core Vocabulary

direct variation, *p. 134*
constant of variation, *p. 134*

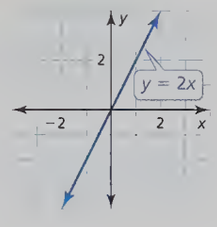
Previous
origin

What You Will Learn

- Determine whether two quantities show direct variation.
- Write direct variation equations.
- Use direct variation equations to solve real-life problems.

Direct Variation

Two quantities x and y show **direct variation** when $y = ax$ and $a \neq 0$. The number a is called the **constant of variation**, and y is said to *vary directly* with x. The equation $y = 2x$ is an example of direct variation, and the constant of variation is 2.



Notice that a direct variation equation $y = ax$ is a linear equation in slope-intercept form, $y = mx + b$, with $m = a$ and $b = 0$. The graph of a direct variation equation is a line with a slope of a that passes through the origin.

EXAMPLE 1 Identifying Direct Variation

Determine whether x and y show direct variation. If so, identify the constant of variation.

a. $2x - 3y = 0$ b. $-x + y = 4$

SOLUTION

a. Solve the equation for y.

$2x - 3y = 0$ Write the equation.
$-3y = -2x$ Subtract $2x$ from each side.
$y = \frac{2}{3}x$ Divide each side by -3 and simplify.

The equation can be rewritten in the form $y = ax$. So, x and y show direct variation. The constant of variation is $\frac{2}{3}$.

b. Solve the equation for y.

$-x + y = 4$ Write the equation.
$y = x + 4$ Add x to each side.

The equation *cannot* be rewritten in the form $y = ax$. So, x and y do *not* show direct variation.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Determine whether x and y show direct variation. If so, identify the constant of variation.

1. $-x + y = 1$
2. $2x + y = 0$
3. $-4x = -5y$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students use the following Core Vocabulary as they read the topic Direct Variation: *direct variation, constant of variation.*

Beginning Repeat the Core Vocabulary to improve pronunciation.
Intermediate Read aloud one sentence with Core Vocabulary.
Advanced Read aloud the text preceding Example 1.
Advanced High Explain a sentence with Core Vocabulary using their own words.

ELPS 3.D.1 Speak using grade-level content area vocabulary in context to internalize new English words.

EXAMPLE 2 Identifying Direct Variation

Determine whether x and y show direct variation. Explain.

a.

x	1	2	3	4
y	−1	1	3	5

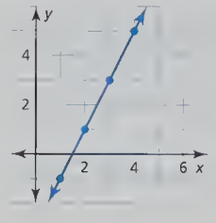
b.

x	−2	0	2	4
y	−1	0	1	2

SOLUTION

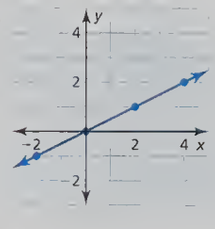
Plot the ordered pairs. Then draw a line through the points.

a.



▶ The line does *not* pass through the origin. So, x and y do *not* show direct variation.

b.



▶ The line passes through the origin. So, x and y show direct variation.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Determine whether x and y show direct variation. Explain.

4.

x	2	3	4	5
y	−4	−6	−8	−10

5.

x	0	1	2	3
y	−2	1	4	7

Writing Direct Variation Equations

The direct variation equation $y = ax$ can be rewritten as $\frac{y}{x} = a$ for $x \neq 0$. So, in a direct variation, the ratio of y to x is constant for all nonzero data pairs (x, y).

EXAMPLE 3 Writing a Direct Variation Equation

The table shows the costs C (in dollars) of downloading s songs from a music website.

Number of songs, s	Cost (dollars), C
3	2.97
5	4.95
7	6.93

a. Explain why C varies directly with s.

b. Write a direct variation equation that relates s and C.

SOLUTION

a. Find the ratio $\frac{C}{s}$ for each data pair (s, C).

$$\frac{2.97}{3} = 0.99, \frac{4.95}{5} = 0.99, \frac{6.93}{7} = 0.99$$

▶ All of the ratios are equal to 0.99. So, C varies directly with s.

b. Because $\frac{C}{s} = 0.99$, the constant of variation is 0.99. So, a direct variation equation is $C = 0.99s$.

APPLYING MATHEMATICS

For real-world data, the ratios may not be exactly equal. You may still be able to use direct variation when the ratios are approximately equal.

Extra Example 2

Determine whether x and y show direct variation. Explain.

a.

x	1	2	3	4
y	−3	−6	−9	−12

x and y show direct variation. The line passes through the origin.

b.

x	−3	0	3	6
y	1	2	3	4

x and y do not show direct variation. The line does not pass through the origin.

Extra Example 3

The table shows the amount A (in dollars) Serina earned for working h hours.

Number of hours, h	Amount earned (dollars), A
9	76.50
15	127.50
20	170.00

a. Explain why A varies directly with h.
All the ratios $\frac{A}{h}$ are equal to 8.5.

b. Write a direct variation equation that relates h and A. $A = 8.5h$

MONITORING PROGRESS ANSWERS

4. yes; The ordered pairs lie on a line that passes through the origin.

5. no; The ordered pairs lie on a line that does not pass through the origin.

English Language Learners

Pair Activity

Pair each English learner with an English speaker. Give each pair two equations to determine whether x and y show a direct variation. Each student works with a different equation, solving for y. Then, they take turns explaining why there is a direct variation or not, while the other student follows along.

Extra Example 4

The number C of calories in a container of orange juice varies directly with the number z of ounces of orange juice in the container. A 12-ounce container of orange juice has 168 calories. Write a direct variation for this situation. How many calories are in a 32-ounce container of the same orange juice? $C = 14z$; 448 calories

MONITORING PROGRESS ANSWERS

6. no; The equation that represents this situation is not of the form $y = ax$.
7. 750 lb

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

6. **WHAT IF?** The website in Example 3 charges a total of \$1.99 for the first 5 songs you download and \$0.99 for each song after the first 5. Is it reasonable to use a direct variation model for this situation? Explain.

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**



The number s of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number w of gallons of water in the tank. A pet shop owner recommends that you add 100 tablespoons of salt to a 20-gallon tank. How many tablespoons of salt should you add to a 30-gallon tank?

USING PRECISE MATHEMATICAL LANGUAGE

The coefficient of variation in Example 4 is a rate of change: 5 tablespoons of sea salt per gallon of water.

SOLUTION

1. **Understand the Problem** You know that a 20-gallon tank requires 100 tablespoons of salt and that the amount of salt varies directly with the capacity of the tank. You are asked to find the amount of salt you should add to a 30-gallon tank.
2. **Make a Plan** Use the direct variation equation $y = ax$ and the given values to write a direct variation equation for this situation. Then solve the equation when $w = 30$.
3. **Solve the Problem**

 Step 1 Write a direct variation equation. Because s varies directly with w, you can use the equation $s = aw$. Also use the fact that $s = 100$ when $w = 20$.

$s = aw$	Write direct variation equation.
$(100) = a(20)$	Substitute.
$5 = a$	Divide each side by 20 and simplify.

 A direct variation equation is $s = 5w$.

 Step 2 Find the number of tablespoons of salt that you should add to a 30-gallon tank. Use the direct variation equation from Step 1.

$s = 5w$	Write direct variation equation.
$s = 5(30)$	Substitute 30 for w.
$s = 150$	Simplify.

 ▶ You should add 150 tablespoons of salt to a 30-gallon tank.

4. **Look Back** Find the ratio $\frac{w}{s}$ for the given data pair and the data calculated in the solution.

 $$\frac{100}{20} = 5, \frac{150}{30} = 5$$

 Both ratios are equal to 5, so the solution makes sense.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. An object that weighs 100 pounds on Earth would weigh just 6 pounds on Pluto. Assume that weight P on Pluto varies directly with weight E on Earth. What would a boulder that weighs 45 pounds on Pluto weigh on Earth?

Closure

- **Muddiest Point:** Ask students to identify, aloud or on a paper to be collected, the muddiest point(s) about the lesson. What was difficult to understand?

3.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Two quantities x and y show _____ when $y = ax$ and $a \neq 0$.
2. **WRITING** A line has a slope of -3 and a y-intercept of 4. Does the equation of the line represent direct variation? Explain.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, determine whether x and y show direct variation. If so, identify the constant of variation. *(See Example 1.)*

3. $y = x$
4. $y = 2x$
5. $y = \frac{1}{2}x - 1$
6. $y = 3x + 2$
7. $4x + y = 1$
8. $-\frac{1}{4}x + y = 0$
9. $-x - 3y = 0$
10. $4 - 6x = 2y$

11. **ERROR ANALYSIS** Describe and correct the error in determining whether x and y show direct variation.

✗ $6x - y = 0$

Because the equation is not in the form $y = ax$, it does not represent direct variation.

12. **ERROR ANALYSIS** Describe and correct the error in identifying the constant of variation for the direct variation equation.

✗ $-5x + 3y = 0$

$3y = 5x$

The constant of variation is 5.

In Exercises 13–16, determine whether x and y show direct variation. Explain. *(See Example 2.)*

13.

x	1	2	3	4	6
y	5	10	15	20	30

14.

x	-3	-1	1	3	5
y	-2	0	2	4	6

15.

x	1	3	5	7	9
y	6	12	18	24	30

16.

x	2	4	6	8	10
y	1	2	4	8	16

In Exercises 17–22, the ordered pair is a solution of a direct variation equation. Write the equation and identify the constant of variation.

17. (5, 6)
18. (2, 1)
19. $(-1, 3)$
20. $(-3, -6)$
21. $(-2, -7)$
22. $(-5, 2)$

In Exercises 23 and 24, determine whether the situation shows direct variation. Explain your reasoning.

23. A canoe rental costs \$20 plus \$5 for each hour of the rental.
24. New carpet costs \$4 per square foot.
25. **MODELING WITH MATHEMATICS** At a recycling center, computers and computer accessories can be recycled for a fee f based on weight w, as shown in the table. *(See Example 3.)*

Weight (pounds), w	10	15	30
Fee (dollars), f	2.50	3.75	7.50

a. Explain why f varies directly with w.

b. Write a direct variation equation that relates w and f.

c. Find the total recycling fee for a computer that weighs 18 pounds and a printer that weighs 10 pounds.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–15 odd, 25, 27, 32, 34–36

Average: 1, 2–28 even, 32, 34–36

Advanced: 1, 2, 8–28 even, 29–36

HOMEWORK CHECK

Basic: 5, 13, 25, 27

Average: 4, 14, 26, 28

Advanced: 8, 16, 26, 28

ANSWERS

1. direct variation
2. no; Direct variation can only be represented by the equation of a line whose y-intercept is 0.
3. yes; 1
4. yes; 2
5. no
6. no
7. no
8. yes; $\frac{1}{4}$
9. yes; $-\frac{1}{3}$
10. no
11. The equation needs to be solved for y; $y = 6x$; x and y show direct variation because the equation can be written in the form $y = ax$.
12. The equation needs to be in the form $y = ax$; $y = \frac{5}{3}x$; The constant of variation is $\frac{5}{3}$.
13. yes; The ordered pairs lie on a line that passes through the origin.
14. no; The ordered pairs lie on a line that does not pass through the origin.
15. no; The ordered pairs lie on a line that does not pass through the origin.
16. no; The ordered pairs do not lie on a line.
17. $y = \frac{6}{5}x$; $\frac{6}{5}$
18. $y = \frac{1}{2}x$; $\frac{1}{2}$
19. $y = -3x$; -3
20. $y = 2x$; 2
21. $y = \frac{7}{2}x$; $\frac{7}{2}$
22. $y = -\frac{2}{5}x$; $-\frac{2}{5}$
23. no; The equation that represents the situation is not of the form $y = ax$.
24. yes; The equation that represents the situation is of the form $y = ax$.
25. a. All of the ratios $\frac{f}{w}$ are equal to 0.25.

 b. $f = 0.25w$

 c. \$7

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

26. **a.** All of the ratios $\frac{p}{\ell}$ are equal to 1.25.

b. $p = 1.25\ell$

c. 24 in.

27. 12 h

28. 1.5 in.

29. yes; The equation is of the form $y = ax$.

30. *Sample answer:* When traveling in a car, as the speed of the car increases, the travel time decreases.

31–36. See Additional Answers.

26. MODELING WITH MATHEMATICS A jewelry store sells gold chain by the inch. The table shows the prices of various lengths of gold chain.

Length (inches), ℓ	7	9	16	18
Price (dollars), p	8.75	11.25	20.00	22.50

a. Explain why p varies directly with ℓ.

b. Write a direct variation equation that relates ℓ and p.

c. You have \$30. What is the longest chain that you can buy?

27. MODELING WITH MATHEMATICS At a company, the number h of vacation hours an employee earns varies directly with the number w of weeks the employee works. An employee who works 2 weeks earns 3 vacation hours. Find the number of vacation hours an employee earns for working 8 weeks. *(See Example 4.)*

28. MODELING WITH MATHEMATICS Landscapers plan to spread a layer of stone on a path. The number s of bags of stone needed varies directly with the depth d (in inches) of the layer. They need 20 bags to spread a layer of stone that is 2 inches deep. How deep will the layer of stone be when they use 15 bags of stone?



29. REASONING The slope of a line is $-\frac{1}{3}$ and the point $(-6, 2)$ lies on the line. Determine whether the equation of the line is a direct variation equation. Explain.

30. THOUGHT PROVOKING Two quantities x and y show *inverse variation* when $y = \frac{a}{x}$ and $a \neq 0$. Give an example of a real-life situation that shows inverse variation.

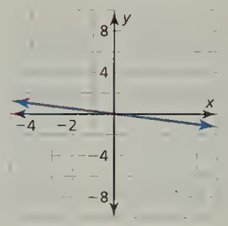
31. REASONING Consider the distance equation $d = rt$, where d is the distance (in feet), r is the rate (in feet per second), and t is the time (in seconds).

a. You run 6 feet per second. Do distance and time vary directly? Explain.

b. You run for 50 seconds. Do distance and rate vary directly? Explain.

c. You run 300 feet. Do rate and time vary directly? Explain.

32. HOW DO YOU SEE IT? Consider the graph shown.



a. Explain why x and y show direct variation.

b. The scale of the x-axis changes. Do x and y still show direct variation? Explain.

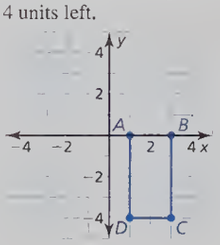
33. CRITICAL THINKING Consider an equation where y varies directly with x. Does x vary directly with y? If so, what is the relationship between the constants of variation?

Maintaining Mathematical Proficiency

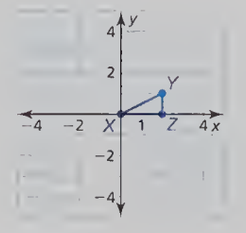
Reviewing what you learned in previous grades and lessons

Find the coordinates of the figure after the transformation. *(Skills Review Handbook)*

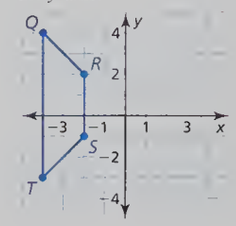
34. Translate the rectangle 4 units left.



35. Dilate the triangle with respect to the origin using a scale factor of 2.



36. Reflect the trapezoid in the y-axis.



Mini-Assessment

Determine whether x and y show direct variation. Explain.

1. $3x - 4y = 0$

x and y show direct variation. The equation can be rewritten in the form $y = ax$.

2.

x	−1	2	3	6
y	6	3	2	−1

x and y do not show direct variation. The line does not pass through the origin.

3. The table shows the amount A (in dollars) Todd earned for washing c cars.

Number of cars, c	Amount earned (dollars), A
3	18.75
5	31.25
8	50.00

a. Explain why A varies directly with c. All the ratios $\frac{A}{c}$ are equal to 6.25.

b. Write a direct variation equation that relates c and A. $A = 6.25c$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

3.7 Transformations of Graphs of Linear Functions

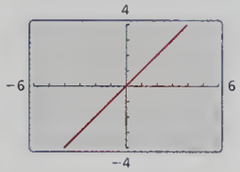
Texas Essential Knowledge and Skills
A.3.E

Essential Question
How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

SELECTING TOOLS
To be proficient in math, you need to use the appropriate tools, including graphs, tables, and technology, to check your results.

EXPLORATION 1 Comparing Graphs of Functions

Work with a partner. The graph of $f(x) = x$ is shown. Sketch the graph of each function, along with f, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?



a. $g(x) = x + 4$
b. $g(x) = x + 2$
c. $g(x) = x - 2$
d. $g(x) = x - 4$

EXPLORATION 2 Comparing Graphs of Functions

Work with a partner. Sketch the graph of each function, along with $f(x) = x$, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?

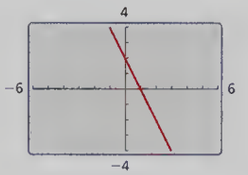
a. $h(x) = \frac{1}{2}x$
b. $h(x) = 2x$
c. $h(x) = -\frac{1}{2}x$
d. $h(x) = -2x$

EXPLORATION 3 Matching Functions with Their Graphs

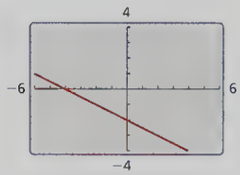
Work with a partner. Match each function with its graph. Use a graphing calculator to check your results. Then use the results of Explorations 1 and 2 to compare the graph of k to the graph of $f(x) = x$.

a. $k(x) = 2x - 4$
b. $k(x) = -2x + 2$
c. $k(x) = \frac{1}{2}x + 4$
d. $k(x) = -\frac{1}{2}x - 2$

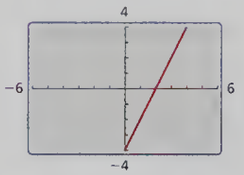
A.



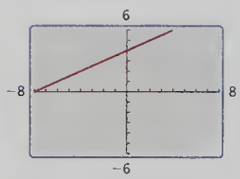
B.



C.



D.

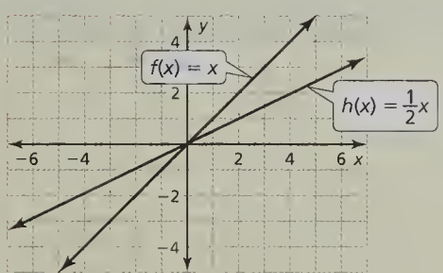


Communicate Your Answer

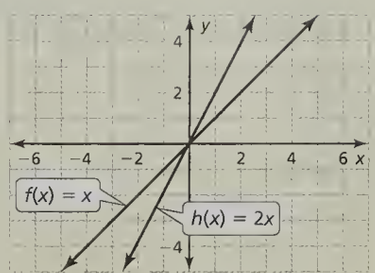
4. How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

2. a.



b.



2c–4. See Additional Answers.

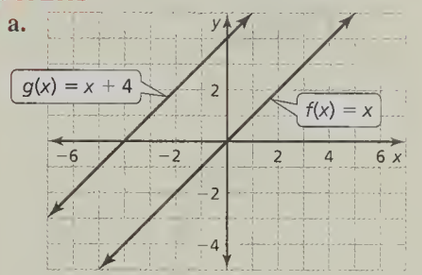
Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

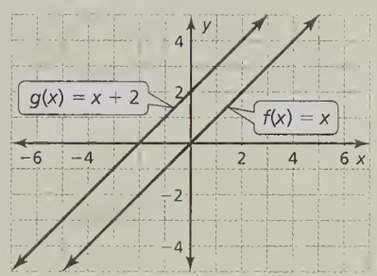
A.3.E The student is expected to determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

ANSWERS

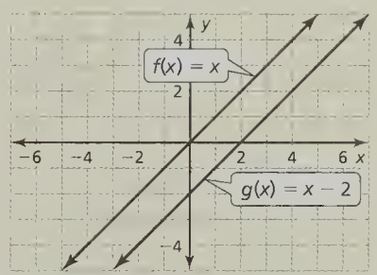
1. a.



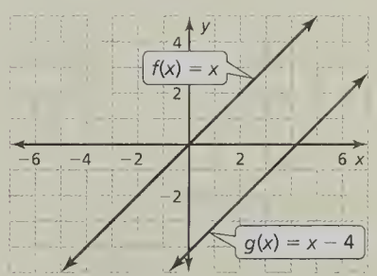
b.



c.



d.

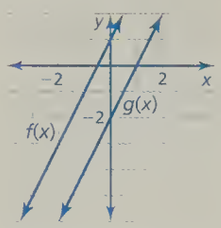


Adding or subtracting a constant to a function causes a vertical shift of the graph.

Extra Example 1

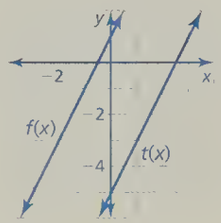
Let $f(x) = 2x + 1$.

a. Graph $g(x) = f(x) - 3$. Describe the transformation from the graph of f to the graph of g.



The graph of g is a vertical translation 3 units down of the graph of f.

b. Graph $t(x) = f(x - 3)$. Describe the transformation from the graph of f to the graph of t.



The graph of t is a horizontal translation 3 units right of the graph of f.

3.7 Lesson

What You Will Learn

- Translate and reflect graphs of linear functions.
- Stretch and shrink graphs of linear functions.
- Combine transformations of graphs of linear functions.

Core Vocabulary

family of functions, *p. 140*
parent function, *p. 140*
transformation, *p. 140*
translation, *p. 140*
reflection, *p. 141*
horizontal shrink, *p. 142*
horizontal stretch, *p. 142*
vertical stretch, *p. 142*
vertical shrink, *p. 142*

Previous
linear function

Translations and Reflections

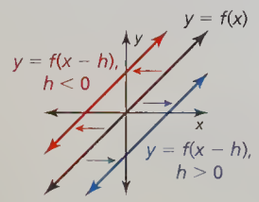
A **family of functions** is a group of functions with similar characteristics. The most basic function in a family of functions is the **parent function**. For nonconstant linear functions, the parent function is $f(x) = x$. The graphs of all other nonconstant linear functions are *transformations* of the graph of the parent function. A **transformation** changes the size, shape, position, or orientation of a graph.

Core Concept

A **translation** is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

Horizontal Translations

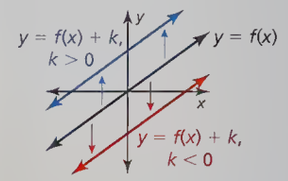
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the *inputs* before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



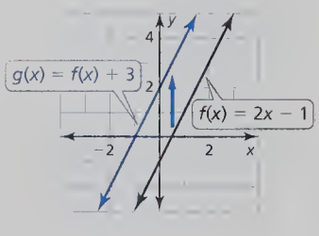
Adding k to the *outputs* shifts the graph down when $k < 0$ and up when $k > 0$.

EXAMPLE 1 Horizontal and Vertical Translations

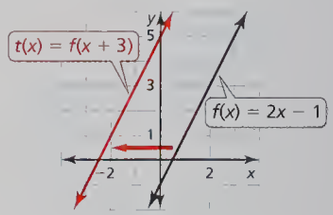
Let $f(x) = 2x - 1$. Graph (a) $g(x) = f(x) + 3$ and (b) $t(x) = f(x + 3)$. Describe the transformations from the graph of f to the graphs of g and t.

SOLUTION

a. The function g is of the form $y = f(x) + k$, where $k = 3$. So, the graph of g is a vertical translation 3 units up of the graph of f.



b. The function t is of the form $y = f(x - h)$, where $h = -3$. So, the graph of t is a horizontal translation 3 units left of the graph of f.



ANALYZING MATHEMATICAL RELATIONSHIPS

In part (a), the output of g is equal to the output of f plus 3.

In part (b), the output of t is equal to the output of f when the input of f is 3 more than the input of t.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Visual

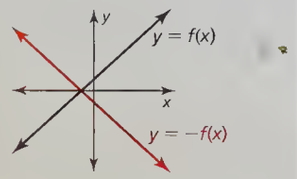
Students may have difficulty remembering which graph is the parent function and which is the transformation. Choose one color for students to use exclusively for graphing the parent function. Have them use any other color to graph the transformation.

Core Concept

A **reflection** is a transformation that flips a graph over a line called the *line of reflection*.

Reflections in the *x*-axis

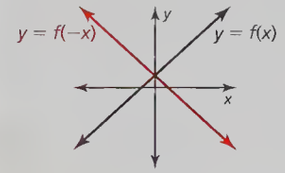
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the outputs by -1 changes their signs.

Reflections in the *y*-axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.



Multiplying the inputs by -1 changes their signs.

STUDY TIP

A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

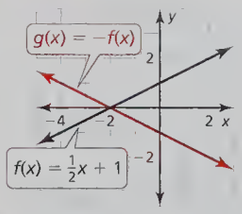
EXAMPLE 2 Reflections in the *x*-axis and the *y*-axis

Let $f(x) = \frac{1}{2}x + 1$. Graph (a) $g(x) = -f(x)$ and (b) $t(x) = f(-x)$. Describe the transformations from the graph of f to the graphs of g and t.

SOLUTION

a. To find the outputs of g, multiply the outputs of f by -1. The graph of g consists of the points $(x, -f(x))$.

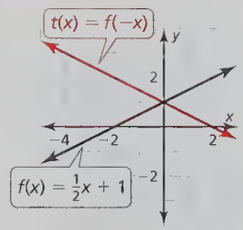
x	-4	-2	0
$f(x)$	-1	0	1
$-f(x)$	1	0	-1



▶ The graph of g is a reflection in the x-axis of the graph of f.

b. To find the outputs of t, multiply the inputs by -1 and then evaluate f. The graph of t consists of the points $(x, f(-x))$.

x	-2	0	2
$-x$	2	0	-2
$f(-x)$	2	1	0



▶ The graph of t is a reflection in the y-axis of the graph of f.

Monitoring Progress

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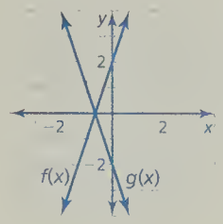
Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

1. $f(x) = 3x + 1$; $g(x) = f(x) - 2$; $h(x) = f(x - 2)$
2. $f(x) = -4x - 2$; $g(x) = -f(x)$; $h(x) = f(-x)$

Extra Example 2

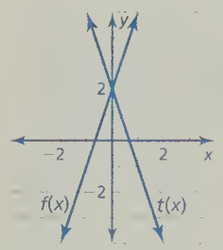
Let $f(x) = 3x + 2$.

a. Graph $g(x) = -f(x)$. Describe the transformation from the graph of f to the graph of g.



The graph of g is a reflection in the x-axis of the graph of f.

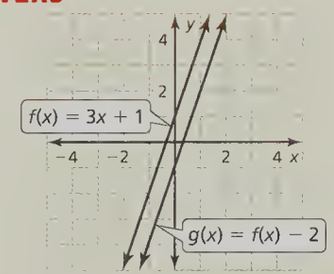
b. Graph $t(x) = f(-x)$. Describe the transformation from the graph of f to the graph of t.



The graph of t is a reflection in the y-axis of the graph of f.

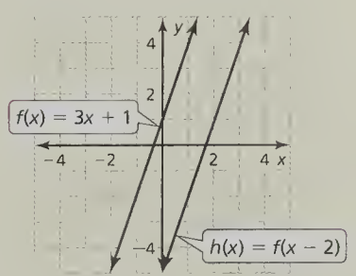
MONITORING PROGRESS ANSWERS

1. a.



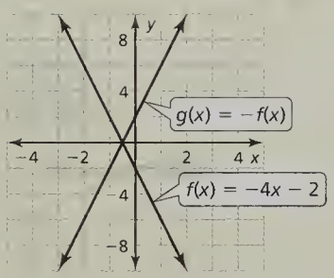
The graph of g is a vertical translation 2 units down of the graph of f.

b.



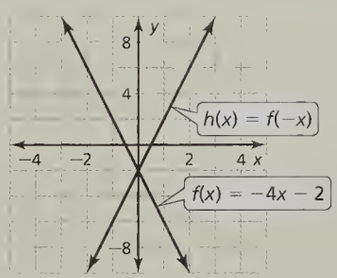
The graph of t is a horizontal translation 2 units right of the graph of f.

2. a.



The graph of g is a reflection in the x-axis of the graph of f.

b.

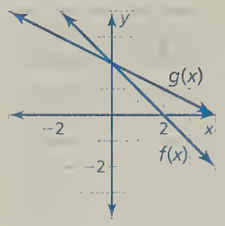


The graph of h is a reflection in the y-axis of the graph of f.

Extra Example 3

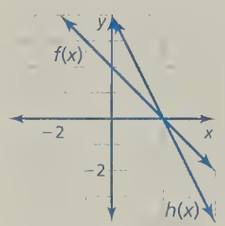
Let $f(x) = -x + 2$.

a. Graph $g(x) = f\left(\frac{1}{2}x\right)$. Describe the transformation from the graph of f to the graph of g.



The graph of g is a horizontal stretch of the graph of f by a factor of $1 \div \frac{1}{2} = 2$.

b. Graph $h(x) = 2f(x)$. Describe the transformation from the graph of f to the graph of h.



The graph of h is a vertical stretch of the graph of f by a factor of 2.

Stretches and Shrinks

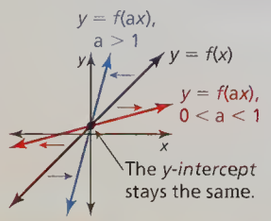
You can transform a function by multiplying all the x-coordinates (inputs) by the same factor a. When $a > 1$, the transformation is a **horizontal shrink** because the graph shrinks toward the y-axis. When $0 < a < 1$, the transformation is a **horizontal stretch** because the graph stretches away from the y-axis. In each case, the y-intercept stays the same.

You can also transform a function by multiplying all the y-coordinates (outputs) by the same factor a. When $a > 1$, the transformation is a **vertical stretch** because the graph stretches away from the x-axis. When $0 < a < 1$, the transformation is a **vertical shrink** because the graph shrinks toward the x-axis. In each case, the x-intercept stays the same.

Core Concept

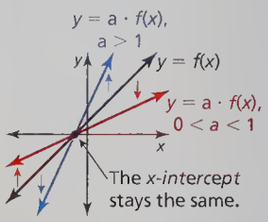
Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



Vertical Stretches and Shrinks

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



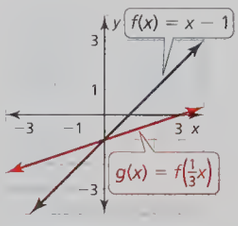
STUDY TIP

The graphs of $y = f(-ax)$ and $y = -a \cdot f(x)$ represent a stretch or shrink *and* a reflection in the x- or y-axis of the graph of $y = f(x)$.

EXAMPLE 3 Horizontal and Vertical Stretches

Let $f(x) = x - 1$. Graph (a) $g(x) = f\left(\frac{1}{3}x\right)$ and (b) $h(x) = 3f(x)$. Describe the transformations from the graph of f to the graphs of g and h.

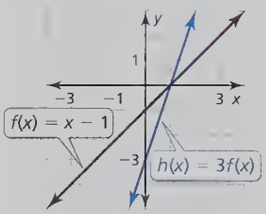
SOLUTION



a. To find the outputs of g, multiply the inputs by $\frac{1}{3}$. Then evaluate f. The graph of g consists of the points $\left(x, f\left(\frac{1}{3}x\right)\right)$.

x	−3	0	3
$\frac{1}{3}(x)$	−1	0	1
$f\left(\frac{1}{3}x\right)$	−2	−1	0

▶ The graph of g is a horizontal stretch of the graph of f by a factor of $1 \div \frac{1}{3} = 3$.



b. To find the outputs of h, multiply the outputs of f by 3. The graph of h consists of the points $(x, 3f(x))$.

x	0	1	2
$f(x)$	−1	0	1
$3f(x)$	−3	0	3

▶ The graph of h is a vertical stretch of the graph of f by a factor of 3.

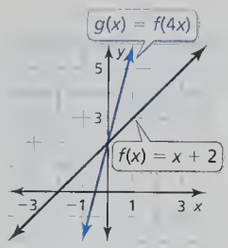
EXAMPLE 4 Horizontal and Vertical Shrinks

Let $f(x) = x + 2$. Graph (a) $g(x) = f(4x)$ and (b) $h(x) = \frac{1}{4}f(x)$. Describe the transformations from the graph of f to the graphs of g and h.

SOLUTION

a. To find the outputs of g, multiply the inputs by 4. Then evaluate f. The graph of g consists of the points $(x, f(4x))$.

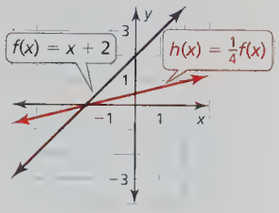
x	-1	0	1
$4x$	-4	0	4
$f(4x)$	-2	2	6



▶ The graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{4}$.

b. To find the outputs of h, multiply the outputs of f by $\frac{1}{4}$. The graph of h consists of the points $\left(x, \frac{1}{4}f(x)\right)$.

x	-2	0	2
$f(x)$	0	2	4
$\frac{1}{4}f(x)$	0	$\frac{1}{2}$	1



▶ The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{4}$.

Monitoring Progress

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Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

3. $f(x) = 4x - 2$; $g(x) = f\left(\frac{1}{2}x\right)$; $h(x) = 2f(x)$

4. $f(x) = -3x + 4$; $g(x) = f(2x)$; $h(x) = \frac{1}{2}f(x)$

Combining Transformations

STUDY TIP

You can perform transformations on the graph of *any* function f using these steps.

Core Concept

Transformations of Graphs

The graph of $y = a \cdot f(x - h) + k$ or the graph of $y = f(ax - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps.

Step 1 Translate the graph of $y = f(x)$ horizontally h units.

Step 2 Use a to stretch or shrink the resulting graph from Step 1.

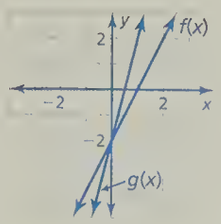
Step 3 Reflect the resulting graph from Step 2 when $a < 0$.

Step 4 Translate the resulting graph from Step 3 vertically k units.

Extra Example 4

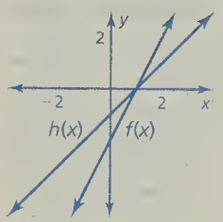
Let $f(x) = 2x - 2$.

a. Graph $g(x) = f(2x)$. Describe the transformation from the graph of f to the graph of g.



The graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{2}$.

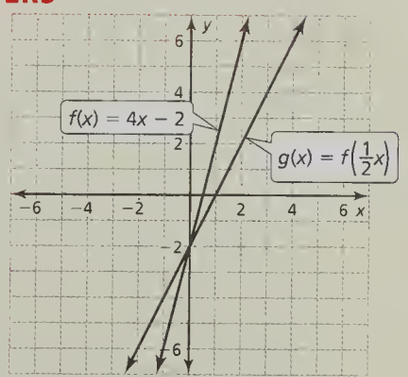
b. Graph $h(x) = \frac{1}{2}f(x)$. Describe the transformation from the graph of f to the graph of h.



The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{2}$.

MONITORING PROGRESS ANSWERS

3. a.

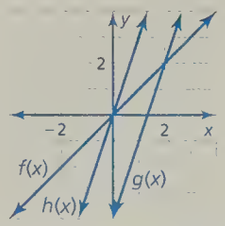


The graph of g is a horizontal stretch of the graph of f by a factor of 2.

3b–4. See Additional Answers.

Extra Example 5

Graph $f(x) = x$ and $g(x) = 3x - 4$. Describe the transformations from the graph of f to the graph of g.



Stretch the graph of f vertically by a factor of 3 to get the graph of $h(x) = 3x$. Translate the graph of h vertically 4 units down to get the graph of $g(x) = 3x - 4$.

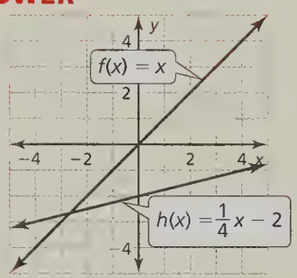
Extra Example 6

In Example 6, the cable company changes the cost to customers from \$60 per month with no installation fee to a new rate of \$45 per month with an installation fee of \$30. The new cost is represented by $s(m) = 45m + 30$. Describe the transformations from the graph of c to the graph of s.

The transformations are a vertical shrink by a factor of $\frac{3}{4}$ and then a vertical translation 30 units up.

MONITORING PROGRESS ANSWER

5.



The transformations are a vertical shrink by a factor of $\frac{1}{4}$ and then a vertical translation 2 units down.

EXAMPLE 5 Combining Transformations

Graph $f(x) = x$ and $g(x) = -2x + 3$. Describe the transformations from the graph of f to the graph of g.

SOLUTION

ANOTHER WAY

You could also rewrite g as $g(x) = f(-2x) + 3$. In this case, the transformations from the graph of f to the graph of g will be different from those in Example 5.

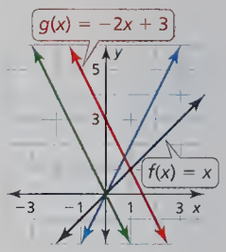
Note that you can rewrite g as $g(x) = -2f(x) + 3$.

Step 1 There is no horizontal translation from the graph of f to the graph of g.

Step 2 Stretch the graph of f vertically by a factor of 2 to get the graph of $h(x) = 2x$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -2x$.

Step 4 Translate the graph of r vertically 3 units up to get the graph of $g(x) = -2x + 3$.

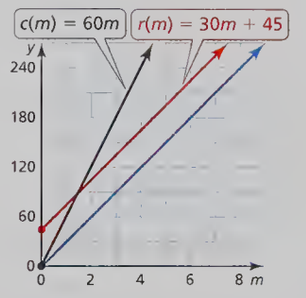


EXAMPLE 6 Solving a Real-Life Problem

A cable company charges customers \$60 per month for its service, with no installation fee. The cost to a customer is represented by $c(m) = 60m$, where m is the number of months of service. To attract new customers, the cable company reduces the monthly fee to \$30 but adds an installation fee of \$45. The cost to a new customer is represented by $r(m) = 30m + 45$, where m is the number of months of service. Describe the transformations from the graph of c to the graph of r.

SOLUTION

Note that you can rewrite r as $r(m) = \frac{1}{2}c(m) + 45$. In this form, you can use the order of operations to get the outputs of r from the outputs of c. First, multiply the outputs of c by $\frac{1}{2}$ to get $h(m) = 30m$. Then add 45 to the outputs of h to get $r(m) = 30m + 45$.



▶ The transformations are a vertical shrink by a factor of $\frac{1}{2}$ and then a vertical translation 45 units up.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

5. Graph $f(x) = x$ and $h(x) = \frac{1}{4}x - 2$. Describe the transformations from the graph of f to the graph of h.

Closure

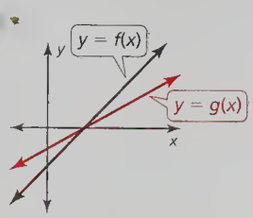
- **Look Back:** Have students *Look Back* over Section 3.7. Have them summarize what they understand about transformations and what helped them learn it.

3.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

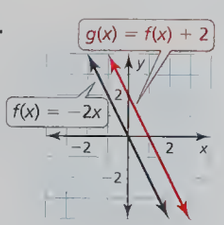
Vocabulary and Core Concept Check

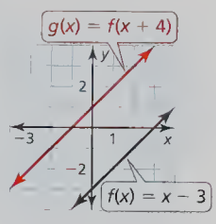
1. **WRITING** Describe the relationship between $f(x) = x$ and all other nonconstant linear functions.
2. **VOCABULARY** Name four types of transformations. Give an example of each and describe how it affects the graph of a function.
3. **WRITING** How does the value of a in the equation $y = f(ax)$ affect the graph of $y = f(x)$? How does the value of a in the equation $y = af(x)$ affect the graph of $y = f(x)$?
4. **REASONING** The functions f and g are linear functions. The graph of g is a vertical shrink of the graph of f. What can you say about the x-intercepts of the graphs of f and g? Is this always true? Explain.



Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, use the graphs of f and g to describe the transformation from the graph of f to the graph of g. *(See Example 1.)*

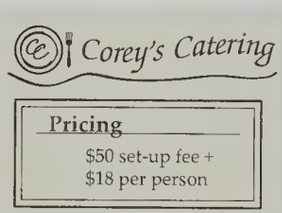
5. 

6. 

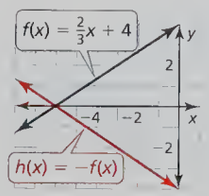
7. $f(x) = \frac{1}{3}x + 3$; $g(x) = f(x) - 3$
8. $f(x) = -3x + 4$; $g(x) = f(x) + 1$
9. $f(x) = -x - 2$; $g(x) = f(x + 5)$
10. $f(x) = \frac{1}{2}x - 5$; $g(x) = f(x - 3)$
11. **MODELING WITH MATHEMATICS** You and a friend start biking from the same location. Your distance d (in miles) after t minutes is given by the function $d(t) = \frac{1}{5}t$. Your friend starts biking 5 minutes after you. Your friend's distance f is given by the function $f(t) = d(t - 5)$. Describe the transformation from the graph of d to the graph of f.

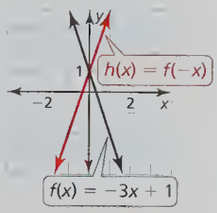


12. **MODELING WITH MATHEMATICS** The total cost C (in dollars) to cater an event with p people is given by the function $C(p) = 18p + 50$. The set-up fee increases by $25. The new total cost T is given by the function $T(p) = C(p) + 25$. Describe the transformation from the graph of C to the graph of T.



In Exercises 13–16, use the graphs of f and h to describe the transformation from the graph of f to the graph of h. *(See Example 2.)*

13. 

14. 

15. $f(x) = -5 - x$; $h(x) = f(-x)$
16. $f(x) = \frac{1}{4}x - 2$; $h(x) = -f(x)$

SUPPORTING English Language Learners

Read Exercise 12 and point out the pricing details from Corey's Catering that are represented by the function C.

Beginning Have students state the original set-up fee and price per person. ($50 and $18)
Intermediate Explain what the 18 and 50 represent in the cost function C.
Advanced/Advanced High Explain how the pricing details are used to determine the original cost function C.

ELPS 4.C.2 Derive meaning of environmental print.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–27 odd, 39–45 odd, 48, 62, 68–73

Average: 1–4, 6–32 even, 36–44 even, 48–52 even, 62, 68–73

Advanced: 1–4, 8–16 even, 20, 22, 26–66 even, 68–73

HOMEWORK CHECK

Basic: 5, 15, 17, 25, 41

Average: 6, 16, 18, 26, 42

Advanced: 8, 16, 20, 28, 42

ANSWERS

1. The graphs of all other nonconstant linear functions are transformations of the graph of $f(x) = x$.
2. translation, reflection, stretch, shrink; *Sample answer:* $y = f(x) + 2$ is a vertical translation 2 units up of the graph of $f(x)$. $y = -f(x)$ is a reflection in the x-axis of the graph of $f(x)$. $y = 2f(x)$ is a vertical stretch by a factor of 2 of the graph of $f(x)$. $y = \frac{1}{2}f(x)$ is a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x)$.
3. causes a horizontal stretch or shrink; causes a vertical stretch or shrink
4. They are the same; yes; *Sample answer:* A point on the x-axis is not affected by a vetical shrink.
5. The graph of g is a vertical translation 2 units up of the graph of f.
6. The graph of g is a horizontal translation 4 units left of the graph of f.
7. The graph of g is a vertical translation 3 units down of the graph of f.
8. The graph of g is a vertical translation 1 unit up of the graph of f.
9. The graph of g is a horizontal translation 5 units left of the graph of f.
10. The graph of g is a horizontal translation 3 units right of the graph of f.
11. The graph of f is a horizontal translation 5 units right of the graph of d.
12. The graph of T is a vertical translation 25 units up of the graph of C.

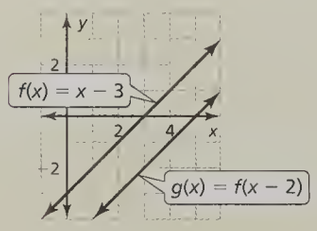
13–16. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

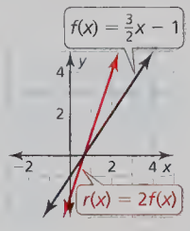
ANSWERS

17. The graph of r is a vertical stretch of the graph of f by a factor of 2.
18. The graph of r is a horizontal stretch of the graph of f by a factor of 4.
19. The graph of r is a horizontal stretch of the graph of f by a factor of 2.
20. The graph of r is a horizontal stretch of the graph of f by a factor of 3.
21. The graph of r is a vertical stretch of the graph of f by a factor of 3.
22. The graph of r is a vertical stretch of the graph of f by a factor of 4.
23. The graph of h is a horizontal shrink of the graph of f by a factor of $\frac{1}{3}$.
24. The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{3}$.
25. The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{6}$.
26. The graph of h is a horizontal shrink of the graph of f by a factor of $\frac{1}{2}$.
27. The graph of h is a horizontal shrink of the graph of f by a factor of $\frac{1}{5}$.
28. The graph of h is a vertical shrink of the graph of f by a factor of $\frac{3}{4}$.
29. The graph of g is a vertical shrink of the graph of f by a factor of $\frac{1}{4}$.
30. The graph of g is a reflection in the x-axis of the graph of f.
31. The graph of g is a horizontal translation 2 units right of the graph of f.
32. The graph of g is a horizontal stretch of the graph of f by a factor of $\frac{3}{2}$.
33. The graph of g is a vertical stretch of the graph of f by a factor of 6.
34. The graph of g is a vertical translation 3 units down of the graph of f.
35. $g(x) = f(x - 2)$
36. $g(x) = f(-x)$
37. $g(x) = 4f(x)$
38. $g(x) = f(5x)$
39. $f(x - 2)$ is a translation to the right, not to the left.

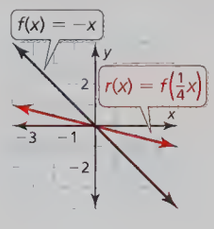


In Exercises 17–22, use the graphs of f and r to describe the transformation from the graph of f to the graph of r. *(See Example 3.)*

17.



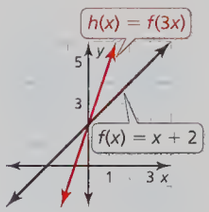
18.



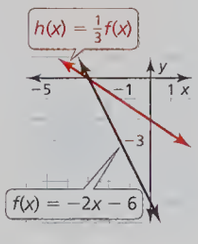
19. $f(x) = -2x - 4$; $r(x) = f\left(\frac{1}{2}x\right)$
20. $f(x) = 3x + 5$; $r(x) = f\left(\frac{1}{3}x\right)$
21. $f(x) = \frac{2}{3}x + 1$; $r(x) = 3f(x)$
22. $f(x) = -\frac{1}{4}x - 2$; $r(x) = 4f(x)$

In Exercises 23–28, use the graphs of f and h to describe the transformation from the graph of f to the graph of h. *(See Example 4.)*

23.



24.



25. $f(x) = 3x - 12$; $h(x) = \frac{1}{6}f(x)$
26. $f(x) = -x + 1$; $h(x) = f(2x)$
27. $f(x) = -2x - 2$; $h(x) = f(5x)$
28. $f(x) = 4x + 8$; $h(x) = \frac{3}{4}f(x)$

In Exercises 29–34, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

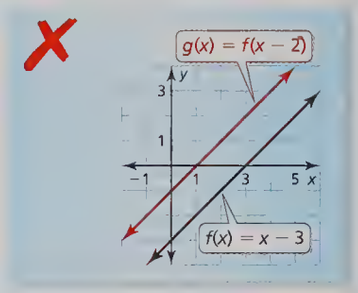
29. $f(x) = x - 2$; $g(x) = \frac{1}{4}f(x)$
30. $f(x) = -4x + 8$; $g(x) = -f(x)$
31. $f(x) = -2x - 7$; $g(x) = f(x - 2)$
32. $f(x) = 3x + 8$; $g(x) = f\left(\frac{2}{3}x\right)$
33. $f(x) = x - 6$; $g(x) = 6f(x)$
34. $f(x) = -x$; $g(x) = f(x) - 3$

In Exercises 35–38, write a function g in terms of f so that the statement is true.

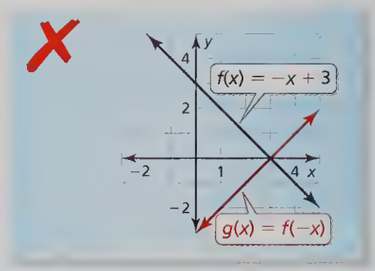
35. The graph of g is a horizontal translation 2 units right of the graph of f.
36. The graph of g is a reflection in the y-axis of the graph of f.
37. The graph of g is a vertical stretch by a factor of 4 of the graph of f.
38. The graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ of the graph of f.

ERROR ANALYSIS **In Exercises 39 and 40, describe and correct the error in graphing g.**

39.



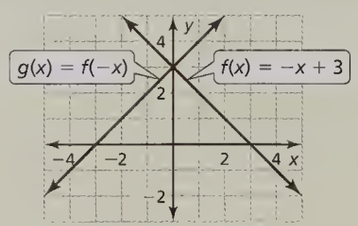
40.



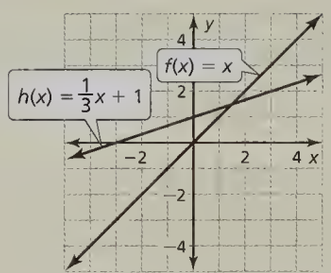
In Exercises 41–46, graph f and h. Describe the transformations from the graph of f to the graph of h. *(See Example 5.)*

41. $f(x) = x$; $h(x) = \frac{1}{3}x + 1$
42. $f(x) = x$; $h(x) = 4x - 2$
43. $f(x) = x$; $h(x) = -3x - 4$
44. $f(x) = x$; $h(x) = -\frac{1}{2}x + 3$
45. $f(x) = 2x$; $h(x) = 6x - 5$
46. $f(x) = 3x$; $h(x) = -3x - 7$

40. $f(-x)$ is a reflection in the y-axis, not the x-axis.



41.



The transformations are a vertical shrink by a factor of $\frac{1}{3}$ then a vertical translation 1 unit up.

42–46. See Additional Answers.

47. MODELING WITH MATHEMATICS The function $t(x) = -4x + 72$ represents the temperature from 5 P.M. to 11 P.M., where x is the number of hours after 5 P.M. The function $d(x) = 4x + 72$ represents the temperature from 10 A.M. to 4 P.M., where x is the number of hours after 10 A.M. Describe the transformation from the graph of t to the graph of d.



48. MODELING WITH MATHEMATICS A school sells T-shirts to promote school spirit. The school's profit is given by the function $P(x) = 8x - 150$, where x is the number of T-shirts sold. During the play-offs, the school increases the price of the T-shirts. The school's profit during the play-offs is given by the function $Q(x) = 16x - 200$, where x is the number of T-shirts sold. Describe the transformations from the graph of P to the graph of Q. *(See Example 6.)*



49. USING STRUCTURE The graph of $g(x) = a \cdot f(x - b) + c$ is a transformation of the graph of the linear function f. Select the word or value that makes each statement true.

reflection	translation	-1
stretch	shrink	0
left	right	1
y-axis	x-axis	

a. The graph of g is a vertical _____ of the graph of f when $a = 4$, $b = 0$, and $c = 0$.

b. The graph of g is a horizontal translation _____ of the graph of f when $a = 1$, $b = 2$, and $c = 0$.

c. The graph of g is a vertical translation 1 unit up of the graph of f when $a = 1$, $b = 0$, and $c =$ ____.

50. USING STRUCTURE The graph of $h(x) = a \cdot f(bx - c) + d$ is a transformation of the graph of the linear function f. Select the word or value that makes each statement true.

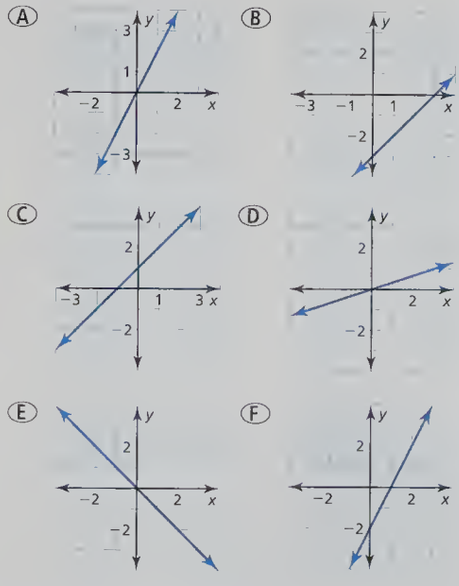
vertical	horizontal	0
stretch	shrink	$\frac{1}{5}$
y-axis	x-axis	5

a. The graph of h is a _____ shrink of the graph of f when $a = \frac{1}{3}$, $b = 1$, $c = 0$, and $d = 0$.

b. The graph of h is a reflection in the _____ of the graph of f when $a = 1$, $b = -1$, $c = 0$, and $d = 0$.

c. The graph of h is a horizontal stretch of the graph of f by a factor of 5 when $a = 1$, $b =$ _____, $c = 0$, and $d = 0$.

51. ANALYZING GRAPHS Which of the graphs are related by only a translation? Explain.



52. ANALYZING RELATIONSHIPS A swimming pool is filled with water by a hose at a rate of 1020 gallons per hour. The amount v (in gallons) of water in the pool after t hours is given by the function $v(t) = 1020t$. How does the graph of v change in each situation?

a. A larger hose is found. Then the pool is filled at a rate of 1360 gallons per hour.

b. Before filling up the pool with a hose, a water truck adds 2000 gallons of water to the pool.

ANSWERS

47. The graph of d is a reflection in the y-axis of the graph of t.

48. The transformations are a horizontal shrink by a factor of $\frac{1}{2}$ then a vertical translation 50 units down.

49. a. stretch

b. right

c. 1

50. a. vertical

b. y-axis

c. $\frac{1}{5}$

51. B and C, A and F; Adding 4 to the y-coordinate of each point on B gives the corresponding y-coordinate on C. Adding 2 to the y-coordinate of each point on F gives you the corresponding y-coordinate on A.

52. a. The new graph is a horizontal shrink of the old graph by a factor of $\frac{3}{4}$.

b. The new graph is a vertical translation of 2000 units up of the old graph.

ANSWERS

53. **a.** The new graph is a vertical translation 10 units up of the old graph.

b. The new graph is a horizontal stretch of the old graph by a factor of 2.

54. *Sample answer:* $g(x) = \frac{1}{4}f(x) + 1$

55–73. See Additional Answers.

Mini-Assessment

Use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

1. $f(x) = 2x - 3$; $g(x) = f(x) - 4$

 The graph of g is a vertical translation 4 units down of the graph of f.

2. $f(x) = 3x$; $g(x) = -f(x)$

 The graph of g is a reflection in the x-axis of the graph of f.

3. $f(x) = -x + 4$; $g(x) = 2f(x)$

 The graph of g is a vertical stretch of the graph of f by a factor of 2.

4. $f(x) = x$; $g(x) = 2x + 3$

 Stretch the graph of f vertically by a factor of 2 to get the graph of $h(x) = 2x$. Translate the graph of h vertically 3 units up to get the graph of $g(x) = 2x + 3$.

5. An Internet service provider charges customers \$80 per month for its service, with no installation fee. The cost to a customer is represented by $c(m) = 80m$, where m is the number of months of service. To attract new customers, the provider reduces the monthly fee to \$60, with an installation fee of \$25. The cost to a new customer is represented by $s(m) = 60m + 25$, where m is the number of months of service. Describe the transformations from the graph of c to the graph of s.

 The transformations are a vertical shrink by a factor of $\frac{3}{4}$ and then a vertical translation 25 units up.

53. ANALYZING RELATIONSHIPS You have \$50 to spend on fabric for a blanket. The amount m (in dollars) of money you have after buying y yards of fabric is given by the function $m(y) = -9.98y + 50$. How does the graph of m change in each situation?



a. You receive an additional \$10 to spend on the fabric.

b. The fabric goes on sale, and each yard now costs \$4.99.

54. THOUGHT PROVOKING Write a function g whose graph passes through the point (4, 2) and is a transformation of the graph of $f(x) = x$.

In Exercises 55–60, graph f and g. Write g in terms of f. Describe the transformation from the graph of f to the graph of g.

55. $f(x) = 2x - 5$; $g(x) = 2x - 8$

56. $f(x) = 4x + 1$; $g(x) = -4x - 1$

57. $f(x) = 3x + 9$; $g(x) = 3x + 15$

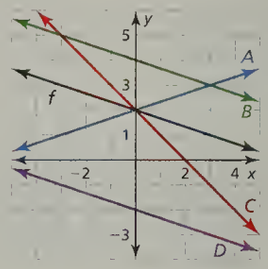
58. $f(x) = -x - 4$; $g(x) = x - 4$

59. $f(x) = x + 2$; $g(x) = \frac{2}{3}x + 2$

60. $f(x) = x - 1$; $g(x) = 3x - 3$

61. REASONING The graph of $f(x) = x + 5$ is a vertical translation 5 units up of the graph of $f(x) = x$. How can you obtain the graph of $f(x) = x + 5$ from the graph of $f(x) = x$ using a horizontal translation?

62. HOW DO YOU SEE IT? Match each function with its graph. Explain your reasoning.

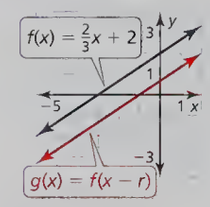


a. $a(x) = f(-x)$ **b.** $g(x) = f(x) - 4$

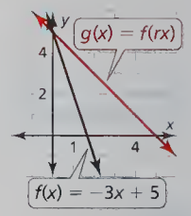
c. $h(x) = f(x) + 2$ **d.** $k(x) = f(3x)$

REASONING In Exercises 63–66, find the value of r.

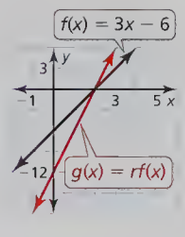
63.



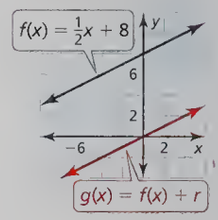
64.



65.



66.



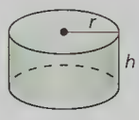
67. CRITICAL THINKING When is the graph of $y = f(x) + w$ the same as the graph of $y = f(x + w)$ for linear functions? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

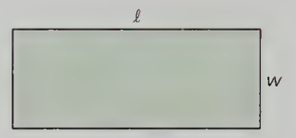
Solve the formula for the indicated variable. *(Section 1.4)*

68. Solve for h.



$V = \pi r^2 h$

69. Solve for w.



$P = 2\ell + 2w$

Solve the inequality. Graph the solution, if possible. *(Section 2.2)*

70. $x - 3 \leq 14$ **71.** $16 < x + 4$ **72.** $-25 \geq x + 7$ **73.** $x - 5 > -15$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

3.4–3.7 What Did You Learn?

Core Vocabulary

standard form, *p. 116*
x-intercept, *p. 117*
y-intercept, *p. 117*
zero of a function, *p. 118*
slope, *p. 124*
rise, *p. 124*
run, *p. 124*
slope-intercept form, *p. 126*
constant function, *p. 126*
direct variation, *p. 134*
constant of variation, *p. 134*
family of functions, *p. 140*
parent function, *p. 140*
transformation, *p. 140*
translation, *p. 140*
reflection, *p. 141*
horizontal shrink, *p. 142*
horizontal stretch, *p. 142*
vertical stretch, *p. 142*
vertical shrink, *p. 142*

Core Concepts

Section 3.4

Horizontal and Vertical Lines, *p. 116*
Using Intercepts to Graph Equations, *p. 117*
Finding Zeros of Functions, *p. 118*

Section 3.5

Slope, *p. 124*
Slope-Intercept Form, *p. 126*

Section 3.6

Identifying Direct Variation Equations, *p. 134*
Writing Direct Variation Equations, *p. 135*

Section 3.7

Horizontal Translations, *p. 140*
Vertical Translations, *p. 140*
Reflections in the *x*-axis, *p. 141*
Reflections in the *y*-axis, *p. 141*
Horizontal Stretches and Shrinks, *p. 142*
Vertical Stretches and Shrinks, *p. 142*
Transformations of Graphs, *p. 143*

Mathematical Thinking

1. Explain how you determined what units of measure to use for the horizontal and vertical axes in Exercise 37 on page 130.
2. Explain your plan for solving Exercise 48 on page 147.

Performance Task

The Cost of a T-Shirt

You receive bids for making T-shirts for your class fundraiser from four companies. To present the pricing information, one company uses a table, one company uses a written description, one company uses an equation, and one company uses a graph. How will you compare the different representations and make the final choice?

To explore the answer to this question and more, go to *BigIdeasMath.com*.



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Dynamic Teaching Tools

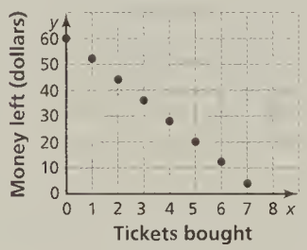
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

ANSWERS

1. The units were given in the statement of the problem. Because t was the independent variable, the units for t went on the horizontal axis. Because d was the dependent variable, the units for d went on the vertical axis.
2. Rewrite Q as $2(8x) - 200$ to match the coefficient in P. Then rewrite the constant term as $-150 - 50$ to get an expression that is an exact match of P. Express Q in terms of P to describe the transformations.

ANSWERS

1. not a function; A vertical line can be drawn through more than one point on the graph in many places.
2. function; Every input has exactly one output.
3. function; Every input has exactly one output.
4. **a.** The amount of money in the bank account is the dependent variable and the hours you babysat is the independent variable.
 b. domain: $0 \le x \le 4$; range: $100 \le y \le 140$
5. linear; As x increases by 5, y decreases by 3. The rate of change is constant.
6. nonlinear; The graph is not a line.
7. **a.** 0, 1, 2, 3, 4, 5, 6, 7; discrete; The number of tickets bought must be a whole number.
 b.



 c. 60, 52, 44, 36, 28, 20, 12, 4

3 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

3.1 Functions *(pp. 89–96)*

Determine whether the relation is a function. Explain.

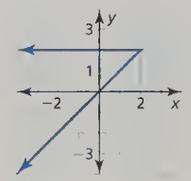
Input, x	2	5	7	9	14
Output, y	5	11	19	12	3

Every input has exactly one output.

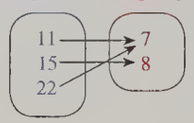
▶ So, the relation is a function.

Determine whether the relation is a function. Explain.

1.



2. Input, x Output, y



3. $y = -x + 6$ with inputs $x = 0$ and $x = 3$

4. The function $y = 10x + 100$ represents the amount y (in dollars) of money in your bank account after you babysit for x hours.
 a. Identify the independent and dependent variables.
 b. You babysit for 4 hours. Find the domain and range of the function.

3.2 Linear Functions *(pp. 97–106)*

Does the table or equation represent a *linear* or *nonlinear* function? Explain.

a.

x	6	10	14	18
y	5	9	14	20

(+4, +4, +4 between the x-values; +4, +5, +6 between the y-values)

As x increases by 4, y increases by different amounts. The rate of change is *not* constant.

▶ So, the function is nonlinear.

b. $y = 3x - 4$

The equation is in the form $y = mx + b$.

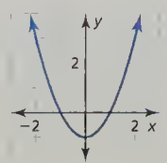
▶ So, the equation represents a linear function.

Does the table or graph represent a *linear* or *nonlinear* function? Explain.

5.

x	2	7	12	17
y	2	−1	−4	−7

6.



7. The function $y = 60 - 8x$ represents the amount y (in dollars) of money you have after buying x movie tickets. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain. (c) Find the range of the function.

3.3 Function Notation *(pp. 107–112)*

a. Evaluate $f(x) = 3x - 9$ when $x = 2$.

$f(x) = 3x - 9$	Write the function.
$f(2) = 3(2) - 9$	Substitute 2 for x.
$= 6 - 9$	Multiply.
$= -3$	Subtract.

▶ When $x = 2$, $f(x) = -3$.

b. For $f(x) = 4x$, find the value of x for which $f(x) = 12$.

$f(x) = 4x$	Write the function.
$12 = 4x$	Substitute 12 for $f(x)$.
$3 = x$	Divide each side by 4.

▶ When $x = 3$, $f(x) = 12$.

Evaluate the function when $x = -3$, 0, and 5.

8. $f(x) = x + 8$ **9.** $g(x) = 4 - 3x$

Find the value of x so that the function has the given value.

10. $k(x) = 7x$, $k(x) = 49$ **11.** $r(x) = -5x - 1$; $r(x) = 19$

3.4 Graphing Linear Equations in Standard Form *(pp. 115–122)*

Use intercepts to graph the equation $2x + 3y = 6$.

Step 1 Find the intercepts.

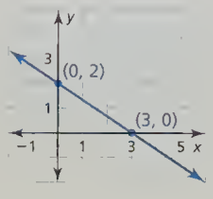
To find the x-intercept, substitute 0 for y and solve for x.

$2x + 3y = 6$
$2x + 3(0) = 6$
$x = 3$

To find the y-intercept, substitute 0 for x and solve for y.

$2x + 3y = 6$
$2(0) + 3y = 6$
$y = 2$

Step 2 Plot the points and draw the line.
The x-intercept is 3, so plot the point (3, 0).
The y-intercept is 2, so plot the point (0, 2).
Draw a line through the points.



Graph the linear equation.

12. $8x - 4y = 16$ **13.** $-12x - 3y = 36$ **14.** $y = -5$ **15.** $x = 6$

Find the zero of the function.

16. $p(x) = 2x - 10$ **17.** $v(x) = 0.25x + 3$ **18.** $b(x) = -8x + 4$

ANSWERS

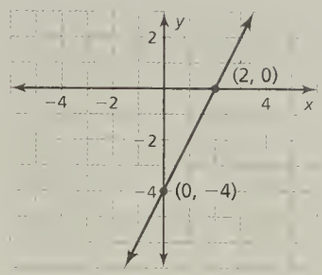
8. 5, 8, 13

9. 13, 4, −11

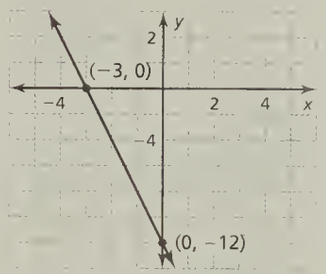
10. $x = 7$

11. $x = -4$

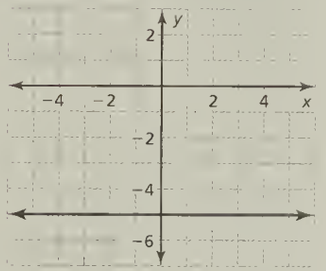
12.



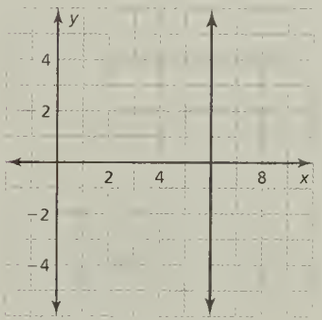
13.



14.



15.



16. 5

17. −12

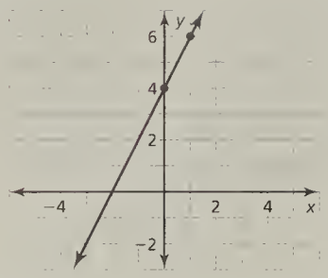
18. $\frac{1}{2}$

ANSWERS

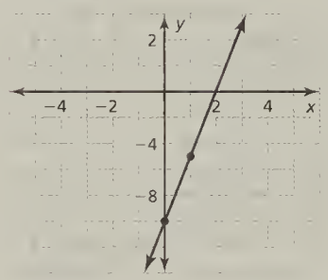
19. $\frac{6}{5}$

20. undefined

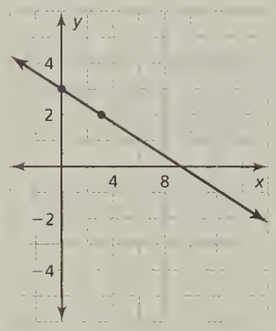
21. 0

22. 

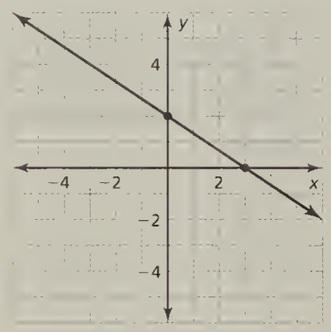
x-intercept: −2

23. 

x-intercept: 2

24. 

x-intercept: 9

25. 

slope: $-\frac{2}{3}$; y-intercept: 2; x-intercept: 3

3.5 Graphing Linear Equations in Slope-Intercept Form *(pp. 123–132)*

a. The points represented by the table lie on a line. How can you find the slope of the line from the table? What is the slope of the line?

Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (1, -7)$ and $(x_2, y_2) = (4, 2)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-7)}{4 - 1} = \frac{9}{3}, \text{ or } 3$$

x	y
1	−7
4	2
7	11
10	20

▶ The slope is 3.

b. Graph $-\frac{1}{2}x + y = 1$. Identify the x-intercept.

Step 1 Rewrite the equation in slope-intercept form.

$$y = \frac{1}{2}x + 1$$

Step 2 Find the slope and the y-intercept.

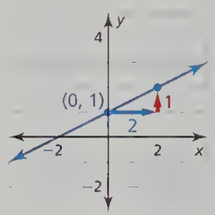
$$m = \frac{1}{2} \text{ and } b = 1$$

Step 3 The y-intercept is 1. So, plot (0, 1).

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$

Plot the point that is 2 units right and 1 unit up from (0, 1). Draw a line through the two points.



▶ The line crosses the x-axis at $(-2, 0)$. So, the x-intercept is -2.

The points represented by the table lie on a line. Find the slope of the line.

19.

x	y
6	9
11	15
16	21
21	27

20.

x	y
3	−5
3	−2
3	5
3	8

21.

x	y
−4	−1
−3	−1
1	−1
9	−1

Graph the linear equation. Identify the x-intercept.

22. $y = 2x + 4$

23. $-5x + y = -10$

24. $x + 3y = 9$

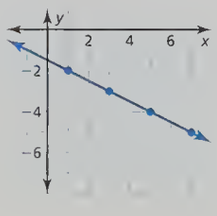
25. A linear function h models a relationship in which the dependent variable decreases 2 units for every 3 units the independent variable increases. Graph h when $h(0) = 2$. Identify the slope, y-intercept, and x-intercept of the graph.

3.6 Modeling Direct Variation *(pp. 133–138)*

a. Determine whether x and y show direct variation. Explain.

x	1	3	5	7
y	−2	−3	−4	−5

Plot the ordered pairs. Then draw a line through the points.



▶ The line does *not* pass through the origin. So, x and y do *not* show direct variation.

b. The table shows the costs C (in dollars) for h hours of repair work on your car. Explain why C varies directly with h. Then write a direct variation equation that relates h and C.

Number of hours, h	Cost (dollars), C
4	360
5	450
6	540

Find the ratio $\frac{C}{h}$ for each data pair (h, C).

$\frac{360}{4} = 90, \frac{450}{5} = 90, \frac{540}{6} = 90$

▶ All of the ratios are equal to 90. So, C varies directly with h.

Because $\frac{C}{h} = 90$, the constant of variation is 90. So, a direct variation equation is $C = 90h$.

Determine whether x and y show direct variation. Explain.

26. $-6x - y = 1$

27. $4x + y = 0$

28.

x	0	1	2	3
y	4	1	−2	−5

29.

x	−4	−2	0	2
y	−6	−3	0	3

30. The table shows the number p of gallons of paint needed to cover w square feet of wall space.

Wall space (ft²), w	Paint (gallons), p
700	2
2100	6
3500	10

a. Explain why p varies directly with w. Then write a direct variation equation that relates w and p.

b. How much paint do you need to cover 1400 square feet of wall space?

ANSWERS

26. no; The equation cannot be written in the form $y = ax$.

27. yes; The equation can be written in the form $y = ax$.

28. no; The ordered pairs lie on a line that does not pass through the origin.

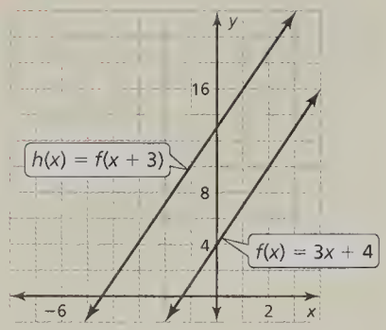
29. yes; The ordered pairs lie on a line that passes through the origin.

30. a. All of the ratios $\frac{p}{w}$ are equal to $\frac{1}{350}$; $p = \frac{1}{350}w$

b. 4 gal

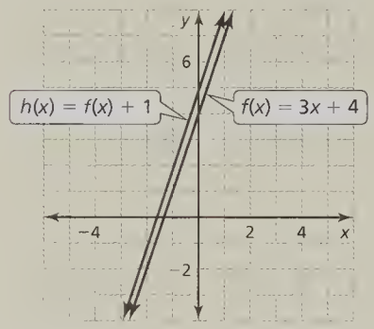
ANSWERS

31.



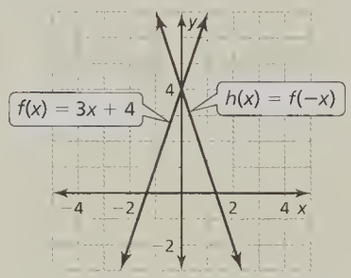
The graph of h is a horizontal translation 3 units left of the graph of f.

32.



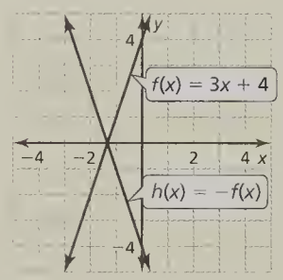
The graph of h is a vertical translation 1 unit up of the graph of f.

33.



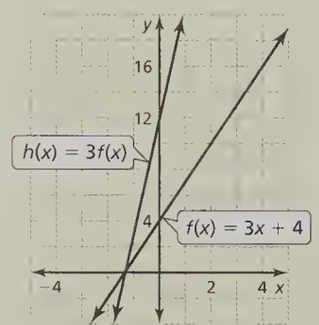
The graph of h is a reflection in the y-axis of the graph of f.

34.



The graph of h is a reflection in the x-axis of the graph of f.

35.

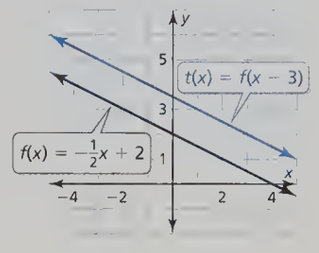


The graph of h is a vertical stretch of the graph of f by a factor of 3.

3.7 Transformations of Graphs of Linear Functions *(pp. 139–148)*

a. Let $f(x) = -\frac{1}{2}x + 2$. Graph $t(x) = f(x - 3)$. Describe the transformation from the graph of f to the graph of t.

The function t is of the form $y = f(x - h)$, where $h = 3$. So, the graph of t is a horizontal translation 3 units right of the graph of f.



b. Graph $f(x) = x$ and $g(x) = -3x - 2$. Describe the transformations from the graph of f to the graph of g.

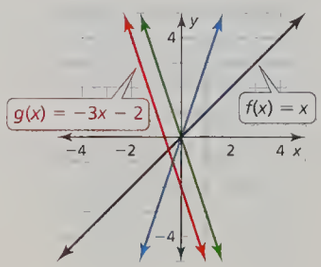
Note that you can rewrite g as $g(x) = -3f(x) - 2$.

Step 1 There is no horizontal translation from the graph of f to the graph of g.

Step 2 Stretch the graph of f vertically by a factor of 3 to get the graph of $h(x) = 3x$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -3x$.

Step 4 Translate the graph of r vertically 2 units down to get the graph of $g(x) = -3x - 2$.



Let $f(x) = 3x + 4$. Graph f and h. Describe the transformation from the graph of f to the graph of h.

31. $h(x) = f(x + 3)$

32. $h(x) = f(x) + 1$

33. $h(x) = f(-x)$

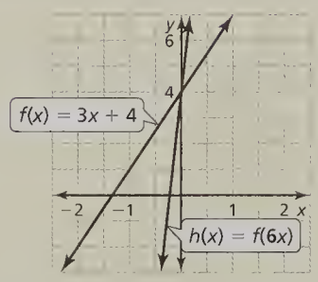
34. $h(x) = -f(x)$

35. $h(x) = 3f(x)$

36. $h(x) = f(6x)$

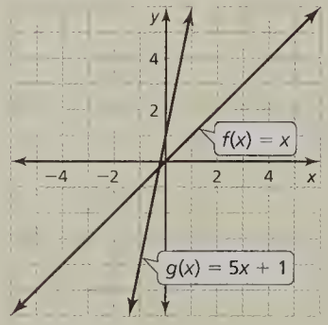
37. Graph $f(x) = x$ and $g(x) = 5x + 1$. Describe the transformations from the graph of f to the graph of g.

36.



The graph of h is a horizontal shrink of the graph of f by a factor of $\frac{1}{6}$.

37.



The transformations are a vertical stretch by a factor of 5, then a vertical translation 1 unit up.

3 Chapter Test

Determine whether the relation is a function. If the relation is a function, determine whether the function is *linear* or *nonlinear*. Explain.

1.

x	-1	0	1	2
y	6	5	9	14

2. $y = -2x + 3$

3. $x = -2$

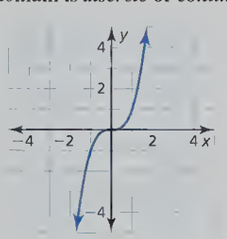
Graph the equation and identify the intercept(s). Find the slope of the line.

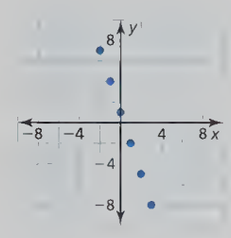
4. $2x - 3y = 6$

5. $y = 4.5$

6. $y = -7x$

Find the domain and range of the function represented by the graph. Determine whether the domain is *discrete* or *continuous*. Explain.

7. 

8. 

Graph f and g. Describe the transformations from the graph of f to the graph of g.

9. $f(x) = 2x + 4$; $g(x) = \frac{1}{2}f(x)$

10. $f(x) = x$; $g(x) = -x + 3$

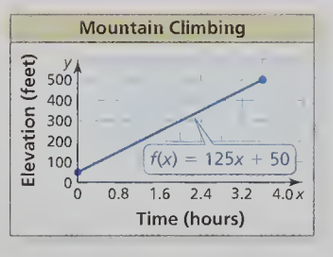
Find the zero of the function.

11. $h(x) = \frac{1}{3}x - 1$

12. $d(x) = -8x - 2$

13. Function A represents the amount of money in a jar based on the number of quarters in the jar. Function B represents your distance from home over time. Compare the domains.

14. A mountain climber is scaling a 500-foot cliff. The graph shows the elevation of the climber over time.



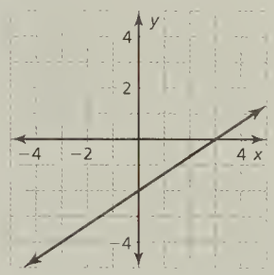
 a. Find and interpret the slope and the y-intercept of the graph.
 b. Explain two ways to find $f(3)$. Then find $f(3)$ and interpret its meaning.
 c. How long does it take the climber to reach the top of the cliff? Justify your answer.

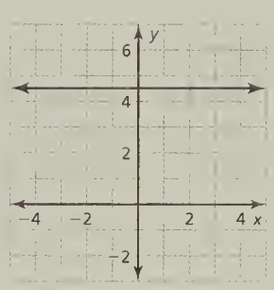
15. Without graphing, compare the slopes and the intercepts of the graphs of the functions $f(x) = x + 1$ and $g(x) = f(2x)$.

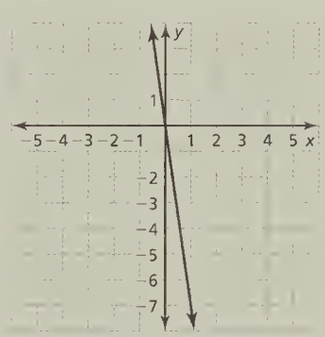
16. You are making cranberry sauce. The number c of cups of fresh cranberries varies directly with the number s of servings of cranberry sauce. A recipe that serves 16 people calls for 4 cups of fresh cranberries. How many cups of fresh cranberries should you use to make 28 servings of cranberry sauce?

If students need help...	If students got it...
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

ANSWERS

1. function; Every input has exactly one output; not linear; As x increases by 1, y changes by different amounts. The rate of change is not constant.
2. function; Every input has exactly one output; linear; It is in the form $y = mx + b$.
3. not a function; The input -2 has infinite output values.
4. 

x-intercept: 3; y-intercept: -2; slope: $\frac{2}{3}$
5. 

no x-intercept; y-intercept: 4.5; slope: 0
6. 

x-intercept: 0; y-intercept: 0; slope: -7
7. domain: all real numbers; range: all real numbers; continuous; The graph is a curve.
8. domain: $-2, -1, 0, 1, 2, 3$; range: $-8, -5, -2, 1, 4, 7$; discrete; The graph is individual points.

9–16. See Additional Answers.

ANSWERS

1. D
2. H
3. 6
4. B
5. G
6. C

3 Standards Assessment

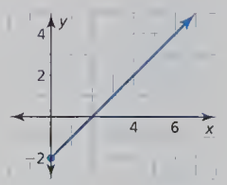
1. Which statement is true for the function whose graph is shown? *(TEKS A.2.A)*

 Ⓐ The domain is $x \leq 0$.

 Ⓑ The domain is $x \leq -2$.

 Ⓒ The range is $y \leq -2$.

 Ⓓ The range is $y \geq -2$.



2. You want to buy a jacket at a clothing store, and you can spend at most \$30. You have a coupon for 10% off any item in the store. Which inequality describes the original prices p (in dollars) of jackets you can buy? *(TEKS A.5.B)*

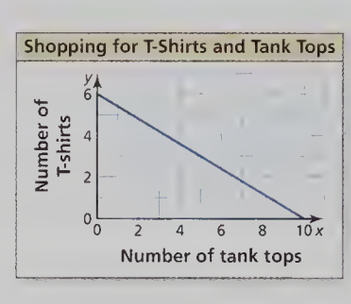
 Ⓕ $p \geq 27.27$

 Ⓖ $p \leq 27.27$

 Ⓗ $p \leq 33.33$

 Ⓙ $p \geq 33.33$

3. **GRIDDED ANSWER** The graph shows the numbers of T-shirts and tank tops you can buy with the amount of money you have. Suppose you buy only T-shirts. What is the greatest number you can buy? *(TEKS A.3.C)*



4. What is the solution of $3x + 24 = 18x - 5$? *(TEKS A.5.A)*

 Ⓐ $x = \frac{15}{29}$

 Ⓑ $x = \frac{29}{15}$

 Ⓒ $x = -\frac{15}{29}$

 Ⓓ $x = \frac{19}{21}$

5. The value of y varies directly with x. Which function represents the relationship between x and y if $y = \frac{10}{3}$ when $x = 60$? *(TEKS A.2.D)*

 Ⓕ $y = 200x$

 Ⓖ $y = \frac{1}{18}x$

 Ⓗ $y = \frac{190}{3}x$

 Ⓙ $y = 18x$

6. On the five science tests you have taken this semester, you received the following scores: 75, 82, 90, 84, and 71. You want a mean score of at least 80 after you take the sixth test. Which inequality describes the scores s you can earn on your sixth test to meet your goal? *(TEKS A.5.B)*

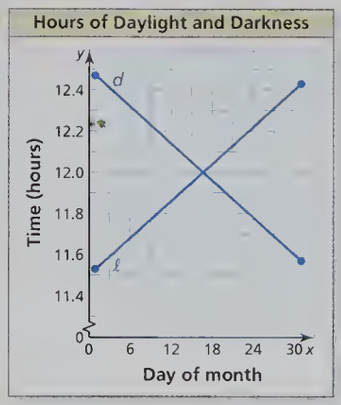
 Ⓐ $s > 76$

 Ⓑ $s > 78$

 Ⓒ $s \geq 78$

 Ⓓ $s \geq 80$

7. The number of hours of daylight in Austin, Texas, during the month of March can be modeled by the function $\ell(x) = 0.03x + 11.5$, where x is the day of the month. The number of hours of darkness can be modeled by d. The graphs of ℓ and d are shown. Which equation describes the relationship between ℓ and d? *(TEKS A.3.E)*



Ⓕ $d(x) = 24 - \ell(x)$

Ⓖ $\ell(x) = 24 + d(x)$

Ⓗ $\ell(x) = 12 - d(x)$

Ⓙ none of the above

8. Solve the literal equation $xy = 2z - 3y$ for y. *(TEKS A.12.E)*

Ⓐ $y = \frac{1}{3x}(2z)$

Ⓑ $y = \frac{2z}{x + 3}$

Ⓒ $y = \frac{1}{3}(2z - xy)$

Ⓓ $y = 2z - x - 3$

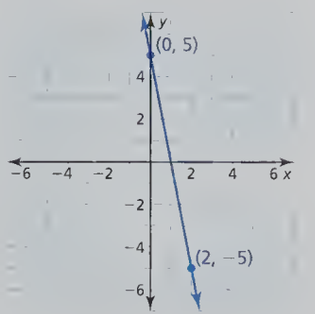
9. What is the slope of the line shown? *(TEKS A.3.A)*

Ⓕ 5

Ⓖ $\frac{1}{5}$

Ⓗ 0

Ⓙ -5



10. For which value of a does the solution of the compound inequality $a < 3x + 8$ *or* $a < -4x - 1$ consist of numbers greater than 5 *or* less than -6? *(TEKS A.5.B)*

Ⓐ 16

Ⓑ 19

Ⓒ 23

Ⓓ 26

11. For the function f, $f(-5) = 3$, and $f(3) = -2$. If $y = f(x)$, what is the value of y when $x = 3$? *(TEKS A.12.B)*

Ⓕ -5

Ⓖ -2

Ⓗ 3

Ⓙ 5

ANSWERS

7. F

8. B

9. J

10. C

11. G

Chapter 4 Pacing Guide

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	2 Days
Quiz	1 Day
Section 5	2 Days
Section 6	2 Days
Section 7	2 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 4	18 Days
Year-to-Date	63 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
4.1	A.2.B, A.2.C, A.3.A
4.2	A.2.B, A.2.C, A.3.A
4.3	A.2.B, A.2.C, A.3.A
4.4	A.2.C, A.2.E, A.2.F, A.2.G, A.3.A
4.5	A.4.C
4.6	A.4.A, A.4.B, A.4.C
4.7	A.12.D

4 Writing Linear Functions

4.1 Writing Equations in Slope-Intercept Form
4.2 Writing Equations in Point-Slope Form
4.3 Writing Equations in Standard Form
4.4 Writing Equations of Parallel and Perpendicular Lines
4.5 Scatter Plots and Lines of Fit
4.6 Analyzing Lines of Fit
4.7 Arithmetic Sequences



Online Auction *(p. 205)*



Old Faithful Geyser *(p. 196)*



Helicopter Rescue *(p. 182)*



School Spirit *(p. 170)*



Renewable Energy *(p. 164)*

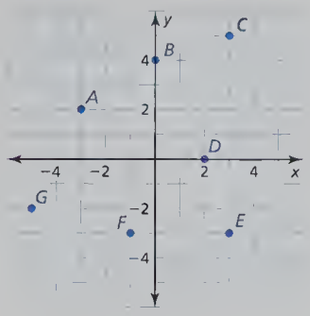
Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

For an overview of this chapter, formative assessment tips, and teaching strategies, see **Laurie's Notes** at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

Using a Coordinate Plane (6.11)

Example 1 What ordered pair corresponds to point A?



Point A is 3 units to the left of the origin and 2 units up. So, the x-coordinate is -3 and the y-coordinate is 2.

▶ The ordered pair $(-3, 2)$ corresponds to point A.

Use the graph to answer the question.

1. What ordered pair corresponds to point G?
2. What ordered pair corresponds to point D?
3. Which point is located in Quadrant I?
4. Which point is located in Quadrant IV?

Rewriting Equations (A.12.E)

Example 2 Solve the equation $3x - 2y = 8$ for y.

$3x - 2y = 8$	Write the equation.
$3x - 2y - 3x = 8 - 3x$	Subtract $3x$ from each side.
$-2y = 8 - 3x$	Simplify.
$\frac{-2y}{-2} = \frac{8 - 3x}{-2}$	Divide each side by -2.
$y = -4 + \frac{3}{2}x$	Simplify.

Solve the equation for y.

5. $x - y = 5$
6. $6x + 3y = -1$
7. $0 = 2y - 8x + 10$
8. $-x + 4y - 28 = 0$
9. $2y + 1 - x = 7x$
10. $y - 4 = 3x + 5y$
11. **ABSTRACT REASONING** Both coordinates of the point (x, y) are multiplied by a negative number. How does this change the location of the point? Be sure to consider points originally located in all four quadrants.

Dynamic Solutions available at *BigIdeasMath.com*

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For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make Notetaking Organizers for the following topics.

- Identifying points in the coordinate plane
- Rewriting equations

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

Texas Essential Knowledge and Skills

6.11 The student applies mathematical process standards to use coordinate geometry to identify locations on a plane. The student is expected to graph points in all four quadrants using ordered pairs of rational numbers.

A.12.E The student is expected to solve mathematic and scientific formulas, and other literal equations, for a specified variable.

ANSWERS

1. $(-5, -2)$
2. $(2, 0)$
3. C
4. E
5. $y = -5 + x$
6. $y = -\frac{1}{3} - 2x$
7. $y = 4x - 5$
8. $y = \frac{1}{4}x + 7$
9. $y = 4x - \frac{1}{2}$
10. $y = -\frac{3}{4}x - 1$
11. Points in Quadrant I move to Quadrant III, points in Quadrant II move to Quadrant IV, points in Quadrant III move to Quadrant I, and points in Quadrant IV move to Quadrant II.

MONITORING PROGRESS ANSWERS

1. $9.40 per hour
2. 26.2 mi/gal
3. about 8.8 h

Mathematical Thinking

Mathematically proficient students use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solutions. **(A.1.B)**

Problem-Solving Strategies

Core Concept

Solve a Simpler Problem

When solving a real-life problem, if the numbers in the problem seem complicated, then try solving a simpler form of the problem. After you have solved the simpler problem, look for a general strategy. Then apply that strategy to the original problem.

EXAMPLE 1 Using a Problem-Solving Strategy

In the deli section of a grocery store, a half pound of sliced roast beef costs $3.19. You buy 1.81 pounds. How much do you pay?

SOLUTION

Step 1 Solve a simpler problem.

Suppose the roast beef costs $3 per half pound, and you buy 2 pounds.

$$\text{Total cost} = \frac{\$3}{1/2 \text{ lb}} \cdot 2 \text{ lb} \qquad \text{Use unit analysis to write a verbal model.}$$

$$= \frac{\$6}{1 \cancel{\text{lb}}} \cdot 2 \cancel{\text{lb}} \qquad \text{Rewrite \$3 per } \tfrac{1}{2} \text{ pound as \$6 per pound.}$$

$$= \$12 \qquad \text{Simplify.}$$

▶ In the simpler problem, you pay $12.

Step 2 Apply the strategy to the original problem.

$$\text{Total cost} = \frac{\$3.19}{1/2 \text{ lb}} \cdot 1.81 \text{ lb} \qquad \text{Use unit analysis to write a verbal model.}$$

$$= \frac{\$6.38}{1 \cancel{\text{lb}}} \cdot 1.81 \cancel{\text{lb}} \qquad \text{Rewrite \$3.19 per } \tfrac{1}{2} \text{ pound as \$6.38 per pound.}$$

$$= \$11.55 \qquad \text{Simplify.}$$

▶ In the original problem, you pay $11.55. ← Your answer is reasonable because you bought about 2 pounds.

Monitoring Progress

1. You work $37\frac{1}{2}$ hours and earn $352.50. What is your hourly wage?
2. You drive 1244.5 miles and use 47.5 gallons of gasoline. What is your car's gas mileage (in miles per gallon)?
3. You drive 236 miles in 4.6 hours. At the same rate, how long will it take you to drive 450 miles?

For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

4.1 Writing Equations in Slope-Intercept Form

Texas Essential Knowledge and Skills
A.2.B
A.2.C
A.3.A

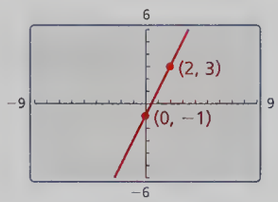
Essential Question Given the graph of a linear function, how can you write an equation of the line?

EXPLORATION 1 Writing Equations in Slope-Intercept Form

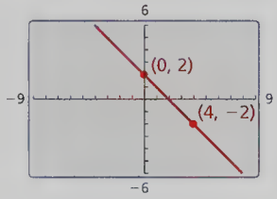
Work with a partner.

- Find the slope and y-intercept of each line.
- Write an equation of each line in slope-intercept form.
- Use a graphing calculator to verify your equation.

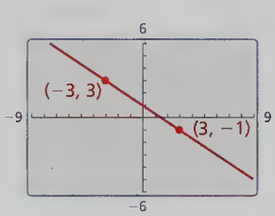
a.



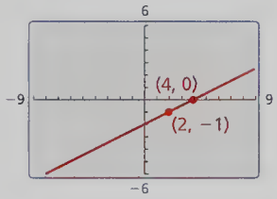
b.



c.



d.



EXPLAINING MATHEMATICAL IDEAS

To be proficient in math, you need to routinely interpret your results in the context of the situation. The reason for studying mathematics is to enable you to model and solve real-life problems.

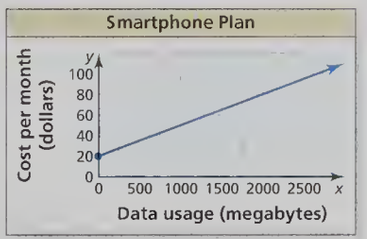
EXPLORATION 2 Mathematical Modeling

Work with a partner. The graph shows the cost of a smartphone plan.

a. What is the y-intercept of the line? Interpret the y-intercept in the context of the problem.

b. Approximate the slope of the line. Interpret the slope in the context of the problem.

c. Write an equation that represents the cost as a function of data usage.



Communicate Your Answer

3. Given the graph of a linear function, how can you write an equation of the line?
4. Give an example of a graph of a linear function that is different from those above. Then use the graph to write an equation of the line.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.B The student is expected to write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

A.2.C The student is expected to write linear equations in two variables given a table of values, a graph, and a verbal description.

A.3.A The student is expected to determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

ANSWERS

1. **a.** slope is 2; y-intercept is -1; $y = 2x - 1$
 b. slope is -1; y-intercept is 2; $y = -x + 2$
 c. slope is $-\frac{2}{3}$; y-intercept is 1; $y = -\frac{2}{3}x + 1$
 d. slope is $\frac{1}{2}$; y-intercept is -2; $y = \frac{1}{2}x - 2$
2. **a** 20; The base cost of the plan is \$20 per month.
 b. *Sample answer:* 0.03; Each megabyte of data costs \$0.03 to use.
 c. *Sample answer:* $C = 0.03x + 20$

3–4. See Additional Answers.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

As students work in pairs to complete the explorations and Communicate Your Answer, circulate through the classroom and listen to their conversations. Ask them to employ the following learning strategies.

Beginning/Advanced Beginning students ask Advanced students to help them discuss ideas they don't understand, using non-verbal cues such as pointing or tracing a figure when necessary. Advanced students use synonyms or alternate phrases to explain. Advanced students request help from teachers or classroom aides as needed.

Intermediate/Advanced High Intermediate students ask Advanced High students to help them discuss ideas they don't understand, using gestures and descriptive language when they don't know a word or phrase. Advanced High students use synonyms or alternate phrases to explain. Advanced High students request help from teachers or classroom aides as needed.

ELPS 1.D Speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution.

Extra Example 1

Write an equation of each line with the given characteristics.

a. slope = 3; y-intercept = -1

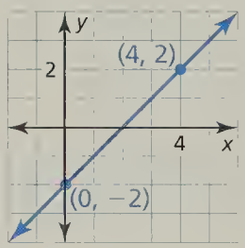
$y = 3x - 1$

b. slope = $\frac{1}{2}$; passes through (2, 3)

$y = \frac{1}{2}x + 2$

Extra Example 2

Write an equation of the line in slope-intercept form.



$y = x - 2$

4.1 Lesson

What You Will Learn

- Write equations in slope-intercept form.
- Use linear equations to solve real-life problems.

Core Vocabulary

linear model, *p. 164*

Previous

slope-intercept form
function
rate

Writing Equations in Slope-Intercept Form

EXAMPLE 1 **Using Slopes and y-Intercepts to Write Equations**

Write an equation of each line with the given characteristics.

a. slope = -3; y-intercept = $\frac{1}{2}$ **b.** slope = 4; passes through $(-2, 5)$

SOLUTION

a. $y = mx + b$ Write the slope-intercept form.

$y = -3x + \frac{1}{2}$ Substitute -3 for m and $\frac{1}{2}$ for b.

▶ An equation is $y = -3x + \frac{1}{2}$.

b. Find the y-intercept.

$y = mx + b$ Write the slope-intercept form.

$5 = 4(-2) + b$ Substitute 4 for m, -2 for x, and 5 for y.

$13 = b$ Solve for b.

Write an equation.

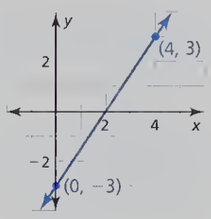
$y = mx + b$ Write the slope-intercept form.

$y = 4x + 13$ Substitute 4 for m and 13 for b.

▶ An equation is $y = 4x + 13$.

EXAMPLE 2 **Using a Graph to Write an Equation**

Write an equation of the line in slope-intercept form.



STUDY TIP

You can use any two points on a line to find the slope.

SOLUTION

Find the slope and y-intercept.

Let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (4, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{4 - 0} = \frac{6}{4}, \text{ or } \frac{3}{2}$$

Because the line crosses the y-axis at $(0, -3)$, the y-intercept is -3.

▶ So, the equation is $y = \frac{3}{2}x - 3$.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

EXAMPLE 3 Using Points to Write Equations

Write an equation of each line that passes through the given points.

a. $(-3, 5), (0, -1)$ **b.** $(0, -5), (8, -5)$

SOLUTION

a. Find the slope and y-intercept.

$$m = \frac{-1 - 5}{0 - (-3)} = -2$$

Because the line crosses the y-axis at $(0, -1)$, the y-intercept is -1.

▶ So, an equation is $y = -2x - 1$.

b. Find the slope and y-intercept.

$$m = \frac{-5 - (-5)}{8 - 0} = 0$$

Because the line crosses the y-axis at $(0, -5)$, the y-intercept is -5.

▶ So, an equation is $y = -5$.

REMEMBER

If f is a function and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x.

EXAMPLE 4 Writing a Linear Function

Write a linear function f with the values $f(0) = 10$ and $f(6) = 34$.

SOLUTION

Step 1 Write $f(0) = 10$ as $(0, 10)$ and $f(6) = 34$ as $(6, 34)$.

Step 2 Find the slope of the line that passes through $(0, 10)$ and $(6, 34)$.

$$m = \frac{34 - 10}{6 - 0} = \frac{24}{6}, \text{ or } 4$$

Step 3 Write an equation of the line. Because the line crosses the y-axis at $(0, 10)$, the y-intercept is 10.

$y = mx + b$ Write the slope-intercept form.

$y = 4x + 10$ Substitute 4 for m and 10 for b.

▶ A function is $f(x) = 4x + 10$.

Monitoring Progress

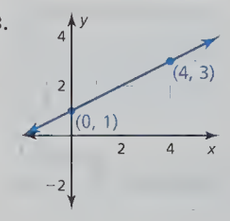
Help in English and Spanish at *BigIdeasMath.com*

Write an equation of the line with the given characteristics.

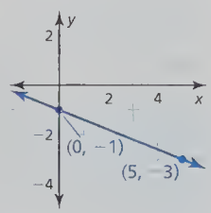
1. slope $= 7$; y-intercept $= 2$
2. slope $= \frac{1}{3}$; passes through $(6, 1)$

Write an equation of the line in slope-intercept form.

3.



4.



5. Write an equation of the line that passes through $(0, -2)$ and $(4, 10)$.
6. Write a linear function g with the values $g(0) = 9$ and $g(8) = 7$.

Extra Example 3

Write an equation of each line that passes through the given points.

a. $(0, -6), (3, 9)$ $y = 5x - 6$

b. $(4, -1), (0, 2)$ $y = -\frac{3}{4}x + 2$

Extra Example 4

Write a linear function f with the values $f(0) = -4$ and $f(3) = 8$. $f(x) = 4x - 4$

MONITORING PROGRESS ANSWERS

1. $y = 7x + 2$
2. $y = \frac{1}{3}x - 1$
3. $y = \frac{1}{2}x + 1$
4. $y = -\frac{2}{5}x - 1$
5. $y = 3x - 2$
6. $g(x) = -\frac{1}{4}x + 9$

Differentiated Instruction

Organization

Have students make and complete a table to describe the methods shown in this section for writing equations in slope-intercept form.

What I Know	How to Find Slope	How to Find y-Intercept
slope and y-intercept		
graph		
two points		
function values		

Extra Example 5

In 2000, the average cost for college tuition in the United States was about \$14,000. In 2010, the cost was about \$18,000. Write a linear model that represents the cost of tuition as a function of the number of years since 2000. Use the model to predict the cost in 2020. Let x represent the number of years since 2000 and let y represent the cost. The linear model is $y = 400x + 14{,}000$. The model predicts tuition will cost about \$22,000 in 2020.

MONITORING PROGRESS ANSWER

7. $y = 5.8x + 248$

Solving Real-Life Problems

A **linear model** is a linear function that models a real-life situation. When a quantity y changes at a constant rate with respect to a quantity x, you can use the equation $y = mx + b$ to model the relationship. The value of m is the constant rate of change, and the value of b is the initial, or starting, value of y.

EXAMPLE 5 **Modeling with Mathematics**



Excluding hydropower, U.S. power plants used renewable energy sources to generate 105 million megawatt hours of electricity in 2007. By 2012, the amount of electricity generated had increased to 219 million megawatt hours. Write a linear model that represents the number of megawatt hours generated by non-hydropower renewable energy sources as a function of the number of years since 2007. Use the model to predict the number of megawatt hours that will be generated in 2017.

SOLUTION

1. **Understand the Problem** You know the amounts of electricity generated in two distinct years. You are asked to write a linear model that represents the amount of electricity generated each year since 2007 and then predict a future amount.
2. **Make a Plan** Break the problem into parts and solve each part. Then combine the results to help you solve the original problem.
 - **Part 1** Define the variables. Find the initial value and the rate of change.
 - **Part 2** Write a linear model and predict the amount in 2017.
3. **Solve the Problem**
 - **Part 1** Let x represent the time (in years) since 2007 and let y represent the number of megawatt hours (in millions). Because time x is defined in years since 2007, 2007 corresponds to $x = 0$ and 2012 corresponds to $x = 5$. Let $(x_1, y_1) = (0, 105)$ and $(x_2, y_2) = (5, 219)$. The initial value is the y-intercept b, which is 105. The rate of change is the slope m.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{219 - 105}{5 - 0} = \frac{114}{5} = 22.8$$

 - **Part 2**

Megawatt hours (millions)	=	Initial value	+	Rate of change	•	Years since 2007	
y	=	105	+	22.8	•	x	
		$y = 105 + 22.8x$					Write the equation.
2017 corresponds to $x = 10$. →		$y = 105 + 22.8(10)$					Substitute 10 for x.
		$y = 333$					Simplify.

 ▶ The linear model is $y = 22.8x + 105$. The model predicts non-hydropower renewable energy sources will generate 333 million megawatt hours in 2017.
4. **Look Back** To check that your model is correct, verify that (0, 105) and (5, 219) are solutions of the equation.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. The corresponding data for electricity generated by hydropower are 248 million megawatt hours in 2007 and 277 million megawatt hours in 2012. Write a linear model that represents the number of megawatt hours generated by hydropower as a function of the number of years since 2007.

Closure

- **Writing Prompt:** For a line that has been graphed in a coordinate plane, you can write the equation by … finding the slope and y-intercept.

4.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A linear function that models a real-life situation is called a _____.
2. **WRITING** Explain how you can use slope-intercept form to write an equation of a line given its slope and y-intercept.

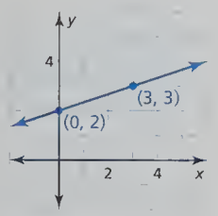
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write an equation of the line with the given characteristics. *(See Example 1.)*

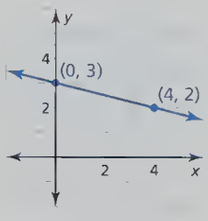
3. slope: 2
 y-intercept: 9
4. slope: 0
 passes through: $(-3, 5)$
5. slope: -3
 passes through: $(2, -6)$
6. slope: -7
 y-intercept: 1
7. slope: $\frac{2}{3}$
 y-intercept: -8
8. slope: $-\frac{3}{4}$
 passes through: $(-8, 0)$

In Exercises 9–12, write an equation of the line in slope-intercept form. *(See Example 2.)*

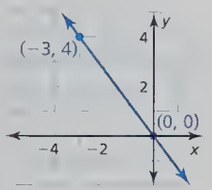
9.



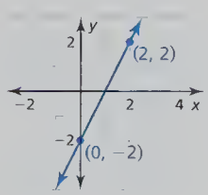
10.



11.



12.



In Exercises 13–18, write an equation of the line that passes through the given points. *(See Example 3.)*

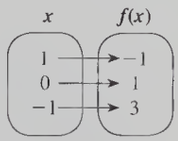
13. $(3, 1), (0, 10)$
14. $(2, 7), (0, -5)$
15. $(2, -4), (0, -4)$
16. $(-6, 0), (0, -24)$
17. $(0, 5), (-1.5, 1)$
18. $(0, 3), (-5, 2.5)$

In Exercises 19–24, write a linear function f with the given values. *(See Example 4.)*

19. $f(0) = 2, f(2) = 4$
20. $f(0) = 7, f(3) = 1$
21. $f(4) = -3, f(0) = -2$
22. $f(5) = -1, f(0) = -5$
23. $f(-2) = 6, f(0) = -4$
24. $f(0) = 3, f(-6) = 3$

In Exercises 25 and 26, write a linear function f with the given values.

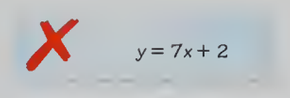
25.



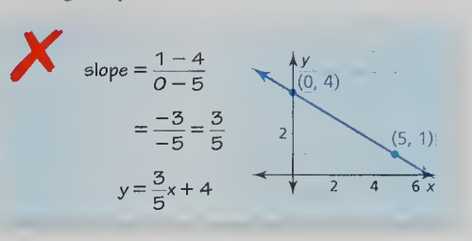
26.

x	$f(x)$
-4	-2
-2	-1
0	0

27. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line with a slope of 2 and a y-intercept of 7.



28. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line shown.



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–23 odd, 27, 29, 34, 36, 38–43

Average: 1, 2–28 even, 29, 30, 34, 36, 38–43

Advanced: 1, 2, 6–12 even, 16, 18, 22–28 even, 29–43

HOMEWORK CHECK

Basic: 3, 11, 13, 19, 29

Average: 6, 12, 14, 20, 29

Advanced: 8, 12, 18, 24, 30

ANSWERS

1. linear model
2. Substitute the slope for m and the y-intercept for b in the slope-intercept form.
3. $y = 2x + 9$
4. $y = 5$
5. $y = -3x$
6. $y = -7x + 1$
7. $y = \frac{2}{3}x - 8$
8. $y = -\frac{3}{4}x - 6$
9. $y = \frac{1}{3}x + 2$
10. $y = -\frac{1}{4}x + 3$
11. $y = -\frac{4}{3}x$
12. $y = 2x - 2$
13. $y = -3x + 10$
14. $y = 6x - 5$
15. $y = -4$
16. $y = -4x - 24$
17. $y = \frac{8}{3}x + 5$
18. $y = \frac{1}{10}x + 3$
19. $f(x) = x + 2$
20. $f(x) = -2x + 7$
21. $f(x) = -\frac{1}{4}x - 2$
22. $f(x) = \frac{4}{5}x - 5$
23. $f(x) = -5x - 4$
24. $f(x) = 3$
25. $f(x) = -2x + 1$
26. $f(x) = \frac{1}{2}x$
27. The slope and the y-intercept were substituted incorrectly; $y = 2x + 7$
28. The x-coordinates and y-coordinates were not substituted in the same order in the slope formula; $slope = \frac{1 - 4}{5 - 0} = -\frac{3}{5}$; $y = -\frac{3}{5}x + 4$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

29. **a.** $y = -0.005t + 3.91$

b. 3.71 min; 3.61 min

30. **a.** $C = 75t + 50$

b. software program; *Sample answer:* The total cost for 12 hours of recording time is \$950, which is more than the cost of the software program.

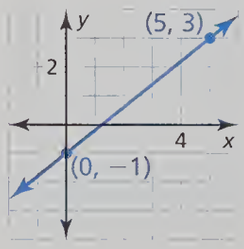
31. no; Because the x-coordinates are the same, the line is vertical and has an undefined slope.

32. *Sample answer:* Jet ski rental costs \$20 for one hour and \$15 for each additional hour.

33–43. See Additional Answers.

Mini-Assessment

1. Write an equation of the line with a slope of -2 and y-intercept of 3.
 $y = -2x + 3$
2. Write an equation of the line in slope-intercept form.



 $y = \frac{4}{5}x - 1$
3. Write an equation of the line that passes through the points $(0, -5)$ and $(4, 7)$. $y = 3x - 5$
4. Write a linear function f with the values $f(0) = 6$ and $f(4) = -6$.
 $f(x) = -3x + 6$
5. In 2006, a company had sales of \$10 million. In 2011, sales were \$12.5 million. Write a linear model that represents the company's sales as a function of the number of years since 2006. Use the model to predict the sales in 2021. The linear model is $y = 0.5x + 10$. The model predicts sales will be \$17.5 million in 2021.

29. MODELING WITH MATHEMATICS In 1960, the world record for the men's mile was 3.91 minutes. In 1980, the record time was 3.81 minutes. *(See Example 5.)*

a. Write a linear model that represents the world record (in minutes) for the men's mile as a function of the number of years since 1960.

b. Use the model to estimate the record time in 2000 and predict the record time in 2020.

30. MODELING WITH MATHEMATICS A recording studio charges musicians an initial fee of \$50 to record an album. Studio time costs an additional \$75 per hour.

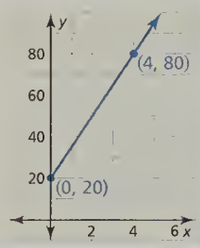
a. Write a linear model that represents the total cost of recording an album as a function of studio time (in hours).

b. Is it less expensive to purchase 12 hours of recording time at the studio or a \$750 music software program that you can use to record on your own computer? Explain.



31. WRITING A line passes through the points $(0, -2)$ and $(0, 5)$. Is it possible to write an equation of the line in slope-intercept form? Justify your answer.

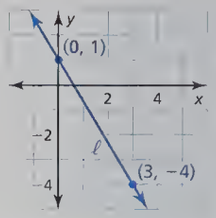
32. THOUGHT PROVOKING
Describe a real-life situation involving a linear function whose graph passes through the points.



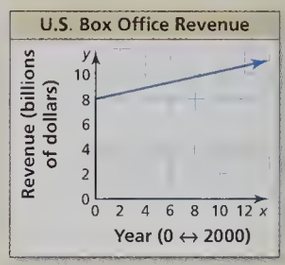
33. REASONING Recall that the standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope-intercept form. Use your answer to find the slope and y-intercept of the graph of the equation $-6x + 5y = 9$.

34. MAKING AN ARGUMENT Your friend claims that given $f(0)$ and any other value of a linear function f, you can write an equation in slope-intercept form that represents the function. Your cousin disagrees, claiming that the two points could lie on a vertical line. Who is correct? Explain.

35. ANALYZING A GRAPH
Line ℓ is a reflection in the x-axis of line k. Write an equation that represents line k.



36. HOW DO YOU SEE IT? The graph shows the approximate U.S. box office revenues (in billions of dollars) from 2000 to 2012, where $x = 0$ represents the year 2000.



a. Estimate the slope and y-intercept of the graph.

b. Interpret your answers in part (a) in the context of the problem.

c. How can you use your answers in part (a) to predict the U.S. box office revenue in 2018?

37. ABSTRACT REASONING Show that the equation of the line that passes through the points $(0, b)$ and $(1, b + m)$ is $y = mx + b$. Explain how you can be sure that the point $(-1, b - m)$ also lies on the line.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. *(Section 1.3)*

38. $3(x - 15) = x + 11$

39. $2(3d + 3) = 7 + 6d$

40. $-5(4 - 3n) = 10(n - 2)$

Determine whether x and y show direct variation. If so, identify the constant of variation. *(Section 3.6)*

41. $y + 6x = 0$

42. $2y + 5x = 4$

43. $4y + x - 3 = 3$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

4.2 Writing Equations in Point-Slope Form

Texas Essential Knowledge and Skills
A.2.B
A.2.C
A.3.A

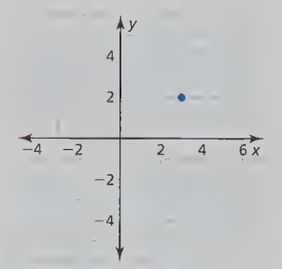
Essential Question
How can you write an equation of a line when you are given the slope and a point on the line?

EXPLORATION 1 Writing Equations of Lines

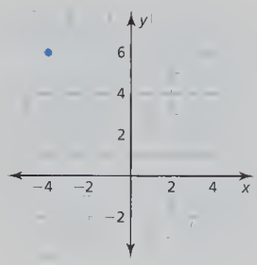
Work with a partner.

- Sketch the line that has the given slope and passes through the given point.
- Find the y-intercept of the line.
- Write an equation of the line.

a. $m = \frac{1}{2}$



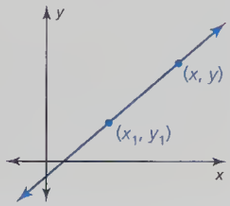
b. $m = -2$



EXPLORATION 2 Writing a Formula

Work with a partner.

The point (x_1, y_1) is a given point on a nonvertical line. The point (x, y) is any other point on the line. Write an equation that represents the slope m of the line. Then rewrite this equation by multiplying each side by the difference of the x-coordinates to obtain the **point-slope form** of a linear equation.



USING TECHNOLOGY

To be proficient in math, you need to understand the feasibility, appropriateness, and limitations of the technological tools at your disposal. For instance, in real-life situations such as the one given in Exploration 3, it may not be feasible to use a square viewing window on a graphing calculator.

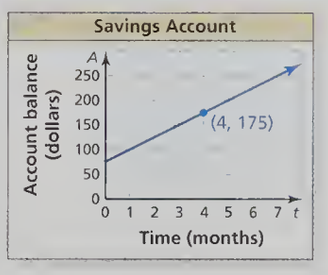
EXPLORATION 3 Writing an Equation

Work with a partner.

For four months, you have saved \$25 per month. You now have \$175 in your savings account.

a. Use your result from Exploration 2 to write an equation that represents the balance A after t months.

b. Use a graphing calculator to verify your equation.



Communicate Your Answer

4. How can you write an equation of a line when you are given the slope and a point on the line?
5. Give an example of how to write an equation of a line when you are given the slope and a point on the line. Your example should be different from those above.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

2. $m = \frac{y - y_1}{x - x_1}$; $y - y_1 = m(x - x_1)$
3. **a.** $A = 25t + 75$
 b. Check students' work.
4. Substitute the slope for m and the coordinates of the point for x_1 and y_1 in $y - y_1 = m(x - x_1)$.
5. *Sample answer:* An equation of the line that passes through the point $(2, -9)$ and has a slope of $-\frac{1}{3}$ is $y + 9 = -\frac{1}{3}(x - 2)$.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

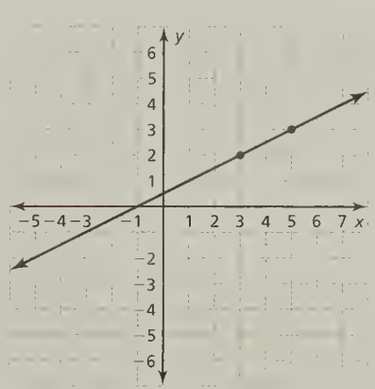
A.2.B The student is expected to write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

A.2.C The student is expected to write linear equations in two variables given a table of values, a graph, and a verbal description.

A.3.A The student is expected to determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

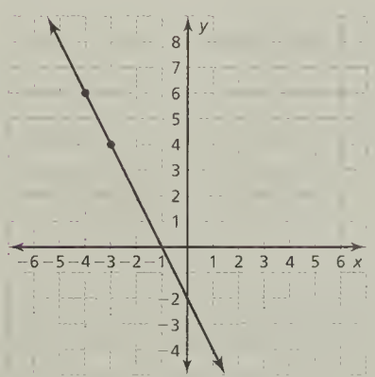
ANSWERS

1. **a.**



$\frac{1}{2}$; $y = \frac{1}{2}x + \frac{1}{2}$

b.



-2; $y = -2x - 2$

Extra Example 1

Identify the slope of the line $y - 3 = 4(x + 1)$. Then identify a point the line passes through. The slope of the line is 4, and the line passes through the point $(-1, 3)$.

Extra Example 2

Write an equation in point-slope form of the line that passes through the point (1, 6) and has a slope of $-\frac{1}{2}$.

$y - 6 = -\frac{1}{2}(x - 1)$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

4.2 Lesson

What You Will Learn

- Write an equation of a line given its slope and a point on the line.
- Write an equation of a line given two points on the line.
- Use linear equations to solve real-life problems.

Core Vocabulary

point-slope form, *p. 168*

Previous

slope-intercept form
function
linear model
rate

Writing Equations of Lines in Point-Slope Form

Given a point on a line and the slope of the line, you can write an equation of the line. Consider the line that passes through (2, 3) and has a slope of $\frac{1}{2}$. Let (x, y) be another point on the line where $x \neq 2$. You can write an equation relating x and y using the slope formula with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (x, y)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Write the slope formula.

$\frac{1}{2} = \frac{y - 3}{x - 2}$ Substitute values.

$\frac{1}{2}(x - 2) = y - 3$ Multiply each side by $(x - 2)$.

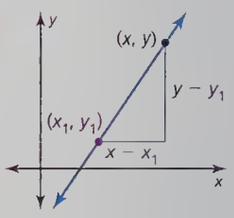
The equation in *point-slope form* is $y - 3 = \frac{1}{2}(x - 2)$.

Core Concept

Point-Slope Form

Words A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**. The line passes through the point (x_1, y_1), and the slope of the line is m.

Algebra $y - y_1 = m(x - x_1)$ (passes through (x_1, y_1); slope)



EXAMPLE 1 Using Point-Slope Form

Identify the slope of the line $y + 2 = -3(x - 2)$. Then identify a point the line passes through.

SOLUTION

The equation is written in point-slope form, $y - y_1 = m(x - x_1)$, where $m = -3$, $x_1 = 2$, and $y_1 = -2$.

▶ So, the slope of the line is -3, and the line passes through the point $(2, -2)$.

EXAMPLE 2 Using a Slope and a Point to Write an Equation

Write an equation in point-slope form of the line that passes through the point (8, 3) and has a slope of $\frac{1}{4}$.

SOLUTION

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 3 = \frac{1}{4}(x - 8)$ Substitute $\frac{1}{4}$ for m, 8 for x_1, and 3 for y_1.

▶ The equation is $y - 3 = \frac{1}{4}(x - 8)$.

English Language Learners

Labels

Explain to students that the name "point-slope form" relates to the information shown in the equation. Have students use colors to label and practice identifying the point and the slope when given equations using this format.

slope

$y - 5 = 2(x - 1)$

point (1, 5)

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. Identify the slope of the line $y - \frac{1}{2} = -2\left(x - \frac{1}{2}\right)$. Then identify a point the line passes through.

Write an equation in point-slope form of the line that passes through the given point and has the given slope.

2. $(3, -1)$; $m = -2$

3. $(4, 0)$; $m = -\frac{2}{3}$

Writing Equations of Lines Given Two Points

When you are given two points on a line, you can write an equation of the line using the following steps.

Step 1 Find the slope of the line.

Step 2 Use the slope and one of the points to write an equation of the line in point-slope form.

ANOTHER WAY

You can use either of the given points to write an equation of the line.

Use $m = -2$ and $(3, -2)$.

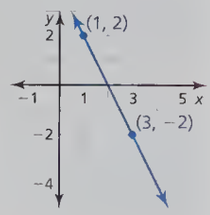
$y - (-2) = -2(x - 3)$

$y + 2 = -2x + 6$

$y = -2x + 4$

EXAMPLE 3 Using Two Points to Write an Equation

Write an equation in slope-intercept form of the line shown.



SOLUTION

Step 1 Find the slope of the line.

$$m = \frac{-2 - 2}{3 - 1} = \frac{-4}{2}, \text{ or } -2$$

Step 2 Use the slope $m = -2$ and the point $(1, 2)$ to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 2 = -2(x - 1)$	Substitute -2 for m, 1 for x_1, and 2 for y_1.
$y - 2 = -2x + 2$	Distributive Property
$y = -2x + 4$	Write in slope-intercept form.

▶ The equation is $y = -2x + 4$.

EXAMPLE 4 Writing a Linear Function

Write a linear function f with the values $f(4) = -2$ and $f(12) = 10$.

SOLUTION

Note that you can rewrite $f(4) = -2$ as $(4, -2)$ and $f(12) = 10$ as $(12, 10)$.

Step 1 Find the slope of the line that passes through $(4, -2)$ and $(12, 10)$.

$$m = \frac{10 - (-2)}{12 - 4} = \frac{12}{8}, \text{ or } 1.5$$

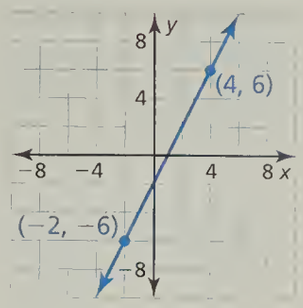
Step 2 Use the slope $m = 1.5$ and the point $(12, 10)$ to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 10 = 1.5(x - 12)$	Substitute 1.5 for m, 12 for x_1, and 10 for y_1.
$y = 1.5x - 8$	Write in slope-intercept form.

▶ A function is $f(x) = 1.5x - 8$.

Extra Example 3

Write an equation in slope-intercept form of the line shown.



$y = 2x - 2$

Extra Example 4

Write a linear function f with the values $f(8) = -1$ and $f(6) = 0$.

$f(x) = -0.5x + 3$

MONITORING PROGRESS ANSWERS

1. -2; *Sample answer:* $\left(\frac{1}{2}, \frac{1}{2}\right)$
2. $y + 1 = -2(x - 3)$
3. $y = -\frac{2}{3}(x - 4)$

Extra Example 5

The table shows the fuel efficiencies of a vehicle at different speeds. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the miles per gallon as a function of the miles per hour.

Miles per hour, x	20	24	30	36
Miles per gallon, y	14.5	17.5	21.2	23.7

The data cannot be modeled by a linear equation, because y does not decrease at a constant rate.

MONITORING PROGRESS ANSWERS

4. $y = 3x + 1$
5. $y = -\frac{1}{4}x - 2$
6. $g(x) = \frac{1}{2}x + 2$
7. $g(x) = -3x + 5$
8. yes; The total cost increases at a constant rate; $C = 42n + 50$

Closure

- **Exit Ticket:** Write an equation of the line with a slope of 2 that passes through the point $(-1, 4)$ in point-slope form and slope-intercept form.
 $y - 4 = 2(x + 1)$; $y = 2x + 6$

Monitoring Progress

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Write an equation in slope-intercept form of the line that passes through the given points.

4. $(1, 4), (3, 10)$
5. $(-4, -1), (8, -4)$

Write a linear function g with the given values.

6. $g(2) = 3, g(6) = 5$
7. $g(-1) = 8, g(2) = -1$

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**



The student council is ordering customized foam hands to promote school spirit. The table shows the cost of ordering different numbers of foam hands. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the cost as a function of the number of foam hands.

Number of foam hands	4	6	8	10	12
Cost (dollars)	34	46	58	70	82

SOLUTION

Step 1 Find the rate of change for consecutive data pairs in the table.

$$\frac{46 - 34}{6 - 4} = 6, \frac{58 - 46}{8 - 6} = 6, \frac{70 - 58}{10 - 8} = 6, \frac{82 - 70}{12 - 10} = 6$$

Because the rate of change is constant, the data are linear. So, use the point-slope form to write an equation that represents the data.

Step 2 Use the constant rate of change (slope) $m = 6$ and the data pair $(4, 34)$ to write an equation. Let C be the cost (in dollars) and n be the number of foam hands.

$C - C_1 = m(n - n_1)$ Write the point-slope form.

$C - 34 = 6(n - 4)$ Substitute 6 for m, 4 for n_1, and 34 for C_1.

$C = 6n + 10$ Write in slope-intercept form.

▶ Because the cost increases at a constant rate, the situation can be modeled by a linear equation. The linear model is $C = 6n + 10$.

Number of months	Total cost (dollars)
3	176
6	302
9	428
12	554

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

8. You pay an installation fee and a monthly fee for Internet service. The table shows the total cost for different numbers of months. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the total cost as a function of the number of months.

SUPPORTING English Language Learners

Read aloud the first sentence of Example 5. Point to the photo and ask students what they think *customized foam hands* means. Explain that *customized* means specially made. Read the first sentence of Monitoring Progress Question 8. Ask if they know what a fee is, and explain that it is the cost or price. Ask if they know the difference between an installation fee and a monthly fee. Explain that they may be able to guess by looking at parts of the words that they may already know (install, month).

Beginning Repeat the phrases modeled by the teacher.

Intermediate Share experiences where students have seen foam hands and describe how they were customized. (sporting events with team logos)

Advanced/Advanced High Share other types of installation fees and monthly fees they may know about. (telephone, wireless, cable TV)

ELPS 2.C.2 Learn new expressions heard during classroom instruction and interactions.

4.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **USING STRUCTURE** Without simplifying, identify the slope of the line given by the equation $y - 5 = -2(x + 5)$. Then identify one point on the line.

2. **WRITING** Explain how you can use the slope formula to write an equation of the line that passes through $(3, -2)$ and has a slope of 4.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, identify the slope of the line. Then identify a point the line passes through. *(See Example 1.)*

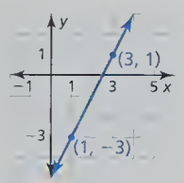
3. $y - 4 = \frac{2}{3}(x - 9)$
4. $y - 1 = -6(x - 3)$
5. $y - 10 = -8\left(x + \frac{1}{4}\right)$
6. $y + 6 = \frac{3}{5}x$

In Exercises 7–14, write an equation in point-slope form of the line that passes through the given point and has the given slope. *(See Example 2.)*

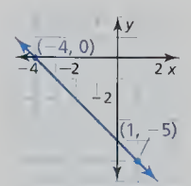
7. $(2, 1)$; $m = 2$
8. $(3, 5)$; $m = -1$
9. $(7, -4)$; $m = -6$
10. $(-8, -2)$; $m = 5$
11. $(9, 0)$; $m = -3$
12. $(0, 2)$; $m = 4$
13. $(-6, 6)$; $m = \frac{3}{2}$
14. $(5, -12)$; $m = -\frac{2}{5}$

In Exercises 15–18, write an equation in slope-intercept form of the line shown. *(See Example 3.)*

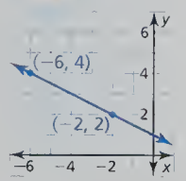
15.



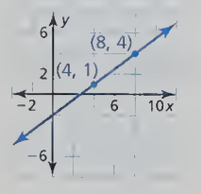
16.



17.



18.



In Exercises 19–24, write an equation in slope-intercept form of the line that passes through the given points.

19. $(7, 2), (2, 12)$
20. $(6, -2), (12, 1)$
21. $(6, -1), (3, -7)$
22. $(-2, 5), (-4, -5)$
23. $(1, -9), (-3, -9)$
24. $(-5, 19), (5, 13)$

In Exercises 25–30, write a linear function f with the given values. *(See Example 4.)*

25. $f(2) = -2, f(1) = 1$
26. $f(5) = 7, f(-2) = 0$
27. $f(-4) = 2, f(6) = -3$
28. $f(-10) = 4, f(-2) = 4$
29. $f(-3) = 1, f(13) = 5$
30. $f(-9) = 10, f(-1) = -2$

In Exercises 31–34, tell whether the data in the table can be modeled by a linear equation. Explain. If possible, write a linear equation that represents y as a function of x. *(See Example 5.)*

31.

x	2	4	6	8	10
y	−1	5	15	29	47

32.

x	−3	−1	1	3	5
y	16	10	4	−2	−8

33.

x	y
0	1.2
1	1.4
2	1.6
4	2

34.

x	y
1	18
2	15
4	12
8	9

35. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line that passes through the point $(1, -5)$ and has a slope of -2.

✗ $y - y_1 = m(x - x_1)$

$y - 5 = -2(x - 1)$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–37 odd, 42, 45–48

Average: 1, 2–38 even, 42, 44–48

Advanced: 1, 2, 12–16 even, 22, 24, 30, 34–48

HOMEWORK CHECK

Basic: 3, 7, 15, 25, 37

Average: 4, 14, 22, 30, 38

Advanced: 12, 16, 30, 34, 38

ANSWERS

1. -2; $(-5, 5)$
2. Substitute 4 for m, 3 for x_1, and -2 for y_1 in the slope formula, eliminate the subscripts for the second point, and solve for y.
3. $\frac{2}{3}$; *Sample answer:* $(9, 4)$
4. -6; *Sample answer:* $(3, 1)$
5. -8; *Sample answer:* $\left(-\frac{1}{4}, 10\right)$
6. $\frac{3}{5}$; *Sample answer:* $(0, -6)$
7. $y - 1 = 2(x - 2)$
8. $y - 5 = -(x - 3)$
9. $y + 4 = -6(x - 7)$
10. $y + 2 = 5(x + 8)$
11. $y = -3(x - 9)$
12. $y - 2 = 4x$
13. $y - 6 = \frac{3}{2}(x + 6)$
14. $y + 12 = -\frac{2}{5}(x - 5)$
15. $y = 2x - 5$
16. $y = -x - 4$
17. $y = -\frac{1}{2}x + 1$
18. $y = \frac{3}{4}x - 2$
19. $y = -2x + 16$
20. $y = \frac{1}{2}x - 5$
21. $y = 2x - 13$
22. $y = 5x + 15$
23. $y = -9$
24. $y = -\frac{3}{5}x + 16$
25. $f(x) = -3x + 4$
26. $f(x) = x + 2$
27. $f(x) = -\frac{1}{2}x$
28. $f(x) = 4$
29. $f(x) = \frac{1}{4}x + \frac{7}{4}$
30. $f(x) = -\frac{3}{2}x - \frac{7}{2}$
31. no; y does not increase at a constant rate.
32. yes, y decreases at a constant rate; $y = -3x + 7$
33. yes; y increases at a constant rate; $y = 0.2x + 1.2$
34. no; y does not decrease at a constant rate.
35. The value substituted for y_1 was 5 instead of -5; $y + 5 = -2(x - 1)$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

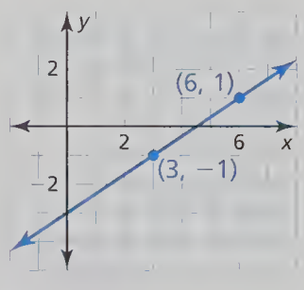
ANSWERS

36. The coordinates substituted into the point-slope form were not from the same point; $y - 2 = \frac{1}{3}(x - 1)$ or $y - 3 = \frac{1}{3}(x - 4)$

37–48. See Additional Answers.

Mini-Assessment

1. Identify the slope of the line $y + 6 = -3(x + 2)$. Then identify a point the line passes through.
 slope -3, point $(-2, -6)$
2. Write an equation in point-slope form of the line that passes through the point $(3, -5)$ and has a slope of 2. $y + 5 = 2(x - 3)$
3. Write an equation in slope-intercept form of the line shown.



 $y = \frac{2}{3}x - 3$
4. Write a linear function f with the values $f(1) = 1$ and $f(-3) = 17$.
 $f(x) = -4x + 5$
5. To ride in a taxi, you pay a start fee and a mileage fee. The table shows the total cost of a taxi ride for different distances. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the cost as a function of the number of miles.

Distance (miles)	4	8	10	12
Cost (dollars)	13	23	28	33

 Yes, the cost increases at a constant rate; $C = 2.5d + 3$

36. ERROR ANALYSIS Describe and correct the error in writing an equation of the line that passes through the points (1, 2) and (4, 3).

✗ $m = \frac{3-2}{4-1} = \frac{1}{3}$ $\quad y - 2 = \frac{1}{3}(x - 4)$

37. MODELING WITH MATHEMATICS You are designing a sticker to advertise your band. A company charges \$225 for the first 1000 stickers and \$80 for each additional 1000 stickers.

a. Write an equation that represents the total cost (in dollars) of the stickers as a function of the number (in thousands) of stickers ordered.

b. Find the total cost of 9000 stickers.

38. MODELING WITH MATHEMATICS You pay a processing fee and a daily fee to rent a beach house. The table shows the total cost of renting the beach house for different numbers of days.

Days	2	4	6	8
Total cost (dollars)	246	450	654	858

a. Can the situation be modeled by a linear equation? Explain.

b. What is the processing fee? the daily fee?

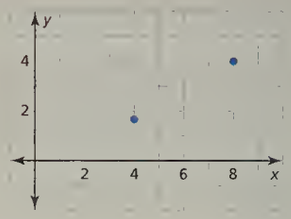
c. You can spend no more than \$1200 on the beach house rental. What is the maximum number of days you can rent the beach house?

39. WRITING Describe two ways to graph the equation $y - 1 = \frac{3}{2}(x - 4)$.

40. THOUGHT PROVOKING The graph of a linear function passes through the point $(12, -5)$ and has a slope of $\frac{2}{5}$. Represent this function in two other ways.

41. REASONING You are writing an equation of the line that passes through two points that are not on the y-axis. Would you use slope-intercept form or point-slope form to write the equation? Explain.

42. HOW DO YOU SEE IT? The graph shows two points that lie on the graph of a linear function.

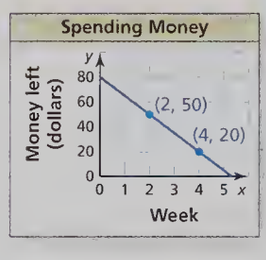


a. Does the y-intercept of the graph of the linear function appear to be positive or negative? Explain.

b. Estimate the coordinates of the two points. How can you use your estimates to confirm your answer in part (a)?

43. CONNECTION TO TRANSFORMATIONS Compare the graph of $y = 2x$ to the graph of $y - 1 = 2(x + 3)$. Make a conjecture about the graphs of $y = mx$ and $y - k = m(x - h)$.

44. COMPARING FUNCTIONS Three siblings each receive money for a holiday and then spend it at a constant weekly rate. The graph describes Sibling A's spending, the table describes Sibling B's spending, and the equation $y = -22.5x + 90$ describes Sibling C's spending. The variable y represents the amount of money left after x weeks.



Week, x	Money left, y
1	\$100
2	\$75
3	\$50
4	\$25

a. Which sibling received the most money? the least money?

b. Which sibling spends money at the fastest rate? the slowest rate?

c. Which sibling runs out of money first? last?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use intercepts to graph the linear equation. *(Section 3.4)*

45. $-4x + 2y = 16$ **46.** $3x + 5y = -15$ **47.** $x - 6y = 24$ **48.** $-7x - 2y = -21$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

4.3 Writing Equations in Standard Form

Texas Essential Knowledge and Skills
A.2.B
A.2.C
A.3.A

Essential Question How can you write the equation of a line in standard form?

EXPLORATION 1 Writing Equations in Standard Form

Work with a partner. So far you have written equations of lines in slope-intercept form and point-slope form. Linear equations can also be written in standard form, $Ax + By = C$. Write each equation in standard form.

a. $y = 3x - 5$ The equation is in slope-intercept form.

[redacted] Subtract $3x$ from each side.

The equation in standard form is [redacted].

b. $y - 4 = -2(x + 6)$ The equation is in slope-intercept form.

[redacted] Distributive Property

[redacted] Add $2x$ to each side.

[redacted] Add 4 to each side.

The equation in standard form is [redacted].

c. $y = 4x + 2$

d. $y = -x - 7$

e. $y - 1 = 3(x + 4)$

f. $y + 8 = -4(x - 3)$

EXPLORATION 2 Finding the Slope and y-Intercept

Work with a partner. The slope and y-intercept of a line are not explicitly known from a linear equation written in standard form. Find the slope and y-intercept of the line represented by each equation.

a. $-6x + 3y = -15$ The equation is in standard form.

[redacted] Add $6x$ to each side.

[redacted] Divide each side by 3.

The slope is [redacted], and the y-intercept is [redacted].

b. $5x - 10y = -40$

c. $-6x - 8y = 56$

MAKING MATHEMATICAL ARGUMENTS
To be proficient in math, you need to justify your conclusions and communicate them to others.

Communicate Your Answer

3. How can you write the equation of a line in standard form?
4. How can you find the slope and y-intercept of a line given the equation of the line in standard form?
5. Consider the graph of $Ax + By = C$.
 a. Does changing the value of A change the slope? Does changing the value of B change the slope? Explain your reasoning.
 b. Does changing the value of A change the y-intercept? Does changing the value of B change the y-intercept? Explain your reasoning.

SUPPORTING English Language Learners

Read aloud the Essential Question. Have students work in pairs of mixed language abilities to complete the explorations.

Beginning/Advanced Advanced students read aloud the instructions and help Beginning students by providing alternate language when necessary. Then they work together to complete the explorations. Beginning students read aloud the red side comments. If needed, Advanced students model the pronunciations by reading the side comments aloud.

Intermediate/Advanced High Advanced High students read aloud the instructions and guide the explorations. Intermediate students read aloud and paraphrase the red side comments.

ELPS 3.B.3 Expand and internalize initial English vocabulary by learning and using routine language needed for classroom communication.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.B The student is expected to write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

A.2.C The student is expected to write linear equations in two variables given a table of values, a graph, and a verbal description.

A.3.A The student is expected to determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

ANSWERS

1. a. $y - 3x = -5$; $-3x + y = -5$
 b. $y - 4 = -2x - 12$;
 $y - 4 + 2x = -12$;
 $y + 2x = -8$; $2x + y = -8$
 c. $-4x + y = 2$
 d. $x + y = -7$
 e. $-3x + y = 13$
 f. $4x + y = 4$
2. a. $3y = 6x - 15$; $y = 2x - 5$; 2; -5
 b. slope: $\frac{1}{2}$; y-intercept: 4
 c. slope: $-\frac{3}{4}$; y-intercept: -7
3. Write the equation in the form $Ax + By = C$.
4. Rewrite the equation in slope-intercept form.
5. a. yes; yes; Changing the value of A changes the numerator of the slope and changing the value of B changes the denominator of the slope.
 b. no; yes; The y-intercept is only affected by the values of B and C because the y-intercept is given by $\frac{C}{B}$.

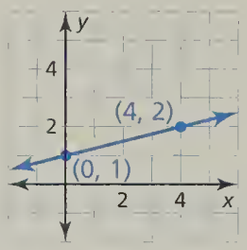
For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Extra Example 1

Write two equations in standard form that are equivalent to $-3x + 6y = 12$. *Sample answers:* $-x + 2y = 4$; $-6x + 12y = 24$

Extra Example 2

Write an equation in standard form of the line shown.



$x - 4y = -4$

MONITORING PROGRESS ANSWERS

1. *Sample answer:* $2x - 2y = 6$, $\frac{1}{3}x - \frac{1}{3}y = 1$
2. $-2x + y = -7$

4.3 Lesson

What You Will Learn

- Write equations in standard form.
- Use linear equations to solve real-life problems.

Core Vocabulary

Previous

standard form
equivalent equation
point-slope form

Writing Equations in Standard Form

Recall that the linear equation $Ax + By = C$ is in standard form, where A, B, and C are real numbers and A and B are not both zero. All linear equations can be written in standard form.

EXAMPLE 1 **Writing Equivalent Equations in Standard Form**

Write two equations in standard form that are equivalent to $2x - 6y = 4$.

REMEMBER

You can produce an equivalent equation by multiplying or dividing each side of an equation by the same nonzero number.

SOLUTION

To write one equivalent equation, multiply each side of the original equation by 2.

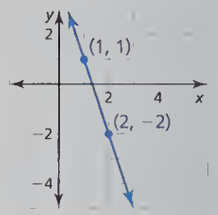
$2(2x - 6y) = 2(4) \rightarrow 4x - 12y = 8$

To write another equivalent equation, divide each side of the original equation by 2.

$\frac{2x - 6y}{2} = \frac{4}{2} \rightarrow x - 3y = 2$

EXAMPLE 2 **Using Two Points to Write an Equation**

Write an equation in standard form of the line shown.



SOLUTION

Step 1 Find the slope of the line.

$m = \frac{1 - (-2)}{1 - 2} = \frac{3}{-1}$, or -3

Step 2 Use the slope $m = -3$ and the point $(1, 1)$ to write an equation in point-slope form.

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 1 = -3(x - 1)$ Substitute -3 for m, 1 for x_1, and 1 for y_1.

Step 3 Write the equation in standard form.

$y - 1 = -3(x - 1)$ Write the equation.

$y - 1 = -3x + 3$ Distributive Property

$3x + y - 1 = 3$ Add $3x$ to each side.

$3x + y = 4$ Add 1 to each side.

An equation is $3x + y = 4$.

ANOTHER WAY

You can use either of the given points to write an equation of the line.

Use $m = -3$ and $(2, -2)$.

$y - (-2) = -3(x - 2)$

$y + 2 = -3x + 6$

$3x + y + 2 = 6$

$3x + y = 4$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. Write two equations in standard form that are equivalent to $x - y = 3$.
2. Write an equation in standard form of the line that passes through $(3, -1)$ and $(2, -3)$.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Inclusion

Some students may think that the only way to write an equivalent equation in standard form is to multiply or divide by 2. Explain that they can multiply or divide by any nonzero number. Emphasize that whatever number they choose, the same nonzero number must be used to multiply or divide both sides of the equation. Students should understand that there are an infinite number of possible equivalent equations for any given equation.

Recall that equations of horizontal lines have the form $y = b$ and equations of vertical lines have the form $x = a$. You cannot write an equation of a vertical line in slope-intercept form or point-slope form because the slope of a vertical line is undefined. However, you can write an equation of a vertical line in standard form.

EXAMPLE 3 Horizontal and Vertical Lines

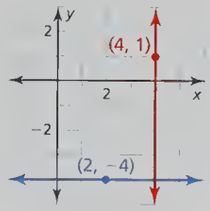
ANOTHER WAY

Using the slope-intercept form to find an equation of the horizontal line gives you $y = 0x - 4$, or $y = -4$.

Write an equation of the specified line.

a. blue line

b. red line



SOLUTION

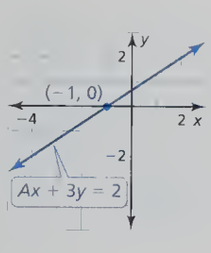
a. The y-coordinate of the given point on the blue line is -4. This means that all points on the line have a y-coordinate of -4.

▶ So, an equation of the line is $y = -4$.

b. The x-coordinate of the given point on the red line is 4. This means that all points on the line have an x-coordinate of 4.

▶ So, an equation of the line is $x = 4$.

EXAMPLE 4 Completing an Equation in Standard Form



Find the missing coefficient in the equation of the line shown. Write the completed equation.

SOLUTION

Step 1 Find the value of A. Substitute the coordinates of the given point for x and y in the equation. Then solve for A.

$Ax + 3y = 2$ Write the equation.

$A(-1) + 3(0) = 2$ Substitute -1 for x and 0 for y.

$-A = 2$ Simplify.

$A = -2$ Divide each side by -1.

Step 2 Complete the equation.

$-2x + 3y = 2$ Substitute -2 for A.

▶ An equation is $-2x + 3y = 2$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write equations of the horizontal and vertical lines that pass through the given point.

3. $(-8, -9)$

4. $(13, -5)$

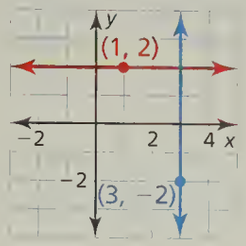
Find the missing coefficient in the equation of the line that passes through the given point. Write the completed equation.

5. $-4x + By = 7; (-1, 1)$

6. $Ax + y = -3; (2, 11)$

Extra Example 3

Write an equation of the specified line.

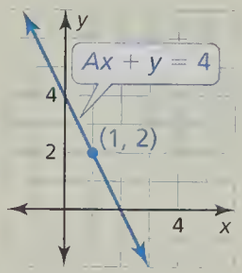


a. blue line $x = 3$

b. red line $y = 2$

Extra Example 4

Find the missing coefficient in the equation of the line shown. Write the completed equation.



$A = 2; 2x + y = 4$

MONITORING PROGRESS ANSWERS

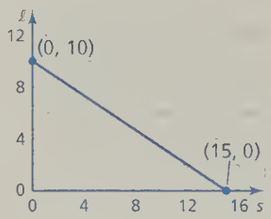
3. $y = -9, x = -8$
4. $y = -5, x = 13$
5. $3; -4x + 3y = 7$
6. $-7; -7x + y = -3$

Extra Example 5

A banquet room has large and small tables. A small table seats 4 people and a large table seats 6 people. The guests at a banquet can fill 9 small tables and 4 large tables.

a. Write an equation in standard form that models the possible combinations of small and large tables that banquet guests can fill. $4s + 6\ell = 60$

b. Graph the equation from part (a).



c. Find four possible combinations.
Sample answer: 0 small and 10 large, 6 small and 6 large, 12 small and 2 large, 15 small and 0 large

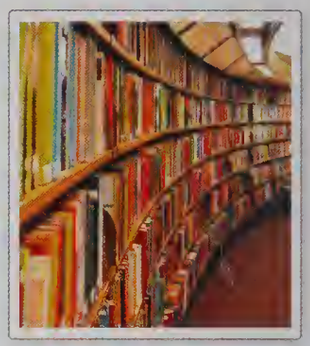
MONITORING PROGRESS ANSWER

7. $8s + 12\ell = 136$; *Sample answer:* 2 small vans and 10 large vans, 5 small vans and 8 large vans, 8 small vans and 6 large vans, and 11 small vans and 4 large vans

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**

Your class is taking a trip to the public library. You can travel in small and large vans. A small van holds 8 people, and a large van holds 12 people. Your class can fill 15 small vans and 2 large vans.



a. Write an equation in standard form that models the possible combinations of small and large vans that your class can fill.

b. Graph the equation from part (a).

c. Find four possible combinations.

SOLUTION

a. Write a verbal model. Then write an equation.

Capacity of small van	·	Number of small vans	+	Capacity of large van	·	Number of large vans	=	People on trip
8	·	s	+	12	·	ℓ	=	p

Because your class can fill 15 small vans and 2 large vans, use (15, 2) to find the value of p.

$8s + 12\ell = p$ Write the equation.

$8(15) + 12(2) = p$ Substitute 15 for s and 2 for ℓ.

$144 = p$ Simplify.

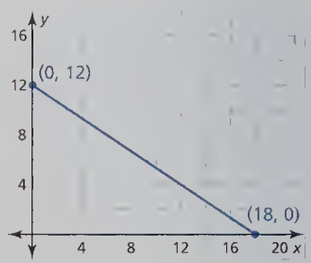
▶ So, the equation $8s + 12\ell = 144$ models the possible combinations.

b. Use intercepts to graph the equation.

Find the intercepts.

Substitute 0 for s.	Substitute 0 for ℓ.
$8(0) + 12\ell = 144$	$8s + 12(0) = 144$
$\ell = 12$	$s = 18$

Plot the points (0, 12) and (18, 0). Connect them with a line segment. For this problem, only whole-number values of s and ℓ make sense.



ANOTHER WAY

Another way to find possible combinations is to substitute values for s or ℓ in the equation and solve for the other variable.

c. The graph passes through (0, 12), (6, 8), (12, 4), and (18, 0). So, four possible combinations are 0 small and 12 large, 6 small and 8 large, 12 small and 4 large, and 18 small and 0 large.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. **WHAT IF?** Eight students decide to not go on the class trip. Write an equation in standard form that models the possible combinations of small and large vans that your class can fill. Find four possible combinations.

Closure

- **Exit Ticket:** Write an equation in standard form of the line that passes through (5, −3) and has a slope of −2. $2x + y = 7$

4.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to write an equation in standard form of a line when two points on the line are given.

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$-3x + 8y = 1$ $\quad y = 5x - 9$ $\quad 6x - 2y = 1$ $\quad x + y = 4$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write two equations in standard form that are equivalent to the given equation. *(See Example 1.)*

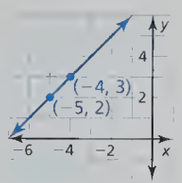
3. $x + y = -10$
4. $5x + 10y = 15$
5. $-x + 2y = 9$
6. $-9x - 12y = 6$
7. $9x - 3y = -12$
8. $-2x + 4y = -5$

In Exercises 9–14, write an equation in standard form of the line that passes through the given point and has the given slope.

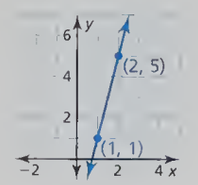
9. $(-3, 2)$; $m = 1$
10. $(4, -1)$; $m = 3$
11. $(0, 5)$; $m = -2$
12. $(-8, 0)$; $m = -4$
13. $(-4, -4)$; $m = -\frac{3}{2}$
14. $(-6, -10)$; $m = \frac{1}{6}$

In Exercises 15–18, write an equation in standard form of the line shown. *(See Example 2.)*

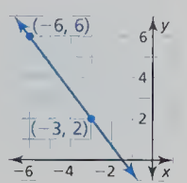
15.



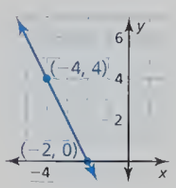
16.



17.



18.

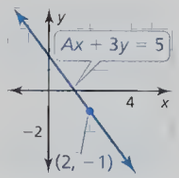


In Exercises 19–22, write equations of the horizontal and vertical lines that pass through the given point. *(See Example 3.)*

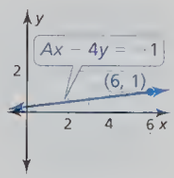
19. $(2, 3)$
20. $(-5, -4)$
21. $(8, -1)$
22. $(-6, 2)$

In Exercises 23–26, find the missing coefficient in the equation of the line shown. Write the completed equation. *(See Example 4.)*

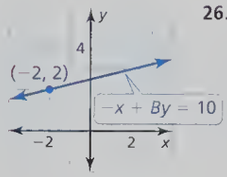
23.



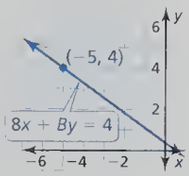
24.



25.



26.



27. **ERROR ANALYSIS** Describe and correct the error in finding the value of A for the equation $Ax - 3y = 5$, when the graph of the equation passes through the point $(1, -4)$.

✗

$A(-4) - 3(1) = 5$

$-4A = 8$

$A = -2$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–27 odd, 28, 29, 32, 36–39

Average: 1, 2–26 even, 27, 28–32 even, 36–39

Advanced: 1, 2, 8–26 even, 27–39

HOMEWORK CHECK

Basic: 3, 15, 19, 23, 29

Average: 4, 16, 20, 24, 30

Advanced: 8, 18, 20, 24, 30

ANSWERS

1. Find the slope of the line. Use the slope and one of the given points to write an equation in point-slope form. Then, rewrite the equation in standard form.
2. $y = 5x - 9$; It is the only equation not written in standard form.
3. *Sample answer:* $2x + 2y = -20$, $3x + 3y = -30$
4. *Sample answer:* $x + 2y = 3$, $10x + 20y = 30$
5. *Sample answer:* $-2x + 4y = 18$, $-3x + 6y = 27$
6. *Sample answer:* $-3x - 4y = 2$, $-18x - 24y = 12$
7. *Sample answer:* $3x - y = -4$, $18x - 6y = -24$
8. *Sample answer:* $-4x + 8y = -10$, $-6x + 12y = -15$
9. $-x + y = 5$
10. $-3x + y = -13$
11. $2x + y = 5$
12. $4x + y = -32$
13. $\frac{3}{2}x + y = -10$
14. $-\frac{1}{6}x + y = -9$
15. $-x + y = 7$
16. $-4x + y = -3$
17. $\frac{4}{3}x + y = -2$
18. $2x + y = -4$
19. $y = 3, x = 2$
20. $y = -4, x = -5$
21. $y = -1, x = 8$
22. $y = 2, x = -6$
23. 4; $4x + 3y = 5$
24. $\frac{1}{2}$; $\frac{1}{2}x - 4y = -1$
25. 4; $-x + 4y = 10$
26. 11; $8x + 11y = 4$
27. -4 was substituted for x instead of y and 1 was substituted for y instead of x; $A = -7$

Dynamic Teaching Tools

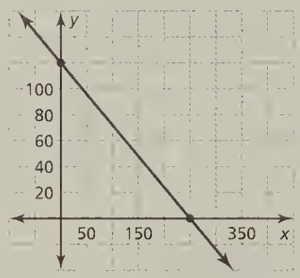
- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

28. no; The equation of a horizontal line is the same in all forms.

29. **a.** $1.20x + 2.50y = 300$

b.



c. *Sample answer:* 0 Vinca plants and 120 Phlox plants, 50 Vinca plants and 96 Phlox plants, 150 Vinca plants and 48 Phlox plants, and 250 Vinca plants and 0 Phlox plants

30–32. See Additional Answers.

33. $\frac{b}{a}x + y = b$

34. **a.** negative and positive; positive and negative

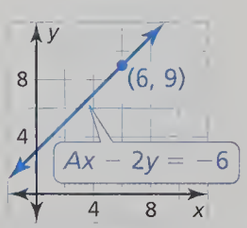
b. multiply B by -1

c. replace x with $(x - h)$ for some constant h

35–39. See Additional Answers.

Mini-Assessment

1. Write two equations in standard form that are equivalent to $x + 3y = 5$. *Sample answers:* $2x + 6y = 10$; $-x - 3y = -5$
2. Write an equation in standard form of the line through the points $(-2, 6)$ and $(1, -6)$. $4x + y = -2$
3. Find the missing coefficient in the equation of the line shown. Write the completed equation.



$A = 2$; $2x - 2y = -6$

28. MAKING AN ARGUMENT Your friend says that you can write an equation of a horizontal line in standard form but not in slope-intercept form or point-slope form. Is your friend correct? Explain.

29. MODELING WITH MATHEMATICS The diagram shows the prices of two types of ground cover plants. A gardener can afford to buy 125 vinca plants and 60 phlox plants. *(See Example 5.)*



a. Write an equation in standard form that models the possible combinations of vinca and phlox plants the gardener can afford to buy.

b. Graph the equation from part (a).

c. Find four possible combinations.

30. MODELING WITH MATHEMATICS One bus ride costs \$0.75. One subway ride costs \$1. A monthly pass for unlimited bus and subway rides costs the same as 36 bus rides plus 36 subway rides.

a. Write an equation in standard form that models the possible combinations of bus and subway rides with the same total cost as the pass.

b. Graph the equation from part (a).

c. You ride the bus 60 times in one month. How many times must you ride the subway for the total cost of the rides to equal the cost of the pass? Explain your reasoning.

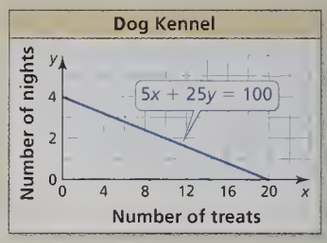
31. WRITING There are three forms of an equation of a line: *slope-intercept*, *point-slope*, and *standard* form. Which form would you prefer to use to do each of the following? Explain.

a. Graph the equation.

b. Find the x-intercept of the graph of the equation.

c. Write an equation of the line given two points on the line.

32. HOW DO YOU SEE IT? A dog kennel charges \$25 per night to board your dog. The kennel also sells dog treats for \$5 each. The graph shows the possible combinations of nights at the kennel and treats that you can buy for \$100.

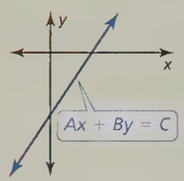


a. List two possible combinations.

b. Interpret the intercepts of the graph.

33. ABSTRACT REASONING Write an equation in standard form of the line that passes through $(a, 0)$ and $(0, b)$, where $a \neq 0$ and $b \neq 0$.

34. THOUGHT PROVOKING Use the graph shown.



a. What are the signs of B and C when A is positive? when A is negative?

b. Explain how to change the equation so that the graph is reflected in the x-axis.

c. Explain how to change the equation so that the graph is translated horizontally.

35. MATHEMATICAL CONNECTIONS Write an equation in standard form that models the possible lengths and widths (in feet) of a rectangle with the same perimeter as a rectangle that is 10 feet wide and 20 feet long. Make a table that shows five possible lengths and widths of the rectangle.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the reciprocal of the number. *(Skills Review Handbook)*

36. 5 **37.** -8 **38.** $-\frac{2}{7}$ **39.** $\frac{3}{2}$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

4.4 Writing Equations of Parallel and Perpendicular Lines

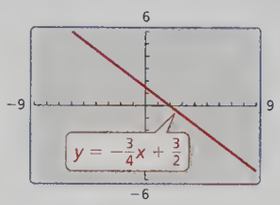
Texas Essential Knowledge and Skills
A.2.C
A.2.E
A.2.F
A.2.G
A.3.A

Essential Question How can you recognize lines that are parallel or perpendicular?

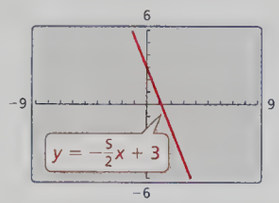
EXPLORATION 1 Recognizing Parallel Lines

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear parallel? How can you tell?

a. $3x + 4y = 6$
$3x + 4y = 12$
$4x + 3y = 12$



b. $5x + 2y = 6$
$2x + y = 3$
$2.5x + y = 5$

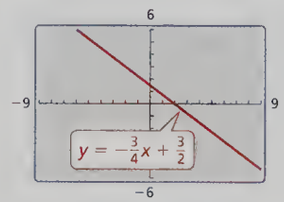


SELECTING TOOLS
To be proficient in math, you need to use a graphing calculator and other available technological tools, as appropriate, to help you explore relationships and deepen your understanding of concepts.

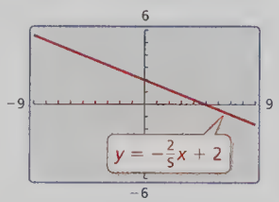
EXPLORATION 2 Recognizing Perpendicular Lines

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear perpendicular? How can you tell?

a. $3x + 4y = 6$
$3x - 4y = 12$
$4x - 3y = 12$



b. $2x + 5y = 10$
$-2x + y = 3$
$2.5x - y = 5$



Communicate Your Answer

3. How can you recognize lines that are parallel or perpendicular?
4. Compare the slopes of the lines in Exploration 1. How can you use slope to determine whether two lines are parallel? Explain your reasoning.
5. Compare the slopes of the lines in Exploration 2. How can you use slope to determine whether two lines are perpendicular? Explain your reasoning.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students read aloud the routine headings used on pages 179–180. (Essential Question, Exploration, Communicate Your Answer, What You Will Learn, Core Vocabulary, Core Concept, Example, Solution, Monitoring Progress)

Beginning/Intermediate Point to each heading as the teacher models it by reading aloud.
Advanced/Advanced High Read each heading aloud and explain its meaning using their own words.

ELPS 4.C.3 Comprehend English vocabulary used routinely in written classroom materials.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.C The student is expected to write linear equations in two variables given a table of values, a graph, and a verbal description.

A.2.E The student is expected to write the equation of a line that contains a given point and is parallel to a given line.

A.2.F The student is expected to write the equation of a line that contains a given point and is perpendicular to a given line.

A.2.G The student is expected to write an equation of a line that is parallel or perpendicular to the X or Y axis and determine whether the slope of the line is zero or undefined.

A.3.A The student is expected to determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

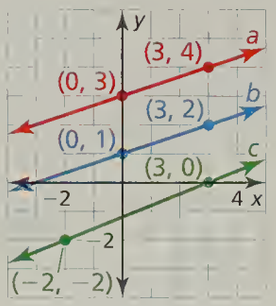
ANSWERS

1–2. See Additional Answers.

3. Parallel lines are always the same distance apart; Perpendicular lines intersect at right angles.
4. Parallel lines have the same slope; The lines with the same slope in Exploration 1 were parallel.
5. Perpendicular lines have slopes that are negative reciprocals of each other; The lines with negative reciprocal slopes in Exploration 2 were perpendicular.

Extra Example 1

Determine which of the lines are parallel.



Lines a and b have the same slope, so they are parallel.

Extra Example 2

Write an equation of the line that passes through (3, 1) and is parallel to the line $y = 3x + 1$. $y = 3x - 8$

MONITORING PROGRESS ANSWERS

1. no; They do not have the same slope.
2. $y = \frac{1}{4}x + 3$

4.4 Lesson

What You Will Learn

- Identify and write equations of parallel lines.
- Identify and write equations of perpendicular lines.

Core Vocabulary

parallel lines, *p. 180*
perpendicular lines, *p. 181*

Previous
reciprocal

Identifying and Writing Equations of Parallel Lines

Core Concept

Parallel Lines and Slopes

Two lines in the same plane that never interseet are **parallel lines**. Nonvertieal lines are parallel if and only if they have the same slope.

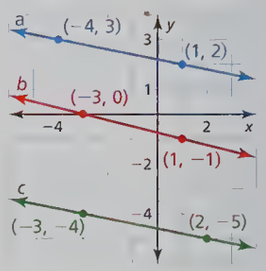
All vertieal lines are parallel.

READING

The phrase "*A if and only if B*" is a way of writing two conditional statements at once. It means that if A is true, then B is true. It also means that if B is true, then A is true.

EXAMPLE 1 Identifying Parallel Lines

Determine whieh of the lines are parallel.



SOLUTION

Find the slope of eaeh line.

Line a: $m = \frac{2 - 3}{1 - (-4)} = -\frac{1}{5}$

Line b: $m = \frac{-1 - 0}{1 - (-3)} = -\frac{1}{4}$

Line c: $m = \frac{-5 - (-4)}{2 - (-3)} = -\frac{1}{5}$

▶ Lines a and c have the same slope, so they are parallel.

EXAMPLE 2 Writing an Equation of a Parallel Line

Write an equation of the line that passes through (5, −4) and is parallel to the line $y = 2x + 3$.

SOLUTION

Step 1 Find the slope of the parallel line. The graph of the given equation has a slope of 2. So, the parallel line that passes through (5, −4) also has a slope of 2.

Step 2 Use the slope-intercept form to find the y-intercept of the parallel line.

$y = mx + b$	Write the slope-intercept form.
$-4 = 2(5) + b$	Substitute 2 for m, 5 for x, and −4 for y.
$-14 = b$	Solve for b.

▶ Using $m = 2$ and $b = -14$, an equation of the parallel line is $y = 2x - 14$.

ANOTHER WAY

You can also use the slope $m = 2$ and the point-slope form to write an equation of the line that passes through (5, −4).

$y - y_1 = m(x - x_1)$
$y - (-4) = 2(x - 5)$
$y = 2x - 14$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. Line a passes through (−5, 3) and (−6, −1). Line b passes through (3, −2) and (2, −7). Are the lines parallel? Explain.
2. Write an equation of the line that passes through (−4, 2) and is parallel to the line $y = \frac{1}{4}x + 1$.

For classroom suggestions on teaching this lesson, see Laurie's Notes at *BigIdeasMath.com*.

Identifying and Writing Equations of Perpendicular Lines

REMEMBER

The product of a nonzero number m and its negative reciprocal is -1:

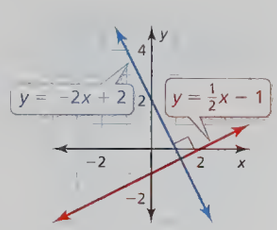
$$m\left(-\frac{1}{m}\right) = -1.$$

Core Concept

Perpendicular Lines and Slopes

Two lines in the same plane that intersect to form right angles are **perpendicular lines**. Nonvertical lines are perpendicular if and only if their slopes are negative reciprocals.

Vertical lines are perpendicular to horizontal lines.



EXAMPLE 3 Identifying Parallel and Perpendicular Lines

Determine which of the lines, if any, are parallel or perpendicular.

Line a: $y = 4x + 2$ Line b: $x + 4y = 3$ Line c: $-8y - 2x = 16$

SOLUTION

Write the equations in slope-intercept form. Then compare the slopes.

Line a: $y = 4x + 2$ Line b: $y = -\frac{1}{4}x + \frac{3}{4}$ Line c: $y = -\frac{1}{4}x - 2$

▶ Lines b and c have slopes of $-\frac{1}{4}$, so they are parallel. Line a has a slope of 4, the negative reciprocal of $-\frac{1}{4}$, so it is perpendicular to lines b and c.

EXAMPLE 4 Writing an Equation of a Perpendicular Line

Write an equation of the line that passes through $(-3, 1)$ and is perpendicular to the line $y = \frac{1}{2}x + 3$.

SOLUTION

Step 1 Find the slope of the perpendicular line. The graph of the given equation has a slope of $\frac{1}{2}$. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line that passes through $(-3, 1)$ is -2.

Step 2 Use the slope $m = -2$ and the point-slope form to write an equation of the perpendicular line that passes through $(-3, 1)$.

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 1 = -2[x - (-3)]$ Substitute -2 for m, -3 for x_1, and 1 for y_1.

$y - 1 = -2x - 6$ Simplify.

$y = -2x - 5$ Write in slope-intercept form.

▶ An equation of the perpendicular line is $y = -2x - 5$.

ANOTHER WAY

You can also use the slope $m = -2$ and the slope-intercept form to write an equation of the line that passes through $(-3, 1)$.

$y = mx + b$

$1 = -2(-3) + b$

$-5 = b$

So, $y = -2x - 5$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

3. Determine which of the lines, if any, are parallel or perpendicular. Explain.

 Line a: $2x + 6y = -3$ Line b: $y = 3x - 8$ Line c: $-6y + 18x = 9$

4. Write an equation of the line that passes through $(-3, 5)$ and is perpendicular to the line $y = -3x - 1$.

Extra Example 3

Determine which of the lines, if any, are parallel or perpendicular.

Line a: $y = \frac{2}{3}x + 3$

Line b: $3x + 2y = 8$

Line c: $3y - 2x = -9$

Lines a and c have slopes of $\frac{2}{3}$, so they are parallel. Line b has a slope of $-\frac{3}{2}$, the negative reciprocal of $\frac{2}{3}$, so it is perpendicular to lines a and c.

Extra Example 4

Write an equation of the line that passes through $(4, -3)$ and is perpendicular to the line $y = 4x - 1$. $y = -\frac{1}{4}x - 2$

MONITORING PROGRESS ANSWERS

3. Lines b and c are parallel; Line a is perpendicular to lines b and c; Lines b and c have the same slope and the slope of line a is the negative reciprocal of the slopes of lines b and c.
4. $y = \frac{1}{3}x + 6$

English Language Learners

Build on Past Knowledge

Remind students that the product of a number and its reciprocal is 1. So the product of a number and its negative reciprocal is -1. Have students check each negative reciprocal of a slope by multiplying it by the original slope.

$$\frac{1}{4}(-4) = \frac{1}{4}\left(\frac{-4}{1}\right) = -1$$

Extra Example 5

a. Write an equation of a line that is parallel to the line $y = 2$. What is the slope of the line?
Sample answer: $y = -1$. The slope of the line is 0.

b. Write an equation of a line that is perpendicular to the line $y = 2$. What is the slope of the line?
Sample answer: $x = 3$. The slope of the line is undefined.

Extra Example 6

Use the graph in Example 6. Another helicopter is at (8, 5). The shortest flight path to the shoreline is one that is perpendicular to the shoreline. Write an equation that represents this path.
$y = \frac{3}{2}x - 7$

MONITORING PROGRESS ANSWERS

5. a. *Sample answer:* $x = 4$; The slope is undefined.
 b. *Sample answer:* $y = 2$; The slope is 0.
6. $y = \frac{3}{2}x - \frac{19}{2}$

REMEMBER
The slope of a horizontal line is 0. The slope of a vertical line is undefined.

EXAMPLE 5 Horizontal and Vertical Lines

Write an equation of a line that is (a) parallel to the x-axis and (b) perpendicular to the x-axis. What is the slope of each line?

SOLUTION

a. The x-axis (the line $y = 0$) is a horizontal line and has a slope of 0. All horizontal lines are parallel. Equations of horizontal lines have the form $y = b$, where b is a constant. Let b equal any number other than 0, such as 5.

▶ An equation of a line parallel to the x-axis is $y = 5$. The slope of the line is 0.

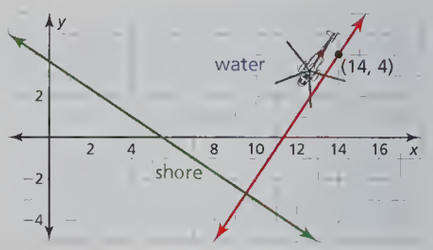
b. The x-axis is a horizontal line. Vertical lines are perpendicular to horizontal lines. The slope of a vertical line is undefined. Equations of vertical lines have the form $x = a$, where a is a constant. Let a equal any number, such as 2.

▶ An equation of a line perpendicular to the x-axis is $x = 2$. The slope of the line is undefined.

EXAMPLE 6 Writing an Equation of a Perpendicular Line



The position of a helicopter search and rescue crew is shown in the graph. The shortest flight path to the shoreline is one that is perpendicular to the shoreline. Write an equation that represents this path.



SOLUTION

Step 1 Find the slope of the line that represents the shoreline. The line passes through points (1, 3) and (4, 1). So, the slope is

$$m = \frac{1 - 3}{4 - 1} = -\frac{2}{3}.$$

Because the shoreline and shortest flight path are perpendicular, the slopes of their respective graphs are negative reciprocals. So, the slope of the graph of the shortest flight path is $\frac{3}{2}$.

Step 2 Use the slope $m = \frac{3}{2}$ and the point-slope form to write an equation of the shortest flight path that passes through (14, 4).

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 4 = \frac{3}{2}(x - 14)$	Substitute $\frac{3}{2}$ for m, 14 for x_1, and 4 for y_1.
$y - 4 = \frac{3}{2}x - 21$	Distributive Property
$y = \frac{3}{2}x - 17$	Write in slope-intercept form.

▶ An equation that represents the shortest flight path is $y = \frac{3}{2}x - 17$.

Monitoring Progress

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5. Write an equation of a line that is (a) parallel to the line $x = 3$ and (b) perpendicular to the line $x = 3$. What is the slope of each line?
6. **WHAT IF?** In Example 6, a boat is traveling perpendicular to the shoreline and passes through (9, 3). Write an equation that represents the path of the boat.

Closure

- **Writing Prompt:** To write an equation of the line that passes through (−2, 4) and is parallel to $y = -3x + 4$, … *Sample answer:* substitute the slope of the parallel line, $m = -3$, and the point into the slope-intercept form to find the y-intercept of the parallel line. Then use the slope and y-intercept to write an equation.

4.4 Exercises

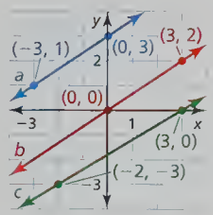
Dynamic Solutions available at *BigIdeasMath.com*

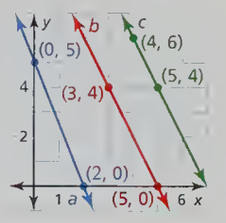
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Nonvertical _____ lines have the same slope.
2. **VOCABULARY** Two lines are perpendicular. The slope of one line is $-\frac{5}{7}$. What is the slope of the other line? Justify your answer.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine which of the lines, if any, are parallel. Explain. *(See Example 1.)*

3. 

4. 

5. Line a passes through $(-1, -2)$ and $(1, 0)$.
 Line b passes through $(4, 2)$ and $(2, -2)$.
 Line c passes through $(0, 2)$ and $(-1, 1)$.

6. Line a passes through $(-1, 3)$ and $(1, 9)$.
 Line b passes through $(-2, 12)$ and $(-1, 14)$.
 Line c passes through $(3, 8)$ and $(6, 10)$.

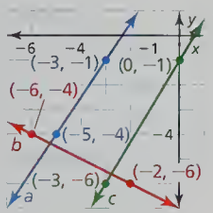
7. Line a: $4y + x = 8$
 Line b: $2y + x = 4$
 Line c: $2y = -3x + 6$

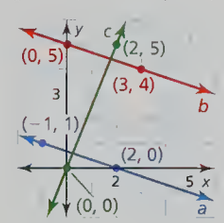
8. Line a: $3y - x = 6$
 Line b: $3y = x + 18$
 Line c: $3y - 2x = 9$

In Exercises 9–12, write an equation of the line that passes through the given point and is parallel to the given line. *(See Example 2.)*

9. $(-1, 3)$; $y = 2x + 2$
10. $(1, 2)$; $y = -5x + 4$
11. $(18, 2)$; $3y - x = -12$
12. $(2, -5)$; $2y = 3x + 10$

In Exercises 13–18, determine which of the lines, if any, are parallel or perpendicular. Explain. *(See Example 3.)*

13. 

14. 

15. Line a passes through $(-2, 1)$ and $(0, 3)$.
 Line b passes through $(4, 1)$ and $(6, 4)$.
 Line c passes through $(1, 3)$ and $(4, 1)$.

16. Line a passes through $(2, 10)$ and $(4, 13)$.
 Line b passes through $(4, 9)$ and $(6, 12)$.
 Line c passes through $(2, 10)$ and $(4, 9)$.

17. Line a: $4x - 3y = 2$
 Line b: $y = \frac{4}{3}x + 2$
 Line c: $4y + 3x = 4$

18. Line a: $y = 6x - 2$
 Line b: $6y = -x$
 Line c: $y + 6x = 1$

In Exercises 19–22, write an equation of the line that passes through the given point and is perpendicular to the given line. *(See Example 4.)*

19. $(7, 10)$; $y = \frac{1}{2}x - 9$
20. $(-4, -1)$; $y = \frac{4}{3}x + 6$
21. $(-3, 3)$; $2y = 8x - 6$
22. $(8, 1)$; $2y + 4x = 12$

In Exercises 23–26, write an equation of a line that is (a) parallel to the given line and (b) perpendicular to the given line. *(See Example 5.)*

23. the y-axis
24. $x = -4$
25. $y = -2$
26. $y = 7$

27. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line that passes through $(1, 3)$ and is parallel to the line $y = \frac{1}{4}x + 2$.

✗
$y - y_1 = m(x - x_1)$
$y - 3 = -4(x - 1)$
$y - 3 = -4x + 4$
$y = -4x + 7$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–29 odd, 33, 34, 39, 40

Average: 1, 2–28 even, 29, 30, 33, 34, 39, 40

Advanced: 1, 2, 6–12 even, 16–28 even, 29–40

HOMEWORK CHECK

Basic: 3, 9, 13, 19, 23

Average: 4, 10, 14, 20, 24

Advanced: 8, 12, 18, 22, 26

ANSWERS

1. parallel
2. $\frac{7}{5}$; $\frac{7}{5}$ is the negative reciprocal of $-\frac{5}{7}$.
3. lines a and b; They have the same slope.
4. lines b and c; They have the same slope.
5. lines a and c; They have the same slope.
6. none; None of the lines have the same slope.
7. none; None of the lines have the same slope.
8. lines a and b; They have the same slope.
9. $y = 2x + 5$
10. $y = -5x + 7$
11. $y = \frac{1}{3}x - 4$
12. $y = \frac{3}{2}x - 8$
13. None are parallel or perpendicular; None of the lines have the same slope or slopes that are negative reciprocals of each other.
14. Lines a and b are parallel; None are perpendicular; Lines a and b have the same slope and none of the lines have slopes that are negative reciprocals of each other.
15. None are parallel; Lines b and c are perpendicular; None of the lines have the same slope and the slope of line b is the negative reciprocal of the slope of line c.
16. Lines a and b are parallel; None are perpendicular; Lines a and b have the same slope and none of the lines have slopes that are negative reciprocals of each other.
17. Lines a and b are parallel; Line c is perpendicular to lines a and b; Lines a and b have the same slope and the slope of line c is the negative reciprocal of the slopes of lines a and b.
18. None are parallel; Lines a and b are perpendicular; None of the lines have the same slope and the slope of line a is the negative reciprocal of the slope of line b.
19. $y = -2x + 24$
20. $y = -\frac{3}{4}x - 4$
21. $y = -\frac{1}{4}x + \frac{9}{4}$
22. $y = \frac{1}{2}x - 3$
23. a. *Sample answer:* $x = 1$
 b. *Sample answer:* $y = -3$
24. a. *Sample answer:* $x = 7$
 b. *Sample answer:* $y = 1$
25. a. *Sample answer:* $y = -6$
 b. *Sample answer:* $x = 2$
26. a. *Sample answer:* $y = 5$
 b. *Sample answer:* $x = 4$
27. Parallel lines have the same slope, not negative reciprocal slopes; $y - 3 = \frac{1}{4}(x - 1)$; $y - 3 = \frac{1}{4}x - \frac{1}{4}$; $y = \frac{1}{4}x + \frac{11}{4}$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

28. Perpendicular lines have negative reciprocal slopes, not just reciprocal slopes; $y - (-5) = -3(x - 4)$; $y + 5 = -3x + 12$; $y = -3x + 7$

29. $y = -\frac{3}{4}x + \frac{3}{2}$

30. $y = \frac{4}{3}x - \frac{1}{3}$

31. a. yes; Opposite sides are parallel.
b. no; Adjacent sides are not perpendicular.

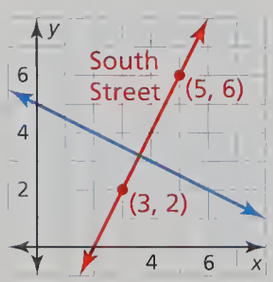
32. $a = -\frac{2}{3}$; $a = 6$

33. no; The lines that form the angle are not perpendicular.

34–40. See Additional Answers.

Mini-Assessment

1. Line a passes through $(-1, -5)$ and $(1, 3)$. Line b passes through $(-3, -7)$ and $(1, 9)$. Line c passes through $(0, -2)$ and $(4, -3)$. Which lines are parallel or perpendicular? Explain. Lines a and b (slopes: 4) are parallel. Line c $\left(\text{slope: } -\frac{1}{4}\right)$ is perpendicular to lines a and b.
2. Write an equation of the line that passes through $(-2, 3)$ and is parallel to the line $y = 2x - 1$. $y = 2x + 7$
3. Write an equation of the line that passes through $(0, -2)$ and is perpendicular to the line $y = \frac{1}{5}x - 2$. $y = -5x - 2$
4. A road is constructed perpendicular to South Street. Write an equation that represents this new road.

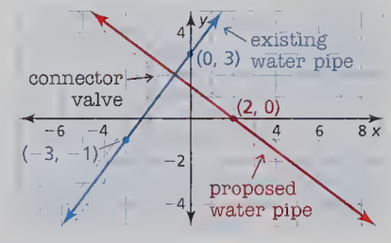


$y = -\frac{1}{2}x + 5$

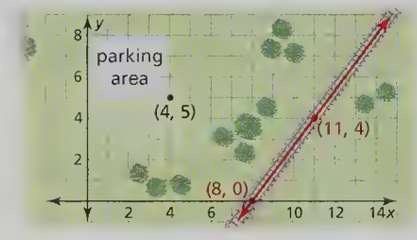
28. ERROR ANALYSIS Describe and correct the error in writing an equation of the line that passes through $(4, -5)$ and is perpendicular to the line $y = \frac{1}{3}x + 5$.

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 3(x - 4)$$
$$y + 5 = 3x - 12$$
$$y = 3x - 17$$

29. MODELING WITH MATHEMATICS A city water department is proposing the construction of a new water pipe, as shown. The new pipe will be perpendicular to the old pipe. Write an equation that represents the new pipe. *(See Example 6.)*



30. MODELING WITH MATHEMATICS A parks and recreation department is constructing a new bike path. The path will be parallel to the railroad tracks shown and pass through the parking area at the point $(4, 5)$. Write an equation that represents the path.



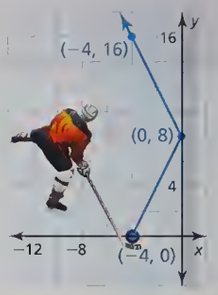
31. MATHEMATICAL CONNECTIONS The vertices of a quadrilateral are $A(2, 2)$, $B(6, 4)$, $C(8, 10)$, and $D(4, 8)$.

a. Is quadrilateral $ABCD$ a parallelogram? Explain.

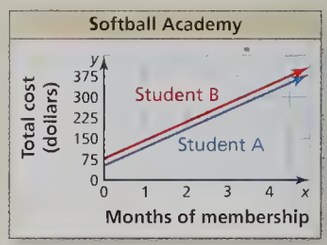
b. Is quadrilateral $ABCD$ a rectangle? Explain.

32. USING STRUCTURE For what value of a are the graphs of $6y = -2x + 4$ and $2y = ax - 5$ parallel? perpendicular?

33. MAKING AN ARGUMENT A hockey puck leaves the blade of a hockey stick, bounces off a wall, and travels in a new direction, as shown. Your friend claims the path of the puck forms a right angle. Is your friend correct? Explain.



34. HOW DO YOU SEE IT? A softball academy charges students an initial registration fee plus a monthly fee. The graph shows the total amounts paid by two students over a 4-month period. The lines are parallel.



a. Did one of the students pay a greater registration fee? Explain.

b. Did one of the students pay a greater monthly fee? Explain.

REASONING In Exercises 35–37, determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

35. Two lines with positive slopes are perpendicular.

36. A vertical line is parallel to the y-axis.

37. Two lines with the same y-intercept are perpendicular.

38. THOUGHT PROVOKING You are designing a new logo for your math club. Your teacher asks you to include at least one pair of parallel lines and at least one pair of perpendicular lines. Sketch your logo in a coordinate plane. Write the equations of the parallel and perpendicular lines.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the relation is a function. Explain. *(Section 3.1)*

39. $(3, 6), (4, 8), (5, 10), (6, 9), (7, 14)$

40. $(-1, 6), (1, 4), (-1, 2), (1, 6), (-1, 5)$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

4.1–4.4 What Did You Learn?

Core Vocabulary

linear model, *p. 164*
point-slope form, *p. 168*
parallel lines, *p. 180*
perpendicular lines, *p. 181*

Core Concepts

Section 4.1

Using Slope-Intercept Form, *p. 162*

Section 4.2

Using Point-Slope Form, *p. 168*

Section 4.3

Writing Equations in Standard Form, *p. 174*

Section 4.4

Parallel Lines and Slopes, *p. 180*
Perpendicular Lines and Slopes, *p. 181*

Mathematical Thinking

1. How can you explain to yourself the meaning of the graph in Exercise 36 on page 166?
2. How did you use the structure of the equations in Exercise 43 on page 172 to make a conjecture?
3. How did you use the diagram in Exercise 33 on page 184 to determine whether your friend was correct?

Study Skills

Getting Actively Involved in Class

If you do not understand something at all and do not even know how to phrase a question, just ask for clarification. You might say something like, "Could you please explain the steps in this problem one more time?"

If your teacher asks for someone to go up to the board, volunteer. The student at the board often receives additional attention and instruction to complete the problem.



Dynamic Teaching Tools

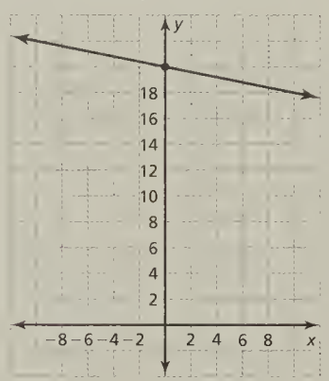
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

ANSWERS

1. The graph shows the increase in U.S. box office revenue over time.
2. The slopes were the same, so the lines were parallel and the constants in the point-slope form indicated similar translations.
3. The points on the diagram were used to determine the slopes of both parts of the path. The slopes were not negative reciprocals, so the lines were not perpendicular.

ANSWERS

1. $y = 5x - 2$
2. $y = -\frac{1}{3}x + 5$
3. $y = 2x + 3$
4. $y - 5 = -2(x + 2)$ or $y + 1 = -2(x - 1)$
5. $y + 2 = \frac{1}{5}(x + 3)$ or $y + 1 = \frac{1}{5}(x - 2)$
6. $y = \frac{4}{3}(x - 1)$ or $y - 4 = \frac{4}{3}(x - 4)$
7. $f(x) = -x + 2$
8. $f(x) = -6$
9. $f(x) = 5x + 13$
10. $2x - y = 6$
11. $x + y = -9$
12. $x = 3$
13. $y = -1$
14. None are parallel; Lines a and b are perpendicular; None of the lines have the same slope and the slope of line a is the negative reciprocal of the slope of line b.
15. Lines b and c are parallel; None are perpendicular; Lines b and c have the same slope and none of the lines have negative reciprocal slopes.
16. a. $y = 3x - 16$
 b. $y = -\frac{1}{3}x + 4$
17. a. $y = -\frac{1}{2}x - 4$
 b. $y = 2x + 1$
18. a. $y = -4x - 16$
 b. $y = \frac{1}{4}x + 1$
19. a. $C = 44m + 48$
 b. \$312
 c. the original company; You can keep the site running for 13 months at the original company. You can keep the site running for 10 months at the new company.
20. yes; The amount of water decreases at a constant rate; $W = -\frac{5}{2}t + 175$
21. a. $0.10x + 0.50y = 10$
 b.



 c. *Sample answer:* 5 black-and-white copies and 19 color copies, 10 black-and-white copies and 18 color copies, 15 black-and-white copies and 17 color copies, and 20 black-and-white copies and 16 color copies

4.1–4.4 Quiz

Write an equation in slope-intercept form of the line with the given characteristics. *(Section 4.1)*

1. passes through: (0, −2) and (1, 3)
2. slope: $-\frac{1}{3}$; passes through: (3, 4)
3. slope: 2; y-intercept: 3

Write an equation in point-slope form of the line that passes through the given points. *(Section 4.2)*

4. (−2, 5), (1, −1)
5. (−3, −2), (2, −1)
6. (1, 0), (4, 4)

Write a linear function f with the given values. *(Section 4.1 and Section 4.2)*

7. $f(0) = 2, f(5) = -3$
8. $f(-1) = -6, f(4) = -6$
9. $f(-3) = -2, f(-2) = 3$

Write an equation in standard form of the line with the given characteristics. *(Section 4.3)*

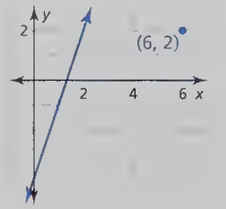
10. passes through: (3, 0) and (5, 4)
11. slope: −1; passes through: (−2, −7)
12. passes through: (3, 4) and (3, 1)
13. slope: 0; passes through: (4, −1)

Determine which of the lines, if any, are parallel or perpendicular. Explain. *(Section 4.4)*

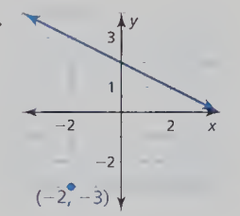
14. Line a passes through (−2, 2) and (2, 1).
 Line b passes through (1, −8) and (3, 0).
 Line c passes through (−4, −3) and (0, −2).
15. Line a: $2x + 6y = -12$
 Line b: $y = \frac{3}{2}x - 5$
 Line c: $3x - 2y = -4$

Write an equation of the line that passes through the given point and is (a) parallel to the given line and (b) perpendicular to the given line. *(Section 4.4)*

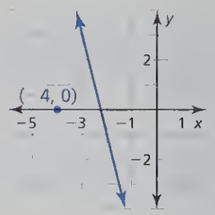
16.



17.



18.



19. A website hosting company charges an initial fee of \$48 to set up a website. The company charges \$44 per month to maintain the website. *(Section 4.1)*
 a. Write a linear model that represents the total cost of setting up and maintaining a website as a function of the number of months it is maintained.
 b. Find the total cost of setting up a website and maintaining it for 6 months.
 c. A different website hosting company charges \$62 per month to maintain a website, but there is no initial set-up fee. You have \$620. At which company can you set up and maintain a website for the greatest amount of time? Explain.
20. The table shows the amount of water remaining in a water tank as it drains. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the amount of water remaining in the tank as a function of time. *(Section 4.2)*

Time (minutes)	Water (gallons)
8	155
10	150
12	145
14	140
16	135

21. You have \$10 on a copy store gift card. The copy store charges \$0.10 for each black-and-white copy and \$0.50 for each color copy. (a) Write an equation in standard form that models the possible combinations of copies you can buy. (b) Graph the equation from part (a). (c) Find four possible combinations. *(Section 4.3)*

4.5 Scatter Plots and Lines of Fit

Texas Essential Knowledge and Skills A.4.C

Essential Question How can you use a scatter plot and a line of fit to make conclusions about data?

A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane.

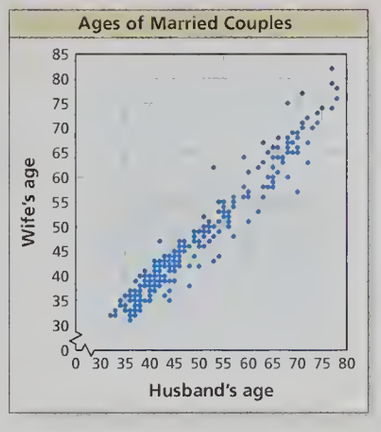
EXPLORATION 1 Finding a Line of Fit

Work with a partner. A survey was taken of 179 married couples. Each person was asked his or her age. The scatter plot shows the results.

a. Draw a line that approximates the data. Write an equation of the line. Explain the method you used.

b. What conclusions can you make from the equation you wrote? Explain your reasoning.

REASONING
To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

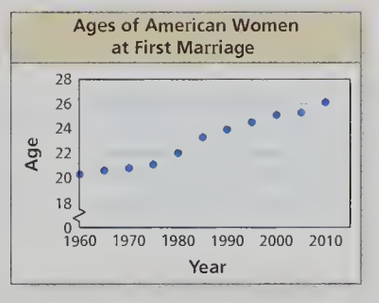


EXPLORATION 2 Finding a Line of Fit

Work with a partner. The scatter plot shows the median ages of American women at their first marriage for selected years from 1960 through 2010.

a. Draw a line that approximates the data. Write an equation of the line. Let x represent the number of years since 1960. Explain the method you used.

b. What conclusions can you make from the equation you wrote?



c. Use your equation to predict the median age of American women at their first marriage in the year 2020.

Communicate Your Answer

3. How can you use a scatter plot and a line of fit to make conclusions about data?
4. Use the Internet or some other reference to find a scatter plot of real-life data that is different from those given above. Then draw a line that approximates the data and write an equation of the line. Explain the method you used.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Explain that passive voice is used to describe something that happens without necessarily stating who makes it happen. Say: *They graphed the data.* Then explain that using passive voice to say the same thing would be: *The data were graphed.* Ask students to listen for examples of passive voice as you read aloud the description of a scatter plot preceding Exploration 1. (are graphed, was taken, was asked) To form the passive voice, a form of the verb *to be* is used (are, was) with the past participle of another verb (graphed, taken, asked).

ELPS 4.C.4 Comprehend English language structures used routinely in written classroom materials.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

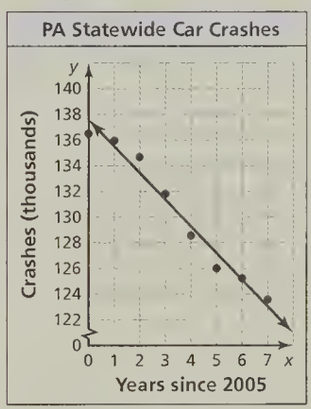
A.4.C The student is expected to write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

ANSWERS

1–2. See Additional Answers.

3. Draw a line of fit on the scatter plot, find its equation, and interpret the slope and y-intercept.

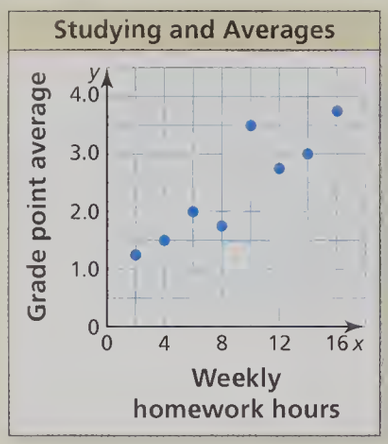
4. *Sample answer:*
The data in the graph is from the Pennsylvania Department of Transportation.



$y = -2.1x + 137.5$; Estimate a line through the middle of the plotted data, then use two points on that line to find the equation.

Extra Example 1

The scatter plot shows the amount of time x (in hours) that 8 students spend on homework each week and their grade point averages y.



a. What is the grade point average of the student who spends 6 hours a week on homework? 2.0

b. How many hours a week does the student who has a 2.75 grade point average spend on homework? 12 hours

c. What tends to happen to the grade point average as the number of hours spent on homework increases? The grade point average increases.

MONITORING PROGRESS ANSWERS

1. 260 cal
2. about 52 g

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

4.5 Lesson

What You Will Learn

- Interpret scatter plots.
- Identify correlations between data sets.
- Use lines of fit to model data.

Core Vocabulary

scatter plot, *p. 188*
correlation, *p. 189*
line of fit, *p. 190*

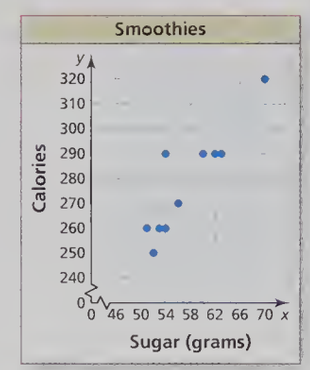
Interpreting Scatter Plots

Core Concept

Scatter Plot

A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane. Scatter plots can show trends in the data.

EXAMPLE 1 **Interpreting a Scatter Plot**



The scatter plot shows the amounts x (in grams) of sugar and the numbers y of calories in 10 smoothies.

a. How many calories are in the smoothie that contains 56 grams of sugar?

b. How many grams of sugar are in the smoothie that contains 320 calories?

c. What tends to happen to the number of calories as the number of grams of sugar increases?

SOLUTION

a. Draw a horizontal line from the point that has an x-value of 56. It crosses the y-axis at 270.

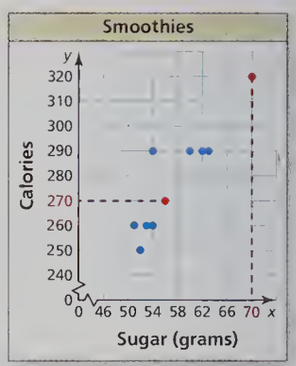
▶ So, the smoothie has 270 calories.

b. Draw a vertical line from the point that has a y-value of 320. It crosses the x-axis at 70.

▶ So, the smoothie has 70 grams of sugar.

c. Looking at the graph, the plotted points go up from left to right.

▶ So, as the number of grams of sugar increases, the number of calories increases.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. How many calories are in the smoothie that contains 51 grams of sugar?
2. How many grams of sugar are in the smoothie that contains 250 calories?

SUPPORTING English Language Learners

Have students perform the following language tasks to show understanding of passive voice.

Beginning Identify the verbs in What You Will Learn. (interpret, identify, use, model) Convert them to present tense passive voice. (is interpreted, is identified, is used, is modeled)

Intermediate Convert the same verbs to past tense passive voice. (was interpreted, was identified, was used, was modeled)

Advanced Convert the verbs to future tense passive voice. (will be interpreted, will be identified, will be used, will be modeled)

Advanced High Who will do the actions in What You Will Learn? (you) Convert the statements into passive voice. (Scatter plots will be interpreted. Correlations between data sets will be identified. Lines of fit will be used. Data will be modeled.)

ELPS 4.C.4 Comprehend English language structures used routinely in written classroom materials.

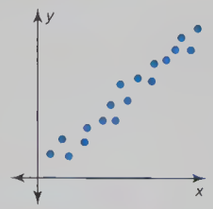
STUDY TIP

You can think of a positive correlation as having a positive slope and a negative correlation as having a negative slope.

Identifying Correlations between Data Sets

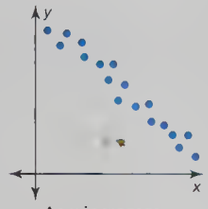
A **correlation** is a relationship between data sets. You can use a scatter plot to describe the correlation between data.

Positive Correlation



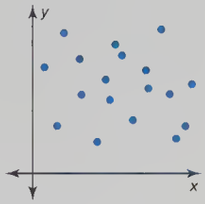
As x increases, y increases.

Negative Correlation



As x increases, y decreases.

No Correlation

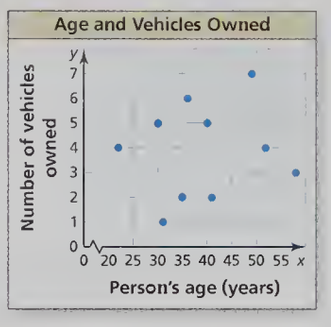


The points show no pattern.

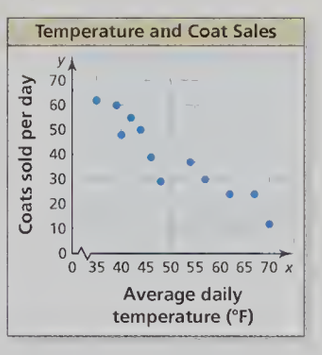
EXAMPLE 2 Identifying Correlations

Tell whether the data show a *positive*, a *negative*, or *no* correlation.

a. age and vehicles owned



b. temperature and coat sales at a store



SOLUTION

a. The points show no pattern. The number of vehicles owned does not depend on a person's age.

▶ So, the scatter plot shows no correlation.

b. As the average temperature increases, the number of coats sold decreases.

▶ So, the scatter plot shows a negative correlation.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Make a scatter plot of the data. Tell whether the data show a *positive*, a *negative*, or *no* correlation.

3.

Temperature (°F), x	82	78	68	87	75	71	92	84
Attendees (thousands), y	4.5	4.0	1.7	5.5	3.8	2.9	4.7	5.3

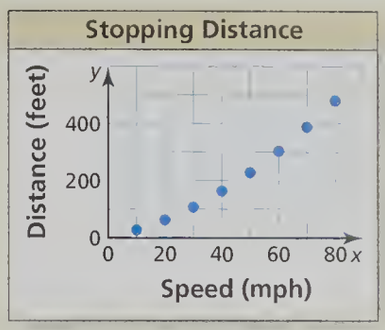
4.

Age of a car (years), x	1	2	3	4	5	6	7	8
Value (thousands), y	$24	$21	$19	$18	$15	$12	$8	$7

Extra Example 2

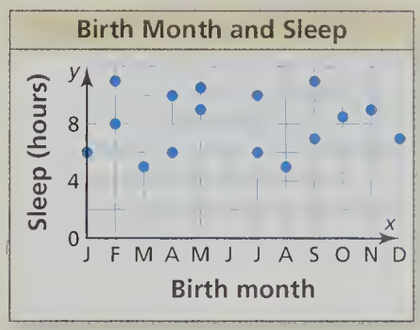
Tell whether the data show a *positive*, a *negative*, or *no* correlation.

a. stopping distance of an automobile



The scatter plot shows a positive correlation.

b. birth month and hours of sleep

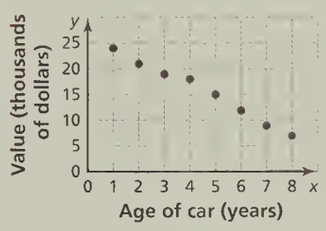


The scatter plot shows no correlation.

MONITORING PROGRESS ANSWERS

3. See Additional Answers.

4.



negative

Differentiated Instruction

Visual

Some students may have difficulty naming the coordinates of points on a scatter plot that are not located on intersecting grid lines. Have these students work with a partner to identify coordinates of points and to plot additional data on a scatter plot.

Extra Example 3

The table shows the salaries y (in thousands of dollars) of 5 architects and the numbers x of years of experience. Write an equation that models the data. Interpret the slope and y-intercept of the line of fit.

x	1	2	3	4	5
y	50	55	56	60	62

Sample answer: Using the points (3, 56) and (5, 62), an equation of the line of fit is $y = 3x + 47$. The slope 3 means the salaries are increasing by about \$3000 each year. The y-intercept 47 means an architect's starting salary is \$47,000.

MONITORING PROGRESS ANSWER

5. *Sample answer:* $y = 0.2x$; The slope of 0.2 means the monthly car payment increases by \$20 for every \$100 increase in monthly income. The y-intercept of 0 means a person with no monthly income has no car payment.

Using Lines of Fit to Model Data

When data show a positive or negative correlation, you can model the *trend* in the data using a line of fit. A **line of fit** is a line drawn on a scatter plot that is close to most of the data points.

STUDY TIP

A line of fit is also called a *trend line*.

Core Concept

Using a Line of Fit to Model Data

Step 1 Make a scatter plot of the data.

Step 2 Decide whether the data can be modeled by a line.

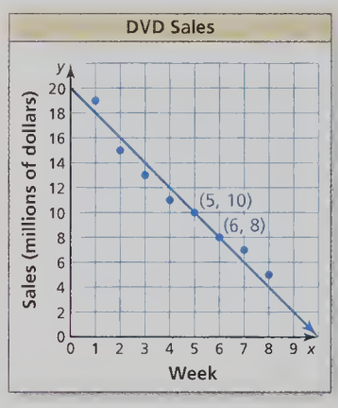
Step 3 Draw a line that appears to fit the data closely. There should be approximately as many points above the line as below it.

Step 4 Write an equation using two points on the line. The points do not have to represent actual data pairs, but they must lie on the line of fit.

EXAMPLE 3 Finding a Line of Fit

The table shows the weekly sales of a DVD and the number of weeks since its release. Write an equation that models the DVD sales as a function of the number of weeks since its release. Interpret the slope and y-intercept of the line of fit.

Week, x	1	2	3	4	5	6	7	8
Sales (millions), y	\$19	\$15	\$13	\$11	\$10	\$8	\$7	\$5



SOLUTION

Step 1 Make a scatter plot of the data.

Step 2 Decide whether the data can be modeled by a line. Because the scatter plot shows a negative correlation, you can fit a line to the data.

Step 3 Draw a line that appears to fit the data closely.

Step 4 Write an equation using two points on the line. Use (5, 10) and (6, 8).

The slope of the line is $m = \frac{8 - 10}{6 - 5} = -2$.

Use the slope $m = -2$ and the point (6, 8) to write an equation of the line.

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 8 = -2(x - 6)$ Substitute -2 for m, 6 for x_1, and 8 for y_1.

$y = -2x + 20$ Solve for y.

An equation of the line of fit is $y = -2x + 20$. The slope of the line is -2. This means the sales are decreasing by about \$2 million each week. The y-intercept is 20. The y-intercept has no meaning in this context because there are no sales in week 0.

Monitoring Progress

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5. The following data pairs show the monthly income x (in dollars) and the monthly car payment y (in dollars) of six people: (2100, 410), (1650, 315), (1950, 405), (1500, 295), (2250, 440), and (1800, 375). Write an equation that models the monthly car payment as a function of the monthly income. Interpret the slope and y-intercept of the line of fit.

Closure

- Have students fit a line for *Monitoring Progress* Questions 3 and/or 4. Check students' work.

4.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

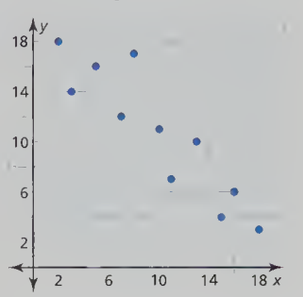
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** When data show a positive correlation, the dependent variable tends to _____ as the independent variable increases.
2. **VOCABULARY** What is a line of fit?

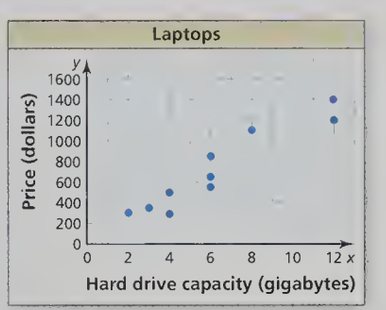
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use the scatter plot to fill in the missing coordinate of the ordered pair.

3. (16, [redacted])
4. (3, [redacted])
5. ([redacted], 12)
6. ([redacted], 17)



7. **INTERPRETING A SCATTER PLOT** The scatter plot shows the hard drive capacities (in gigabytes) and the prices (in dollars) of 10 laptops. *(See Example 1.)*

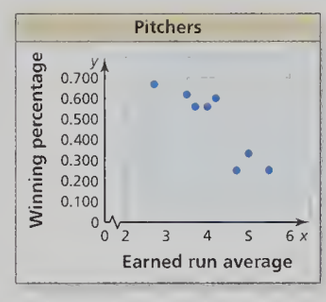


a. What is the price of the laptop with a hard drive capacity of 8 gigabytes?

b. What is the hard drive capacity of the $1200 laptop?

c. What tends to happen to the price as the hard drive capacity increases?

8. **INTERPRETING A SCATTER PLOT** The scatter plot shows the earned run averages and the winning percentages of eight pitchers on a baseball team.



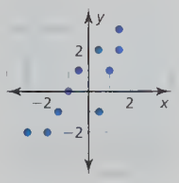
a. What is the winning percentage of the pitcher with an earned run average of 4.2?

b. What is the earned run average of the pitcher with a winning percentage of 0.33?

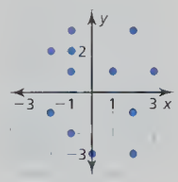
c. What tends to happen to the winning percentage as the earned run average increases?

In Exercises 9–12, tell whether x and y show a *positive*, a *negative*, or *no* correlation. *(See Example 2.)*

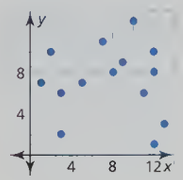
9.



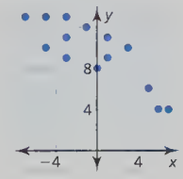
10.



11.



12.



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–15 odd, 18, 22, 25–28

Average: 1, 2–18 even, 22, 25–28

Advanced: 1, 2, 8–16 even, 17–28

HOMEWORK CHECK

Basic: 7, 9, 13, 15

Average: 8, 10, 14, 16

Advanced: 8, 12, 14, 16

ANSWERS

1. increase
2. a line drawn on a scatter plot that is close to most of the data points
3. 6
4. 14
5. 7
6. 8
7. a. $1100
 b. 12 GB
 c. increases
8. a. 0.600
 b. 5
 c. decreases
9. positive
10. no
11. no
12. negative

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

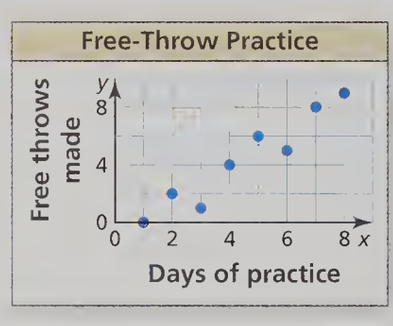
13. 

no

14–28. See Additional Answers.

Mini-Assessment

1. The scatter plot shows the days x of practice and the numbers y of free throws made during practice.

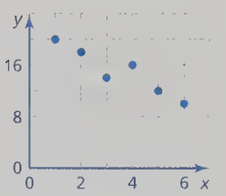


 a. How many free throws were made on day 5 of practice?
 6 free throws

 b. On which day were 5 free throws made? Day 6

2. Make a scatter plot of the data. Tell whether x and y show a *positive*, a *negative*, or *no* correlation.

x	1	2	3	4	5	6
y	20	18	14	16	12	10



a negative correlation

3. Write an equation that models the data in Question 2. *Sample answer:* $y = -2x + 22$, using points (1, 20) and (2, 18)

In Exercises 13 and 14, make a scatter plot of the data. Tell whether x and y show a *positive*, a *negative*, or *no* correlation.

13.

x	3.1	2.2	2.5	3.7	3.9	1.5	2.7	2.0
y	1	0	1	2	0	2	3	2

14.

x	3	4	5	6	7	8	9	10
y	67	67	50	33	25	21	19	4

15. **MODELING WITH MATHEMATICS** The table shows the world birth rates y (number of births per 1000 people) x years since 1960. *(See Example 3.)*

x	0	10	20	30	40	50
y	35.4	33.6	28.3	27.0	22.4	20.0

 a. Write an equation that models the birthrate as a function of the number of years since 1960.

 b. Interpret the slope and y-intercept of the line of fit.

16. **MODELING WITH MATHEMATICS** The table shows the total earnings y (in dollars) of a food server who works x hours.

x	0	1	2	3	4	5	6
y	0	18	40	62	77	85	113

 a. Write an equation that models the server's earnings as a function of the number of hours the server works.

 b. Interpret the slope and y-intercept of the line of fit.

17. **OPEN-ENDED** Give an example of a real-life data set that shows a negative correlation.

18. **MAKING AN ARGUMENT** Your friend says that the data in the table show a negative correlation because the dependent variable y is decreasing. Is your friend correct? Explain.

x	14	12	10	8	6	4	2
y	4	1	0	−1	−2	−4	−5

19. **USING TOOLS** Use a ruler or a yardstick to find the heights and arm spans of five people.

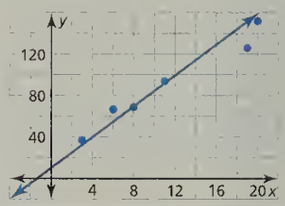
 a. Make a scatter plot using the data you collected. Then draw a line of fit for the data.

 b. Interpret the slope and y-intercept of the line of fit.

20. **THOUGHT PROVOKING** A line of fit for a scatter plot is given by the equation $y = 5x + 20$. Describe a real-life data set that could be represented by the scatter plot.

21. **WRITING** When is data best displayed in a scatter plot, rather than another type of display, such as a bar graph or circle graph?

22. **HOW DO YOU SEE IT?** The scatter plot shows part of a data set and a line of fit for the data set. Four data points are missing. Choose possible coordinates for these data points.



23. **REASONING** A data set has no correlation. Is it possible to find a line of fit for the data? Explain.

24. **ANALYZING RELATIONSHIPS** Make a scatter plot of the data in the tables. Describe the relationship between the variables. Is it possible to fit a line to the data? If so, write an equation of the line. If not, explain why.

x	−12	−9	−7	−4	−3	−1
y	150	76	50	15	10	1

x	2	5	6	7	9	15
y	5	22	37	52	90	226

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the function when $x = -3$, 0, and 4. *(Section 3.3)*

25. $g(x) = 6x$

26. $h(x) = -10x$

27. $f(x) = 5x - 8$

28. $v(x) = 14 - 3x$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

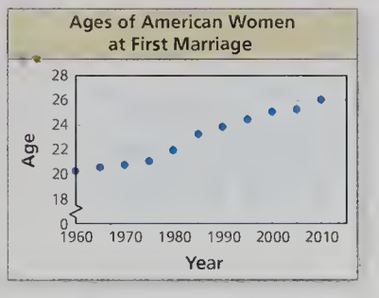
4.6 Analyzing Lines of Fit

Essential Question How can you *analytically* find a line of best fit for a scatter plot?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.4.A
A.4.B
A.4.C

EXPLORATION 1 Finding a Line of Best Fit

Work with a partner.
The scatter plot shows the median ages of American women at their first marriage for selected years from 1960 through 2010. In Exploration 2 in Section 4.5, you approximated a line of fit graphically. To find the line of *best* fit, you can use a computer, spreadsheet, or graphing calculator that has a *linear regression* feature.



a. The data from the scatter plot is shown in the table. Note that 0, 5, 10, and so on represent the numbers of years since 1960. What does the ordered pair (25, 23.3) represent?

L1	L2	L3
0	20.3	
5	20.6	
10	20.8	
15	21.1	
20	22	
25	23.3	
30	23.9	
35	24.5	
40	25.1	
45	25.3	
50	26.1	

L1(55)=		

b. Use the *linear regression* feature to find an equation of the line of best fit. You should obtain results such as those shown below.

```
LinReg
 y=ax+b
 a=.1261818182
 b=19.84545455
 r²=.9738676804
 r=.986847344
```

MAKING MATHEMATICAL ARGUMENTS
To be proficient in math, you need to reason inductively about data.

c. Write an equation of the line of best fit. Compare your result with the equation you obtained in Exploration 2 in Section 4.5.

Communicate Your Answer

2. How can you *analytically* find a line of best fit for a scatter plot?
3. The data set relates the number of chirps per second for striped ground crickets and the outside temperature in degrees Fahrenheit. Make a scatter plot of the data. Then find an equation of the line of best fit. Use your result to estimate the outside temperature when there are 19 chirps per second.

Chirps per second	20.0	16.0	19.8	18.4	17.1
Temperature (°F)	88.6	71.6	93.3	84.3	80.6

Chirps per second	14.7	15.4	16.2	15.0	14.4
Temperature (°F)	69.7	69.4	83.3	79.6	76.3

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.4.A The student is expected to calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association.

A.4.B The student is expected to compare and contrast association and causation in real-world problems.

A.4.C The student is expected to write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

ANSWERS

1. a. The median age of American women at their first marriage in 1985 was 23.3.
 b. Check students' work.
 c. $y = 0.13x + 19.8$; The equations are similar.
2. Use a computer, spreadsheet, or graphing calculator that has a linear regression feature.
3. See Additional Answers.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

As students work in pairs to complete the exploration and Communicate Your Answer, circulate through the classroom and listen to their conversations. Ask them to employ the following learning strategies.

Beginning/Advanced Beginning students ask Advanced students to help them discuss ideas they don't understand, using non-verbal cues such as pointing or tracing a figure when necessary. Advanced students use synonyms or alternate phrases to explain. Advanced students request help from teachers or classroom aides as needed.

Intermediate/Advanced High Intermediate students ask Advanced High students to help them discuss ideas they don't understand, using gestures and descriptive language when they don't know a word or phrase. Advanced High students use synonyms or alternate phrases to explain. Advanced High students request help from teachers or classroom aides as needed.

ELPS 1.D Speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution.

Extra Example 1

The table shows the number y of gloves sold for each amount x (in inches) of snowfall. The equation $y = 1.5x + 3.5$ models the data. Is the model a good fit?

Snowfall (inches), x	Gloves Sold, y
1	5
2	8
3	7
4	10
5	11
6	12
7	14

The points (x, residual) are evenly dispersed about the horizontal axis. So, the equation $y = 1.5x + 3.5$ is a good fit.

4.6 Lesson

What You Will Learn

- Use residuals to determine how well lines of fit model data.
- Use technology to find lines of best fit.
- Distinguish between correlation and causation.

Core Vocabulary

residual, *p. 194*
linear regression, *p. 195*
line of best fit, *p. 195*
correlation coefficient, *p. 195*
interpolation, *p. 197*
extrapolation, *p. 197*
causation, *p. 197*

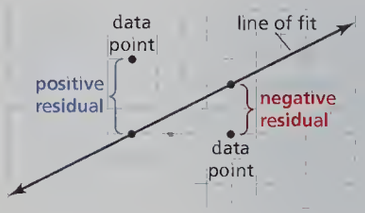
Analyzing Residuals

One way to determine how well a line of fit models a data set is to analyze *residuals*.

Core Concept

Residuals

A **residual** is the difference of the y-value of a data point and the corresponding y-value found using the line of fit. A residual can be positive, negative, or zero.



A scatter plot of the residuals shows how well a model fits a data set. If the model is a good fit, then the absolute values of the residuals are relatively small, and the residual points will be more or less evenly dispersed about the horizontal axis. If the model is not a good fit, then the residual points will form some type of pattern that suggests the data are not linear. Wildly scattered residual points suggest that the data might have no correlation.

EXAMPLE 1 Using Residuals

Week, x	Sales (millions), y
1	\$19
2	\$15
3	\$13
4	\$11
5	\$10
6	\$8
7	\$7
8	\$5

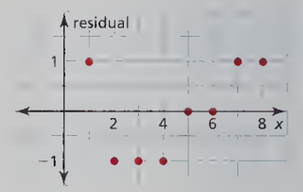
In Example 3 in Section 4.5, the equation $y = -2x + 20$ models the data in the table shown. Is the model a good fit?

SOLUTION

Step 1 Calculate the residuals. Organize your results in a table.

Step 2 Use the points (x, residual) to make a scatter plot.

x	y	y-Value from model	Residual
1	19	18	$19 - 18 = 1$
2	15	16	$15 - 16 = -1$
3	13	14	$13 - 14 = -1$
4	11	12	$11 - 12 = -1$
5	10	10	$10 - 10 = 0$
6	8	8	$8 - 8 = 0$
7	7	6	$7 - 6 = 1$
8	5	4	$5 - 4 = 1$



- The points are evenly dispersed about the horizontal axis. So, the equation $y = -2x + 20$ is a good fit.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Auditory

Some students may benefit from working with a partner to find the residuals. Students can take turns using the equation to find each residual, describing each step in the process to their partner.

EXAMPLE 2 Using Residuals

The table shows the ages x and salaries y (in thousands of dollars) of eight employees at a company. The equation $y = 0.2x + 38$ models the data. Is the model a good fit?

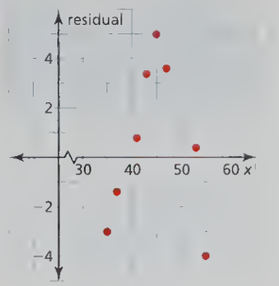
Age, x	35	37	41	43	45	47	53	55
Salary, y	42	44	47	50	52	51	49	45

SOLUTION

Step 1 Calculate the residuals. Organize your results in a table.

Step 2 Use the points (x, residual) to make a scatter plot.

x	y	y-Value from model	Residual
35	42	45.0	$42 - 45.0 = -3.0$
37	44	45.4	$44 - 45.4 = -1.4$
41	47	46.2	$47 - 46.2 = 0.8$
43	50	46.6	$50 - 46.6 = 3.4$
45	52	47.0	$52 - 47.0 = 5.0$
47	51	47.4	$51 - 47.4 = 3.6$
53	49	48.6	$49 - 48.6 = 0.4$
55	45	49.0	$45 - 49.0 = -4.0$



▶ The residual points form a ∩-shaped pattern, which suggests the data are not linear. So, the equation $y = 0.2x + 38$ does not model the data well.

Monitoring Progress

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1. The table shows the attendances y (in thousands) at an amusement park from 2005 to 2014, where $x = 0$ represents the year 2005. The equation $y = -9.8x + 850$ models the data. Is the model a good fit?

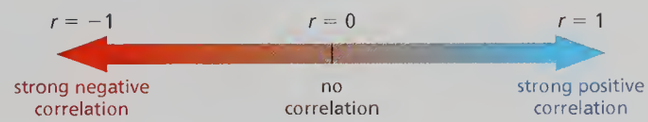
Year, x	0	1	2	3	4	5	6	7	8	9
Attendance, y	850	845	828	798	800	792	785	781	775	760

STUDY TIP

You know how to use two points to find an equation of a line of fit. When finding an equation of the line of best fit, every point in the data set is used.

Finding Lines of Best Fit

Graphing calculators use a method called **linear regression** to find a precise line of fit called a **line of best fit**. This line best models a set of data. A calculator often gives a value r, called the **correlation coefficient**. This value tells whether the correlation is positive or negative and how closely the equation models the data. Values of r range from -1 to 1. When r is close to 1 or -1, there is a strong correlation between the variables. As r gets closer to 0, the correlation becomes weaker.



Extra Example 2

The table shows the high temperatures on Monday for eight weeks. The equation $y = 7x - 8$ models the data. Is the model a good fit?

Week	High Temperature (°F)
1	48
2	43
3	42
4	40
5	45
6	56
7	67
8	76

No, the residual points form a V-shaped pattern, which suggests the data are not linear. So, the equation $y = 7x - 8$ is not a good fit.

MONITORING PROGRESS ANSWER

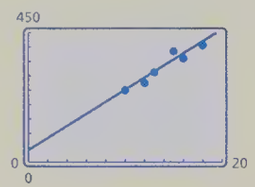
1. no

Extra Example 3

The table shows the number x of teachers and the number y of students in each of several schools.

Teachers, x	Students, y
10	250
12	275
13	312
16	360
15	385
18	406

a. Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window. *Sample answer:* $y = 21x + 42$

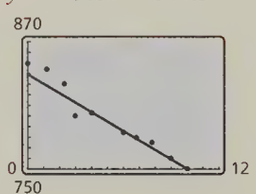


b. Identify and interpret the correlation coefficient. about 0.960; The relationship between the number of students and the number of teachers has a strong positive correlation and the equation closely models the data.

c. Interpret the slope and y-intercept of the line of best fit. The slope 21 means the number of students increases by about 21 for each additional teacher. The y-intercept 42 has no meaning in this context because a school would not have 0 teachers.

MONITORING PROGRESS ANSWERS

2. a. $y = -9.6x + 844.5$



b. -0.964; strong negative correlation

c. The slope of -9.6 means the attendance decreases by about 9600 each year. The y-intercept of 844.5 means the attendance in 2005 was about 844,500.



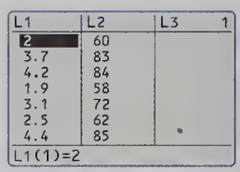
EXAMPLE 3 Finding a Line of Best Fit Using Technology

The table shows the durations x (in minutes) of several eruptions of the geyser Old Faithful and the times y (in minutes) until the next eruption. (a) Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window. (b) Identify and interpret the correlation coefficient. (c) Interpret the slope and y-intercept of the line of best fit.

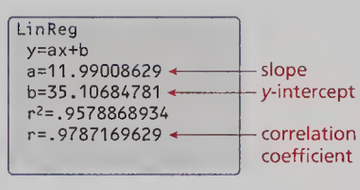
Duration, x	2.0	3.7	4.2	1.9	3.1	2.5	4.4	3.9
Time, y	60	83	84	58	72	62	85	85

SOLUTION

a. Step 1 Enter the data from the table into two lists.



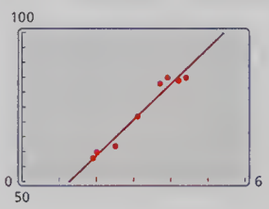
Step 2 Use the *linear regression* feature. The values in the equation can be rounded to obtain $y = 12.0x + 35$.



PRECISION

Be sure to analyze the data values to select an appropriate viewing window for your graph.

Step 3 Enter the equation $y = 12.0x + 35$ into the calculator. Then plot the data and graph the equation in the same viewing window.



b. The correlation coefficient is about 0.979. This means that the relationship between the durations and the times until the next eruption has a strong positive correlation and the equation closely models the data, as shown in the graph.

c. The slope of the line is 12. This means the time until the next eruption increases by about 12 minutes for each minute the duration increases. The y-intercept is 35, but it has no meaning in this context because the duration cannot be 0 minutes.

Monitoring Progress

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2. Use the data in Monitoring Progress Question 1. (a) Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window. (b) Identify and interpret the correlation coefficient. (c) Interpret the slope and y-intercept of the line of best fit.

Using a graph or its equation to *approximate* a value between two known values is called **interpolation**. Using a graph or its equation to *predict* a value outside the range of known values is called **extrapolation**. In general, the farther removed a value is from the known values, the less confidence you can have in the accuracy of the prediction.

STUDY TIP

To approximate or predict an unknown value, you can evaluate the model algebraically or graph the model with a graphing calculator and use the *trace* feature.

EXAMPLE 4 Interpolating and Extrapolating Data

Refer to Example 3. Use the equation of the line of best fit.

a. Approximate the duration before a time of 77 minutes.

b. Predict the time after an eruption lasting 5.0 minutes.

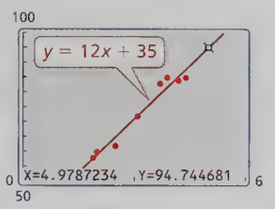
SOLUTION

a. $y = 12.0x + 35$ Write the equation.

$77 = 12.0x + 35$ Substitute 77 for y.

$3.5 = x$ Solve for x.

▶ An eruption lasts about 3.5 minutes before a time of 77 minutes.



b. Use a graphing calculator to graph the equation. Use the *trace* feature to find the value of y when $x \approx 5.0$, as shown.

▶ A time of about 95 minutes will follow an eruption of 5.0 minutes.

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3. Refer to Monitoring Progress Question 2. Use the equation of the line of best fit to predict the attendance at the amusement park in 2017.

Correlation and Causation

READING

A causal relationship exists when one variable causes a change in another variable.

When a change in one variable causes a change in another variable, it is called **causation**. Causation produces a strong correlation between the two variables. The converse is *not* true. In other words, correlation does not imply causation.

EXAMPLE 5 Identifying Correlation and Causation

Tell whether a correlation is likely in the situation. If so, tell whether there is a causal relationship. Explain your reasoning.

a. time spent exercising and the number of calories burned

b. the number of banks and the population of a city

SOLUTION

a. There is a positive correlation and a causal relationship because the more time you spend exercising, the more calories you burn.

b. There may be a positive correlation but no causal relationship. Building more banks will not cause the population to increase.

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4. Is there a correlation between time spent playing video games and grade point average? If so, is there a causal relationship? Explain your reasoning.

Extra Example 4

Refer to Extra Example 3. Use the equation of the line of best fit.

a. Approximate the number of teachers when there are 336 students.
about 14 teachers

b. Predict the number of students when there are 20 teachers.
about 462 students

Extra Example 5

Tell whether a correlation is likely in the situation. If so, tell whether there is a causal relationship. Explain your reasoning.

a. time spent babysitting and the amount of money earned
There is a positive correlation and a causal relationship because the more time you spend babysitting, the more money you earn.

b. eating organic food and getting a good score on a test
There may be a positive correlation but no causal relationship. Eating organic food will not cause you to get a good score on a test.

MONITORING PROGRESS ANSWERS

3. 729,300

4. There may be a negative correlation; A causal relationship is possible because the more time you spend playing video games, the less time you spend studying.

Closure

- **Exit Ticket:** Find an equation of the line of best fit for the data in the table. Use the equation to estimate the cumulative snowfall in week 5.

Number of weeks	1	2	3	4	6	7
Cumulative snowfall (in.)	11	18	32	52	73	82

$y = 12.5x - 3$; 59.5 inches

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–21 odd, 26, 28, 32, 33

Average: 1–3, 4–26 even, 27, 28, 32, 33

Advanced: 1–4, 8–28 even, 29–33

HOMEWORK CHECK

Basic: 5, 9, 17, 19, 21

Average: 6, 10, 18, 20, 22

Advanced: 8, 14, 18, 20, 24

ANSWERS

1. when the actual y-value is greater than the y-value from the model; when the actual y-value is less than the y-value from the model.
2. If the model is a good fit, then the absolute values of the residuals are relatively small and the residual points will be more or less evenly dispersed about the horizontal axis. If the model is not a good fit, then the residual points will form some type of pattern that suggests the data are not linear.
3. Interpolation is using a graph or its equation to approximate a value between two known values, and extrapolation is using a graph or its equation to predict a value outside the range of known values.
4. $r = -0.09$; It is the only one that suggests no correlation.
5. no; The residual points are not evenly dispersed about the horizontal axis.
6. yes; The residual points are evenly dispersed about the horizontal axis.
7. yes; The residual points are evenly dispersed about the horizontal axis.
8. no; The residual points form a nonlinear pattern.
9. yes; The residual points are evenly dispersed about the horizontal axis.
10. no; The residual points are not evenly dispersed about the horizontal axis.
11. $y = 2.1x - 8$; $r = 0.980$; strong positive correlation
12. $y = -1.3x + 8$; $r = -0.886$; strong negative correlation
13. $y = 1.4x + 16$; $r = 0.999$; strong positive correlation
14. $y = -x + 11$; $r = -0.444$; weak negative correlation

4.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** When is a residual positive? When is it negative?
2. **WRITING** Explain how you can use residuals to determine how well a line of fit models a data set.
3. **VOCABULARY** Compare interpolation and extrapolation.
4. **WHICH ONE DOESN'T BELONG?** Which correlation coefficient does *not* belong with the other three? Explain your reasoning.

$r = -0.98$ $r = 0.96$ $r = -0.09$ $r = 0.97$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, use residuals to determine whether the model is a good fit for the data in the table. Explain. *(See Examples 1 and 2.)*

5. $y = 4x - 5$

x	−4	−3	−2	−1	0	1	2	3	4
y	−18	−13	−10	−7	−2	0	6	10	15

6. $y = 6x + 4$

x	1	2	3	4	5	6	7	8	9
y	13	14	23	26	31	42	45	52	62

7. $y = -1.3x + 1$

x	−8	−6	−4	−2	0	2	4	6	8
y	9	10	5	8	−1	1	−4	−12	−7

8. $y = -0.5x - 2$

x	4	6	8	10	12	14	16	18	20
y	−1	−3	−6	−8	−10	−10	−10	−9	−9

9. **ANALYZING RESIDUALS** The table shows the growth y (in inches) of an elk's antlers during week x. The equation $y = -0.7x + 6.8$ models the data. Is the model a good fit? Explain.

Week, x	1	2	3	4	5
Growth, y	6.0	5.5	4.7	3.9	3.3

10. **ANALYZING RESIDUALS** The table shows the approximate numbers y (in thousands) of movie tickets sold from January to June for a theater. In the table, $x = 1$ represents January. The equation $y = 1.3x + 27$ models the data. Is the model a good fit? Explain.

Month, x	Ticket sales, y
1	27
2	28
3	36
4	28
5	32
6	35

In Exercises 11–14, use a graphing calculator to find an equation of the line of best fit for the data. Identify and interpret the correlation coefficient.

11.

x	0	1	2	3	4	5	6	7
y	−8	−5	−2	−1	−1	2	5	8

12.

x	−4	−2	0	2	4	6	8	10
y	17	7	8	1	5	−2	2	−8

13.

x	−15	−10	−5	0	5	10	15	20
y	−4	2	7	16	22	30	37	43

14.

x	5	6	7	8	9	10	11	12
y	12	−2	8	3	−1	−4	6	0

ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in interpreting the graphing calculator display.**

```
LinReg
y=ax+b
a=-4.47
b=23.16
r²=.9989451055
r=-.9994724136
```

15. ✗ An equation of the line of best fit is $y = 23.16x - 4.47$.

16. ✗ The data have a strong positive correlation.

17. **MODELING WITH MATHEMATICS** The table shows the total numbers y of people who reported an earthquake x minutes after it ended. *(See Example 3.)*

Minutes, x	People, y
1	10
2	100
3	400
4	900
5	1400
6	1800
7	2100

a. Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window.

b. Identify and interpret the correlation coefficient.

c. Interpret the slope and y-intercept of the line of best fit.

18. **MODELING WITH MATHEMATICS** The table shows the numbers y of people who volunteer at an animal shelter on each day x.

Day, x	1	2	3	4	5	6	7	8
People, y	9	5	13	11	10	11	19	12

a. Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window.

b. Identify and interpret the correlation coefficient.

c. Interpret the slope and y-intercept of the line of best fit.

19. **MODELING WITH MATHEMATICS** The table shows the mileages x (in thousands of miles) and the selling prices y (in thousands of dollars) of several used automobiles of the same year and model. *(See Example 4.)*

Mileage, x	22	14	18	30	8	24
Price, y	16	17	17	14	18	15

a. Use a graphing calculator to find an equation of the line of best fit.

b. Identify and interpret the correlation coefficient.



c. Interpret the slope and y-intercept of the line of best fit.

d. Approximate the mileage of an automobile that costs \$15,500.

e. Predict the price of an automobile with 6000 miles.

20. **MODELING WITH MATHEMATICS** The table shows the lengths x and costs y of several sailboats.

Length (feet), x	Cost (thousands of dollars), y
27	94
18	56
25	58
32	123
18	60
26	87
36	145

a. Use a graphing calculator to find an equation of the line of best fit.

b. Identify and interpret the correlation coefficient.

c. Interpret the slope and y-intercept of the line of best fit.

d. Approximate the cost of a sailboat that is 20 feet long.

e. Predict the length of a sailboat that costs \$147,000.

In Exercises 21–24, tell whether a correlation is likely in the situation. If so, tell whether there is a causal relationship. Explain your reasoning. *(See Example 5.)*

21. the amount of time spent talking on a cell phone and the remaining battery life

22. the height of a toddler and the size of the toddler's vocabulary

23. the number of hats you own and the size of your head

24. the weight of a dog and the length of its tail

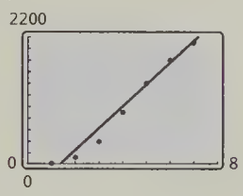
Dynamic Teaching Tools
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

ANSWERS

15. The slope and y-intercept were reversed; $y = -4.47x + 23.16$

16. The data do not have a strong positive correlation; The data have a strong negative correlation.

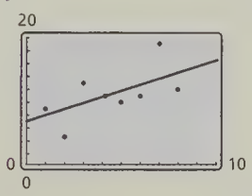
17. a. $y = 381x - 566$



b. $r = 0.989$; strong positive correlation

c. The slope of 381 means the number of people who reported an earthquake increased by about 381 each minute after the earthquake ended. The y-intercept of -566 has no meaning in this context because the number of people cannot be negative.

18. a. $y = x + 7$



b. $r = 0.619$; weak positive correlation

c. The slope of 1 means the number of people who volunteer increases by about 1 each day. The y-intercept of 6.75 has no meaning in this context because there is no day 0.

19. a. $y = -0.2x + 20$

b. $r = -0.968$; strong negative correlation

c. The slope of -0.2 means the selling price decreases by about \$200 for every increase in mileage of 1000 miles. The y-intercept of 19.7 has no meaning in this context because a used car cannot have 0 mileage.

d. 22,500 mi

e. \$18,800

20. a. $y = 4.9x - 38$

b. $r = 0.936$; strong positive correlation

c. The slope of 4.9 means the cost of a sailboat increases by about \$4900 for each foot the length of the sailboat increases. The y-intercept of -37.7 has no meaning in this context because the length of a sailboat cannot be negative.

d. \$60,000

e. about 38 ft

21. There is a negative correlation and a causal relationship because the more time you spend talking on the phone, the less charge there is left in the battery.

22. There may be a positive correlation but no causal relationship. The height of a toddler does not determine the toddler's vocabulary.

23. A correlation is unlikely. The number of hats you own is not related to the size of your head.

24. There may be a positive correlation but no causal relationship. The weight of a dog does not determine the length of its tail.

ANSWERS

25. *Sample answer:* ACT math score and SAT math score

26–33. See Additional Answers.

Mini-Assessment

1. Is the model $y = 2x - 3$ a good fit for the data in the table? Explain.

x	1	3	5	7	9
y	0	3	8	11	14

Yes; The residual points are evenly dispersed about the horizontal axis.

2. Tell whether a correlation is likely in the situation: the number of cars in a store parking lot and the number of people in the store. If so, tell whether there is a causal relationship. Explain. There is a positive correlation and a causal relationship because the more cars there are in the parking lot, the more people there are in the store.

The table shows the distance x (in miles) Tom rode his bicycle and the time y (in minutes) of each ride.

Distance (miles), x	Time (minutes), y
3	15
5	27
9	42
10	50
12	58
15	80

3. Use a graphing calculator to find an equation of the line of best fit for the data. Identify and interpret the correlation coefficient. $y = 5x - 1$; about 0.993; a strong positive correlation between distance traveled and time riding

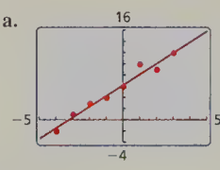
4. Use the equation of the line of best fit to approximate the distance for a 30-minute ride. about 6.2 miles

5. Predict the time for a 20-mile ride. about 99 minutes

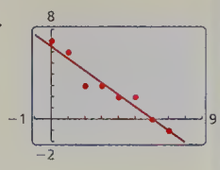
25. OPEN-ENDED Describe a data set that has a strong correlation but does not have a causal relationship.

26. HOW DO YOU SEE IT? Match each graph with its correlation coefficient. Explain your reasoning.

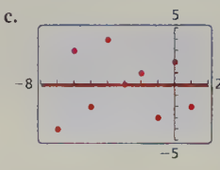
a.



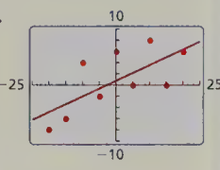
b.



c.



d.



A. $r = 0$ B. $r = 0.98$

C. $r = -0.97$ D. $r = 0.69$

27. ANALYZING RELATIONSHIPS The table shows the grade point averages y of several students and the numbers x of hours they spend watching television each week.

Hours, x	Grade point average, y
10	3.0
5	3.4
3	3.5
12	2.7
20	2.1
15	2.8
8	3.0
4	3.7
16	2.5

a. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

b. Interpret the slope and y-intercept of the line of best fit.

c. Another student watches about 14 hours of television each week. Approximate the student's grade point average.

d. Do you think there is a causal relationship between time spent watching television and grade point average? Explain.

28. MAKING AN ARGUMENT A student spends 2 hours watching television each week and has a grade point average of 2.4. Your friend says including this information in the data set in Exercise 27 will weaken the correlation. Is your friend correct? Explain.

29. USING MODELS Refer to Exercise 17.

a. Predict the total numbers of people who reported an earthquake 9 minutes and 15 minutes after it ended.

b. The table shows the actual data. Describe the accuracy of your extrapolations in part (a).

Minutes, x	9	15
People, y	2750	3200

30. THOUGHT PROVOKING A data set consists of the numbers x of people at Beach 1 and the numbers y of people at Beach 2 recorded daily for 1 week. Sketch a possible graph of the data set. Describe the situation shown in the graph and give a possible correlation coefficient. Determine whether there is a causal relationship. Explain.

31. COMPARING METHODS The table shows the numbers y (in billions) of text messages sent each year in a five-year period, where $x = 1$ represents the first year in the five-year period.

Year, x	1	2	3	4	5
Text messages (billions), y	241	601	1360	1806	2206

a. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

b. Is there a causal relationship? Explain your reasoning.

c. Calculate the residuals. Then make a scatter plot of the residuals and interpret the results.

d. Compare the methods you used in parts (a) and (c) to determine whether the model is a good fit. Which method do you prefer? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the table represents a *linear* or *nonlinear* function. Explain. *(Section 3.2)*

32.

x	5	6	7	8
y	−4	4	−4	4

33.

x	2	4	6	8
y	13	8	3	−2

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

4.7 Arithmetic Sequences

Texas Essential Knowledge and Skills
A.12.D

Essential Question
How can you use an arithmetic sequence to describe a pattern?

An **arithmetic sequence** is an ordered list of numbers in which the difference between each pair of consecutive **terms**, or numbers in the list, is the same.

EXPLORATION 1 Describing a Pattern

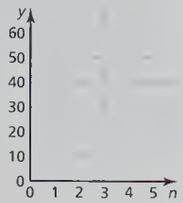
Work with a partner. Use the figures to complete the table. Plot the points given by your completed table. Describe the pattern of the y-values.

ANALYZING MATHEMATICAL RELATIONSHIPS
To be proficient in math, you need to look closely to discern patterns and structure.

a. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



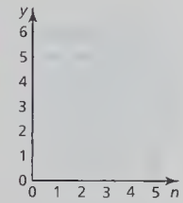
Number of stars, n	1	2	3	4	5
Number of sides, y					



b. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



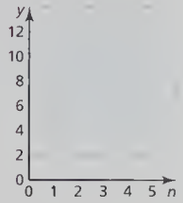
n	1	2	3	4	5
Number of circles, y					



c. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



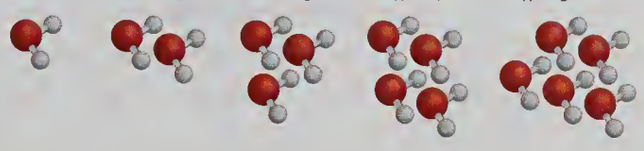
Number of rows, n	1	2	3	4	5
Number of dots, y					



Communicate Your Answer

2. How can you use an arithmetic sequence to describe a pattern? Give an example from real life.
3. In chemistry, water is called H_2O because each molecule of water has two hydrogen atoms and one oxygen atom. Describe the pattern shown below. Use the pattern to determine the number of atoms in 23 molecules.

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



SUPPORTING English Language Learners

Read aloud the Essential Question and description of an arithmetic sequence. Have students work in pairs of mixed language abilities to complete the exploration.

Beginning/Advanced Advanced students read aloud the instructions and help Beginning students by providing alternative language when necessary. Then they work together to complete the exploration. Beginning students state the answers from their tables, reading aloud the labels from the tables.

Intermediate/Advanced High Advanced High students read aloud the instructions and guide the explorations. Intermediate students explain their answers.

ELPS 3.B.3 Expand and internalize initial English vocabulary by learning and using routine language needed for classroom communication.

Dynamic Teaching Tools

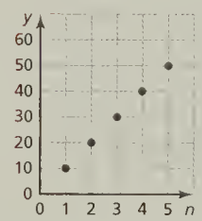
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.12.D The student is expected to write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms.

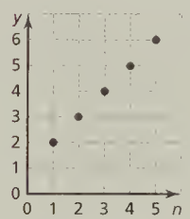
ANSWERS

1. a. 10, 20, 30, 40, 50



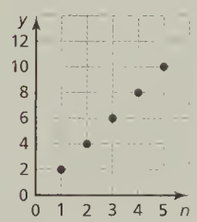
The y-value increases by 10 each time.

b. 2, 3, 4, 5, 6



The y-value increases by 1 each time.

c. 2, 4, 6, 8, 10



The y-value increases by 2 each time.

2. An arithmetic sequence can describe a pattern in which the difference between consecutive terms is the same; *Sample answer:* The amount of money earned for selling candy bars at \$2 each.
3. Each molecule added to the group increases the number of atoms by 3; 69 atoms

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Extra Example 1

Write the next three terms of the arithmetic sequence.

$-2, 6, 14, 22, \ldots$

The next three terms are 30, 38, and 46.

MONITORING PROGRESS ANSWERS

1. 36, 48, 60
2. 1.8, 2.2, 2.6
3. $3, 2\frac{3}{4}, 2\frac{1}{2}$

4.7 Lesson

What You Will Learn

- Write the terms of arithmetic sequences.
- Graph arithmetic sequences.
- Write arithmetic sequences as functions.

Core Vocabulary

sequence, *p. 202*
term, *p. 202*
arithmetic sequence, *p. 202*
common difference, *p. 202*

Previous

point-slope form
function notation

Writing the Terms of Arithmetic Sequences

A **sequence** is an ordered list of numbers. Each number in a sequence is called a **term**. Each term a_n has a specific position n in the sequence.

$5, \quad 10, \quad 15, \quad 20, \quad 25, \ldots, a_n, \ldots$

1st position | 3rd position | nth position

Core Concept

Arithmetic Sequence

In an **arithmetic sequence**, the difference between each pair of consecutive terms is the same. This difference is called the **common difference**. Each term is found by adding the common difference to the previous term.

$5, \quad 10, \quad 15, \quad 20, \ldots$ Terms of an arithmetic sequence

$+5 \quad +5 \quad +5$ ← common difference

READING

An ellipsis (. . .) is a series of dots that indicates an intentional omission of information. In mathematics, the . . . notation means "and so forth." The ellipsis indicates that there are more terms in the sequence that are not shown.

EXAMPLE 1 Extending an Arithmetic Sequence

Write the next three terms of the arithmetic sequence.

$-7, -14, -21, -28, \ldots$

SOLUTION

Use a table to organize the terms and find the pattern.

Position	1	2	3	4
Term	−7	−14	−21	−28

$+(-7) \quad +(-7) \quad +(-7)$

Each term is 7 less than the previous term. So, the common difference is −7.

Add −7 to a term to find the next term.

Position	1	2	3	4	5	6	7
Term	−7	−14	−21	−28	−35	−42	−49

$+(-7) \quad +(-7) \quad +(-7)$

▶ The next three terms are −35, −42, and −49.

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Write the next three terms of the arithmetic sequence.

1. $-12, 0, 12, 24, \ldots$
2. $0.2, 0.6, 1, 1.4, \ldots$
3. $4, 3\frac{3}{4}, 3\frac{1}{2}, 3\frac{1}{4}, \ldots$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Auditory

Students may benefit from articulating the process for extending an arithmetic sequence by using the common difference. Have students take turns working at the board with one of several tables showing the terms of a sequence. Students should calculate the common difference and then use it to find the next three terms, explaining each step in the process.

Graphing Arithmetic Sequences

To graph a sequence, let a term's position number n in the sequence be the x-value. The term a_n is the corresponding y-value. Plot the ordered pairs (n, a_n).

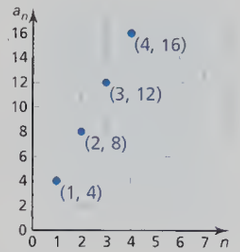
EXAMPLE 2 Graphing an Arithmetic Sequence

Graph the arithmetic sequence 4, 8, 12, 16, ... What do you notice?

SOLUTION

Make a table. Then plot the ordered pairs (n, a_n).

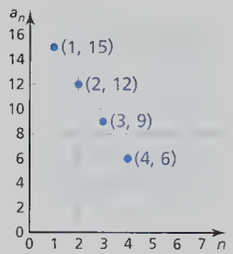
Position, n	Term, a_n
1	4
2	8
3	12
4	16



▶ The points lie on a line.

EXAMPLE 3 Identifying an Arithmetic Sequence from a Graph

Does the graph represent an arithmetic sequence? Explain.



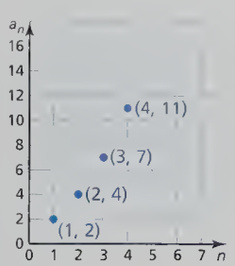
SOLUTION

Make a table to organize the ordered pairs. Then determine whether there is a common difference.

Position, n	1	2	3	4
Term, a_n	15	12	9	6
	+(−3)	+(−3)	+(−3)	

Each term is 3 less than the previous term. So, the common difference is −3.

▶ Consecutive terms have a common difference of −3. So, the graph represents the arithmetic sequence 15, 12, 9, 6, ...



Monitoring Progress

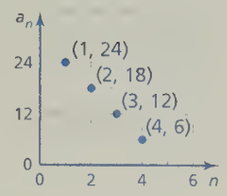
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Graph the arithmetic sequence. What do you notice?

4. 3, 6, 9, 12, . . .
5. 4, 2, 0, −2, . . .
6. 1, 0.8, 0.6, 0.4, . . .
7. Does the graph shown represent an arithmetic sequence? Explain.

Extra Example 2

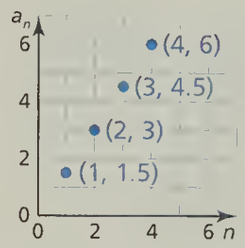
Graph the arithmetic sequence 24, 18, 12, 6, ... What do you notice?



The points lie on a line.

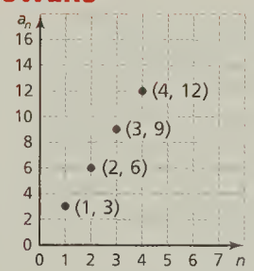
Extra Example 3

Does the graph represent an arithmetic sequence? Explain.

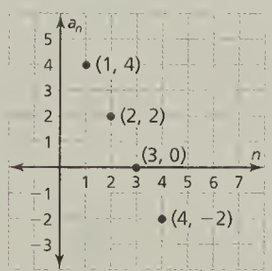


Consecutive terms have a common difference of 1.5. So, the graph represents the arithmetic sequence 1.5, 3, 4.5, 6, ...

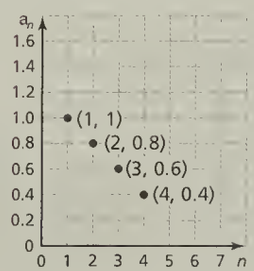
MONITORING PROGRESS ANSWERS

4. 

The points lie on a line.

5. 

The points lie on a line.

6. 

The points lie on a line.

7. no; Consecutive terms do not have a common difference.

Extra Example 4

Write an equation for the nth term of the arithmetic sequence 7, 3, −1, −5, ... Then find a_{30}. $a_n = -4n + 11$; The 30th term of the arithmetic sequence is −109.

MONITORING PROGRESS ANSWERS

8. $a_n = n + 3$; 28
9. $a_n = 8n$; 200
10. $a_n = -n + 2$; −23

Writing Arithmetic Sequences as Functions

Because consecutive terms of an arithmetic sequence have a common difference, the sequence has a constant rate of change. So, the points represented by any arithmetic sequence lie on a line. You can use the first term and the common difference to write a linear function that describes an arithmetic sequence. Let $a_1 = 4$ and $d = 3$.

Position, n	Term, a_n	Written using a_1 and d	Numbers
1	first term, a_1	a_1	4
2	second term, a_2	$a_1 + d$	$4 + 3 = 7$
3	third term, a_3	$a_1 + 2d$	$4 + 2(3) = 10$
4	fourth term, a_4	$a_1 + 3d$	$4 + 3(3) = 13$
⋮	⋮	⋮	⋮
n	nth term, a_n	$a_1 + (n - 1)d$	$4 + (n - 1)(3)$

ANOTHER WAY

An *arithmetic sequence* is a linear function whose domain is the set of positive integers. You can think of d as the slope and $(1, a_1)$ as a point on the graph of the function. An equation in point-slope form for the function is

$a_n - a_1 = d(n - 1)$.

This equation can be rewritten as

$a_n = a_1 + (n - 1)d$.

Core Concept

Equation for an Arithmetic Sequence

Let a_n be the nth term of an arithmetic sequence with first term a_1 and common difference d. The nth term is given by

$$a_n = a_1 + (n - 1)d.$$

EXAMPLE 4 Finding the nth Term of an Arithmetic Sequence

Write an equation for the nth term of the arithmetic sequence 14, 11, 8, 5, ... Then find a_{50}.

STUDY TIP

Notice that the equation in Example 4 is of the form $y = mx + b$, where y is replaced by a_n and x is replaced by n.

SOLUTION

The first term is 14, and the common difference is −3.

$a_n = a_1 + (n - 1)d$ — Equation for an arithmetic sequence

$a_n = 14 + (n - 1)(-3)$ — Substitute 14 for a_1 and −3 for d.

$a_n = -3n + 17$ — Simplify.

Use the equation to find the 50th term.

$a_n = -3n + 17$ — Write the equation.

$a_{50} = -3(50) + 17$ — Substitute 50 for n.

$= -133$ — Simplify.

▶ The 50th term of the arithmetic sequence is −133.

Monitoring Progress

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Write an equation for the nth term of the arithmetic sequence. Then find a_{25}.

8. 4, 5, 6, 7, . . .
9. 8, 16, 24, 32, . . .
10. 1, 0, −1, −2, . . .

You can rewrite the equation for an arithmetic sequence with first term a_1 and common difference d in function notation by replacing a_n with $f(n)$.

$$f(n) = a_1 + (n - 1)d$$

The domain of the function is the set of positive integers.

EXAMPLE 5 Writing Real-Life Functions



Online bidding for a purse increases by \$5 for each bid after the \$60 initial bid.

Bid number	1	2	3	4
Bid amount	\$60	\$65	\$70	\$75

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. The winning bid is \$105. How many bids were there?

SOLUTION

a. The first term is 60, and the common difference is 5.

$f(n) = a_1 + (n - 1)d$ Function for an arithmetic sequence

$f(n) = 60 + (n - 1)5$ Substitute 60 for a_1 and 5 for d.

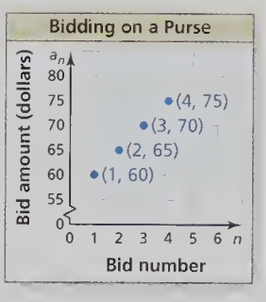
$f(n) = 5n + 55$ Simplify.

▶ The function $f(n) = 5n + 55$ represents the arithmetic sequence.

b. Make a table. Then plot the ordered pairs (n, a_n).

REMEMBER
The domain is the set of positive integers.

Bid number, n	Bid amount, a_n
1	60
2	65
3	70
4	75



c. Use the function to find the value of n for which $f(n) = 105$.

$f(n) = 5n + 55$ Write the function.

$105 = 5n + 55$ Substitute 105 for $f(n)$.

$10 = n$ Solve for n.

▶ There were 10 bids.

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Games	Total cost
1	\$7
2	\$9
3	\$11
4	\$13

11. A carnival charges \$2 for each game after you pay a \$5 entry fee.

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. How many games can you play when you take \$29 to the carnival?

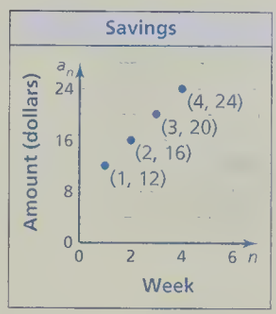
Extra Example 5

Juan saves \$12 the first week of the year. He will increase the amount he saves each week by \$4.

Week	1	2	3	4
Amount	\$12	\$16	\$20	\$24

a. Write a function that represents the arithmetic sequence. $f(n) = 4n + 8$

b. Graph the function.

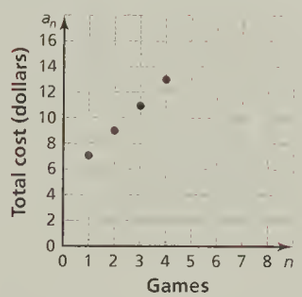


c. Juan's goal is to save \$68 in one week. In which week will he save that amount? Week 15

MONITORING PROGRESS ANSWERS

11. a. $f(n) = 2n + 5$

b.



c. 12 games

Closure

- **3-2-1:** Distribute a printed 3-2-1 reflection sheet and give time for students to reflect on their learning and to write.
 - 3 new things (concepts, skills, procedures, …) I learned in this chapter
 - 2 things (concepts, skills, procedures, …) I am still struggling with
 - 1 thing that will help me tomorrow

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–25 odd, 33–39 odd, 45, 50, 54, 58–60

Average: 1, 2–40 even, 46–50 even, 54, 58–60

Advanced: 1, 2, 16, 22, 26–48 even, 50–60

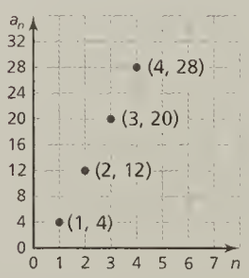
HOMEWORK CHECK

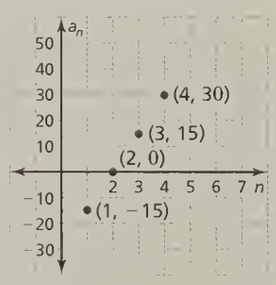
Basic: 11, 17, 23, 33, 45

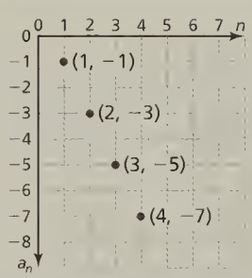
Average: 12, 18, 24, 34, 46

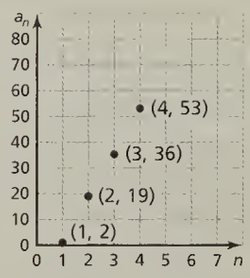
Advanced: 16, 22, 26, 38, 46

ANSWERS

1. The graph of an arithmetic sequence is a graph of a linear function whose domain is the set of positive integers.
2. Find the difference between the terms a_2 and a_4; 6; 3
3. 15, 28, 41
4. 12, 6, 0
5. 5
6. -25
7. 4
8. $-\frac{1}{3}$
9. -1.5
10. 9
11. 31, 34, 37
12. 45, 56, 67
13. 36, 41, 46
14. $-60, -90, -120$
15. $0.1, -0.2, -0.5$
16. $\frac{1}{6}, 0, -\frac{1}{6}$
17. 

18. 

19. 

20. 

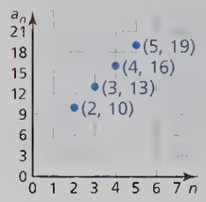
21–31. See Additional Answers.

4.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Describe the graph of an arithmetic sequence.
2. **DIFFERENT WORDS, SAME QUESTION** Consider the arithmetic sequence represented by the graph. Which is different? Find "both" answers.



Find the slope of the linear function.

Find the difference between consecutive terms of the arithmetic sequence.

Find the difference between the terms a_2 and a_4.

Find the common difference of the arithmetic sequence.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, write the next three terms of the arithmetic sequence.

3. First term: 2
 Common difference: 13
4. First term: 18
 Common difference: −6

In Exercises 5–10, find the common difference of the arithmetic sequence.

5. 13, 18, 23, 28, . . .
6. 175, 150, 125, 100, . . .
7. $-16, -12, -8, -4, \ldots$
8. $4, 3\frac{2}{3}, 3\frac{1}{3}, 3, \ldots$
9. 6.5, 5, 3.5, 2, . . .
10. $-16, -7, 2, 11, \ldots$

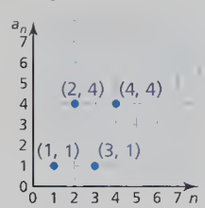
In Exercises 11–16, write the next three terms of the arithmetic sequence. *(See Example 1.)*

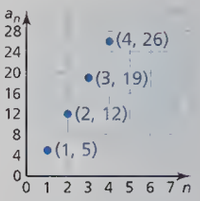
11. 19, 22, 25, 28, . . .
12. 1, 12, 23, 34, . . .
13. 16, 21, 26, 31, . . .
14. 60, 30, 0, −30, . . .
15. 1.3, 1, 0.7, 0.4, . . .
16. $\frac{5}{6}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \ldots$

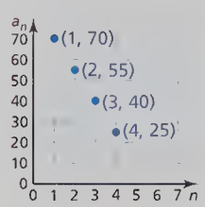
In Exercises 17–22, graph the arithmetic sequence. *(See Example 2.)*

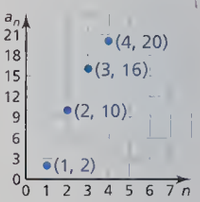
17. 4, 12, 20, 28, . . .
18. $-15, 0, 15, 30, \ldots$
19. $-1, -3, -5, -7, \ldots$
20. 2, 19, 36, 53, . . .
21. $0, 4\frac{1}{2}, 9, 13\frac{1}{2}, \ldots$
22. 6, 5.25, 4.5, 3.75, . . .

In Exercises 23–26, determine whether the graph represents an arithmetic sequence. Explain. *(See Example 3.)*

23. 

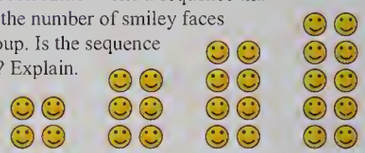
24. 

25. 

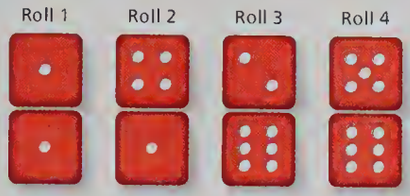
26. 

In Exercises 27–30, determine whether the sequence is arithmetic. If so, find the common difference.

27. 13, 26, 39, 52, . . .
28. 5, 9, 14, 20, . . .
29. 48, 24, 12, 6, . . .
30. 87, 81, 75, 69, . . .
31. **FINDING A PATTERN** Write a sequence that represents the number of smiley faces in each group. Is the sequence arithmetic? Explain.



32. FINDING A PATTERN Write a sequence that represents the sum of the numbers in each roll. Is the sequence arithmetic? Explain.



In Exercises 33–38, write an equation for the nth term of the arithmetic sequence. Then find a_{10}. *(See Example 4.)*

33. $-5, -4, -3, -2, \ldots$

34. $-6, -9, -12, -15, \ldots$

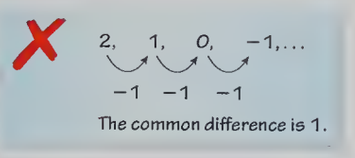
35. $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$

36. $100, 110, 120, 130, \ldots$

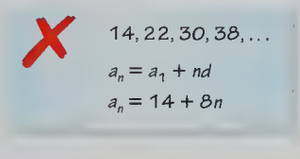
37. $10, 0, -10, -20, \ldots$

38. $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \ldots$

39. ERROR ANALYSIS Describe and correct the error in finding the common difference of the arithmetic sequence.



40. ERROR ANALYSIS Describe and correct the error in writing an equation for the nth term of the arithmetic sequence.



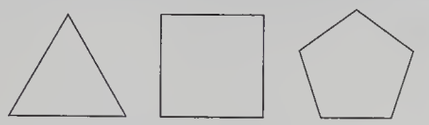
41. NUMBER SENSE The first term of an arithmetic sequence is 3. The common difference of the sequence is 1.5 times the first term. Write the next three terms of the sequence. Then graph the sequence.

42. NUMBER SENSE The first row of a dominoes display has 10 dominoes. Each row after the first has two more dominoes than the row before it. Write the first five terms of the sequence that represents the number of dominoes in each row. Then graph the sequence.



REPEATED REASONING In Exercises 43 and 44, (a) draw the next three figures in the sequence and (b) describe the 20th figure in the sequence.

43.



44.



45. MODELING WITH MATHEMATICS The total number of babies born in a country each minute after midnight January 1st can be estimated by the sequence shown in the table. *(See Example 5.)*

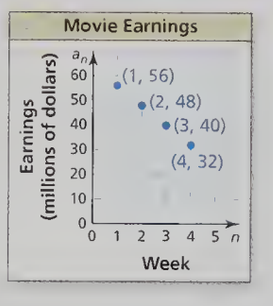
Minutes after midnight January 1st	1	2	3	4
Total babies born	5	10	15	20

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. Estimate how many minutes after midnight January 1st it takes for 100 babies to be born.

46. MODELING WITH MATHEMATICS The amount of money a movie earns each week after its release can be approximated by the sequence shown in the graph.



a. Write a function that represents the arithmetic sequence.

b. In what week does the movie earn \$16 million?

c. How much money does the movie earn overall?

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ANSWERS

32. 2, 5, 8, 11; yes; Consecutive terms have a common difference of 3.

33. $a_n = n - 6$; 4

34. $a_n = -3n - 3$; -33

35. $a_n = \frac{1}{2}n$; 5

36. $a_n = 10n + 90$; 190

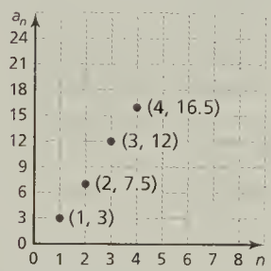
37. $a_n = -10n + 20$; -80

38. $a_n = \frac{1}{7}n + \frac{2}{7}$; $1\frac{5}{7}$;

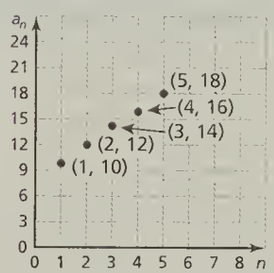
39. -1 is added each time, not 1; The common difference is -1.

40. n should be $n - 1$;
$a_n = a_1 + (n - 1)d$;
$a_n = 14 + (n - 1)8$; $a_n = 8n + 6$

41. 7.5, 12, 16.5



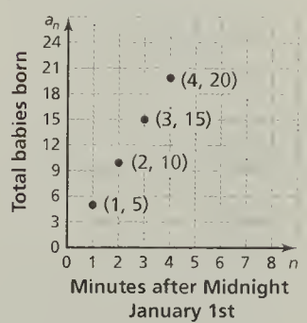
42. 10, 12, 14, 16, 18



43–44. See Additional Answers.

45. a. $f(n) = 5n$

b.



c. 20 min

46. a. $f(n) = -8n + 64$

b. week 6

c. \$224 million

ANSWERS

47. arithmetic sequence; $f(n) = 4n - 3$; 117

48. not an arithmetic sequence

49. discrete; discrete

50. no; If the first term of the sequence is negative and the output values constantly increase, or if the first term of the sequence is positive and the output values constantly decrease, the range contains positive and negative numbers.

51. *Sample answer:* $-12, -15, -18, -21$; $a_n = -3n - 9$; 12, 9, 6, 3; $a_n = -3n + 15$

52. *Sample answer:* The number of people who have ridden a ride at an amusement park that seats 25 people at a time, when every seat is filled each time.

53. 119

54. **a.** yes; Consecutive terms have a common difference of 12,500.

b. Write a function that represents an arithmetic sequence with $a_1 = 20{,}000$ and $d = 12{,}500$, then find the value of the function at 6.

55. $f(n) = 6n + 17$

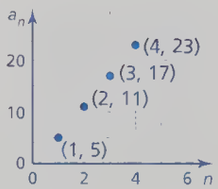
56. 7 min

57–60. See Additional Answers.

Mini-Assessment

1. Write the next three terms of the arithmetic sequence.
 $-4, -8, -12, -16, \ldots$
 $-20, -24$, and -28

2. Graph the arithmetic sequence.
 5, 11, 17, 23, ...



3. Write an equation for the nth term of the arithmetic sequence $-7, 1, 9, 17, \ldots$. Then find a_{20}.
 $a_n = 8n - 15$; 145

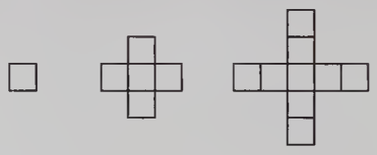
4. Dorrie increases the number of sit-ups she does each week by 8 after doing 10 sit-ups the first week.

Week	1	2	3	4
Sit-Ups	10	18	26	34

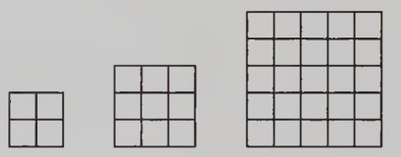
Write a function that represents the arithmetic sequence. $f(n) = 8n + 2$

MATHEMATICAL CONNECTIONS **In Exercises 47 and 48, each small square represents 1 square inch. Determine whether the areas of the figures form an arithmetic sequence. If so, write a function f that represents the arithmetic sequence and find $f(30)$.**

47.



48.



49. REASONING Is the domain of an arithmetic sequence discrete or continuous? Is the range of an arithmetic sequence discrete or continuous?

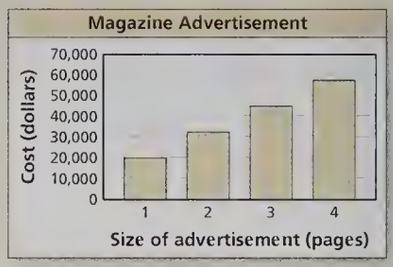
50. MAKING AN ARGUMENT Your friend says that the range of a function that represents an arithmetic sequence always contains only positive numbers or only negative numbers. Your friend claims this is true because the domain is the set of positive integers and the output values either constantly increase or constantly decrease. Is your friend correct? Explain.

51. OPEN-ENDED Write the first four terms of two different arithmetic sequences with a common difference of -3. Write an equation for the nth term of each sequence.

52. THOUGHT PROVOKING Describe an arithmetic sequence that models the numbers of people in a real-life situation.

53. REPEATED REASONING Firewood is stacked in a pile. The bottom row has 20 logs, and the top row has 14 logs. Each row has one more log than the row above it. How many logs are in the pile?

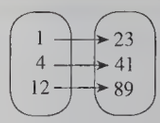
54. HOW DO YOU SEE IT? The bar graph shows the costs of advertising in a magazine.



a. Does the graph represent an arithmetic sequence? Explain.

b. Explain how you would estimate the cost of a six-page advertisement in the magazine.

55. REASONING Write a function f that represents the arithmetic sequence shown in the mapping diagram.



56. PROBLEM SOLVING A train stops at a station every 12 minutes starting at 6:00 A.M. You arrive at the station at 7:29 A.M. How long must you wait for the train?

57. ABSTRACT REASONING Let x be a constant. Determine whether each sequence is an arithmetic sequence. Explain.

a. $x + 6, 3x + 6, 5x + 6, 7x + 6, \ldots$

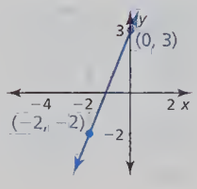
b. $x + 1, 3x + 1, 9x + 1, 27x + 1, \ldots$

Maintaining Mathematical Proficiency

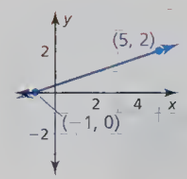
Reviewing what you learned in previous grades and lessons

Find the slope of the line. *(Section 3.5)*

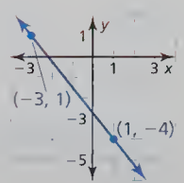
58.



59.



60.



If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

4.5–4.7 What Did You Learn?

Core Vocabulary

scatter plot, *p. 188*
correlation, *p. 189*
line of fit, *p. 190*
residual, *p. 194*
linear regression, *p. 195*
line of best fit, *p. 195*
correlation coefficient, *p. 195*
interpolation, *p. 197*
extrapolation, *p. 197*
causation, *p. 197*
sequence, *p. 202*
term, *p. 202*
arithmetic sequence, *p. 202*
common difference, *p. 202*

Core Concepts

Section 4.5

Scatter Plot, *p. 188*
Identifying Correlations, *p. 189*
Using a Line of Fit to Model Data, *p. 190*

Section 4.6

Residuals, *p. 194*
Lines of Best Fit, *p. 195*
Correlation and Causation, *p. 197*

Section 4.7

Arithmetic Sequence, *p. 202*
Equation for an Arithmetic Sequence, *p. 204*

Mathematical Thinking

1. What resources can you use to help you answer Exercise 17 on page 192?
2. What calculations are repeated in Exercises 11–16 on page 206? When finding a term such as a_{50}, is there a general method or shortcut you can use instead of repeating calculations?

Performance Task

Any Beginning

With so many ways to represent a linear relationship, where do you start? Use what you know to move between equations, graphs, tables, and contexts.

To explore the answer to this question and more, go to ***BigIdeasMath.com***.



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- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

1. *Sample answer:* the Internet, an almanac
2. addition; yes, substitute the values of a_1 and d in the formula $a_n = a_1 + (n - 1)d$, and then substitute 50 for n to calculate a_{50}.

ANSWERS

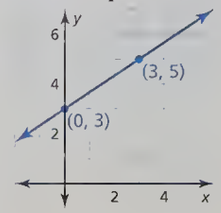
1. $y = -\frac{1}{2}x + 1$
2. $y - 7 = -(x - 4)$
3. $f(x) = x - 5$
4. $f(x) = -4$
5. $f(x) = -\frac{5}{3}x + 18$

4 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

4.1 Writing Equations in Slope-Intercept Form *(pp. 161–166)*

Write an equation of the line in slope-intercept form.



Find the slope and y-intercept.

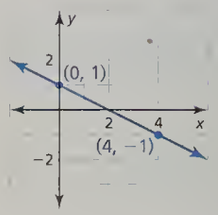
Let $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (3, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{3 - 0} = \frac{2}{3}$$

Because the line crosses the y-axis at $(0, 3)$, the y-intercept is 3.

▶ So, the equation is $y = \frac{2}{3}x + 3$.

1. Write an equation of the line in slope-intercept form.



4.2 Writing Equations in Point-Slope Form *(pp. 167–172)*

Write an equation in point-slope form of the line that passes through the point $(-1, -8)$ and has a slope of 3.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - (-8) = 3[x - (-1)]$	Substitute 3 for m, -1 for x_1, and -8 for y_1.
$y + 8 = 3(x + 1)$	Simplify.

▶ The equation is $y + 8 = 3(x + 1)$.

2. Write an equation in point-slope form of the line that passes through the point $(4, 7)$ and has a slope of -1.

Write a linear function f with the given values.

3. $f(10) = 5, f(2) = -3$ **4.** $f(3) = -4, f(5) = -4$ **5.** $f(6) = 8, f(9) = 3$

4.3 Writing Equations in Standard Form *(pp. 173–178)*

Write an equation in standard form of the line that passes through the point $(-1, 1)$ and has a slope of -2.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 1 = -2[x - (-1)]$	Substitute -2 for m, -1 for x_1, and 1 for y_1.
$2x + y = -1$	Simplify. Collect variable terms on one side and constants on the other.

▶ The equation is $2x + y = -1$.

6. Write two equations in standard form that are equivalent to $5x + y = -10$.

Write an equation in standard form of the line with the given characteristics.

7. slope: -4
 passes through: $(-2, 7)$

8. passes through: $(-1, -5)$ and $(3, 7)$

Write equations of the horizontal and vertical lines that pass through the given point.

9. $(2, -12)$

10. $(-4, 3)$

4.4 Writing Equations of Parallel and Perpendicular Lines *(pp. 179–184)*

Determine which of the lines, if any, are parallel or perpendicular.

Line a: $y = 2x + 3$ **Line b: $2y + x = 5$** **Line c: $4y - 8x = -4$**

Write the equations in slope-intercept form. Then compare the slopes.

Line a: $y = 2x + 3$ Line b: $y = -\frac{1}{2}x + \frac{5}{2}$ Line c: $y = 2x - 1$

▶ Lines a and c have slopes of 2, so they are parallel. Line b has a slope of $-\frac{1}{2}$, the negative reciprocal of 2, so it is perpendicular to lines a and c.

Determine which of the lines, if any, are parallel or perpendicular. Explain.

11. Line a passes through $(0, 4)$ and $(4, 3)$.
 Line b passes through $(0, 1)$ and $(4, 0)$.
 Line c passes through $(2, 0)$ and $(4, 4)$.

12. Line a: $2x - 7y = 14$
 Line b: $y = \frac{7}{2}x - 8$
 Line c: $2x + 7y = -21$

13. Write an equation of the line that passes through $(1, 5)$ and is parallel to the line $y = -4x + 2$.

14. Write an equation of the line that passes through $(2, -3)$ and is perpendicular to the line $y = -2x - 3$.

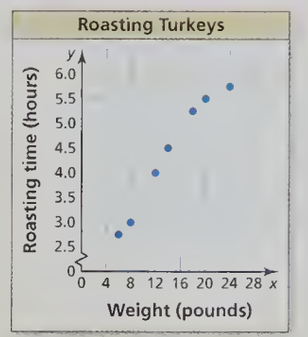
15. Write an equation of a line that is (a) parallel and (b) perpendicular to the line $x = 4$.

4.5 Scatter Plots and Lines of Fit *(pp. 187–192)*

The scatter plot shows the roasting times (in hours) and weights (in pounds) of seven turkeys. Tell whether the data show a *positive*, a *negative*, or *no* correlation.

As the weight of a turkey increases, the roasting time increases.

▶ So, the scatter plot shows a positive correlation.



Use the scatter plot in the example.

16. What is the roasting time for a 12-pound turkey?

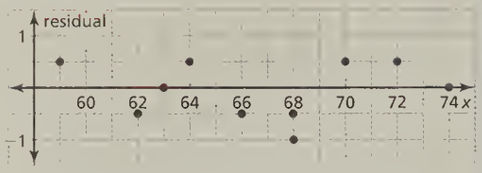
17. Write an equation that models the roasting time as a function of the weight of a turkey. Interpret the slope and y-intercept of the line of fit.

ANSWERS

6. $10x + 2y = -20$, $x + \frac{1}{5}y = -2$
7. $4x + y = -1$
8. $3x - y = 2$
9. $y = -12$, $x = 2$
10. $y = 3$, $x = -4$
11. Lines a and b are parallel; None of the lines are perpendicular; Lines a and b have the same slope and none of them have negative reciprocal slopes.
12. None of the lines are parallel; Lines b and c are perpendicular; None of the lines have the same slope and the slope of line b is the negative reciprocal of the slope of line c.
13. $y = -4x + 9$
14. $y = \frac{1}{2}x - 4$
15. a. *Sample answer:* $x = 6$
 b. *Sample answer:* $y = 4$
16. $4h$
17. *Sample answer:* $y = \frac{1}{5}x + 2$; The slope of $\frac{1}{5}$ means the roasting time increases by about $\frac{1}{5}$ hour for each pound the weight of the turkey increases. The y-intercept of 2 has no meaning in this context because the weight of the turkey cannot be 0 pounds.

ANSWERS

18.



19. a. 65 in.
 b. 6.5
20. no; Height does not determine shoe size.
21. $a_n = -n + 12$; -18
22. $a_n = 6n$; 180
23. $a_n = 3n - 12$; 78

4.6 Analyzing Lines of Fit *(pp. 193–200)*

The table shows the heights x (in inches) and shoe sizes y of several students. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

Height, x	64	62	70	63	72	68	66	74	68	59
Shoe Size, y	9	7	12	8	13	9.5	9	13.5	10	6.5

Step 1 Enter the data from the table into two lists.

Step 2 Use the *linear regression* feature.

```
LinReg
 y=ax+b
 a=.4989919355
 b=-23.4828629
 r²=.9477256904
 r=.9735120392
```

▶ An equation of the line of best fit is $y = 0.50x - 23.5$. The correlation coefficient is about 0.974. This means that the relationship between the heights and the shoe sizes has a strong positive correlation and the equation closely models the data.

18. Make a scatter plot of the residuals to verify that the model in the example is a good fit.
19. Use the data in the example. (a) Approximate the height of a student whose shoe size is 9. (b) Predict the shoe size of a student whose height is 60 inches.
20. Is there a causal relationship in the data in the example? Explain.

4.7 Arithmetic Sequences *(pp. 201–208)*

Write an equation for the nth term of the arithmetic sequence $-3, -5, -7, -9, \ldots$. Then find a_{20}.

The first term is -3, and the common difference is -2.

$a_n = a_1 + (n - 1)d$ — Equation for an arithmetic sequence

$a_n = -3 + (n - 1)(-2)$ — Substitute -3 for a_1 and -2 for d.

$a_n = -2n - 1$ — Simplify.

Use the equation to find the 20th term.

$a_{20} = -2(20) - 1$ — Substitute 20 for n.

$= -41$ — Simplify.

▶ The 20th term of the arithmetic sequence is -41.

Write an equation for the nth term of the arithmetic sequence. Then find a_{30}.

21. 11, 10, 9, 8, . . .
22. 6, 12, 18, 24, . . .
23. $-9, -6, -3, 0, \ldots$

4 Chapter Test

Write an equation in standard form of the line with the given characteristics.

1. slope $= \frac{1}{2}$; y-intercept $= -6$
2. passes through $(1, 5)$ and $(3, -5)$

Write an equation in slope-intercept form of the line with the given characteristics.

3. slope $= \frac{2}{5}$; y-intercept $= -7$
4. slope $= -4$; passes through $(-2, 16)$
5. passes through $(0, 6)$ and $(3, -3)$
6. passes through $(5, -7)$ and $(10, -7)$
7. parallel to the line $y = 3x - 1$; passes through $(-2, -8)$
8. perpendicular to the line $y = \frac{1}{4}x - 9$; passes through $(1, 1)$

Write an equation in point-slope form of the line with the given characteristics.

9. slope $= 10$; passes through $(6, 2)$
10. passes through $(-3, 2)$ and $(6, -1)$
11. The first row of an auditorium has 42 seats. Each row after the first has three more seats than the row before it.
 a. Find the number of seats in Row 25.
 b. Which row has 90 seats?
12. The table shows the amount x (in dollars) spent on advertising for a neighborhood festival and the attendance y of the festival for several years.
 a. Make a scatter plot of the data. Describe the correlation.
 b. Write an equation that models the attendance as a function of the amount spent on advertising.
 c. Interpret the slope and y-intercept of the line of fit.

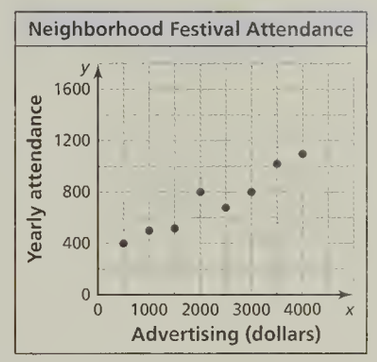
Advertising (dollars), x	Yearly attendance, y
500	400
1000	550
1500	550
2000	800
2500	650
3000	800
3500	1050
4000	1100

13. Consider the data in the table in Exercise 12.
 a. Use a graphing calculator to find an equation of the line of best fit.
 b. Identify and interpret the correlation coefficient.
 c. What would you expect the scatter plot of the residuals to look like?
 d. Is there a causal relationship in the data? Explain your reasoning.
 e. Predict the amount that must be spent on advertising to get 2000 people to attend the festival.
14. Let a, b, c, and d be constants. Determine which of the lines, if any, are parallel or perpendicular. Explain.

 Line 1: $y - c = ax$ Line 2: $ay = -x - b$ Line 3: $ax + y = d$
15. You are buying ribbon to make costumes for a school play. Organza ribbon costs \$0.08 per yard. Satin ribbon costs \$0.04 per yard. (a) Write an equation in standard form that models the possible combinations of yards of ribbon you can buy for \$5. (b) Graph the equation from part (a). (c) Find four possible combinations.

If students need help...	If students got it...
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

ANSWERS

1. $\frac{1}{2}x - y = 6$
2. $5x + y = 10$
3. $y = \frac{2}{5}x - 7$
4. $y = -4x + 8$
5. $y = -3x + 6$
6. $y = -7$
7. $y = 3x - 2$
8. $y = -4x + 5$
9. $y - 2 = 10(x - 6)$
10. $y - 2 = -\frac{1}{3}(x + 3)$ or $y + 1 = -\frac{1}{3}(x - 6)$
11. a. 114
 b. row 17
12. a. 

 positive correlation
 b. *Sample answer:* $y = \frac{1}{5}x + 300$
 c. *Sample answer:* The slope of $\frac{1}{5}$ means the attendance increases by about 1 person for every additional \$5 spent on advertising. The y-intercept of 300 means that if no money is spent on advertising, about 300 people attend the festival.
13. a. $y = 0.19x + 309$
 b. $r = 0.943$; strong positive correlation
 c. more or less evenly dispersed about the horizontal axis
 d. There may be a causal relationship in the data but the correlation may be caused by other factors, such as the quality of the attractions each year.
 e. \$8900
14. None of the lines are parallel; Lines 1 and 2 are perpendicular; None of the lines have the same slope, but the slope of line 1 is the negative reciprocal of the slope of line 2.
15. See Additional Answers.

ANSWERS

1. C
2. J
3. C
4. −8
5. J

4 Standards Assessment

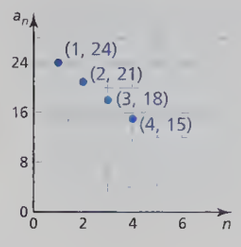
1. Which function represents the arithmetic sequence shown in the graph? *(TEKS A.12.D)*

 Ⓐ $f(n) = 15 + 3n$

 Ⓑ $f(n) = 4 - 3n$

 Ⓒ $f(n) = 27 - 3n$

 Ⓓ $f(n) = 24 - 3n$



2. In which situation is both a correlation and causation likely? *(TEKS A.4.B)*

 Ⓕ the number of pairs of shoes a person owns and the person's shoe size

 Ⓖ the number of cell phones and the number of taxis in a city

 Ⓗ a person's IQ and the time it takes the person to run 50 meters

 Ⓙ the price of a pair of pants and the number sold

3. Which inequality is *not* equivalent to $4x + 1 < 17$? *(TEKS A.5.B)*

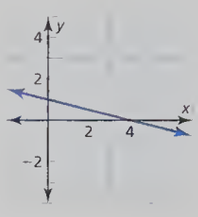
 Ⓐ $3x + 6 < 2x + 10$

 Ⓑ $-6x + 14 > -10$

 Ⓒ $-x - 5 > 2x + 7$

 Ⓓ $5x - 5 < x + 11$

4. **GRIDDED ANSWER** The equation of the line shown can be written in the standard form $-2x + By = -8$. What is the value of B? *(TEKS A.2.C)*



5. The cost to rent a movie at a video store can be modeled by a linear function. The table shows the total costs of renting a movie for different lengths of time. Based on the table, which statement is true? *(TEKS A.3.B)*

 Ⓕ The slope is 1.5. It represents the initial cost.

 Ⓖ The slope is 1.5. It represents the rate per night.

 Ⓗ The slope is 1.25. It represents the initial cost.

 Ⓙ The slope is 1.25. It represents the rate per night.

Nights rented, x	Total cost (dollars), y
1	1.50
2	2.75
3	4.00
4	5.25
5	6.50

6. Which equation represents the line that passes through (0, 0) and is parallel to the line that passes through (2, 3) and (6, 1)? *(TEKS A.2.E, TEKS A.3.A)*

Ⓐ $y = \frac{1}{2}x$ Ⓑ $y = -\frac{1}{2}x$

Ⓒ $y = -2x$ Ⓓ $y = 2x$

7. In which step below does a mistake first appear in solving the equation $-2(4 - x) + 5x = 6(2x - 3)$? *(TEKS A.5.A, TEKS A.10.D)*

Step 1 $-8 - 2x + 5x = 12x - 18$
Step 2 $-8 + 3x = 12x - 18$
Step 3 $10 = 9x$
Step 4 $x = \frac{10}{9}$

Ⓕ Step 1 Ⓖ Step 2

Ⓗ Step 3 Ⓙ Step 4

8. The table shows the daily high temperatures x (in degrees Fahrenheit) and the numbers y of frozen fruit bars sold on eight randomly selected days. Which of the following is a reasonable equation to model the data? *(TEKS A.4.C)*

Temperature (°F), x	54	60	68	72	78	84	92	98
Frozen fruit bars, y	40	120	180	260	280	260	220	180

Ⓐ $y = 3x - 50$ Ⓑ $y = -3x + 50$

Ⓒ $y = x - 150$ Ⓓ $y = -x + 150$

9. The ordered pairs $(-4, 7)$, $(-2, 6)$, $(0, 3)$, $(-2, 1)$, $(2, -1)$, and $(4, 1)$ represent a relation. Based on the relation, which statement is true? *(TEKS A.12.A)*

Ⓕ The relation is a function.

Ⓖ The relation is not a function because one output is paired with two inputs.

Ⓗ The relation is not a function because one input is paired with two outputs.

Ⓙ The relation is not a function because the rate of change is not constant.

10. Which of the following is enough information to write an equation of a line that does not pass through the origin? *(TEKS A.2.B)*

I. two points on the line
II. the slope of the line
III. both intercepts of the line
IV. the slope and x-intercept of the line

Ⓐ I and II only Ⓑ I, III, and IV only

Ⓒ I and IV only Ⓓ I, II, and IV only

ANSWERS

6. B
7. F
8. A
9. H
10. B

Chapter 5 Pacing Guide

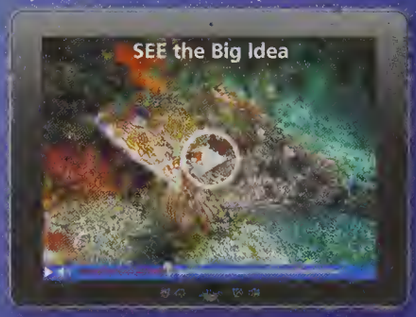
Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	2 Days
Quiz	1 Day
Section 5	2 Days
Section 6	2 Days
Section 7	3 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 5	19 Days
Year-to-Date	82 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
5.1	A.2.I, A.3.F, A.3.G, A.5.C
5.2	A.2.I, A.5.C
5.3	A.2.I, A.5.C
5.4	A.2.I, A.3.F, A.5.C
5.5	A.5.A
5.6	A.2.H, A.3.D
5.7	A.3.H

5 Solving Systems of Linear Equations

5.1 Solving Systems of Linear Equations by Graphing
5.2 Solving Systems of Linear Equations by Substitution
5.3 Solving Systems of Linear Equations by Elimination
5.4 Solving Special Systems of Linear Equations
5.5 Solving Equations by Graphing
5.6 Linear Inequalities in Two Variables
5.7 Systems of Linear Inequalities



Fishing *(p. 265)*


Fruit Salad *(p. 255)*


Pets *(p. 250)*


Drama Club *(p. 228)*


Roofing Contractor *(p. 222)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

For an overview of this chapter, formative assessment tips, and teaching strategies, see Laurie's Notes at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

Graphing Linear Functions (A.3.C)

Example 1 Graph $3 + y = \frac{1}{2}x$.

Step 1 Rewrite the equation in slope-intercept form.

$$y = \frac{1}{2}x - 3$$

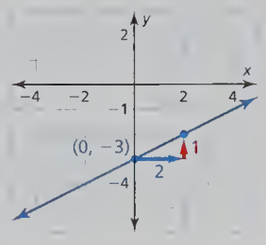
Step 2 Find the slope and the y-intercept.

$$m = \frac{1}{2} \text{ and } b = -3$$

Step 3 The y-intercept is -3. So, plot $(0, -3)$.

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$



Plot the point that is 2 units right and 1 unit up from $(0, -3)$. Draw a line through the two points.

Graph the equation.

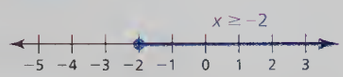
1. $y + 4 = x$
2. $6x - y = -1$
3. $4x + 5y = 20$
4. $-2y + 12 = -3x$

Solving and Graphing Linear Inequalities (7.10.B, A.5.B)

Example 2 Solve $2x - 17 \leq 8x - 5$. Graph the solution.

$2x - 17 \leq 8x - 5$	Write the inequality.
$+5 \quad +5$	Add 5 to each side.
$2x - 12 \leq 8x$	Simplify.
$-2x \quad -2x$	Subtract $2x$ from each side.
$-12 \leq 6x$	Simplify.
$\frac{-12}{6} \leq \frac{6x}{6}$	Divide each side by 6.
$-2 \leq x$	Simplify.

▶ The solution is $x \geq -2$.



Solve the inequality. Graph the solution.

5. $m + 4 > 9$
6. $24 \leq -6t$
7. $2a - 5 \leq 13$
8. $-5z + 1 < -14$
9. $4k - 16 < k + 2$
10. $7w + 12 \geq 2w - 3$
11. **ABSTRACT REASONING** The graphs of the linear functions g and h have different slopes. The value of both functions at $x = a$ is b. When g and h are graphed in the same coordinate plane, what happens at the point (a, b)?

Dynamic Solutions available at *BigIdeasMath.com*

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For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make Process Diagrams for the following topics.

- Graphing a linear function using slope-intercept form
- Solving and graphing a linear inequality

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

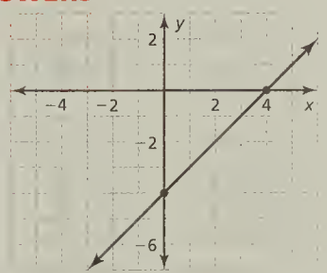
Texas Essential Knowledge and Skills

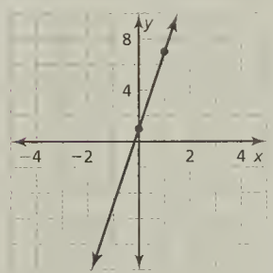
A.3.C The student is expected to graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

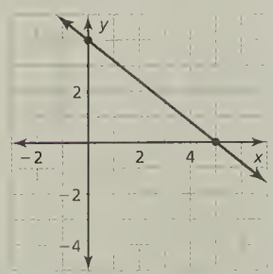
7.10.B The student is expected to represent solutions for one-variable, two-step equations and inequalities on number lines.

A.5.B The student is expected to solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

1. 

2. 

3. 

4–11. See Additional Answers.

MONITORING PROGRESS ANSWERS

1. $(0, -3)$
2. $\left(\frac{3}{2}, -\frac{1}{2}\right)$
3. $(2, 2)$

Mathematical Thinking

Mathematically proficient students select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems. **(A.1.C)**

Using a Graphing Calculator

Core Concept

Finding the Point of Intersection

You can use a graphing calculator to find the point of intersection, if it exists, of the graphs of two linear equations.

1. Enter the equations into a graphing calculator.
2. Graph the equations in an appropriate viewing window, so that the point of intersection is visible.
3. Use the *intersect* feature of the graphing calculator to find the point of intersection.

EXAMPLE 1 **Using a Graphing Calculator**

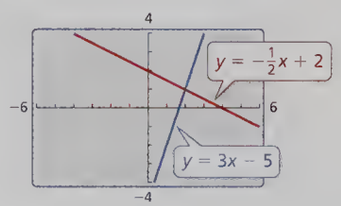
Use a graphing calculator to find the point of intersection, if it exists, of the graphs of the two linear equations.

$y = -\frac{1}{2}x + 2$ Equation 1

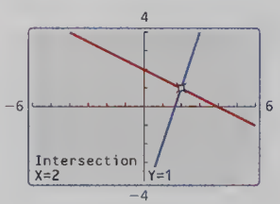
$y = 3x - 5$ Equation 2

SOLUTION

The slopes of the lines are not the same, so you know that the lines intersect. Enter the equations into a graphing calculator. Then graph the equations in an appropriate viewing window.



Use the *intersect* feature to find the point of intersection of the lines.



▶ The point of intersection is $(2, 1)$.

Monitoring Progress

Use a graphing calculator to find the point of intersection of the graphs of the two linear equations.

1. $y = -2x - 3$
 $y = \frac{1}{2}x - 3$
2. $y = -x + 1$
 $y = x - 2$
3. $3x - 2y = 2$
 $2x - y = 2$

For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

5.1 Solving Systems of Linear Equations by Graphing

Texas Essential Knowledge and Skills
A.2.I
A.3.F
A.3.G
A.5.C

Essential Question How can you solve a system of linear equations?

EXPLORATION 1 Writing a System of Linear Equations

Work with a partner. Your family opens a bed-and-breakfast. They spend \$600 preparing a bedroom to rent. The cost to your family for food and utilities is \$15 per night. They charge \$75 per night to rent the bedroom.

a. Write an equation that represents the costs.

$$\begin{matrix}\text{Cost, } C \\ \text{(in dollars)}\end{matrix} = \begin{matrix}\text{\$15 per} \\ \text{night}\end{matrix} \cdot \begin{matrix}\text{Number of} \\ \text{nights, } x\end{matrix} + \$600$$

b. Write an equation that represents the revenue (income).

$$\begin{matrix}\text{Revenue, } R \\ \text{(in dollars)}\end{matrix} = \begin{matrix}\text{\$75 per} \\ \text{night}\end{matrix} \cdot \begin{matrix}\text{Number of} \\ \text{nights, } x\end{matrix}$$

c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

APPLYING MATHEMATICS
To be proficient in math, you need to identify important quantities in real-life problems and map their relationships using tools such as diagrams, tables, and graphs.

EXPLORATION 2 Using a Table or Graph to Solve a System

Work with a partner. Use the cost and revenue equations from Exploration 1 to determine how many nights your family needs to rent the bedroom before recovering the cost of preparing the bedroom. This is the *break-even point*.

a. Copy and complete the table.

x (nights)	0	1	2	3	4	5	6	7	8	9	10	11
C (dollars)												
R (dollars)												

b. How many nights does your family need to rent the bedroom before breaking even?

c. In the same coordinate plane, graph the cost equation and the revenue equation from Exploration 1.

d. Find the point of intersection of the two graphs. What does this point represent? How does this compare to the break-even point in part (b)? Explain.

Communicate Your Answer

3. How can you solve a system of linear equations? How can you check your solution?

4. Solve each system by using a table or sketching a graph. Explain why you chose each method. Use a graphing calculator to check each solution.

a. $y = -4.3x - 1.3$
$y = 1.7x + 4.7$

b. $y = x$
$y = -3x + 8$

c. $y = -x - 1$
$y = 3x + 5$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

4. a. $(-1, 3)$; *Sample answer:* table because decimals are difficult to graph

b. $(2, 2)$; *Sample answer:* graph because it is easier to draw

c. $\left(-\frac{3}{2}, \frac{1}{2}\right)$; *Sample answer:* graph because it is easier to draw

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.I The student is expected to write systems of two linear equations given a table of values, a graph, and a verbal description.

A.3.F The student is expected to graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

A.3.G The student is expected to estimate graphically the solutions to systems of two linear equations with two variables in real-world problems.

A.5.C The student is expected to solve systems of two linear equations with two variables for mathematical and real-world problems.

ANSWERS

1. a. $C = 15x + 600$

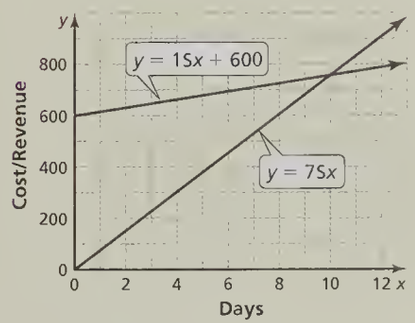
b. $R = 75x$

c. $C = 15x + 600$
$R = 75x$

2. a. See Additional Answers.

b. 10 nights

c.



d. (10, 750); the point where both functions have the same x- and y-values; It is the same point; The break-even point is the point where both functions have the same x- and y-values.

3. Graph both equations and find the point of intersection; Substitute the x-coordinate for x in both of the equations and verify that both results are the y-coordinate of the point of intersection.

Extra Example 1

Tell whether the ordered pair is a solution of the system of linear equations.

a. (5, 1)

$x + y = 6$

$2x - y = 9$

Because the ordered pair (5, 1) is a solution of each equation, it is a solution of the linear system.

b. (−1, −5)

$y = -x + 2$

$y = 3x - 2$

The ordered pair (−1, −5) is *not* a solution of the first equation. So, it is *not* a solution of the linear system.

MONITORING PROGRESS ANSWERS

1. no
2. yes

5.1 Lesson

What You Will Learn

- Check solutions of systems of linear equations.
- Solve systems of linear equations by graphing.
- Use systems of linear equations to solve real-life problems.

Core Vocabulary

system of linear equations, *p. 220*

solution of a system of linear equations, *p. 220*

Previous

linear equation

ordered pair

Systems of Linear Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown below.

$x + y = 7$ Equation 1

$2x - 3y = -11$ Equation 2

A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

EXAMPLE 1 Checking Solutions

Tell whether the ordered pair is a solution of the system of linear equations.

a. $(2, 5)$; $x + y = 7$ Equation 1, $2x - 3y = -11$ Equation 2

b. $(-2, 0)$; $y = -2x - 4$ Equation 1, $y = x + 4$ Equation 2

SOLUTION

a. Substitute 2 for x and 5 for y in each equation.

Equation 1: $x + y = 7$; $2 + 5 \stackrel{?}{=} 7$; $7 = 7$ ✓

Equation 2: $2x - 3y = -11$; $2(2) - 3(5) \stackrel{?}{=} -11$; $-11 = -11$ ✓

READING: A system of linear equations is also called a *linear system*.

▶ Because the ordered pair (2, 5) is a solution of each equation, it is a solution of the linear system.

b. Substitute -2 for x and 0 for y in each equation.

Equation 1: $y = -2x - 4$; $0 \stackrel{?}{=} -2(-2) - 4$; $0 = 0$ ✓

Equation 2: $y = x + 4$; $0 \stackrel{?}{=} -2 + 4$; $0 \neq 2$ ✗

▶ The ordered pair $(-2, 0)$ is a solution of the first equation, but it is not a solution of the second equation. So, $(-2, 0)$ is *not* a solution of the linear system.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Tell whether the ordered pair is a solution of the system of linear equations.

1. $(1, -2)$; $2x + y = 0$, $-x + 2y = 5$

2. $(1, 4)$; $y = 3x + 1$, $y = -x + 5$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Read aloud the topic Systems of Linear Equations. Point out that students should already be familiar with most of the words used in the explanation, but their use and arrangement here conveys new meaning.

Beginning Repeat the individual words of each highlighted phrase. Then repeat the entire phrase.
Intermediate/Advanced Read aloud the definitions of the highlighted phrases.
Advanced High Explain the definitions using their own words.

ELPS 1.F Use accessible language and learn new and essential language in the process.

Solving Systems of Linear Equations by Graphing

The solution of a system of linear equations is the point of intersection of the graphs of the equations.

Core Concept

Solving a System of Linear Equations by Graphing

Step 1 Graph each equation in the same coordinate plane.

Step 2 Estimate the point of intersection.

Step 3 Check the point from Step 2 by substituting for x and y in each equation of the original system.

REMEMBER

Note that the linear equations are in slope-intercept form. You can use the method presented in Section 3.5 to graph the equations.

EXAMPLE 2 **Solving a System of Linear Equations by Graphing**

Solve the system of linear equations by graphing.

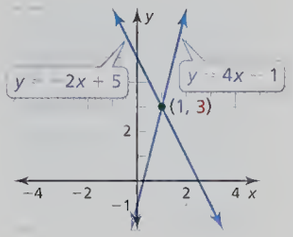
$y = -2x + 5$ Equation 1

$y = 4x - 1$ Equation 2

SOLUTION

Step 1 Graph each equation.

Step 2 Estimate the point of intersection.
The graphs appear to intersect at $(1, 3)$.



Step 3 Check your point from Step 2.

Equation 1

$y = -2x + 5$

$3 \stackrel{?}{=} -2(1) + 5$

$3 = 3$ ✓

Equation 2

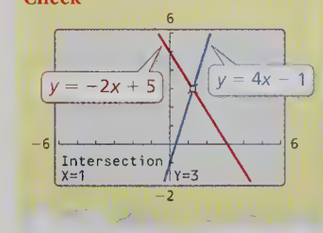
$y = 4x - 1$

$3 \stackrel{?}{=} 4(1) - 1$

$3 = 3$ ✓

▶ The solution is $(1, 3)$.

Check



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the system of linear equations by graphing.

3. $y = x - 2$
$y = -x + 4$

4. $y = \frac{1}{2}x + 3$
$y = -\frac{3}{2}x - 5$

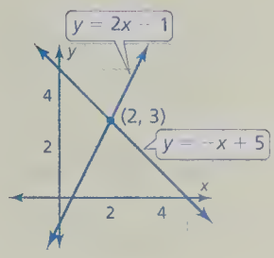
5. $2x + y = 5$
$3x - 2y = 4$

Extra Example 2

Solve the system of linear equations by graphing.

$y = -x + 5$

$y = 2x - 1$



The solution is $(2, 3)$.

MONITORING PROGRESS ANSWERS

3. $(3, 1)$

4. $(-4, 1)$

5. $(2, 1)$

Differentiated Instruction

Visual

When graphing a system of equations, some students may benefit from using color to distinguish between the equations in the system. Have students use one color for the first equation and its graph and a different color for the second equation and its graph.

Extra Example 3

For the first tour on Monday, a museum sells 4 child tickets and 8 adult tickets for $128. For the second tour, the museum sells 6 adult tickets for $72. Find the price of one child ticket and the price of one adult ticket. child ticket: $8; adult ticket: $12

MONITORING PROGRESS ANSWERS

6. 12 math problems, 6 science problems

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**



A roofing contractor buys 30 bundles of shingles and 4 rolls of roofing paper for $1040. In a second purchase (at the same prices), the contractor buys 8 bundles of shingles for $256. Find the price per bundle of shingles and the price per roll of roofing paper.

SOLUTION

1. **Understand the Problem** You know the total price of each purchase and how many of each item were purchased. You are asked to find the price of each item.
2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.
3. **Solve the Problem**

Words $30 \cdot \text{Price per bundle} + 4 \cdot \text{Price per roll} = 1040$

$8 \cdot \text{Price per bundle} + 0 \cdot \text{Price per roll} = 256$

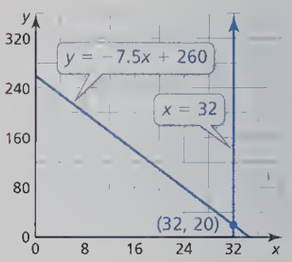
Variables Let x be the price (in dollars) per bundle and let y be the price (in dollars) per roll.

System $30x + 4y = 1040$ Equation 1

$8x = 256$ Equation 2

Step 1 Graph each equation. Note that only the first quadrant is shown because x and y must be positive.

Step 2 Estimate the point of intersection. The graphs appear to intersect at (32, 20).



Step 3 Check your point from Step 2.

Equation 1	Equation 2
$30x + 4y = 1040$	$8x = 256$
$30(32) + 4(20) \stackrel{?}{=} 1040$	$8(32) \stackrel{?}{=} 256$
$1040 = 1040$ ✓	$256 = 256$ ✓

▶ The solution is (32, 20). So, the price per bundle of shingles is $32, and the price per roll of roofing paper is $20.

4. **Look Back** You can use estimation to check that your solution is reasonable. A bundle of shingles costs about $30. So, 30 bundles of shingles and 4 rolls of roofing paper (at $20 per roll) cost about 30(30) + 4(20) = $980, and 8 bundles of shingles costs about 8(30) = $240. These prices are close to the given values, so the solution seems reasonable.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

6. You have a total of 18 math and science exercises for homework. You have six more math exercises than science exercises. How many exercises do you have in each subject?

Closure

- **Phone Call:** Write a brief script for a phone conversation with a friend who was not in class today. Explain what a system of linear equations is and how to solve a system of linear equations by graphing.

5.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Do the equations $5y - 2x = 18$ and $6x = -4y - 10$ form a system of linear equations? Explain.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the system of linear equations $-4x + 2y = 4$ and $4x - y = -6$. Which is different? Find "both" answers.

Solve the system of linear equations.

Solve each equation for y.

Find the point of intersection of the graphs of the equations.

Find an ordered pair that is a solution of each equation in the system.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell whether the ordered pair is a solution of the system of linear equations. *(See Example 1.)*

3. $(2, 6)$; $\begin{aligned} x + y &= 8 \\ 3x - y &= 0 \end{aligned}$

4. $(8, 2)$; $\begin{aligned} x - y &= 6 \\ 2x - 10y &= 4 \end{aligned}$

5. $(-1, 3)$; $\begin{aligned} y &= -7x - 4 \\ y &= 8x + 5 \end{aligned}$

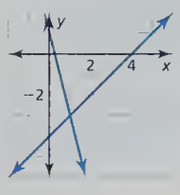
6. $(-4, -2)$; $\begin{aligned} y &= 2x + 6 \\ y &= -3x - 14 \end{aligned}$

7. $(-2, 1)$; $\begin{aligned} 6x + 5y &= -7 \\ 2x - 4y &= -8 \end{aligned}$

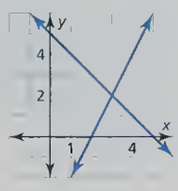
8. $(5, -6)$; $\begin{aligned} 6x + 3y &= 12 \\ 4x + y &= 14 \end{aligned}$

In Exercises 9–12, use the graph to solve the system of linear equations. Check your solution.

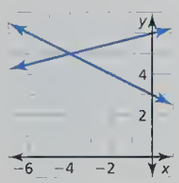
9. $\begin{aligned} x - y &= 4 \\ 4x + y &= 1 \end{aligned}$



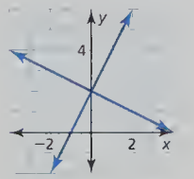
10. $\begin{aligned} x + y &= 5 \\ y - 2x &= -4 \end{aligned}$



11. $\begin{aligned} 6y + 3x &= 18 \\ -x + 4y &= 24 \end{aligned}$



12. $\begin{aligned} 2x - y &= -2 \\ 2x + 4y &= 8 \end{aligned}$



In Exercises 13–20, solve the system of linear equations by graphing. *(See Example 2.)*

13. $\begin{aligned} y &= -x + 7 \\ y &= x + 1 \end{aligned}$

14. $\begin{aligned} y &= -x + 4 \\ y &= 2x - 8 \end{aligned}$

15. $\begin{aligned} y &= \tfrac{1}{3}x + 2 \\ y &= \tfrac{2}{3}x + 5 \end{aligned}$

16. $\begin{aligned} y &= \tfrac{3}{4}x - 4 \\ y &= -\tfrac{1}{2}x + 11 \end{aligned}$

17. $\begin{aligned} 9x + 3y &= -3 \\ 2x - y &= -4 \end{aligned}$

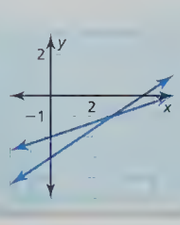
18. $\begin{aligned} 4x - 4y &= 20 \\ y &= -5 \end{aligned}$

19. $\begin{aligned} x - 4y &= -4 \\ -3x - 4y &= 12 \end{aligned}$

20. $\begin{aligned} 3y + 4x &= 3 \\ x + 3y &= -6 \end{aligned}$

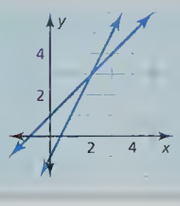
ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in solving the system of linear equations.**

21. ✗



The solution of the linear system $x - 3y = 6$ and $2x - 3y = 3$ is $(3, -1)$.

22. ✗



The solution of the linear system $y = 2x - 1$ and $y = x + 1$ is $x = 2$.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–21 odd, 27, 32–36

Average: 1, 2–28 even, 32–36

Advanced: 1, 2, 6–10 even, 18–28 even, 29–36

HOMEWORK CHECK

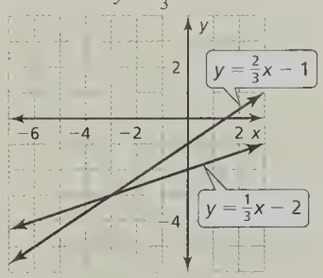
Basic: 3, 9, 13, 27

Average: 4, 10, 14, 28

Advanced: 6, 10, 18, 28

ANSWERS

1. yes; They are two equations in the same variables.
2. Solve each equation for y; $y = 2x + 2$, $y = 4x + 6$; $(-2, -2)$
3. yes
4. no
5. no
6. yes
7. yes
8. yes
9. $(1, -3)$
10. $(3, 2)$
11. $(-4, 5)$
12. $(0, 2)$
13. $(3, 4)$
14. $(4, 0)$
15. $(-9, -1)$
16. $(12, 5)$
17. $(-1, 2)$
18. $(0, -5)$
19. $(-4, 0)$
20. $(3, -3)$
21. The graph of the second equation should be $y = \frac{2}{3}x - 1$.



$(-3, -3)$

22. The solution needs to be an ordered pair; (2, 3)

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

23. (10, 5)

24. (5, 5)

25. (1.2, 1.4)

26. (0.5, 0.5)

27. 30 min on the elliptical trainer, 10 min on the stationary bike

28. 12 small candles, 16 large candles

29. $A = 18x - 18$; $P = 6x + 6$; (2, 18); The value of the perimeter and the area of the rectangle will both be 18 when $x = 2$.

30–33. See Additional Answers.

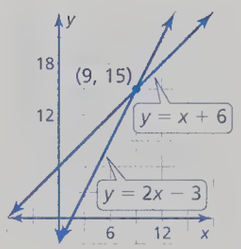
34. $y = -x + 4$

35. $y = 2x + 3$

36. $y = -3x + 20$

Mini-Assessment

1. Tell whether the ordered pair $(-2, -2)$ is a solution of the system of linear equations.
 $2x - y = -2$
 $x - 2y = 2$
 $(-2, -2)$ is a solution of the system.
2. Solve the system of linear equations by graphing.
 $y = 2x - 3$
 $y = x + 6$



 The solution is (9, 15).
3. Rob has a total of 14 pens and markers. He has six more markers than pens. How many of each item does Rob have? 10 markers and 4 pens

USING TOOLS In Exercises 23–26, use a graphing calculator to solve the system of linear equations.

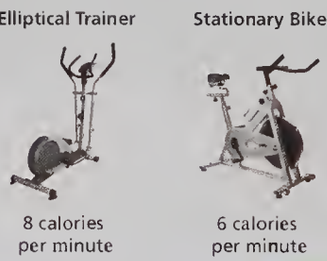
23. $0.2x + 0.4y = 4$
$-0.6x + 0.6y = -3$

24. $-1.6x - 3.2y = -24$
$2.6x + 2.6y = 26$

25. $-7x + 6y = 0$
$0.5x + y = 2$

26. $4x - y = 1.5$
$2x + y = 1.5$

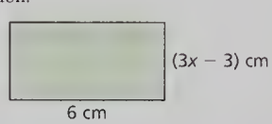
27. MODELING WITH MATHEMATICS You have 40 minutes to exercise at the gym, and you want to burn 300 calories total using both machines. How much time should you spend on each machine? *(See Example 3.)*



28. MODELING WITH MATHEMATICS You sell small and large candles at a craft fair. You collect \$144 selling a total of 28 candles. How many of each type of candle did you sell?



29. MATHEMATICAL CONNECTIONS Write a linear equation that represents the area and a linear equation that represents the perimeter of the rectangle. Solve the system of linear equations by graphing. Interpret your solution.



30. THOUGHT PROVOKING Your friend's bank account balance (in dollars) is represented by the equation $y = 25x + 250$, where x is the number of months. Graph this equation. After 6 months, you want to have the same account balance as your friend. Write a linear equation that represents your account balance. Interpret the slope and y-intercept of the line that represents your account balance.

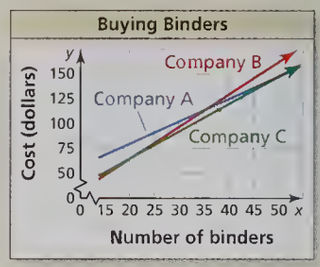
31. COMPARING METHODS Consider the equation $x + 2 = 3x - 4$.

a. Solve the equation using algebra.

b. Solve the system of linear equations $y = x + 2$ and $y = 3x - 4$ by graphing.

c. How is the linear system and the solution in part (b) related to the original equation and the solution in part (a)?

32. HOW DO YOU SEE IT? A teacher is purchasing binders for students. The graph shows the total costs of ordering x binders from three different companies.



a. For what numbers of binders are the costs the same at two different companies? Explain.

b. How do your answers in part (a) relate to systems of linear equations?

33. MAKING AN ARGUMENT You and a friend are going hiking but start at different locations. You start at the trailhead and walk 5 miles per hour. Your friend starts 3 miles from the trailhead and walks 3 miles per hour.



a. Write and graph a system of linear equations that represents this situation.

b. Your friend says that after an hour of hiking you will both be at the same location on the trail. Is your friend correct? Use the graph from part (a) to explain your answer.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the literal equation for y. *(Section 1.4)*

34. $10x + 5y = 5x + 20$

35. $9x + 18 = 6y - 3x$

36. $\frac{3}{4}x + \frac{1}{4}y = 5$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

5.2 Solving Systems of Linear Equations by Substitution

Texas Essential Knowledge and Skills
A.2.I
A.5.C

Essential Question How can you use substitution to solve a system of linear equations?

EXPLORATION 1 Using Substitution to Solve Systems

Work with a partner. Solve each system of linear equations using two methods.

Method 1 Solve for x first.
Solve for x in one of the equations. Substitute the expression for x into the other equation to find y. Then substitute the value of y into one of the original equations to find x.

Method 2 Solve for y first.
Solve for y in one of the equations. Substitute the expression for y into the other equation to find x. Then substitute the value of x into one of the original equations to find y.

Is the solution the same using both methods? Explain which method you would prefer to use for each system.

a. $x + y = -7$
$-5x + y = 5$

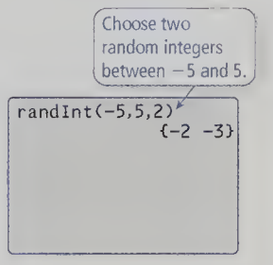
b. $x - 6y = -11$
$3x + 2y = 7$

c. $4x + y = -1$
$3x - 5y = -18$

EXPLORATION 2 Writing and Solving a System of Equations

Work with a partner.

a. Write a random ordered pair with integer coordinates. One way to do this is to use a graphing calculator. The ordered pair generated at the right is $(-2, -3)$.



b. Write a system of linear equations that has your ordered pair as its solution.

c. Exchange systems with your partner and use one of the methods from Exploration 1 to solve the system. Explain your choice of method.

USING PRECISE MATHEMATICAL LANGUAGE
To be proficient in math, you need to communicate precisely with others.

Communicate Your Answer

3. How can you use substitution to solve a system of linear equations?

4. Use one of the methods from Exploration 1 to solve each system of linear equations. Explain your choice of method. Check your solutions.

a. $x + 2y = -7$
$2x - y = -9$

b. $x - 2y = -6$
$2x + y = -2$

c. $-3x + 2y = -10$
$-2x + y = -6$

d. $3x + 2y = 13$
$x - 3y = -3$

e. $3x - 2y = 9$
$-x - 3y = 8$

f. $3x - y = -6$
$4x + 5y = 11$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Read aloud the Essential Question and instructions for Exploration 1 as students read along. Have students close their books and listen as you reread individual portions of what was read. Ask them to restate in their own words what was said, or ask questions to clarify what they don't understand. Then have them complete the exploration in groups of mixed language abilities.

Intermediate Explain what *Work with a partner* instructs students to do.
Beginning Restate what the bold headings in Methods 1 and 2 say to do.
Advanced Restate the process described for Method 1.
Advanced High Restate the process described for Method 2.

ELPS 2.D.1 Monitor understanding of spoken language during classroom instruction and interactions.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.I The student is expected to write systems of two linear equations given a table of values, a graph, and a verbal description.

A.5.C The student is expected to solve systems of two linear equations with two variables for mathematical and real-world problems.

ANSWERS

1. **a.** $(-2, -5)$; yes; *Sample answer:* Method 2 because both equations can be solved for y easily.
 b. $(1, 2)$; yes; *Sample answer:* Method 1 because the first equation can be solved for x easily.
 c. $(-1, 3)$; yes; *Sample answer:* Method 2 because the first equation can be solved for y easily.
2. **a.** *Sample answer:* $(2, 4)$
 b. *Sample answer:* $y = x + 2$ and $3x - 4y = -10$
 c. *Sample answer:* $y = x + 2$ and $3x - 4y = -10$; $(2, 4)$; Because the first equation is already solved for y, substitute that expression for y in the second equation.
3. Solve one of the equations for one of the variables. Substitute the expression for that variable into the other equation to find the value of the other variable. Substitute this value into one of the original equations to find the value of the remaining variable.
4. **a.** $(-5, -1)$; *Sample answer:* The first equation can be solved for x easily.
 b. $(-2, 2)$; *Sample answer:* The first equation can be solved for x easily.
 c. $(2, -2)$; *Sample answer:* The second equation can be solved for y easily.

4. d–f. See Additional Answers.

Extra Example 1

Solve the system of linear equations by substitution.

$y = -x + 3$

$3y + 5x = -1$

The solution is $(-5, 8)$.

MONITORING PROGRESS ANSWERS

1. $(-2, 8)$
2. $\left(\frac{1}{2}, -\frac{3}{4}\right)$
3. $(-1, 1)$

5.2 Lesson

What You Will Learn

- Solve systems of linear equations by substitution.
- Use systems of linear equations to solve real-life problems.

Core Vocabulary

Previous

system of linear equations
solution of a system of linear equations

Solving Linear Systems by Substitution

Another way to solve a system of linear equations is to use substitution.

Core Concept

Solving a System of Linear Equations by Substitution

Step 1 Solve one of the equations for one of the variables.

Step 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3 Substitute the value from Step 2 into one of the original equations and solve.

EXAMPLE 1 Solving a System of Linear Equations by Substitution

Solve the system of linear equations by substitution.

$y = -2x - 9$ Equation 1

$6x - 5y = -19$ Equation 2

SOLUTION

Step 1 Equation 1 is already solved for y.

Step 2 Substitute $-2x - 9$ for y in Equation 2 and solve for x.

$6x - 5y = -19$ Equation 2

$6x - 5(-2x - 9) = -19$ Substitute $-2x - 9$ for y.

$6x + 10x + 45 = -19$ Distributive Property

$16x + 45 = -19$ Combine like terms.

$16x = -64$ Subtract 45 from each side.

$x = -4$ Divide each side by 16.

Step 3 Substitute -4 for x in Equation 1 and solve for y.

$y = -2x - 9$ Equation 1

$= -2(-4) - 9$ Substitute -4 for x.

$= 8 - 9$ Multiply.

$= -1$ Subtract.

▶ The solution is $(-4, -1)$.

Check

Equation 1

$y = -2x - 9$

$-1 \stackrel{?}{=} -2(-4) - 9$

$-1 = -1$ ✓

Equation 2

$6x - 5y = -19$

$6(-4) - 5(-1) \stackrel{?}{=} -19$

$-19 = -19$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the system of linear equations by substitution. Check your solution.

1. $y = 3x + 14$
 $y = -4x$
2. $3x + 2y = 0$
 $y = \frac{1}{2}x - 1$
3. $x = 6y - 7$
 $4x + y = -3$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Inclusion

Some students may have difficulty deciding which equation to use for the expression to be substituted. Tell them to look first for an equation that is already solved for x or y and use that equation. If neither equation is written like that, then they should look for an equation that has a variable with a coefficient of 1 or -1 and solve for that variable.

EXAMPLE 2 Solving a System of Linear Equations by Substitution

ANOTHER WAY

You could also begin by solving for x in Equation 1, solving for y in Equation 2, or solving for x in Equation 2.

Solve the system of linear equations by substitution.

$-x + y = 3$ Equation 1

$3x + y = -1$ Equation 2

SOLUTION

Step 1 Solve for y in Equation 1.

$y = x + 3$ Revised Equation 1

Step 2 Substitute $x + 3$ for y in Equation 2 and solve for x.

$3x + y = -1$ Equation 2

$3x + (x + 3) = -1$ Substitute $x + 3$ for y.

$4x + 3 = -1$ Combine like terms.

$4x = -4$ Subtract 3 from each side.

$x = -1$ Divide each side by 4.

Step 3 Substitute -1 for x in Equation 1 and solve for y.

$-x + y = 3$ Equation 1

$-(-1) + y = 3$ Substitute -1 for x.

$y = 2$ Subtract 1 from each side.

▶ The solution is $(-1, 2)$.

Algebraic Check

Equation 1

$-x + y = 3$

$-(-1) + 2 \stackrel{?}{=} 3$

$3 = 3$ ✓

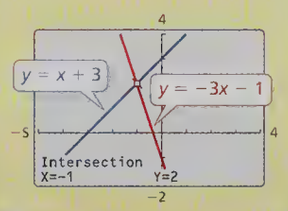
Equation 2

$3x + y = -1$

$3(-1) + 2 \stackrel{?}{=} -1$

$-1 = -1$ ✓

Graphical Check



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the system of linear equations by substitution. Check your solution.

4. $x + y = -2$
$-3x + y = 6$

5. $-x + y = -4$
$4x - y = 10$

6. $2x - y = -5$
$3x - y = 1$

7. $x - 2y = 7$
$3x - 2y = 3$

Extra Example 2

Solve the system of linear equations by substitution.

$x - y = -4$

$2x + y = 4$

The solution is $(0, 4)$.

MONITORING PROGRESS ANSWERS

4. $(-2, 0)$

5. $(2, -2)$

6. $(6, 17)$

7. $\left(-2, -\frac{9}{2}\right)$

Extra Example 3

There are a total of 124 students in the band. There are 12 more boys than girls in the band. Write a system of linear equations that represents this situation. How many boys and how many girls are in the band?

$x + y = 124$

$x = y + 12$

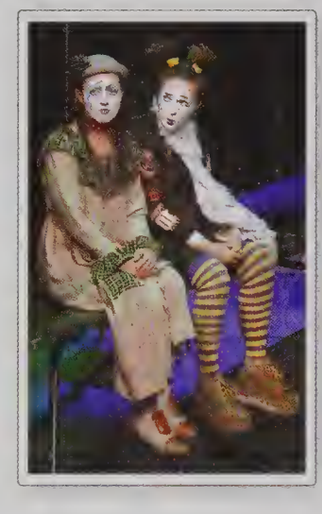
The solution is (68, 56). So, there are 68 boys and 56 girls in the band.

MONITORING PROGRESS ANSWERS

8. $x + y = 64$ and $x = y + 10$; 37 drama club students, 27 yearbook club students

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**



A drama club earns \$1040 from a production. An adult ticket costs twice as much as a student ticket. Write a system of linear equations that represents this situation. What is the price of each type of ticket?

Tickets sold	
Type	Number
adult	64
student	132

SOLUTION

1. **Understand the Problem** You know the amount earned, the total numbers of adult and student tickets sold, and the relationship between the price of an adult ticket and the price of a student ticket. You are asked to write a system of linear equations that represents the situation and find the price of each type of ticket.
2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.
3. **Solve the Problem**

Words $64 \cdot$ Adult ticket price $+ 132 \cdot$ Student ticket price $= 1040$

Adult ticket price $= 2 \cdot$ Student ticket price

Variables Let x be the price (in dollars) of an adult ticket and let y be the price (in dollars) of a student ticket.

System $64x + 132y = 1040$ Equation 1

$x = 2y$ Equation 2

Step 1 Equation 2 is already solved for x.

Step 2 Substitute $2y$ for x in Equation 1 and solve for y.

$64x + 132y = 1040$	Equation 1
$64(2y) + 132y = 1040$	Substitute $2y$ for x.
$260y = 1040$	Simplify.
$y = 4$	Simplify.

Step 3 Substitute 4 for y in Equation 2 and solve for x.

$x = 2y$	Equation 2
$x = 2(4)$	Substitute 4 for y.
$x = 8$	Simplify.

▶ The solution is (8, 4). So, an adult ticket costs \$8 and a student ticket costs \$4.

STUDY TIP

You can use either of the original equations to solve for x. However, using Equation 2 requires fewer calculations.

4. **Look Back** To check that your solution is correct, substitute the values of x and y into both of the original equations and simplify.

$64(8) + 132(4) = 1040$ $\quad$ $8 = 2(4)$

$1040 = 1040$ ✓ $\quad$ $8 = 8$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

8. There are a total of 64 students in a drama club and a yearbook club. The drama club has 10 more students than the yearbook club. Write a system of linear equations that represents this situation. How many students are in each club?

Closure

- **Exit Ticket:** Solve the system by substitution and by graphing.

$y = 3x + 1$

$y = x + 3$

(1, 4); check students' graphs.

5.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Describe how to solve a system of linear equations by substitution.
2. **NUMBER SENSE** When solving a system of linear equations by substitution, how do you decide which variable to solve for in Step 1?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell which equation you would choose to solve for one of the variables. Explain.

3. $x + 4y = 30$
 $x - 2y = 0$
4. $3x - y = 0$
 $2x + y = -10$
5. $5x + 3y = 11$
 $5x - y = 5$
6. $3x - 2y = 19$
 $x + y = 8$
7. $x - y = -3$
 $4x + 3y = -5$
8. $3x + 5y = 25$
 $x - 2y = -6$

In Exercises 9–16, solve the sytem of linear equations by substitution. Check your solution. *(See Examples 1 and 2.)*

9. $x = 17 - 4y$
 $y = x - 2$
10. $6x - 9 = y$
 $y = -3x$
11. $x = 16 - 4y$
 $3x + 4y = 8$
12. $-5x + 3y = 51$
 $y = 10x - 8$
13. $2x = 12$
 $x - 5y = -29$
14. $2x - y = 23$
 $x - 9 = -1$
15. $5x + 2y = 9$
 $x + y = -3$
16. $11x - 7y = -14$
 $x - 2y = -4$

17. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $8x + 2y = -12$ and $5x - y = 4$.

✗
Step 1 $5x - y = 4$
$-y = -5x + 4$
$y = 5x - 4$

Step 2 $5x - (5x - 4) = 4$
$5x - 5x + 4 = 4$
$4 = 4$

18. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $4x + 2y = 6$ and $3x + y = 9$.

✗
Step 1 $3x + y = 9$
$y = 9 - 3x$

Step 2 $4x + 2(9 - 3x) = 6$
$4x + 18 - 6x = 6$
$-2x = -12$
$x = 6$

Step 3 $3x + y = 9$
$3x + 6 = 9$
$3x = 3$
$x = 1$

19. **MODELING WITH MATHEMATICS** A farmer plants corn and wheat on a 180-acre farm. The farmer wants to plant three times as many acres of corn as wheat. Write a system of linear equations that represents this situation. How many acres of each crop should the farmer plant? *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** A company that offers tubing trips down a river rents tubes for a person to use and "cooler" tubes to carry food and water. A group spends \$270 to rent a total of 15 tubes. Write a system of linear equations that represents this situation. How many of each type of tube does the group rent?



20. $x + y = 15$ and $20x + 12.5y = 270$; 11 one-person tubes, 4 cooler tubes

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–19 odd, 25, 30, 32, 36–41

Average: 1, 2–26 even, 30, 32, 36–41

Advanced: 1, 2, 8–28 even, 29–41

HOMEWORK CHECK

Basic: 3, 9, 13, 19

Average: 6, 10, 14, 20

Advanced: 8, 12, 14, 20

ANSWERS

1. Solve one of the equations for one of the variables. Substitute the expression for that variable into the other equation to find the value of the other variable. Substitute this value into one of the original equations to find the value of the remaining variable.
2. *Sample answer:* Look for the equation that has a variable with a coefficient of 1 or −1.
3. *Sample answer:* $x - 2y = 0$; This equation can be solved for x easily.
4. *Sample answer:* $2x + y = -10$; This equation can be solved for y easily.
5. *Sample answer:* $5x - y = 5$; This equation can be solved for y easily.
6. *Sample answer:* $x + y = 8$; This equation can be solved for x easily.
7. *Sample answer:* $x - y = -3$; This equation can be solved for x easily.
8. *Sample answer:* $x - 2y = -6$; This equation can be solved for x easily.
9. $(5, 3)$
10. $(1, -3)$
11. $(-4, 5)$
12. $(3, 22)$
13. $(6, 7)$
14. $(8, -7)$
15. $(5, -8)$
16. $(0, 2)$
17. In Step 2, the expression for y needs to be substituted in the other equation; $8x + 2(5x - 4) = -12$, $8x + 10x - 8 = -12$, $18x - 8 = -12$, $18x = -4$, $x = -\frac{2}{9}$
18. In Step 3, 6 should be substituted for x, not for y; $3(6) + y = 9$, $18 + y = 9$, $y = -9$
19. $x + y = 180$ and $x = 3y$; 135 acres of corn, 45 acres of wheat

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

21. *Sample answer:* $y = x + 2$ and $x + 2y = 13$
22. *Sample answer:* $y - 2x = 12$ and $x - 2y = -18$
23. *Sample answer:* $x = \frac{1}{3}y$ and $x + y = -16$
24. *Sample answer:* $2x + y = 5$ and $2x - y = 55$
25. 8 five-point problems, 30 two-point problems
26. 80 shares of Stock A, 120 shares of Stock B
27. **a.** $x + y + 90 = 180$
 b. $x = 67, y = 23$
28. **a.** $x + 2y - 18 = 180$
 b. $x = 86, y = 56$
29. $a = 4, b = 5$
30. yes; Both equations only have 1 variable and the variables are different.

31–41. See Additional Answers.

Mini-Assessment

Solve the system of linear equations by substitution.

1. $x = 5 - y$
 $2x + 5y = 4$ (7, −2)
2. $x - y = -2$
 $3x - y = 2$ (2, 4)
3. A theater earns $3640 from a concert. An orchestra ticket costs three times as much as a balcony ticket.

Ticket Type	Number Sold
orchestra	148
balcony	76

Write a system of linear equations that represents this situation. What is the price of each type of ticket?

$148x + 76y = 3640$
$x = 3y$

The solution is (21, 7); orchestra ticket: $21, balcony ticket: $7.

In Exercises 21–24, write a system of linear equations that has the ordered pair as its solution.

21. (3, 5)
22. (−2, 8)
23. (−4, −12)
24. (15, −25)

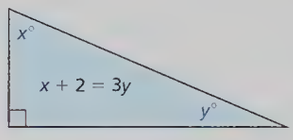
25. **PROBLEM SOLVING** A math test is worth 100 points and has 38 problems. Each problem is worth either 5 points or 2 points. How many problems of each point value are on the test?

26. **PROBLEM SOLVING** An investor owns shares of Stock A and Stock B. The investor owns a total of 200 shares with a total value of $4000. How many shares of each stock does the investor own?

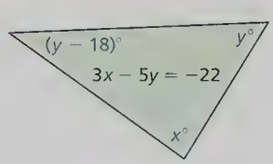
Stock	Price
A	$9.50
B	$27.00

MATHEMATICAL CONNECTIONS **In Exercises 27 and 28, (a) write an equation that represents the sum of the angle measures of the triangle and (b) use your equation and the equation shown to find the values of x and y.**

27.



28.



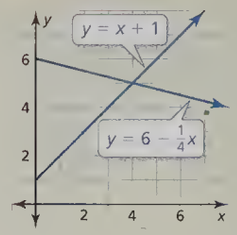
29. **REASONING** Find the values of a and b so that the solution of the linear system is (−9, 1).

$ax + by = -31$ Equation 1
$ax - by = -41$ Equation 2

30. **MAKING AN ARGUMENT** Your friend says that given a linear system with an equation of a horizontal line and an equation of a vertical line, you cannot solve the system by substitution. Is your friend correct? Explain.

31. **OPEN-ENDED** Write a system of linear equations in which (3, −5) is a solution of Equation 1 but not a solution of Equation 2, and (−1, 7) is a solution of the system.

32. **HOW DO YOU SEE IT?** The graphs of two linear equations are shown.



 a. At what point do the lines appear to intersect?
 b. Could you solve a system of linear equations by substitution to check your answer in part (a)? Explain.

33. **REPEATED REASONING** A radio station plays a total of 272 pop, rock, and hip-hop songs during a day. The number of pop songs is 3 times the number of rock songs. The number of hip-hop songs is 32 more than the number of rock songs. How many of each type of song does the radio station play?

34. **THOUGHT PROVOKING** You have $2.65 in coins. Write a system of equations that represents this situation. Use variables to represent the number of each type of coin.

35. **NUMBER SENSE** The sum of the digits of a two-digit number is 11. When the digits are reversed, the number increases by 27. Find the original number.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the sum or difference. (*Skills Review Handbook*)

36. $(x - 4) + (2x - 7)$
37. $(5y - 12) + (-5y - 1)$
38. $(t - 8) - (t + 15)$
39. $(6d + 2) - (3d - 3)$
40. $4(m + 2) + 3(6m - 4)$
41. $2(5v + 6) - 6(-9v + 2)$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

5.3 Solving Systems of Linear Equations by Elimination

Texas Essential Knowledge and Skills
A.2.I
A.5.C

Essential Question How can you use elimination to solve a system of linear equations?

EXPLORATION 1 Writing and Solving a System of Equations

Work with a partner. You purchase a drink and a sandwich for $4.50. Your friend purchases a drink and five sandwiches for $16.50. You want to determine the price of a drink and the price of a sandwich.

a. Let x represent the price (in dollars) of one drink. Let y represent the price (in dollars) of one sandwich. Write a system of equations for the situation. Use the following verbal model.

$$\begin{array}{c}\text{Number}\\ \text{of drinks}\end{array} \cdot \begin{array}{c}\text{Price}\\ \text{per drink}\end{array} + \begin{array}{c}\text{Number of}\\ \text{sandwiches}\end{array} \cdot \begin{array}{c}\text{Price per}\\ \text{sandwich}\end{array} = \begin{array}{c}\text{Total}\\ \text{price}\end{array}$$

Label one of the equations Equation 1 and the other equation Equation 2.

b. Subtract Equation 1 from Equation 2. Explain how you can use the result to solve the system of equations. Then find and interpret the solution.

USING PROBLEM-SOLVING STRATEGIES
To be proficient in math, you need to monitor and evaluate your progress and change course using a different solution method, if necessary.

EXPLORATION 2 Using Elimination to Solve Systems

Work with a partner. Solve each system of linear equations using two methods.

Method 1 Subtract. Subtract Equation 2 from Equation 1. Then use the result to solve the system.

Method 2 Add. Add the two equations. Then use the result to solve the system.

Is the solution the same using both methods? Which method do you prefer?

a. $3x - y = 6$
$3x + y = 0$

b. $2x + y = 6$
$2x - y = 2$

c. $x - 2y = -7$
$x + 2y = 5$

EXPLORATION 3 Using Elimination to Solve a System

Work with a partner.

$2x + y = 7$ Equation 1
$x + 5y = 17$ Equation 2

a. Can you eliminate a variable by adding or subtracting the equations as they are? If not, what do you need to do to one or both equations so that you can?

b. Solve the system individually. Then exchange solutions with your partner and compare and check the solutions.

Communicate Your Answer

4. How can you use elimination to solve a system of linear equations?

5. When can you add or subtract the equations in a system to solve the system? When do you have to multiply first? Justify your answers with examples.

6. In Exploration 3, why can you multiply an equation in the system by a constant and not change the solution of the system? Explain your reasoning.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Read aloud the Essential Question. Have students work in pairs of mixed language abilities to complete Exploration 1.

Beginning/Advanced Advanced students read aloud the problem setup and instructions in part (a). Beginning students read aloud the elements of the given verbal model. They work together to complete part (a). Advanced students provide the explanation for part (b).

Intermediate/Advanced High Intermediate students read aloud the problem setup. Advanced High students read part (a). They work together to complete part (a) and provide the explanation for part (b).

ELPS 3.E Share information in cooperative learning interactions.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.2.I The student is expected to write systems of two linear equations given a table of values, a graph, and a verbal description.

A.5.C The student is expected to solve systems of two linear equations with two variables for mathematical and real-world problems.

ANSWERS

1. **a.** Equation 1: $x + y = 4.5$; Equation 2: $x + 5y = 16.5$
 b. $4y = 12$; Solve the resulting equation for y. Then substitute the value of y into one of the original equations and solve for x; (1.5, 3); Drinks cost $1.50 and sandwiches cost $3.
2. **a.** (1, −3); yes; *Sample answer:* method 2
 b. (2, 2); yes; *Sample answer:* method 2
 c. (−1, 3); yes; *Sample answer:* method 1
3. **a.** no; *Sample answer:* Multiply each side of Equation 2 by –2.
 b. (2, 3)
4. Multiply, if necessary, one or both equations by a constant so one pair of like terms has the same or opposite coefficients. Add or subtract the equations to eliminate one of the variables. Solve the resulting equation. Then substitute the value of the variable found into one of the original equations and solve for the other variable.
5. when one of the variables has the same or opposite coefficients in both equations; *Sample answer:* $x + 3y = 2$ and $-x + 5y = 7$; when neither of the variables has the same nor opposite coefficients in both equations; *Sample answer:* $3x - 2y = 12$ and $2x - 3y = -15$
6. Multiplication Property of Equality; Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Extra Example 1

Solve the system of linear equations by elimination.

$-2x + 3y = 4$

$2x - y = -8$

The solution is $(-5, -2)$.

5.3 Lesson

What You Will Learn

- Solve systems of linear equations by elimination.
- Use systems of linear equations to solve real-life problems.

Core Vocabulary

Previous

coefficient

Solving Linear Systems by Elimination

Core Concept

Solving a System of Linear Equations by Elimination

Step 1 Multiply, if necessary, one or both equations by a constant so at least one pair of like terms has the same or opposite coefficients.

Step 2 Add or subtract the equations to eliminate one of the variables.

Step 3 Solve the resulting equation.

Step 4 Substitute the value from Step 3 into one of the original equations and solve for the other variable.

You can use elimination to solve a system of equations because replacing one equation in the system with the sum of that equation and a multiple of the other produces a system that has the same solution. Here is why.

System 1

$a = b$ Equation 1

$c = d$ Equation 2

System 2

$a + kc = b + kd$ Equation 3

$c = d$ Equation 2

Consider System 1. In this system, a and c are algebraic expressions, and b and d are constants. Begin by multiplying each side of Equation 2 by a constant k. By the Multiplication Property of Equality, $kc = kd$. You can rewrite Equation 1 as Equation 3 by adding kc on the left and kd on the right. You can rewrite Equation 3 as Equation 1 by subtracting kc on the left and kd on the right. Because you can rewrite either system as the other, System 1 and System 2 have the same solution.

EXAMPLE 1 Solving a System of Linear Equations by Elimination

Solve the system of linear equations by elimination.

$3x + 2y = 4$ Equation 1

$3x - 2y = -4$ Equation 2

SOLUTION

Step 1 Because the coefficients of the y-terms are opposites, you do not need to multiply either equation by a constant.

Step 2 Add the equations.

$3x + 2y = 4$ Equation 1

$3x - 2y = -4$ Equation 2

$6x = 0$ Add the equations.

Step 3 Solve for x.

$6x = 0$ Resulting equation from Step 2

$x = 0$ Divide each side by 6.

Step 4 Substitute 0 for x in one of the original equations and solve for y.

$3x + 2y = 4$ Equation 1

$3(0) + 2y = 4$ Substitute 0 for x.

$y = 2$ Solve for y.

▶ The solution is $(0, 2)$.

Check

Equation 1

$3x + 2y = 4$

$3(0) + 2(2) \stackrel{?}{=} 4$

$4 = 4$ ✓

Equation 2

$3x - 2y = -4$

$3(0) - 2(2) \stackrel{?}{=} -4$

$-4 = -4$ ✓

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Pair Activity

Pair each English learner with an English speaker. Ask one student to solve a system of equations by elimination and the other student to describe the steps in the process. Have students alternate solving and describing the process as they continue to solve several systems of equations.

EXAMPLE 2 Solving a System of Linear Equations by Elimination

Solve the system of linear equations by elimination.

$-10x + 3y = 1$ Equation 1

$-5x - 6y = 23$ Equation 2

SOLUTION

Step 1 Multiply Equation 2 by -2 so that the coefficients of the x-terms are opposites.

$$\begin{aligned} -10x + 3y &= 1 \\ -5x - 6y &= 23 \end{aligned} \quad \xrightarrow{\text{Multiply by } -2.} \quad \begin{aligned} -10x + 3y &= 1 \\ 10x + 12y &= -46 \end{aligned}$$

Equation 1

Revised Equation 2

Step 2 Add the equations.

$$\begin{aligned} -10x + 3y &= 1 \\ 10x + 12y &= -46 \\ \hline 15y &= -45 \end{aligned}$$

Equation 1

Revised Equation 2

Add the equations.

Step 3 Solve for y.

$15y = -45$ Resulting equation from Step 2

$y = -3$ Divide each side by 15.

Step 4 Substitute -3 for y in one of the original equations and solve for x.

$-5x - 6y = 23$ Equation 2

$-5x - 6(-3) = 23$ Substitute -3 for y.

$-5x + 18 = 23$ Multiply.

$-5x = 5$ Subtract 18 from each side.

$x = -1$ Divide each side by -5.

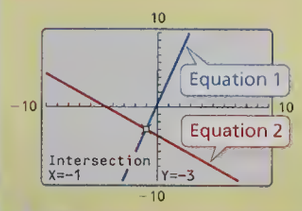
▶ The solution is $(-1, -3)$.

ANOTHER WAY

To use subtraction to eliminate one of the variables, multiply Equation 2 by 2 and then subtract the equations.

$$\begin{aligned} -10x + 3y &= 1 \\ -(-10x - 12y &= 46) \\ \hline 15y &= -45 \end{aligned}$$

Check



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the system of linear equations by elimination. Check your solution.

1. $3x + 2y = 7$, $-3x + 4y = 5$
2. $x - 3y = 24$, $3x + y = 12$
3. $x + 4y = 22$, $4x + y = 13$

Concept Summary

Methods for Solving Systems of Linear Equations

Method	When to Use
Graphing *(Lesson 5.1)*	To estimate solutions
Substitution *(Lesson 5.2)*	When one of the variables in one of the equations has a coefficient of 1 or -1
Elimination *(Lesson 5.3)*	When at least one pair of like terms has the same or opposite coefficients
Elimination (Multiply First) *(Lesson 5.3)*	When one of the variables cannot be eliminated by adding or subtracting the equations

Extra Example 2

Solve the system of linear equations by elimination.

$3x + 2y = -2$

$-6x - 5y = 2$

The solution is $(-2, 2)$.

MONITORING PROGRESS ANSWERS

1. $(1, 2)$
2. $(6, -6)$
3. $(2, 5)$

Extra Example 3

The graph represents the Candle Shop's monthly store and online sales. During what month were the sales equal? What were the store and online sales for that month?



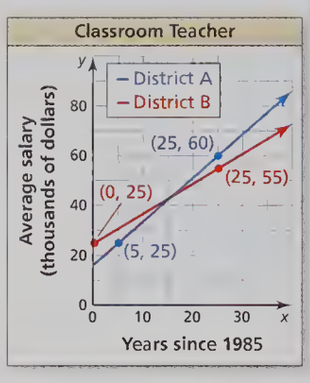
May; $28,000

MONITORING PROGRESS ANSWERS

4. *Sample answer:* $-x + 2y = 16$ and $-3x + y = -8$; (6.4, 11.2)

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**



The graph represents the average salaries of classroom teachers in two school districts. During what year were the average salaries in the two districts equal? What was the average salary in both districts in that year?

SOLUTION

1. **Understand the Problem** You know two points on each line in the graph. You are asked to determine the year in which the average salaries were equal and then determine the average salary in that year.
2. **Make a Plan** Use the points in the graph to write a system of linear equations. Then solve the system of linear equations.
3. **Solve the Problem** Find the slope of each line.

District A: $m = \frac{60 - 25}{25 - 5} = \frac{35}{20} = \frac{7}{4}$ District B: $m = \frac{55 - 25}{25 - 0} = \frac{30}{25} = \frac{6}{5}$

Use each slope and a point on each line to write equations of the lines.

District A		District B
$y - y_1 = m(x - x_1)$	Write the point-slope form.	$y - y_1 = m(x - x_1)$
$y - 25 = \frac{7}{4}(x - 5)$	Substitute for m, x_1, and y_1.	$y - 25 = \frac{6}{5}(x - 0)$
$-7x + 4y = 65$	Write in standard form.	$-6x + 5y = 125$

System $-7x + 4y = 65$ Equation 1

$-6x + 5y = 125$ Equation 2

Step 1 Multiply Equation 1 by -5. Multiply Equation 2 by 4.

$-7x + 4y = 65$ **Multiply by −5.** $35x - 20y = -325$ Revised Equation 1

$-6x + 5y = 125$ **Multiply by 4.** $-24x + 20y = 500$ Revised Equation 2

Step 2 Add the equations.

$$\begin{array}{rrl} 35x - 20y = & -325 & \text{Revised Equation 1} \\ -24x + 20y = & 500 & \text{Revised Equation 2} \\ \hline 11x = & 175 & \text{Add the equations.} \end{array}$$

Step 3 Solving the equation $11x = 175$ gives $x = \frac{175}{11} \approx 15.9$.

Step 4 Substitute $\frac{175}{11}$ for x in one of the original equations and solve for y.

$-7x + 4y = 65$ Equation 1

$-7\left(\frac{175}{11}\right) + 4y = 65$ Substitute $\frac{175}{11}$ for x.

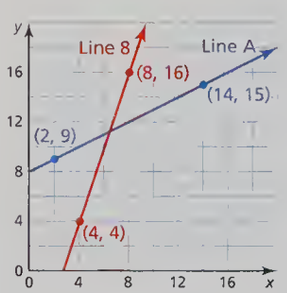
$y \approx 44$ Solve for y.

▶ The solution is about (15.9, 44). Because $x \approx 15.9$ corresponds to the year 2000, the average salary in both districts was about $44,000 in 2000.

4. **Look Back** Using the graph, the point of intersection appears to be about (15, 45). So, the solution of (15.9, 44) is reasonable.

STUDY TIP

In Example 3, both equations are multiplied by a constant so that the coefficients of the y-terms are opposites.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

4. Write and solve a system of linear equations represented by the graph at the left.

Closure

- **Example/Non-example:** Students have learned to solve systems of linear equations by graphing, substitution, and elimination. Although any linear system can be solved by any of the three methods, the approach selected usually depends on what the system looks like. For each of the three methods, have partners write a system that they would (example) and would not (non-example) solve using that method. When presented, listen for justification of why students would or would not use that method for the system written. Give feedback as needed.

5.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **OPEN-ENDED** Give an example of a system of linear equations that can be solved by first adding the equations to eliminate one variable.

2. **WRITING** Explain how to solve the system of linear equations by elimination.

$$\begin{aligned} 2x - 3y &= -4 \quad \text{Equation 1} \\ -5x + 9y &= 7 \quad \text{Equation 2} \end{aligned}$$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the system of linear equations by elimination. Check your solution. *(See Example 1.)*

3. $x + 2y = 13$
 $-x + y = 5$

4. $9x + y = 2$
 $-4x - y = -17$

5. $5x + 6y = 50$
 $x - 6y = -26$

6. $-x + y = 4$
 $x + 3y = 4$

7. $-3x - 5y = -7$
 $-4x + 5y = 14$

8. $4x - 9y = -21$
 $-4x - 3y = 9$

9. $-y - 10 = 6x$
 $5x + y = -10$

10. $3x - 30 = y$
 $7y - 6 = 3x$

In Exercises 11–18, solve the system of linear equations by elimination. Check your solution. *(See Examples 2 and 3.)*

11. $x + y = 2$
 $2x + 7y = 9$

12. $8x - 5y = 11$
 $4x - 3y = 5$

13. $11x - 20y = 28$
 $3x + 4y = 36$

14. $10x - 9y = 46$
 $-2x + 3y = 10$

15. $4x - 3y = 8$
 $5x - 2y = -11$

16. $-2x - 5y = 9$
 $3x + 11y = 4$

17. $9x + 2y = 39$
 $6x + 13y = -9$

18. $12x - 7y = -2$
 $8x + 11y = 30$

19. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $5x - 7y = 16$ and $x + 7y = 8$.

✗

$$\begin{aligned} 5x - 7y &= 16 \\ x + 7y &= 8 \\ \hline 4x &= 24 \\ x &= 6 \end{aligned}$$

20. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $4x + 3y = 8$ and $x - 2y = -13$.

✗

$4x + 3y = 8$

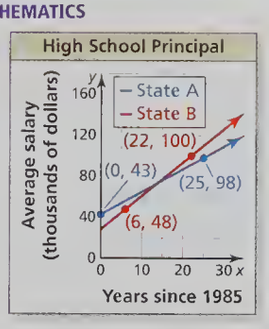
$x - 2y = -13$ **Multiply by −4.** $\rightarrow$

$$\begin{aligned} 4x + 3y &= 8 \\ -4x + 8y &= -13 \\ \hline 11y &= -5 \\ y &= \frac{-5}{11} \end{aligned}$$

21. **MODELING WITH MATHEMATICS** A service center charges a fee of x dollars for an oil change plus y dollars per quart of oil used. A sample of its sales record is shown. Write a system of linear equations that represents this situation. Find the fee and cost per quart of oil.

	A	B	C
1	Customer	Oil Tank Size (quarts)	Total Cost
2	A	5	$22.45
3	B	7	$25.45

22. **MODELING WITH MATHEMATICS** The graph represents the average salaries of high school principals in two states. During what year were the average salaries in the two states equal? What was the average salary in both states in that year?



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–21 odd, 22, 23, 28, 29, 36–42

Average: 1, 2–28 even, 29, 34, 36–42

Advanced: 1, 2, 8, 10, 16–26 even, 27–42

HOMEWORK CHECK

Basic: 3, 11, 15, 21

Average: 4, 12, 16, 22

Advanced: 8, 16, 18, 22

ANSWERS

1. *Sample answer:* $x + 3y = 5$ and $-x + 4y = 10$
2. *Sample answer:* Multiply each side of Equation 1 by 3. Add the equations to eliminate y. Solve the resulting equation for x. Substitute the value of x into one of the original equations and solve for y.
3. $(1, 6)$
4. $(-3, 29)$
5. $(4, 5)$
6. $(-2, 2)$
7. $(-1, 2)$
8. $(-3, 1)$
9. $(0, -10)$
10. $(12, 6)$
11. $(1, 1)$
12. $(2, 1)$
13. $(8, 3)$
14. $(19, 16)$
15. $(-7, -12)$
16. $(-17, 5)$
17. $(5, -3)$
18. $(1, 2)$
19. $5x + x \neq 4x$; $6x = 24$, $x = 4$
20. The right side of the second equation did not get multiplied by –4; $-4x + 8y = 52$, $11y = 60$, $y = \frac{60}{11}$
21. $x + 5y = 22.45$ and $x + 7y = 25.45$; \$14.95 fee, \$1.50 per quart of oil
22. 1998; about \$73,400

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

23. $(2, -1)$; *Sample answer:* elimination because y has the same coefficient in both equations

24. $(-5, -3)$; *Sample answer:* substitution because the second equation is already solved for x

25. $(4, 6)$; *Sample answer:* substitution because the second equation is already solved for y

26. $\left(\frac{1}{3}, -\frac{2}{3}\right)$; *Sample answer:* substitution because y has a coefficient of 1 in the first equation

27. 4 and -4; These values yield x terms with either the same or opposite coefficients.

28. **a.** *Sample answer:* 10 breakfast, 15 lunch

b. $y = x + 5$ and $x + y = 25$

c. Solve the linear system in part (b) using substitution and compare the solution to the answers in part (a).

29. no; *Sample answer:* The system $8x - 5y = 11$ and $4x - 3y = 5$ can be solved by elimination in fewer steps than it can be solved by substitution.

30. *Sample answer:* $3x + 4y = 12$ and $3x - 4y = 6$

31. **a.** 2ℓ and $2w = 18$ and $6\ell + 4w = 46$; length: 5 in.; width: 4 in.

b. length: 15 in.; width: 8 in.

32–42. See Additional Answers.

In Exercises 23–26, solve the system of linear equations using any method. Explain why you chose the method.

23. $3x + 2y = 4$
$2y = 8 - 5x$

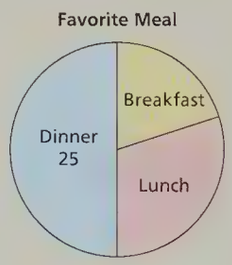
24. $-6y + 2 = -4x$
$y - 2 = x$

25. $y - x = 2$
$y = -\frac{1}{4}x + 7$

26. $3x + y = \frac{1}{3}$
$2x - 3y = \frac{8}{3}$

27. WRITING For what values of a can you solve the linear system $ax + 3y = 2$ and $4x + 5y = 6$ by elimination without multiplying first? Explain.

28. HOW DO YOU SEE IT? The circle graph shows the results of a survey in which 50 students were asked about their favorite meal.



a. Estimate the numbers of students who chose breakfast and lunch.

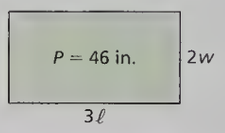
b. The number of students who chose lunch was 5 more than the number of students who chose breakfast. Write a system of linear equations that represents the numbers of students who chose breakfast and lunch.

c. Explain how you can solve the linear system in part (b) to check your answers in part (a).

29. MAKING AN ARGUMENT Your friend says that any system of equations that can be solved by elimination can be solved by substitution in an equal or fewer number of steps. Is your friend correct? Explain.

30. THOUGHT PROVOKING Write a system of linear equations that can be added to eliminate a variable *or* subtracted to eliminate a variable.

31. MATHEMATICAL CONNECTIONS A rectangle has a perimeter of 18 inches. A new rectangle is formed by doubling the width w and tripling the length ℓ, as shown. The new rectangle has a perimeter P of 46 inches.



a. Write and solve a system of linear equations to find the length and width of the original rectangle.

b. Find the length and width of the new rectangle.

32. CRITICAL THINKING Refer to the discussion of System 1 and System 2 on page 232. Without solving, explain why the two systems shown have the same solution.

System 1		System 2	
$3x - 2y = 8$	Equation 1	$5x = 20$	Equation 3
$x + y = 6$	Equation 2	$x + y = 6$	Equation 2

33. PROBLEM SOLVING You are making 6 quarts of fruit punch for a party. You have bottles of 100% fruit juice and 20% fruit juice. How many quarts of each type of juice should you mix to make 6 quarts of 80% fruit juice?

34. PROBLEM SOLVING A motorboat takes 40 minutes to travel 20 miles downstream. The return trip takes 60 minutes. What is the speed of the current?

35. CRITICAL THINKING Solve for x, y, and z in the system of equations. Explain your steps.

$x + 7y + 3z = 29$ Equation 1
$3z + x - 2y = -7$ Equation 2
$5y = 10 - 2x$ Equation 3

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*. *(Section 1.3)*

36. $5d - 8 = 1 + 5d$

37. $9 + 4t = 12 - 4t$

38. $3n + 2 = 2(n - 3)$

39. $-3(4 - 2v) = 6v - 12$

Write an equation of the line that passes through the given point and is parallel to the given line. *(Section 4.4)*

40. $(4, -1)$; $y = -2x + 7$

41. $(0, 6)$; $y = 5x - 3$

42. $(-5, -2)$; $y = \frac{2}{3}x + 1$

Mini-Assessment

Solve the system of linear equations by elimination.

1. $3x + y = 14$
$4x - y = 0$
$(2, 8)$

2. $2x + y = 7$
$-3x - 2y = -9$
$(5, -3)$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

5.4 Solving Special Systems of Linear Equations

Texas Essential Knowledge and Skills
A.2.I
A.3.F
A.5.C

Essential Question Can a system of linear equations have no solution or infinitely many solutions?

EXPLORATION 1 Using a Table to Solve a System

Work with a partner. You invest $450 for equipment to make skateboards. The materials for each skateboard cost $20. You sell each skateboard for $20.

a. Write the cost and revenue equations. Then copy and complete the table for your cost C and your revenue R.

x (skateboards)	0	1	2	3	4	5	6	7	8	9	10
C (dollars)											
R (dollars)											

APPLYING MATHEMATICS
To be proficient in math, you need to interpret mathematical results in real-life contexts.

b. When will your company break even? What is wrong?

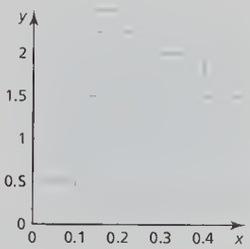
EXPLORATION 2 Writing and Analyzing a System

Work with a partner. A necklace and matching bracelet have two types of beads. The necklace has 40 small beads and 6 large beads and weighs 10 grams. The bracelet has 20 small beads and 3 large beads and weighs 5 grams. The threads holding the beads have no significant weight.

a. Write a system of linear equations that represents the situation. Let x be the weight (in grams) of a small bead and let y be the weight (in grams) of a large bead.

b. Graph the system in the coordinate plane shown. What do you notice about the two lines?

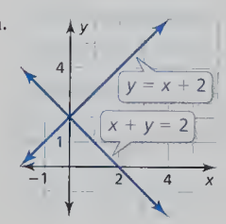
c. Can you find the weight of each type of bead? Explain your reasoning.

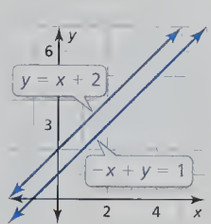


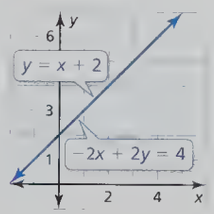
Communicate Your Answer

3. Can a system of linear equations have no solution or infinitely many solutions? Give examples to support your answers.

4. Does the system of linear equations represented by each graph have *no solution*, *one solution*, or *infinitely many solutions*? Explain.

a.


b.


c.


For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

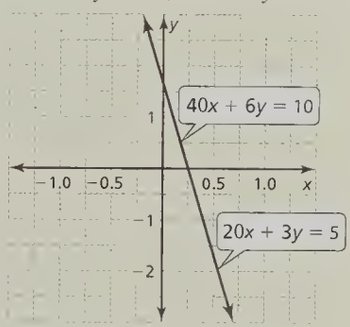
Texas Essential Knowledge and Skills

A.2.I The student is expected to write systems of two linear equations given a table of values, a graph, and a verbal description.

A.3.F The student is expected to graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

A.5.C The student is expected to solve systems of two linear equations with two variables for mathematical and real-world problems.

ANSWERS

1. See Additional Answers.
2. a. $40x + 6y = 10$; $20x + 3y = 5$
 b.


 They are the same line.
 c. no; Because the equations are equivalent, there are infinitely many solutions.
3. yes; *Sample answer:* The system $y = 3x$ and $y = 3x + 1$ has no solution. The system $y = 3x$ and $2y = 6x$ has infinitely many solutions.
4. a. one solution; The lines intersect at one point.
 b. no solution; The lines do not intersect.
 c. infinitely many solutions; The lines are the same line.

Extra Example 1

Solve the system of linear equations.

$y = 3x + 2$

$y = 3x - 1$

The equation $-1 = 2$ is never true. So, the system of linear equations has no solution.

5.4 Lesson

What You Will Learn

- Determine the numbers of solutions of linear systems.
- Use linear systems to solve real-life problems.

Core Vocabulary

Previous

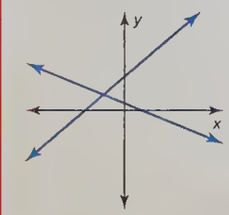
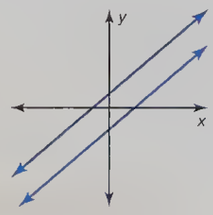
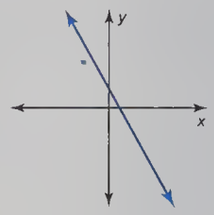
parallel

The Numbers of Solutions of Linear Systems

Core Concept

Solutions of Systems of Linear Equations

A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.

One solution	No solution	Infinitely many solutions
		
The lines intersect.	The lines are parallel.	The lines are the same.

ANOTHER WAY

You can solve some linear systems by inspection. In Example 1, notice you can rewrite the system as

$-2x + y = 1$

$-2x + y = -5$.

This system has no solution because $-2x + y$ cannot be equal to both 1 and -5.

EXAMPLE 1 **Solving a System: No Solution**

Solve the system of linear equations.

$y = 2x + 1$ Equation 1

$y = 2x - 5$ Equation 2

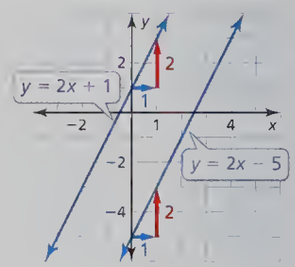
SOLUTION

Method 1 Solve by graphing.

Graph each equation.

The lines have the same slope and different y-intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.



- So, the system of linear equations has no solution.

Method 2 Solve by substitution.

Substitute $2x - 5$ for y in Equation 1.

$y = 2x + 1$ Equation 1

$2x - 5 = 2x + 1$ Substitute $2x - 5$ for y.

$-5 = 1$ ✗ Subtract $2x$ from each side.

- The equation $-5 = 1$ is never true. So, the system of linear equations has no solution.

STUDY TIP

A linear system with no solution is called an *inconsistent system*.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Group Activity

Form groups of English learners and English speakers. Give each group three large index cards and have students write one of the following phrases on each card: *one solution*, *no solution*, and *infinitely many solutions*. One at a time, show students graphs of linear systems that illustrate all three types of solutions. Have each group discuss the graph and then hold up the card with the correct description for the solution of the system.

ANOTHER WAY

You can also solve the linear system by graphing. The lines have the same slope and the same y-intercept. So, the lines are the same, which means all points on the line are solutions of both equations.

EXAMPLE 2 **Solving a System: Infinitely Many Solutions**

Solve the system of linear equations.

$-2x + y = 3$ Equation 1

$-4x + 2y = 6$ Equation 2

SOLUTION

Solve by elimination.

Step 1 Multiply Equation 1 by -2.

$-2x + y = 3$ **Multiply by −2.** $4x - 2y = -6$ Revised Equation 1

$-4x + 2y = 6$ $-4x + 2y = 6$ Equation 2

Step 2 Add the equations.

$4x - 2y = -6$ Revised Equation 1

$-4x + 2y = 6$ Equation 2

$0 = 0$ Add the equations.

▶ The equation $0 = 0$ is always true. So, the solutions are all the points on the line $-2x + y = 3$. The system of linear equations has infinitely many solutions.

STUDY TIP

A linear system with infinitely many solutions is called a *consistent dependent system*.

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Solve the system of linear equations.

1. $x + y = 3$
 $2x + 2y = 6$
2. $y = -x + 3$
 $2x + 2y = 4$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

Cost (dollars)		
Hours	Hall A	Hall B
0	75	100
1	175	200
2	275	300
3	375	400

An athletic director is comparing the costs of renting two banquet halls for an awards banquet. Write a system of linear equations that represents this situation. If the cost patterns continue, will the cost of Hall A ever equal the cost of Hall B?

SOLUTION

Words Total cost = Cost per hour • Number of hours + Initial cost

Variables Let y be the cost (in dollars) and let x be the number of hours.

System $y = 100x + 75$ Equation 1 - Cost of Hall A

$y = 100x + 100$ Equation 2 - Cost of Hall B

The equations are in slope-intercept form. The graphs of the equations have the same slope but different y-intercepts. There is no solution because the lines are parallel.

▶ So, the cost of Hall A will never equal the cost of Hall B.

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3. **WHAT IF?** What happens to the solution in Example 3 when the cost per hour for Hall A is \$125?

Extra Example 2

Solve the system of linear equations.

$x - 3y = 6$

$3x - 9y = 18$

The equation $0 = 0$ is always true. The solutions are all the points on the line $x - 3y = 6$. So, the system of linear equations has infinitely many solutions.

Extra Example 3

A family is comparing the costs of renting two vacation cabins. The costs include a one-time fee of \$50 or \$80 for housekeeping. Write a system of linear equations that represents this situation. If the cost patterns continue, will the cost of Cabin A ever equal the cost of Cabin B?

	Cost (dollars)	
Days	Cabin A	Cabin B
0	50	80
1	175	205
2	300	330
3	425	455

$y = 125x + 50$

$y = 125x + 80$

No, the cost of Cabin A will never equal the cost of Cabin B.

MONITORING PROGRESS ANSWERS

1. infinitely many solutions
2. no solution
3. Hall A and Hall B each cost \$200 for 1 hour of use.

SUPPORTING English Language Learners

Have students read Example 3. Explain that the table and the different parts of the solution help students understand the process that is being described.

Advanced High Read aloud the problem setup.
Intermediate Read aloud the verbal model and the description of the variables in the solution.
Beginning Read aloud the equations of the system.
Advanced Read aloud the conclusion and the answer.

ELPS 4.E Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

Extra Example 4

Use the diagrams in Example 3. The perimeter of the trapezoidal piece of land is 72 kilometers. The perimeter of the rectangular piece of land is 216 kilometers. Write and solve a system of linear equations to find the values of x and y.

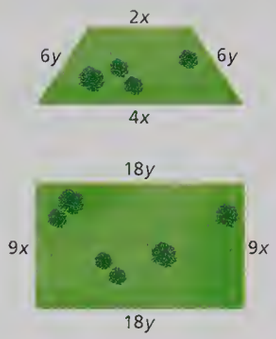
$6x + 12y = 72$

$18x + 36y = 216$

The system has infinitely many solutions.

MONITORING PROGRESS ANSWERS

4. The system has no solution; The lines still have the same slope, but they now have different y-intercepts, so they are parallel.



EXAMPLE 4 Modeling with Mathematics

The perimeter of the trapezoidal piece of land is 48 kilometers. The perimeter of the rectangular piece of land is 144 kilometers. Write and solve a system of linear equations to find the values of x and y.

SOLUTION

1. **Understand the Problem** You know the perimeter of each piece of land and the side lengths in terms of x or y. You are asked to write and solve a system of linear equations to find the values of x and y.
2. **Make a Plan** Use the figures and the definition of perimeter to write a system of linear equations that represents the problem. Then solve the system of linear equations.
3. **Solve the Problem**

Perimeter of trapezoid		Perimeter of rectangle	
$2x + 4x + 6y + 6y = 48$		$9x + 9x + 18y + 18y = 144$	
$6x + 12y = 48$	Equation 1	$18x + 36y = 144$	Equation 2

System $6x + 12y = 48$ Equation 1

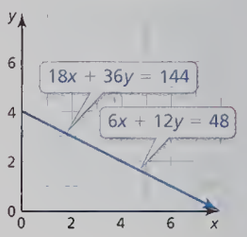
$18x + 36y = 144$ Equation 2

Method 1 Solve by graphing.

Graph each equation.

The lines have the same slope and the same y-intercept. So, the lines are the same.

In this context, x and y must be positive. Because the lines are the same, all the points on the line in Quadrant I are solutions of both equations.



▶ So, the system of linear equations has infinitely many solutions.

Method 2 Solve by elimination.

Multiply Equation 1 by -3 and add the equations.

$6x + 12y = 48$	Multiply by -3.	$-18x - 36y = -144$	Revised Equation 1
$18x + 36y = 144$		$18x + 36y = 144$	Equation 2
		$0 = 0$	Add the equations.

▶ The equation $0 = 0$ is always true. In this context, x and y must be positive. So, the solutions are all the points on the line $6x + 12y = 48$ in Quadrant I. The system of linear equations has infinitely many solutions.

4. **Look Back** Choose a few of the ordered pairs (x, y) that are solutions of Equation 1. You should find that no matter which ordered pairs you choose, they will also be solutions of Equation 2. So, *infinitely many solutions* seems reasonable.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

4. **WHAT IF?** What happens to the solution in Example 4 when the perimeter of the trapezoidal piece of land is 96 kilometers? Explain.

Closure

- **Exit Ticket:**
 Write a system of equations that has no solution. *Sample answer:* $2x + 3y = 4$; $4x + 6y = 12$
 Write a system of equations that has infinitely many solutions. *Sample answer:* $x - 2y = 6$; $-2x + 4y = -12$

5.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

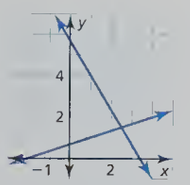
1. **REASONING** Is it possible for a system of linear equations to have exactly two solutions? Explain.
2. **WRITING** Compare the graph of a system of linear equations that has infinitely many solutions and the graph of a system of linear equations that has no solution.

Monitoring Progress and Modeling with Mathematics

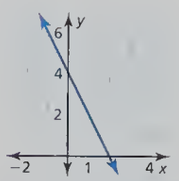
In Exercises 3–8, match the system of linear equations with its graph. Then determine whether the system has *one solution*, *no solution*, or *infinitely many solutions*.

3. $-x + y = 1$
 $x - y = 1$
4. $2x - 2y = 4$
 $-x + y = -2$
5. $2x + y = 4$
 $-4x - 2y = -8$
6. $x - y = 0$
 $5x - 2y = 6$
7. $-2x + 4y = 1$
 $3x - 6y = 9$
8. $5x + 3y = 17$
 $x - 3y = -2$

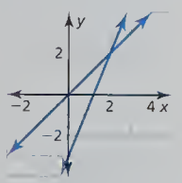
A.



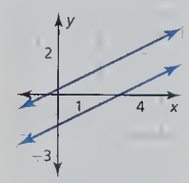
B.



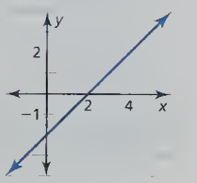
C.



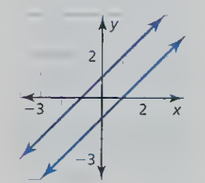
D.



E.



F.



In Exercises 9–16, solve the system of linear equations. *(See Examples 1 and 2.)*

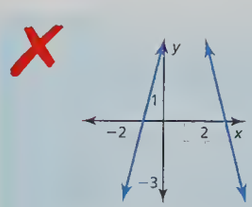
9. $y = -2x - 4$
 $y = 2x - 4$
10. $y = -6x - 8$
 $y = -6x + 8$
11. $3x - y = 6$
 $-3x + y = -6$
12. $-x + 2y = 7$
 $x - 2y = 7$
13. $4x + 4y = -8$
 $-2x - 2y = 4$
14. $15x - 5y = -20$
 $-3x + y = 4$
15. $9x - 15y = 24$
 $6x - 10y = -16$
16. $3x - 2y = -5$
 $4x + 5y = 47$

In Exercises 17–22, use only the slopes and *y*-intercepts of the graphs of the equations to determine whether the system of linear equations has *one solution*, *no solution*, or *infinitely many solutions*. Explain.

17. $y = 7x + 13$
 $-21x + 3y = 39$
18. $y = -6x - 2$
 $12x + 2y = -6$
19. $4x + 3y = 27$
 $4x - 3y = -27$
20. $-7x + 7y = 1$
 $2x - 2y = -18$
21. $-18x + 6y = 24$
 $3x - y = -2$
22. $2x - 2y = 16$
 $3x - 6y = 30$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in solving the system of linear equations.**

23.



$-4x + y = 4$
$4x + y = 12$

The lines do not intersect. So, the system has no solution.

24.

$y = 3x - 8$
$y = 3x - 12$

The lines have the same slope. So, the system has infinitely many solutions.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–25 odd, 26, 30, 32–35

Average: 1, 2–24 even, 25–27, 30, 32–35

Advanced: 1, 2, 8, 14–24 even, 25–35

HOMEWORK CHECK

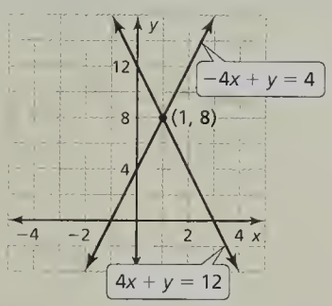
Basic: 3, 9, 17, 25, 26

Average: 4, 10, 14, 25, 26

Advanced: 8, 14, 18, 25, 26

ANSWERS

1. no; Two lines cannot intersect at exactly two points.
2. The graph of a system of linear equations that has infinitely many solutions is a single line and the graph of a system of linear equations that has no solution is two parallel lines.
3. F; no solution
4. E; infinitely many solutions
5. B; infinitely many solutions
6. C; one solution
7. D; no solution
8. A; one solution
9. $(0, -4)$
10. no solution
11. infinitely many solutions
12. no solution
13. infinitely many solutions
14. infinitely many solutions
15. no solution
16. $(3, 7)$
17. infinitely many solutions; The lines have the same slope and the same *y*-intercept, so they are the same line.
18. no solution; The lines have the same slope but different *y*-intercepts, so they are parallel.
19. one solution; The lines have different slopes, so they will intersect.
20. no solution; The lines have the same slope but different *y*-intercepts, so they are parallel.
21. no solution; The lines have the same slope but different *y*-intercepts, so they are parallel.
22. one solution; The lines have different slopes, so they will intersect.
23. The lines are not parallel, so they must intersect.



$(1, 8)$

24. The lines have the same slope but different *y*-intercepts, so the system has no solution.

Dynamic Teaching Tools

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- Dynamic Classroom with Dynamic Investigations

ANSWERS

25. $d = 6t + 3$ and $d = 6t + 1$; no; The system has no solution.

26. $3f + 4a = 6$ and $\frac{9}{2}f + 6a = 9$; infinitely many solutions

27–35. See Additional Answers.

Mini-Assessment

Solve the system of linear equations.

1. $y = 4x - 2$
 $y = 4x - 4$
 no solution

2. $x - y = -2$
 $-4x + 4y = 8$
 infinitely many solutions

3. A homeowner is comparing the costs of hiring two plumbers. The costs include a one-time travel charge of \$50 or \$75. Write a system of linear equations that represents this situation. If the cost patterns continue, will the cost of Plumber A ever equal the cost of Plumber B?

	Cost (dollars)	
Hours	Plumber A	Plumber B
0	50	75
1	100	125
2	150	175
3	200	225

 $y = 50x + 50$; $y = 50x + 75$
 The cost of Plumber A will never equal the cost of Plumber B.

4. A small fruit basket costs \$8 and a large fruit basket costs \$20. The small basket contains 6 apples and 4 pears. The large basket contains 18 apples and 12 pears. Write a system of linear equations to represent this situation. Can you find the unit price of each fruit? Explain. $6x + 4y = 8$; $18x + 12y = 20$; The system has no solution, so you cannot find the unit price of each fruit.

25. MODELING WITH MATHEMATICS The table shows the distances two groups have traveled at different times during a canoeing excursion. The groups continue traveling at their current rates until they reach the same destination. Let d be the distance traveled and t be the time since 1 P.M. Write a system of linear equations that represents this situation. Will Group B catch up to Group A before reaching the destination? Explain. *(See Example 3.)*

	Distance Traveled (miles)			
	1 P.M.	2 P.M.	3 P.M.	4 P.M.
Group A	3	9	15	21
Group B	1	7	13	19

26. MODELING WITH MATHEMATICS A \$6-bag of trail mix contains 3 cups of dried fruit and 4 cups of almonds. A \$9-bag contains $4\frac{1}{2}$ cups of dried fruit and 6 cups of almonds. Write and solve a system of linear equations to find the price of 1 cup of dried fruit and 1 cup of almonds. *(See Example 4.)*

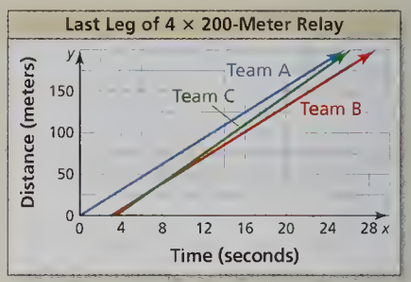
27. PROBLEM SOLVING A train travels from New York City to Washington, D.C., and then back to New York City. The table shows the number of tickets purchased for each leg of the trip. The cost per ticket is the same for each leg of the trip. Is there enough information to determine the cost of one coach ticket? Explain.

Destination	Coach tickets	Business class tickets	Money collected (dollars)
Washington, D.C.	150	80	22,860
New York City	170	100	27,280

28. THOUGHT PROVOKING Write a system of three linear equations in two variables so that any two of the equations have exactly one solution, but the entire system of equations has no solution.

29. REASONING In a system of linear equations, one equation has a slope of 2 and the other equation has a slope of $-\frac{1}{3}$. How many solutions does the system have? Explain.

30. HOW DO YOU SEE IT? The graph shows information about the last leg of a 4 × 200-meter relay for three relay teams. Team A's runner ran about 7.8 meters per second, Team B's runner ran about 7.8 meters per second, and Team C's runner ran about 8.8 meters per second.



a. Estimate the distance at which Team C's runner passed Team B's runner.

b. If the race was longer, could Team C's runner have passed Team A's runner? Explain.

c. If the race was longer, could Team B's runner have passed Team A's runner? Explain.

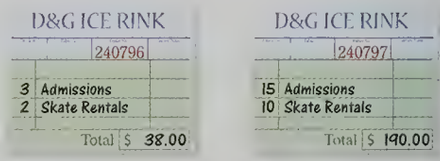
31. ABSTRACT REASONING Consider the system of linear equations $y = ax + 4$ and $y = bx - 2$, where a and b are real numbers. Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

a. The system has infinitely many solutions.

b. The system has no solution.

c. When $a < b$, the system has one solution.

32. MAKING AN ARGUMENT One admission to an ice skating rink costs x dollars, and renting a pair of ice skates costs y dollars. Your friend says she can determine the exact cost of one admission and one skate rental. Is your friend correct? Explain.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations by graphing. *(Section 5.1)*

33. $y = x - 6$
$y = -x + 10$

34. $y = 2x + 3$
$y = -3x - 7$

35. $2x + y = 6$
$3x - 2y = 16$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

5.1–5.4 What Did You Learn?

Core Vocabulary

system of linear equations, *p. 220*

solution of a system of linear equations, *p. 220*

Core Concepts

Section 5.1

Solving a System of Linear Equations by Graphing, *p. 221*

Section 5.2

Solving a System of Linear Equations by Substitution, *p. 226*

Section 5.3

Solving a System of Linear Equations by Elimination, *p. 232*

Section 5.4

Solutions of Systems of Linear Equations, *p. 238*

Mathematical Thinking

1. Describe the given information in Exercise 33 on page 230 and your plan for finding the solution.
2. Describe another real-life situation similar to Exercise 21 on page 235 and the mathematics that you can apply to solve the problem.
3. What question(s) can you ask your friend to help her understand the error in the statement she made in Exercise 32 on page 242?

Study Skills

Analyzing Your Errors

Study Errors

What Happens: You do not study the right material or you do not learn it well enough to remember it on a test without resources such as notes.

How to Avoid This Error: Take a practice test. Work with a study group. Discuss the topics on the test with your teacher. Do not try to learn a whole chapter's worth of material in one night.



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- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

1. The given information is the total number of songs played and the number of pop and hip-hop songs in relation to rock songs. The solution can be found by writing a system of linear equations and then solving the system using the substitution method.
2. *Sample answer:* A store sells candy bars for x dollars and eight-packs of candy bars for y dollars. Person A buys 4 candy bars and 2 eight-packs for $18. Person B buys 3 candy bars and 5 eight-packs for $38. To solve, write and solve a system of linear equations that represents this situation.
3. *Sample answer:* Are the equations derived from the two receipts equations for different lines?

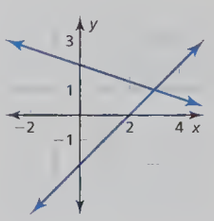
ANSWERS

1. (3, 1)
2. (−2, −2)
3. (0, 1)
4. (−22, −26)
5. (2, −3)
6. (6, 1)
7. (2, 2)
8. (4, −1)
9. (2, 3)
10. no solution
11. (2, 2)
12. infinitely many solutions
13. **a.** $y = 4x + 14$ and $y = 6x + 8$
 b. (3, 26); Both trees will be 26 in. tall after 3 years.
14. **a.** 1 h at 55 mi/h, 2 h at 40 mi/h
 b. 55 mi; 80 mi
15. $7t + 3f = 26$ and $t + f = 6$; 2 touchdowns, 4 field goals

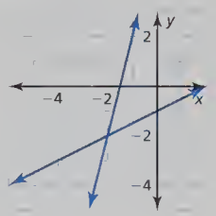
5.1–5.4 Quiz

Use the graph to solve the system of linear equations. Check your solution. *(Section 5.1)*

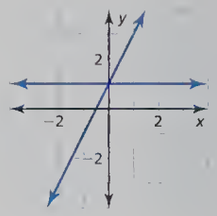
1. $y = -\frac{1}{3}x + 2$
 $y = x - 2$



2. $y = \frac{1}{2}x - 1$
 $y = 4x + 6$



3. $y = 1$
 $y = 2x + 1$



Solve the system of linear equations by substitution. Check your solution. *(Section 5.2)*

4. $y = x - 4$
 $-2x + y = 18$
5. $2y + x = -4$
 $y - x = -5$
6. $3x - 5y = 13$
 $x + 4y = 10$

Solve the system of linear equations by elimination. Check your solution. *(Section 5.3)*

7. $x + y = 4$
 $-3x - y = -8$
8. $x + 3y = 1$
 $5x + 6y = 14$
9. $2x - 3y = -5$
 $5x + 2y = 16$

Solve the system of linear equations. *(Section 5.4)*

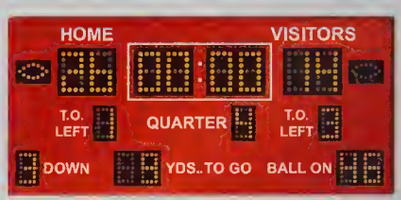
10. $x - y = 1$
 $x - y = 6$
11. $6x + 2y = 16$
 $2x - y = 2$
12. $3x - 3y = -2$
 $-6x + 6y = 4$

13. You plant a spruce tree that grows 4 inches per year and a hemlock tree that grows 6 inches per year. The initial heights are shown. *(Section 5.1)*
 a. Write a system of linear equations that represents this situation.
 b. Solve the system by graphing. Interpret your solution.



14. It takes you 3 hours to drive to a concert 135 miles away. You drive 55 miles per hour on highways and 40 miles per hour on the rest of the roads. *(Section 5.1, Section 5.2, and Section 5.3)*
 a. How much time do you spend driving at each speed?
 b. How many miles do you drive on highways? the rest of the roads?

15. In a football game, all of the home team's points are from 7-point touchdowns and 3-point field goals. The team scores six times. Write and solve a system of linear equations to find the numbers of touchdowns and field goals that the home team scores. *(Section 5.1, Section 5.2, and Section 5.3)*



5.5 Solving Equations by Graphing

Texas Essential Knowledge and Skills
A.5.A

Essential Question How can you use a system of linear equations to solve an equation with variables on both sides?

Previously, you learned how to use algebra to solve equations with variables on both sides. Another way is to use a system of linear equations.

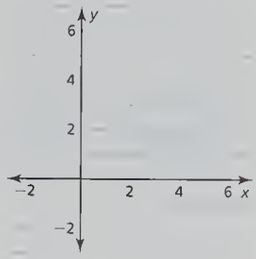
EXPLORATION 1 Solving an Equation by Graphing

Work with a partner. Solve $2x - 1 = -\frac{1}{2}x + 4$ by graphing.

a. Use the left side to write a linear equation. Then use the right side to write another linear equation.

b. Graph the two linear equations from part (a). Find the x-value of the point of intersection. Check that the x-value is the solution of

$$2x - 1 = -\frac{1}{2}x + 4.$$



c. Explain why this "graphical method" works.

SELECTING TOOLS
To be proficient in math, you need to consider the available tools, which may include pencil and paper or a graphing calculator, when solving a mathematical problem.

EXPLORATION 2 Solving Equations Algebraically and Graphically

Work with a partner. Solve each equation using two methods.

Method 1 Use an algebraic method.

Method 2 Use a graphical method.

Is the solution the same using both methods?

a. $\frac{1}{2}x + 4 = -\frac{1}{4}x + 1$

b. $\frac{2}{3}x + 4 = \frac{1}{3}x + 3$

c. $-\frac{2}{3}x - 1 = \frac{1}{3}x - 4$

d. $\frac{4}{5}x + \frac{7}{5} = 3x - 3$

e. $-x + 2.5 = 2x - 0.5$

f. $-3x + 1.5 = x + 1.5$

Communicate Your Answer

3. How can you use a system of linear equations to solve an equation with variables on both sides?

4. Compare the algebraic method and the graphical method for solving a linear equation with variables on both sides. Describe the advantages and disadvantages of each method.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Read aloud the Essential Question and Exploration 1 as students read along. Have students close their books and listen as you reread individual portions of what was read. Ask them to restate in their own words what was said, or have them ask questions to clarify what they don't understand. Then have them complete the exploration in groups of mixed language abilities.

Advanced High Explain the introduction that precedes Exploration 1.
Beginning Explain what *Work with a partner* instructs students to do.
Intermediate Restate the instructions in part (a).
Advanced Restate the instructions in parts (b) and (c).

ELPS 2.D.1 Monitor understanding of spoken language during classroom instruction and interactions.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

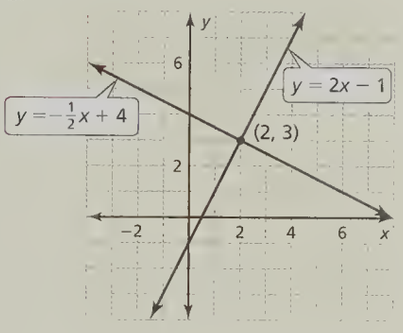
Texas Essential Knowledge and Skills

A.5.A The student is expected to solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ANSWERS

1. a. $y = 2x - 1$; $y = -\frac{1}{2}x + 4$

b.



$x = 2$

c. The two sides of the equation are equal to each other. If you set one side of the equation equal to y, the transitive property allows you to set the other side of the equation equal to y.

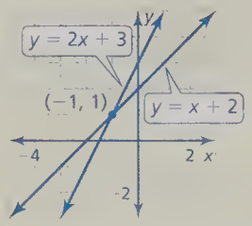
2. a. $x = -4$; yes
b. $x = -3$; yes
c. $x = 3$; yes
d. $x = 2$; yes
e. $x = 1$; yes
f. $x = 0$; yes

3. Write two linear equations setting y equal to each side. Solve the system of linear equations. The x-value of the solution of the system of linear equations is the solution of the equation.

4. The algebraic method will always give the exact solution, but it is difficult to work with fractions as coefficients. The graphical method is easier with fractional coefficients of x because these are the slopes of the lines, but solutions may sometimes be estimates, especially when the solution does not fall on a grid line.

Extra Example 1

Solve $2x + 3 = x + 2$ by graphing. Check your solution.



$x = -1$

MONITORING PROGRESS ANSWERS

1. $x = -2$
2. $x = \frac{1}{2}$

5.5 Lesson

What You Will Learn

- Solve linear equations by graphing.
- Use linear equations to solve real-life problems.

Core Vocabulary

Previous
absolute value equation

Solving Linear Equations by Graphing

You can use a system of linear equations to solve an equation with variables on both sides.

Core Concept

Solving Linear Equations by Graphing

Step 1 To solve the equation $ax + b = cx + d$, write two linear equations.

$$ax + b = cx + d$$

$y = ax + b$ and $y = cx + d$

Step 2 Graph the system of linear equations. The x-value of the solution of the system of linear equations is the solution of the equation $ax + b = cx + d$.

EXAMPLE 1 Solving an Equation by Graphing

Solve $-x + 1 = 2x - 5$ by graphing. Check your solution.

SOLUTION

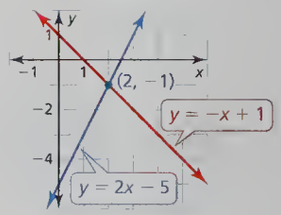
Step 1 Write a system of linear equations using each side of the original equation.

$$-x + 1 = 2x - 5$$

$y = -x + 1$ $\qquad$ $y = 2x - 5$

Step 2 Graph the system.

$y = -x + 1$ Equation 1
$y = 2x - 5$ Equation 2



The graphs intersect at $(2, -1)$.

So, the solution of the equation is $x = 2$.

Check

$$-x + 1 = 2x - 5$$
$$-(2) + 1 \stackrel{?}{=} 2(2) - 5$$
$$-1 = -1 \checkmark$$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation by graphing. Check your solution.

1. $\frac{1}{2}x - 3 = 2x$
2. $-4 + 9x = -3x + 2$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Organization

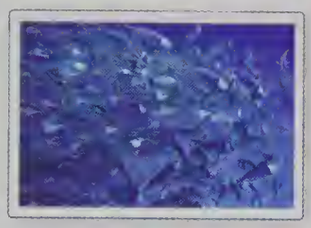
Have students write in their notebooks the steps for solving an equation by graphing a system of related linear equations.

Step 1: Use each side of the original equation to write a system of linear equations.
Step 2: Graph the system.
Step 3: Find the x-value of the point of intersection. Use it to write the solution.

Students should include a note to check their results by substitution into the original equation.

Solving Real-Life Problems

EXAMPLE 2 **Modeling with Mathematics**



You are an ecologist studying the populations of two types of fish in a lake. Use the information in the table to predict when the populations of the two types of fish will be equal.

Type	Current population	Change per year
A	5750	−250
B	2825	400

SOLUTION

1. **Understand the Problem** You know the current population of each type of fish and the annual change in each population. You are asked to determine when the populations of the two types of fish will be equal.
2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation by graphing.
3. **Solve the Problem**

Words

Type A: (Change per year) · Years + Current population = Type B: (Change per year) · Years + Current population

Variable Let x be the number of years.

Equation $-250x + 5750 = 400x + 2825$

Solve the equation by graphing.

Step 1 Write a system of linear equations using each side of the original equation.

$-250x + 5750 = 400x + 2825$

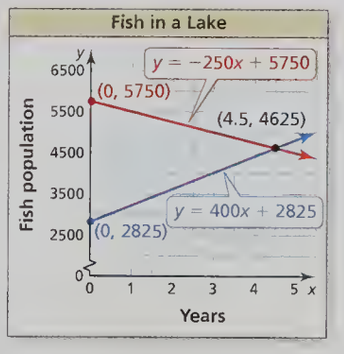
$y = -250x + 5750$ $\qquad$ $y = 400x + 2825$

Step 2 Graph the system.

$y = -250x + 5750$ Equation 1

$y = 400x + 2825$ Equation 2

The graphs appear to intersect at (4.5, 4625). Check this solution in each equation of the linear system, as shown.



Check

$y = -250x + 5750$

$4625 \stackrel{?}{=} -250(4.5) + 5750$

$4625 = 4625$ ✓

$y = 400x + 2825$

$4625 \stackrel{?}{=} 400(4.5) + 2825$

$4625 = 4625$ ✓

▶ So, the populations of the two types of fish will be equal in 4.5 years.

4. **Look Back** To check that your solution is correct, verify that $x = 4.5$ is the solution of the original equation.

$-250(4.5) + 5750 = 400(4.5) + 2825$

$4625 = 4625$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

3. **WHAT IF?** Type C has a current population of 3500 and grows by 500 each year. Predict when the population of Type C will be equal to the population of Type A.

Extra Example 2

You are a biologist studying the populations of two frog species in a swamp. Use the information in the table to predict when the two populations will be equal.

Frog species	Current population	Change per year
A	4250	−300
B	1000	350

The populations of the two species will be equal in 5 years.

MONITORING PROGRESS ANSWERS

3. 3 yr

Extra Example 3

Your family needs to rent a car for a week while on vacation. Company B charges \$3 per mile plus a flat fee of \$150 per week. Company D charges \$4 per mile plus a flat fee of \$75 per week. After how many miles are the total costs the same at Company B and Company D? The total costs are the same after 75 miles.

MONITORING PROGRESS ANSWERS

4. 200 mi



EXAMPLE 3 **Modeling with Mathematics**

Your family needs to rent a car for a week while on vacation. Company A charges \$3.25 per mile plus a flat fee of \$125 per week. Company B charges \$3 per mile plus a flat fee of \$150 per week. After how many miles of travel are the total costs the same at both companies?

SOLUTION

1. **Understand the Problem** You know the costs of renting a car from two companies. You are asked to determine how many miles of travel will result in the same total costs at both companies.
2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation by graphing.
3. **Solve the Problem**

Words

$$\underbrace{\text{Cost per mile} \cdot \text{Miles} + \text{Flat fee}}_{\text{Company A}} = \underbrace{\text{Cost per mile} \cdot \text{Miles} + \text{Flat fee}}_{\text{Company B}}$$

Variable Let x be the number of miles traveled.

Equation $3.25x + 125 = 3x + 150$

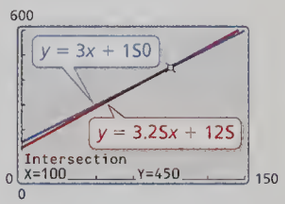
Solve the equation by graphing.

Step 1 Write a system of linear equations using each side of the original equation.

$$3.25x + 125 = 3x + 150$$

$y = 3.25x + 125$ $\qquad$ $y = 3x + 150$

Step 2 Use a graphing calculator to graph the system.



Because the graphs intersect at (100, 450), the solution of the equation is $x = 100$.

▶ So, the total costs are the same after 100 miles.

4. **Look Back** One way to check your solution is to solve the equation algebraically, as shown.

Check

$$3.25x + 125 = 3x + 150$$
$$0.25x + 125 = 150$$
$$0.25x = 25$$
$$x = 100$$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

4. **WHAT IF?** Company C charges \$3.30 per mile plus a flat fee of \$115 per week. After how many miles are the total costs the same at Company A and Company C?

Closure

- **Exit Ticket:** Solve the equation $3x - 4 = \frac{1}{2}x + 1$ by graphing. Check your solution. $x = 2$

5.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

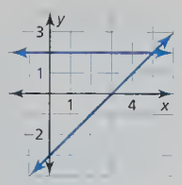
Vocabulary and Core Concept Check

1. **REASONING** The graphs of the equations $y = 3x - 20$ and $y = -2x + 10$ intersect at the point $(6, -2)$. Without solving, find the solution of the equation $3x - 20 = -2x + 10$.
2. **WRITING** Consider the equation $ax + b = cx + d$, where $c = 0$. Can you solve the equation by graphing a system of linear equations? Explain.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, use the graph to solve the equation. Check your solution.

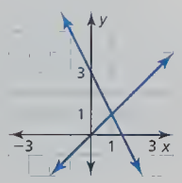
3. $x - 3 = 2$



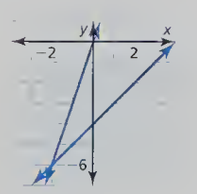
4. $-3 = 4x + 1$



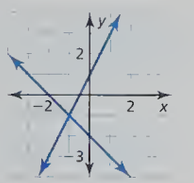
5. $-2x + 3 = x$



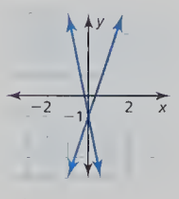
6. $x - 4 = 3x$



7. $2x + 1 = -x - 2$



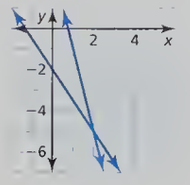
8. $-5x - 1 = 3x - 1$



9. $-x - 1 = \frac{1}{3}x + 3$



10. $-\frac{3}{2}x - 2 = -4x + 3$



In Exercises 11–18, solve the equation by graphing. Check your solution. *(See Example 1.)*

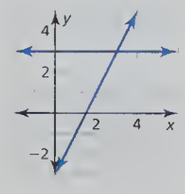
11. $x + 4 = -x$
12. $4x = x + 3$
13. $x + 5 = -2x - 4$
14. $-2x + 6 = 5x - 1$
15. $\frac{1}{2}x - 2 = 9 - 5x$
16. $-5 + \frac{1}{4}x = 3x + 6$
17. $5x - 7 = 2(x + 1)$
18. $-6(x + 4) = -3x - 6$

In Exercises 19–24, solve the equation by graphing. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*.

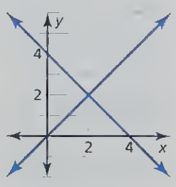
19. $3x - 1 = -x + 7$
20. $5x - 4 = 5x + 1$
21. $-4(2 - x) = 4x - 8$
22. $-2x - 3 = 2(x - 2)$
23. $-x - 5 = -\frac{1}{3}(3x + 5)$
24. $\frac{1}{2}(8x + 3) = 4x + \frac{3}{2}$

In Exercises 25–28, write an equation that has the same solution as the linear system represented by the graph.

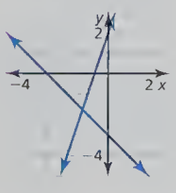
25.



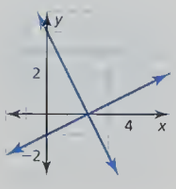
26.



27.



28.



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–31 odd, 34, 36, 40–47

Average: 1, 2–36 even, 37, 40–47

Advanced: 1, 2, 10, 16, 18, 24, 28, 29–47

HOMEWORK CHECK

Basic: 3, 11, 19, 31

Average: 4, 12, 20, 32

Advanced: 10, 16, 24, 32

ANSWERS

1. $x = 6$
2. yes; Graph the equations $y = ax + b$ and $y = d$. The solution is the x-value of the point of intersection.
3. $x = 5$
4. $x = -1$
5. $x = 1$
6. $x = -2$
7. $x = -1$
8. $x = 0$
9. $x = -3$
10. $x = 2$
11. $x = -2$
12. $x = 1$
13. $x = -3$
14. $x = 1$
15. $x = 2$
16. $x = -4$
17. $x = 3$
18. $x = -6$
19. $x = 2$; one solution
20. no solution
21. all real numbers; infinitely many solutions
22. $x = \frac{1}{4}$; one solution
23. no solution
24. all real numbers; infinitely many solutions
25. $2x - 3 = 3$
26. $-x + 4 = x$
27. $-x - 3 = 3x + 2$
28. $-2x + 4 = \frac{1}{2}x - 1$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

29. $x = -2$

30. $x = 1.8$

31. 75 guests

32. 4 human years

33. 30 sec

34. no; Only the x-coordinate is the solution, so the solution is $x = 4$.

35. *Sample answer:* $m = 1, b = 8$

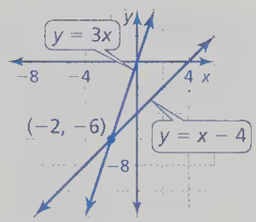
36. **a.** *Sample answer:* (7, 5.5)

b. *Sample answer:* After about 7 years, the company's revenue and expenses will each be about \$5.5 million.

37–47. See Additional Answers.

Mini-Assessment

1. Solve $x - 4 = 3x$ by graphing. Check your solution.



$x = -2$

2. You are an ecologist studying the populations of two bird species in a forest. Use the information in the table to predict when the two populations will be equal.

Bird species	Current population	Change per year
A	1200	50
B	1270	30

3.5 years

3. You need to hire an electrician to install new wiring in your house. Electrician A charges \$75 plus \$45 per hour. Electrician B charges \$100 plus \$40 per hour. For how many hours of work are the total costs the same for both electricians? 5 hours

USING TOOLS **In Exercises 29 and 30, use a graphing calculator to solve the equation.**

29. $0.7x + 0.5 = -0.2x - 1.3$

30. $2.1x + 0.6 = -1.4x + 6.9$

31. MODELING WITH MATHEMATICS You need to hire a catering company to serve meals to guests at a wedding reception. Company A charges \$500 plus \$20 per guest. Company B charges \$800 plus \$16 per guest. For how many guests are the total costs the same at both companies? *(See Examples 2 and 3.)*

32. MODELING WITH MATHEMATICS Your dog is 16 years old in dog years. Your cat is 28 years old in cat years. For every human year, your dog ages by 7 dog years and your cat ages by 4 cat years. In how many human years will both pets be the same age in their respective types of years?



33. MODELING WITH MATHEMATICS You and a friend race across a field to a fence. Your friend has a 50-meter head start. The equations shown represent you and your friend's distances d (in meters) from the fence t seconds after the race begins. Find the time at which you catch up to your friend.

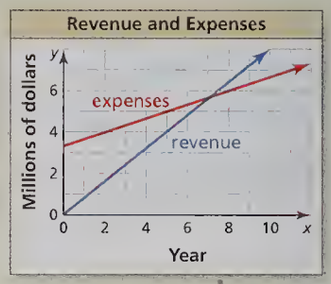
You: $d = -5t + 200$

Your friend: $d = -3\frac{1}{3}t + 150$

34. MAKING AN ARGUMENT The graphs of $y = -x + 4$ and $y = 2x - 8$ intersect at the point (4, 0). So, your friend says the solution of the equation $-x + 4 = 2x - 8$ is (4, 0). Is your friend correct? Explain.

35. OPEN-ENDED Find values for m and b so that the solution of the equation $mx + b = -2x - 1$ is $x = -3$.

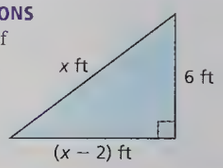
36. HOW DO YOU SEE IT? The graph shows the total revenue and expenses of a company x years after it opens for business.



a. Estimate the point of intersection of the graphs.

b. Interpret your answer in part (a).

37. MATHEMATICAL CONNECTIONS The value of the perimeter of the triangle (in feet) is equal to the value of the area of the triangle (in square feet). Use a graph to find x.



38. THOUGHT PROVOKING A car has an initial value of \$20,000 and decreases in value at a rate of \$1500 per year. Describe a different car that will be worth the same amount as this car in exactly 5 years. Specify the initial value and the rate at which the value decreases.

39. ABSTRACT REASONING Use a graph to determine the sign of the solution of the equation $ax + b = cx + d$ in each situation.

a. $0 < b < d$ and $a < c$ **b.** $d < b < 0$ and $a < c$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph the inequality. *(Section 2.1)*

40. $y > 5$ **41.** $x \leq -2$ **42.** $n \geq 9$ **43.** $c < -6$

Use the graphs of f and g to describe the transformation from the graph of f to the graph of g. *(Section 3.7)*

44. $f(x) = x - 5$; $g(x) = f(x + 2)$ **45.** $f(x) = 6x$; $g(x) = -f(x)$

46. $f(x) = -2x + 1$; $g(x) = f(4x)$ **47.** $f(x) = \frac{1}{2}x - 2$; $g(x) = f(x - 1)$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

5.6 Linear Inequalities in Two Variables

Texas Essential Knowledge and Skills
A.2.H
A.3.D

Essential Question

Essential Question How can you write and graph a linear inequality in two variables?

A **solution of a linear inequality in two variables** is an ordered pair (x, y) that makes the inequality true. The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

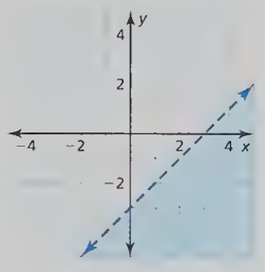
EXPLORATION 1 Writing a Linear Inequality in Two Variables

Work with a partner.

a. Write an equation represented by the dashed line.

b. The solutions of an inequality are represented by the shaded region. In words, describe the solutions of the inequality.

c. Write an inequality represented by the graph. Which inequality symbol did you use? Explain your reasoning.

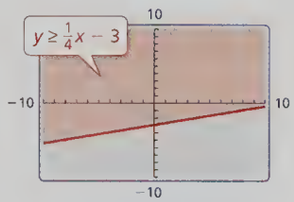


EXPLORATION 2 Using a Graphing Calculator

Work with a partner. Use a graphing calculator to graph $y \geq \frac{1}{4}x - 3$.

a. Enter the equation $y = \frac{1}{4}x - 3$ into your calculator.

b. The inequality has the symbol $\geq$. So, the region to be shaded is above the graph of $y = \frac{1}{4}x - 3$, as shown. Verify this by testing a point in this region, such as (0, 0), to make sure it is a solution of the inequality.



Because the inequality symbol is *greater than or equal to*, the line is solid and not dashed. Some graphing calculators always use a solid line when graphing inequalities. In this case, you have to determine whether the line should be solid or dashed, based on the inequality symbol used in the original inequality.

SELECTING TOOLS
To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

EXPLORATION 3 Graphing Linear Inequalities in Two Variables

Work with a partner. Graph each linear inequality in two variables. Explain your steps. Use a graphing calculator to check your graphs.

a. $y > x + 5$ b. $y \leq -\frac{1}{2}x + 1$ c. $y \geq -x - 5$

Communicate Your Answer

4. How can you write and graph a linear inequality in two variables?
5. Give an example of a real-life situation that can be modeled using a linear inequality in two variables.

Dynamic Teaching Tools
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- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

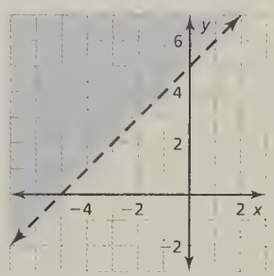
Texas Essential Knowledge and Skills

A.2.H The student is expected to write linear inequalities in two variables given a table of values, a graph, and a verbal description.

A.3.D The student is expected to graph the solution set of linear inequalities in two variables on the coordinate plane.

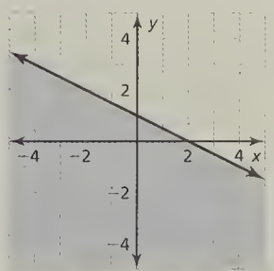
ANSWERS

1. a. $y = x - 3$
 b. all ordered pairs below the graph of $y = x - 3$
 c. $y < x - 3$; $<$; *Sample answer:* The point (4, 0) is in the shaded region, and to make the inequality true for that point the $<$ symbol is needed.
2. $0 \geq -3$
3. a.



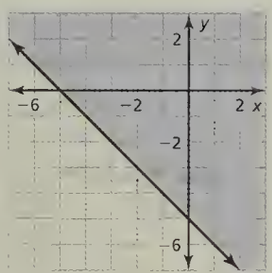
Sample answer: Graph $y = x + 5$ with a dashed line. Test the point (0, 0), which does not make the inequality true. Shade the half-plane that does not contain the point (0, 0).

b.



Sample answer: Graph $y = -\frac{1}{2}x + 1$ with a solid line. Test the point (0, 0), which does make the inequality true. Shade the half-plane that contains the point (0, 0).

3. c.



Sample answer: Graph $y = -x - 5$ with a solid line. Test the point (0, 0), which does make the inequality true. Shade the half-plane that contains the point (0, 0).

4. Write an equation represented by the boundary line. Use $<$ or $>$ for a dashed line. Use $\leq$ or $\geq$ for a solid line. Use $<$ or $\leq$ when the half-plane below the boundary line is shaded. Use $>$ or $\geq$ when the half-plane above the boundary line is shaded; Graph the boundary line for the inequality. Test a point that is not on the boundary line to determine whether it is a solution of the inequality. When the test point is a solution, shade the half-plane that contains the point. When the test point is not a solution, shade the half-plane that does not contain the point.
5. See Additional Answers.

Extra Example 1

Tell whether the ordered pair is a solution of the inequality.

a. $3x - y < 2$; $(-2, 2)$

$(-2, 2)$ is a solution of the inequality.

b. $4x - y \geq 5$; $(1, 3)$

$(1, 3)$ is *not* a solution of the inequality.

MONITORING PROGRESS ANSWERS

1. no
2. no
3. yes
4. yes

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

5.6 Lesson

Core Vocabulary

linear inequality in two variables, *p. 252*
solution of a linear inequality in two variables, *p. 252*
graph of a linear inequality, *p. 252*
half-planes, *p. 252*

Previous
ordered pair

What You Will Learn

- Check solutions of linear inequalities.
- Graph linear inequalities in two variables.
- Write linear inequalities in two variables.
- Use linear inequalities to solve real-life problems.

Linear Inequalities

A **linear inequality in two variables**, x and y, can be written as

$$ax + by < c \qquad ax + by \leq c \qquad ax + by > c \qquad ax + by \geq c$$

where a, b, and c are real numbers. A **solution of a linear inequality in two variables** is an ordered pair (x, y) that makes the inequality true.

EXAMPLE 1 Checking Solutions

Tell whether the ordered pair is a solution of the inequality.

a. $2x + y < -3$; $(-1, 9)$ b. $x - 3y \geq 8$; $(2, -2)$

SOLUTION

a. $2x + y < -3$ Write the inequality.

$2(-1) + 9 \stackrel{?}{<} -3$ Substitute -1 for x and 9 for y.

$7 \not< -3$ ✗ Simplify. 7 is *not* less than -3.

▶ So, $(-1, 9)$ is *not* a solution of the inequality.

b. $x - 3y \geq 8$ Write the inequality.

$2 - 3(-2) \stackrel{?}{\geq} 8$ Substitute 2 for x and -2 for y.

$8 \geq 8$ ✓ Simplify. 8 is equal to 8.

▶ So, $(2, -2)$ is a solution of the inequality.

Monitoring Progress

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Tell whether the ordered pair is a solution of the inequality.

1. $x + y > 0$; $(-2, 2)$
2. $4x - y \geq 5$; $(0, 0)$
3. $5x - 2y \leq -1$; $(-4, -1)$
4. $-2x - 3y < 15$; $(5, -7)$

Graphing Linear Inequalities in Two Variables

The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

READING

A dashed boundary line means that points on the line are *not* solutions. A solid boundary line means that points on the line are solutions.

All solutions of $y < 2x$ lie on one side of the *boundary line* $y = 2x$.



The boundary line divides the coordinate plane into two **half-planes**. The shaded half-plane is the graph of $y < 2x$.

SUPPORTING English Language Learners

Point out that What You Will Learn states what is taught in the section, the yellow highlighting lets you know what vocabulary is most important, the red side comments explain the steps of the process, and the graph helps you visualize what is being described.

Beginning Use short phrases to describe What You Will Learn. (Possibilities: check solutions, graph/write linear inequalities, solve real-life problems)
Intermediate State the Core Vocabulary used in context on the page.
Advanced Explain the process described by the red side comments.
Advanced High Explain what the graph shows.

ELPS 4.F.3 Use visual and contextual support to develop vocabulary needed to comprehend increasingly challenging language.

Core Concept

Graphing a Linear Inequality in Two Variables

Step 1 Graph the boundary line for the inequality. Use a dashed line for $<$ or $>$. Use a solid line for $\leq$ or $\geq$.

Step 2 Test a point that is not on the boundary line to determine whether it is a solution of the inequality.

Step 3 When the test point is a solution, shade the half-plane that contains the point. When the test point is *not* a solution, shade the half-plane that does *not* contain the point.

STUDY TIP

It is often convenient to use the origin as a test point. However, you must choose a different test point when the origin is on the boundary line.

EXAMPLE 2 Graphing a Linear Inequality in One Variable

Graph $y \leq 2$ in a coordinate plane.

SOLUTION

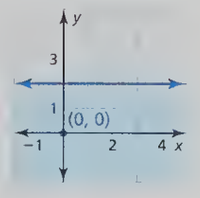
Step 1 Graph $y = 2$. Use a solid line because the inequality symbol is $\leq$.

Step 2 Test $(0, 0)$.

$y \leq 2$ Write the inequality.

$0 \leq 2$ ✓ Substitute.

Step 3 Because $(0, 0)$ is a solution, shade the half-plane that contains $(0, 0)$.



EXAMPLE 3 Graphing a Linear Inequality in Two Variables

Graph $-x + 2y > 2$ in a coordinate plane.

SOLUTION

Step 1 Graph $-x + 2y = 2$, or $y = \frac{1}{2}x + 1$. Use a dashed line because the inequality symbol is $>$.

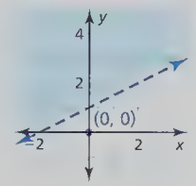
Step 2 Test $(0, 0)$.

$-x + 2y > 2$ Write the inequality.

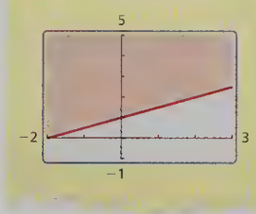
$-(0) + 2(0) \stackrel{?}{>} 2$ Substitute.

$0 \not> 2$ ✗ Simplify.

Step 3 Because $(0, 0)$ is *not* a solution, shade the half-plane that does *not* contain $(0, 0)$.



Check



Monitoring Progress

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Graph the inequality in a coordinate plane.

5. $y > -1$
6. $x \leq -4$
7. $x + y \leq -4$
8. $x - 2y < 0$

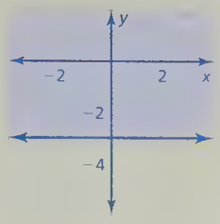
English Language Learners

Build on Past Knowledge

Remind students that when they graph inequalities on a number line, they use an open circle when the inequality symbol is $<$ or $>$. They use a closed circle when the inequality symbol is $\leq$ or $\geq$. Explain that when graphing inequalities in a coordinate plane, they will use a dashed line when the inequality symbol is $<$ or $>$, and a solid line when the inequality symbol is $\leq$ or $\geq$.

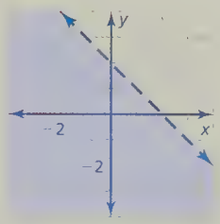
Extra Example 2

Graph $y \geq -3$ in a coordinate plane.



Extra Example 3

Graph $x + y < 2$ in a coordinate plane.

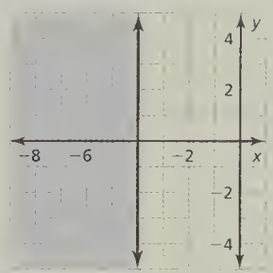


MONITORING PROGRESS ANSWERS

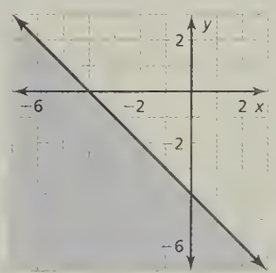
5.



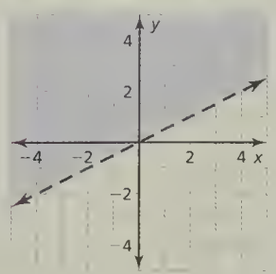
6.



7.

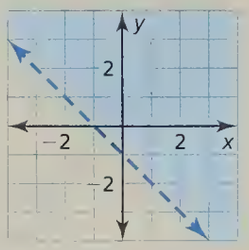


8.



Extra Example 4

Write an inequality that represents the graph.



$y > -x - 1$

Extra Example 5

A tour helicopter can carry 6 passengers with a total weight of no more than 1000 pounds. The table shows the weight of each passenger on the helicopter. Write an inequality that represents the weights of passengers B and D.

Passenger	Weight (pounds)
A	150
B	x
C	200
D	y
E	122
F	168

$x + y \leq 360$

MONITORING PROGRESS ANSWERS

9. $y > x + 3$

10. $x + y > 19$

Writing Linear Inequalities in Two Variables

Core Concept

Writing a Linear Inequality in Two Variables Using a Graph

Write an equation in slope-intercept form of the boundary line.

If the shaded half-plane is *above* the boundary line, then

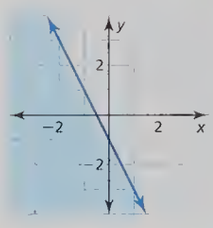
- replace $=$ with $>$ when the boundary line is *dashed*.
- replace $=$ with $\geq$ when the boundary line is *solid*.

If the shaded half-plane is *below* the boundary line, then

- replace $=$ with $<$ when the boundary line is *dashed*.
- replace $=$ with $\leq$ when the boundary line is *solid*.

		Boundary Line	
		Dashed	Solid
Half-plane	Above	$>$	$\geq$
	Below	$<$	$\leq$

EXAMPLE 4 Writing a Linear Inequality Using a Graph



Write an inequality that represents the graph.

SOLUTION

The boundary line has a slope of -2 and a y-intercept of -1. So, an equation of the boundary line is $y = -2x - 1$. The shaded half-plane is *below* the boundary line and the boundary line is *solid*. So, replace $=$ with $\leq$.

▶ The inequality $y \leq -2x - 1$ represents the graph.

EXAMPLE 5 Writing a Linear Inequality Using a Table

Flavor	Quantity (gallons)
vanilla	20
chocolate	15
strawberry	x
banana	y
sherbet	18

An ice cream truck can carry at most 75 gallons of ice cream. The table shows the inventory on the truck. Write an inequality that represents the numbers of gallons of strawberry and banana ice cream on the truck.

SOLUTION

Use the table to write an inequality that represents the problem.

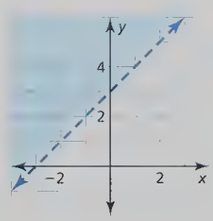
$20 + 15 + x + y + 18 \leq 75$ The total number of gallons is less than or equal to 75.

$x + y \leq 22$ Isolate the variable terms on one side.

▶ The inequality $x + y \leq 22$ represents the numbers of gallons of strawberry and banana ice cream on the truck.

Monitoring Progress

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9. Write an inequality that represents the graph.

10. For a weight loss competition, the blue team needs to lose more than 55 pounds to win. Write an inequality that represents the amounts (in pounds) of weight Team Members B and E must lose so that the blue team wins.

Blue team member	A	B	C	D	E	F
Weight lost (pounds)	10	y	8	13	x	5

Solving Real-Life Problems



EXAMPLE 6 Modeling with Mathematics

You can spend at most \$10 on grapes and apples for a fruit salad. Grapes cost \$2.50 per pound, and apples cost \$1 per pound. Write and graph an inequality that represents the amounts of grapes and apples you can buy. Identify and interpret two solutions of the inequality.

SOLUTION

1. **Understand the Problem** You know the most that you can spend and the prices per pound for grapes and apples. You are asked to write and graph an inequality and then identify and interpret two solutions.
2. **Make a Plan** Use a verbal model to write an inequality that represents the problem. Then graph the inequality. Use the graph to identify two solutions. Then interpret the solutions.
3. **Solve the Problem**

Words Cost per pound of grapes • Pounds of grapes + Cost per pound of apples • Pounds of apples ≤ Amount you can spend

Variables Let x be pounds of grapes and y be pounds of apples.

Inequality $2.50 \cdot x + 1 \cdot y \leq 10$

Step 1 Graph $2.5x + y = 10$, or $y = -2.5x + 10$. Use a solid line because the inequality symbol is ≤. Restrict the graph to positive values of x and y because negative values do not make sense in this real-life context.

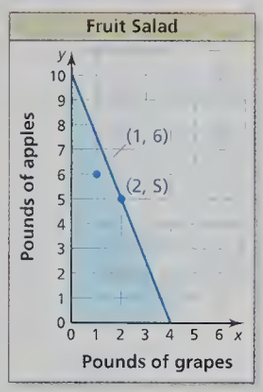
Step 2 Test (0, 0).

$2.5x + y \leq 10$ Write the inequality.

$2.5(0) + 0 \stackrel{?}{\leq} 10$ Substitute.

$0 \leq 10$ ✓ Simplify.

Step 3 Because (0, 0) is a solution, shade the half-plane that contains (0, 0).



Check

$2.5x + y \leq 10$

$2.5(1) + 6 \stackrel{?}{\leq} 10$

$8.5 \leq 10$ ✓

$2.5x + y \leq 10$

$2.5(2) + 5 \stackrel{?}{\leq} 10$

$10 \leq 10$ ✓

▶ One possible solution is (1, 6) because it lies in the shaded half-plane. Another possible solution is (2, 5) because it lies on the solid line. So, you can buy 1 pound of grapes and 6 pounds of apples, or 2 pounds of grapes and 5 pounds of apples.

4. **Look Back** Check your solutions by substituting them into the original inequality, as shown.

Monitoring Progress

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11. You can spend at most \$12 on red peppers and tomatoes for salsa. Red peppers cost \$4 per pound, and tomatoes cost \$3 per pound. Write and graph an inequality that represents the amounts of red peppers and tomatoes you can buy. Identify and interpret two solutions of the inequality.

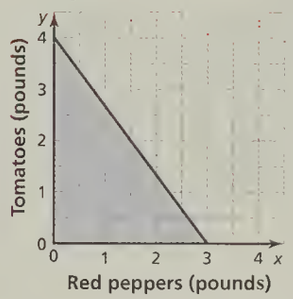
Extra Example 6

You can spend at most \$9 on potatoes and carrots for stew. Potatoes cost \$3 per pound, and carrots cost \$1.50 per pound. Write an inequality that represents the amounts of potatoes and carrots you can buy. Identify and interpret two solutions of the inequality.

$3x + 1.50y \leq 9$; *Sample answer:* (1, 3) and (2, 2). You can buy 1 pound of potatoes and 3 pounds of carrots, or 2 pounds of potatoes and 2 pounds of carrots.

MONITORING PROGRESS ANSWERS

11. $y \leq -\frac{4}{3}x + 4$



Sample answer: (3, 0), You can buy 3 pounds of red peppers and no tomatoes; (1, 1), You can buy 1 pound of red peppers and 1 pound of tomatoes.

Closure

- **Phone Message:** Write a brief phone message that you would leave for a friend who missed today's class. Explain how to graph a linear inequality in two variables. *Sample answer:* First, graph the boundary line. If the inequality symbol is ≤ or ≥, then use a solid line. Otherwise, use a dashed line. Test a point on one side of the line by substituting the x-value and y-value into the inequality. If the point makes the inequality true, then shade the half of the plane that contains the point. Otherwise, shade the half of the plane on the other side of the line.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–39 odd, 43, 50, 55–58

Average: 1, 2–44 even, 48, 50, 55–58

Advanced: 1, 2, 8, 10, 16–18, 24, 30–34 even, 40–46 even, 48–58

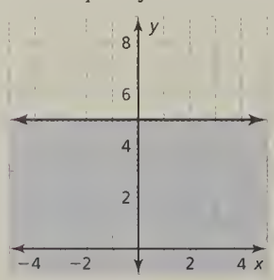
HOMEWORK CHECK

Basic: 3, 19, 25, 33, 39

Average: 4, 20, 26, 34, 40

Advanced: 8, 24, 30, 34, 44

ANSWERS

1. Substitute the values into the inequality and verify that the statement is true.
2. The graph of a linear equation in two variables is a line. The graph of a linear inequality in two variables has a line as a boundary and a shaded half-plane.
3. yes
4. no
5. no
6. no
7. yes
8. yes
9. no
10. yes
11. no
12. yes
13. yes
14. no
15. no
16. no
17. no; (12, 14) is not a solution of the inequality.
18. yes; (20, 18) is a solution of the inequality.
19. 

5.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** How can you tell whether an ordered pair is a solution of a linear inequality?
2. **WRITING** Compare the graph of a linear inequality in two variables with the graph of a linear equation in two variables.

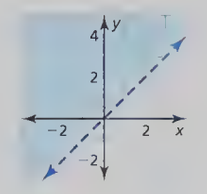
Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, tell whether the ordered pair is a solution of the inequality. *(See Example 1.)*

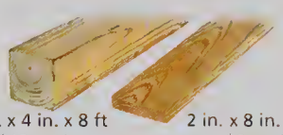
3. $x + y < 7; (2, 3)$
4. $x - y \leq 0; (5, 2)$
5. $x + 3y \geq -2; (-9, 2)$
6. $8x + y > -6; (-1, 2)$
7. $-6x + 4y \leq 6; (-3, -3)$
8. $3x - 5y \geq 2; (-1, -1)$
9. $-x - 6y > 12; (-8, 2)$
10. $-4x - 8y < 15; (-6, 3)$

In Exercises 11–16, tell whether the ordered pair is a solution of the inequality whose graph is shown.

11. $(0, -1)$
12. $(-1, 3)$
13. $(1, 4)$
14. $(0, 0)$
15. $(3, 3)$
16. $(2, 1)$



17. **MODELING WITH MATHEMATICS** A carpenter has at most \$250 to spend on lumber. The inequality $8x + 12y \leq 250$ represents the numbers x of 2-by-8 boards and the numbers y of 4-by-4 boards the carpenter can buy. Can the carpenter buy twelve 2-by-8 boards and fourteen 4-by-4 boards? Explain.



4 in. x 4 in. x 8 ft
\$12 each

2 in. x 8 in. x 8 ft
\$8 each

18. **MODELING WITH MATHEMATICS** The inequality $3x + 2y \geq 93$ represents the numbers x of multiple-choice questions and the numbers y of matching questions you can answer correctly to receive an A on a test. You answer 20 multiple-choice questions and 18 matching questions correctly. Do you receive an A on the test? Explain.

In Exercises 19–24, graph the inequality in a coordinate plane. *(See Example 2.)*

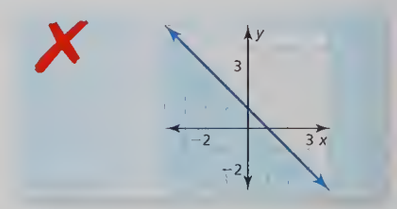
19. $y \leq 5$
20. $y > 6$
21. $x < 2$
22. $x \geq -3$
23. $y > -7$
24. $x < 9$

In Exercises 25–30, graph the inequality in a coordinate plane. *(See Example 3.)*

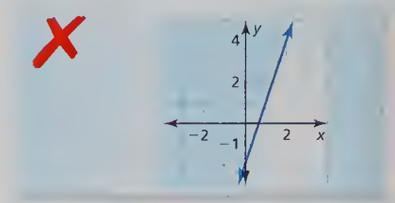
25. $y > -2x - 4$
26. $y \leq 3x - 1$
27. $-4x + y < -7$
28. $3x - y \geq 5$
29. $5x - 2y \leq 6$
30. $-x + 4y > -12$

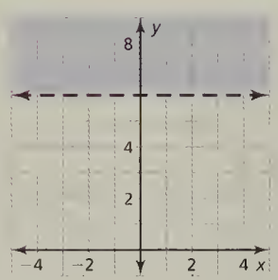
ERROR ANALYSIS **In Exercises 31 and 32, describe and correct the error in graphing the inequality.**

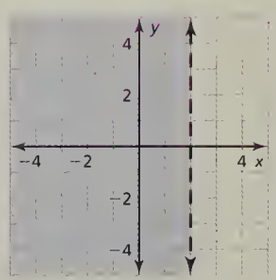
31. $y < -x + 1$



32. $y \leq 3x - 2$

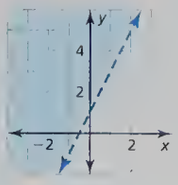


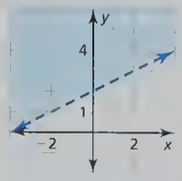
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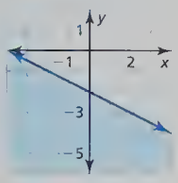
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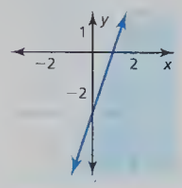
22–32. See Additional Answers.

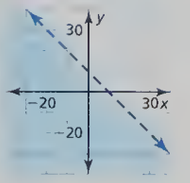
In Exercises 33–38, write an inequality that represents the graph. *(See Example 4.)*

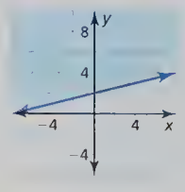
33. 

34. 

35. 

36. 

37. 

38. 

39. **MODELING WITH MATHEMATICS** A restaurant employee works at least 40 hours per week. The table shows the employee's weekly work schedule. Write an inequality that represents the numbers of hours the employee spends cooking and washing dishes. *(See Example 5.)*

Task	Hours
cleaning	6
cooking	x
ordering inventory	2
stocking food	3
washing dishes	y

40. **MODELING WITH MATHEMATICS** You are allowed to watch TV for less than 10 hours total on school nights. The table shows the numbers of hours you watch TV. Write an inequality that represents the numbers of hours you can watch TV on Sunday and Wednesday.

School night	Hours
Sunday	x
Monday	1.5
Tuesday	0.5
Wednesday	y
Thursday	2.5

41. **MODELING WITH MATHEMATICS** A department store offers a 10% discount on any purchase over $200. The table shows the costs of items in a purchase at the store. Write an inequality that represents the costs of the shirt and shoes that will qualify the purchase for the discount.

Item	Cost
backpack	$54.99
jeans	$39.99
shirt	x
shoes	y
video game	$49.95

ERROR ANALYSIS **In Exercises 42 and 43, describe and correct the error in writing the inequality.**

42. 

✗ $y > -3x + 2$

43. You eat no more than 2500 calories a day. The table shows the numbers of calories you eat at each meal.

Meal	Calories
breakfast	770
lunch	x
snack	150
dinner	y
snack	250

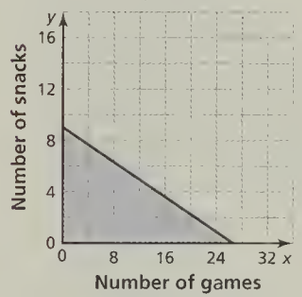
✗ $x + y \leq 3670$

44. **MODELING WITH MATHEMATICS** You have at most $20 to spend at an arcade. Arcade games cost $0.75 each, and snacks cost $2.25 each. Write and graph an inequality that represents the numbers of games you can play and snacks you can buy. Identify and interpret two solutions of the inequality. *(See Example 6.)*

Dynamic Teaching Tools
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

ANSWERS

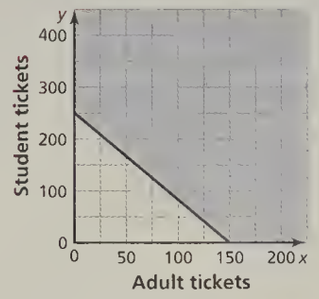
33. $y > 2x + 1$
34. $y > \frac{1}{2}x + 2$
35. $y \leq -\frac{1}{2}x - 2$
36. $y \leq 3x - 3$
37. $y < -x + 10$
38. $y \geq \frac{1}{4}x + 2$
39. $x + y \geq 29$
40. $x + y < 5.5$
41. $x + y > 55.07$
42. The symbol $<$ should be used; $y < -3x + 2$
43. The calories for breakfast and both snacks were added to 2500 instead of subtracted; $x + y \leq 1330$
44. $0.75x + 2.25y \leq 20$



Sample answer: (12, 4), You can play 12 games and buy 4 snacks; (5, 2), You can play 5 games and buy 2 snacks.

ANSWERS

45. $10x + 6y \geq 1500$

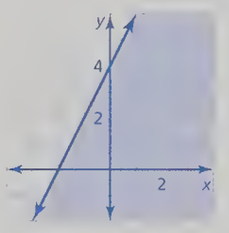


Sample answer: (150, 0), 150 adult tickets were sold and no student tickets were sold; (100, 300), 100 adult tickets were sold and 300 student tickets were sold.

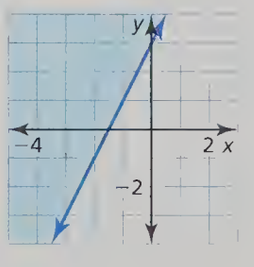
46–58. See Additional Answers.

Mini-Assessment

1. Tell whether the ordered pair is a solution of the inequality.
 $4x + y > 3$; (2, 4)
 (2, 4) is a solution.
2. Graph $2x - y \geq -4$ in a coordinate plane.



3. Write an inequality that represents the graph.



$y \geq 2x + 3$

4. You have at most \$5 to spend on notebooks and markers. Markers cost \$1.00 each, and notebooks cost \$1.25 each. Write an inequality that represents this situation. Identify and interpret two solutions of the inequality.
 $x + 1.25y \leq 5$; *Sample answer:* (1, 3) and (5, 0). You can buy 1 marker and 3 notebooks, or 5 markers and 0 notebooks.

45. MODELING WITH MATHEMATICS A drama club must sell at least \$1500 worth of tickets to cover the expenses of producing a play. Write and graph an inequality that represents how many adult and student tickets the club must sell. Identify and interpret two solutions of the inequality.



46. MODELING WITH MATHEMATICS A shipping company is delivering ceramic pots. The company earns \$1 for each unbroken ceramic pot and is fined \$4 for each broken ceramic pot. The company wants to earn at least \$2000 for this delivery. Write and graph an inequality that represents the numbers of unbroken and broken ceramic pots the company must deliver. Identify and interpret two solutions of the inequality.

47. MODELING WITH MATHEMATICS A clothing store sells T-shirts for \$20 and skirts for \$25. The store wants to sell a minimum of \$800 worth of T-shirts and skirts. Write and graph an inequality that represents the numbers of T-shirts and skirts the store must sell. Identify and interpret two solutions of the inequality.

48. PROBLEM SOLVING Large boxes weigh 75 pounds, and small boxes weigh 40 pounds.

a. Write and graph an inequality that represents the numbers of large and small boxes a 200-pound delivery person can take on the elevator.

b. Explain why some solutions of the inequality might not be practical in real life.



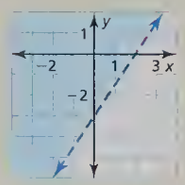
49. WRITING Can you always use (0, 0) as a test point when graphing an inequality? Explain.

50. HOW DO YOU SEE IT? Match each inequality with its graph.

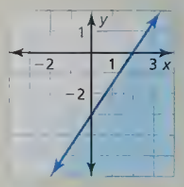
a. $3x - 2y \leq 6$ **b.** $3x - 2y < 6$

c. $3x - 2y > 6$ **d.** $3x - 2y \geq 6$

A.



B.



C.



D.



51. REASONING When graphing a linear inequality in two variables, why must you choose a test point that is *not* on the boundary line?

52. THOUGHT PROVOKING Write a linear inequality in two variables that has the following two properties.

- (0, 0), (0, −1), and (0, 1) are not solutions.
- (1, 1), (3, −1), and (−1, 3) are solutions.

CRITICAL THINKING **In Exercises 53 and 54, write and graph an inequality whose graph is described by the given information.**

53. The points (2, 5) and (−3, −5) lie on the boundary line. The points (6, 5) and (−2, −3) are solutions of the inequality.

54. The points (−7, −16) and (1, 8) lie on the boundary line. The points (−7, 0) and (3, 14) are *not* solutions of the inequality.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the next three terms of the arithmetic sequence. *(Section 4.7)*

55. 0, 8, 16, 24, 32, . . .

56. −5, −8, −11, −14, −17, . . .

57. 3.6, 2.8, 2, 1.2, 0.4, . . .

58. $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

5.7 Systems of Linear Inequalities

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.3.H

Essential Question How can you graph a system of linear inequalities?

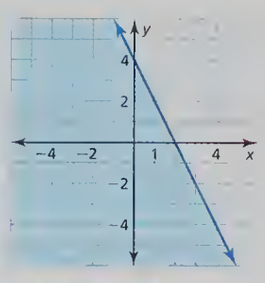
EXPLORATION 1 Graphing Linear Inequalities

Work with a partner. Match each linear inequality with its graph. Explain your reasoning.

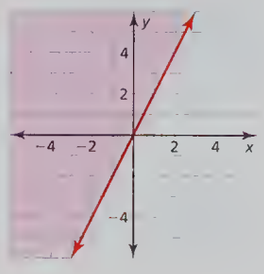
$2x + y \leq 4$ Inequality 1

$2x - y \leq 0$ Inequality 2

A.



B.



EXPLORATION 2 Graphing a System of Linear Inequalities

Work with a partner. Consider the linear inequalities given in Exploration 1.

$2x + y \leq 4$ Inequality 1

$2x - y \leq 0$ Inequality 2

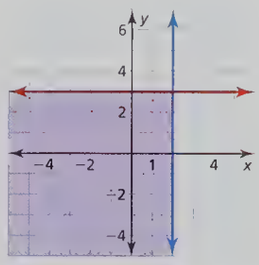
USING PROBLEM-SOLVING STRATEGIES
To be proficient in math, you need to explain to yourself the meaning of a problem.

a. Use two different colors to graph the inequalities in the same coordinate plane. What is the result?

b. Describe each of the shaded regions of the graph. What does the unshaded region represent?

Communicate Your Answer

3. How can you graph a system of linear inequalities?
4. When graphing a system of linear inequalities, which region represents the solution of the system?
5. Do you think all systems of linear inequalities have a solution? Explain your reasoning.
6. Write a system of linear inequalities represented by the graph.



For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Dynamic Teaching Tools

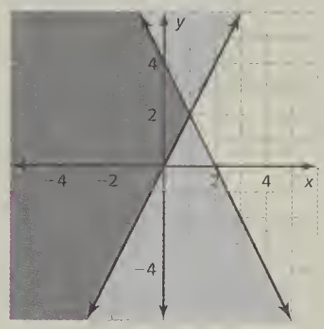
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.3.H The student is expected to graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

ANSWERS

1. Inequality 1: A; The graph has a boundary line of $y = -2x + 4$; Inequality 2: B; The graph has a boundary line of $y = 2x$.
2. a.



Parts of the two half-planes overlap.

b. One shaded region is only shaded the first color, one shaded region is only shaded the second color, and one shaded region is shaded both colors; the values where both inequalities are false

3. Graph each inequality in the same coordinate plane. Find the intersection of the half-planes that are solutions of the inequalities. This intersection is the graph of the system.
4. the region where the shaded half-planes of the inequalities overlap
5. no; When the boundary lines are parallel, it is possible the shaded regions will not overlap.
6. $x \leq 2$ and $y \leq 3$

Extra Example 1

Tell whether each ordered pair is a solution of the system of linear inequalities.

$y > 3x$

$y \leq -x - 2$

a. $(-2, -1)$

$(-2, -1)$ is a solution of the system.

b. $(0, 4)$

$(0, 4)$ is *not* a solution of the system.

MONITORING PROGRESS ANSWERS

1. no
2. yes

5.7 Lesson

What You Will Learn

- Check solutions of systems of linear inequalities.
- Graph systems of linear inequalities.
- Write systems of linear inequalities.
- Use systems of linear inequalities to solve real-life problems.

Core Vocabulary

system of linear inequalities, *p. 260*

solution of a system of linear inequalities, *p. 260*

graph of a system of linear inequalities, *p. 261*

Previous

linear inequality in two variables

Systems of Linear Inequalities

A **system of linear inequalities** is a set of two or more linear inequalities in the same variables. An example is shown below.

$y < x + 2$ Inequality 1

$y \geq 2x - 1$ Inequality 2

A **solution of a system of linear inequalities** in two variables is an ordered pair that is a solution of each inequality in the system.

EXAMPLE 1 Checking Solutions

Tell whether each ordered pair is a solution of the system of linear inequalities.

$y < 2x$ Inequality 1

$y \geq x + 1$ Inequality 2

a. $(3, 5)$ **b.** $(-2, 0)$

SOLUTION

a. Substitute 3 for x and 5 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$5 \overset{?}{<} 2(3)$	$5 \overset{?}{\geq} 3 + 1$
$5 < 6$ ✓	$5 \geq 4$ ✓

▶ Because the ordered pair $(3, 5)$ is a solution of each inequality, it is a solution of the system.

b. Substitute -2 for x and 0 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$0 \overset{?}{<} 2(-2)$	$0 \overset{?}{\geq} -2 + 1$
$0 \not< -4$ ✗	$0 \geq -1$ ✓

▶ Because $(-2, 0)$ is not a solution of each inequality, it is *not* a solution of the system.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Tell whether the ordered pair is a solution of the system of linear inequalities.

1. $(-1, 5)$; $y < 5$, $y > x - 4$

2. $(1, 4)$; $y \geq 3x + 1$, $y > x - 1$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Kinesthetic

Some students may benefit by using graphing calculators or a graphing software program to check solutions of systems of inequalities. Students can enter the inequalities, view their graphs, and determine whether or not the given points are in the intersection of the half-planes.

Graphing Systems of Linear Inequalities

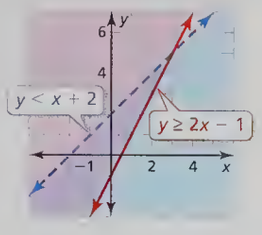
The **graph of a system of linear inequalities** is the graph of all the solutions of the system.

Core Concept

Graphing a System of Linear Inequalities

Step 1 Graph each inequality in the same coordinate plane.

Step 2 Find the intersection of the half-planes that are solutions of the inequalities. This intersection is the graph of the system.



Check

Verify that $(-3, 1)$ is a solution of each inequality.

Inequality 1

$y \leq 3$

$1 \leq 3$ ✓

Inequality 2

$y > x + 2$

$1 \stackrel{?}{>} -3 + 2$

$1 > -1$ ✓

EXAMPLE 2 Graphing a System of Linear Inequalities

Graph the system of linear inequalities.

$y \leq 3$ Inequality 1

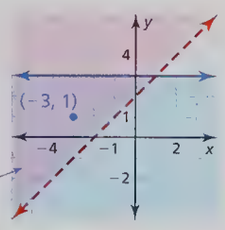
$y > x + 2$ Inequality 2

SOLUTION

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. One solution is $(-3, 1)$.

The solution is the purple-shaded region.



EXAMPLE 3 Graphing a System of Linear Inequalities: No Solution

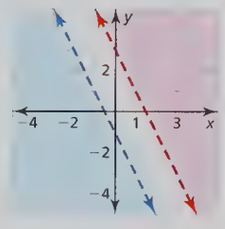
Graph the system of linear inequalities.

$2x + y < -1$ Inequality 1

$2x + y > 3$ Inequality 2

SOLUTION

Step 1 Graph each inequality.



Step 2 Find the intersection of the half-planes. Notice that the lines are parallel, and the half-planes do not intersect.

▶ So, the system has no solution.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Graph the system of linear inequalities.

3. $y \geq -x + 4$
 $x + y \leq 0$

4. $y > 2x - 3$
 $y \geq \frac{1}{2}x + 1$

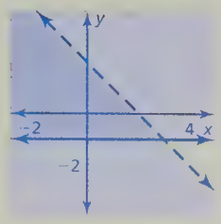
5. $-2x + y < 4$
 $2x + y > 4$

Extra Example 2

Graph the system of linear inequalities.

$y \geq -1$

$y < -x + 2$

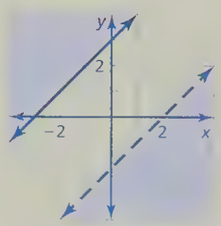


Extra Example 3

Graph the system of linear inequalities.

$x - y > 2$

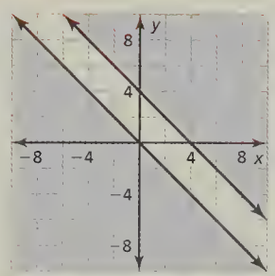
$x - y \leq -3$



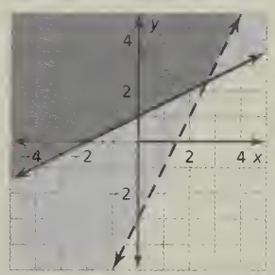
The system has no solution.

MONITORING PROGRESS ANSWERS

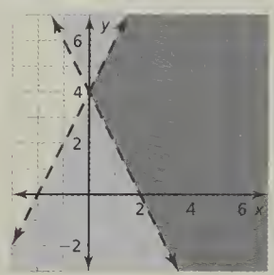
3.



4.



5.



SUPPORTING English Language Learners

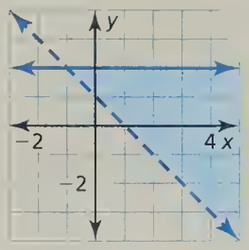
Have students read Example 2. Explain that the steps of the solution help students understand the process described. Point out that the Check shown at the left provides support for verifying the solution.

Beginning Read aloud the headings.
Intermediate Read aloud the problem, including the system of inequalities.
Advanced Read aloud the steps of the solution.
Advanced High Read aloud and interpret the Check shown at the left.

ELPS 4.E Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

Extra Example 4

Write a system of linear inequalities represented by the graph.

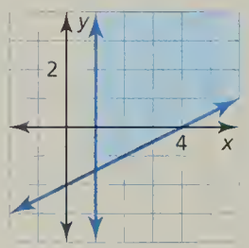


$y \leq 2$

$y > -x + 1$

Extra Example 5

Write a system of linear inequalities represented by the graph.



$x \geq 1$

$y \geq \frac{1}{2}x - 2$

MONITORING PROGRESS ANSWERS

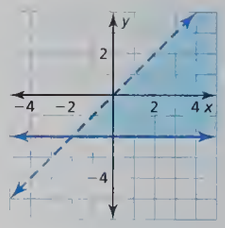
6. $y < -x + 2$ and $x < 3$

7. $y \geq -\frac{1}{2}x + 1$ and $y \leq 2x - 3$

Writing Systems of Linear Inequalities

EXAMPLE 4 Writing a System of Linear Inequalities

Write a system of linear inequalities represented by the graph.



SOLUTION

Inequality 1 The horizontal boundary line passes through $(0, -2)$. So, an equation of the line is $y = -2$. The shaded region is *above* the *solid* boundary line, so the inequality is $y \geq -2$.

Inequality 2 The slope of the other boundary line is 1, and the y-intercept is 0. So, an equation of the line is $y = x$. The shaded region is *below* the *dashed* boundary line, so the inequality is $y < x$.

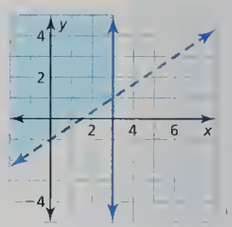
▶ The system of linear inequalities represented by the graph is

$y \geq -2$ Inequality 1

$y < x$. Inequality 2

EXAMPLE 5 Writing a System of Linear Inequalities

Write a system of linear inequalities represented by the graph.



SOLUTION

Inequality 1 The vertical boundary line passes through $(3, 0)$. So, an equation of the line is $x = 3$. The shaded region is to the *left* of the *solid* boundary line, so the inequality is $x \leq 3$.

Inequality 2 The slope of the other boundary line is $\frac{2}{3}$, and the y-intercept is -1. So, an equation of the line is $y = \frac{2}{3}x - 1$. The shaded region is *above* the *dashed* boundary line, so the inequality is $y > \frac{2}{3}x - 1$.

▶ The system of linear inequalities represented by the graph is

$x \leq 3$ Inequality 1

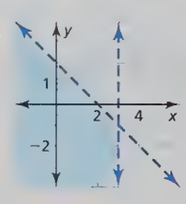
$y > \frac{2}{3}x - 1$. Inequality 2

Monitoring Progress

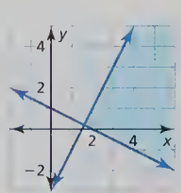
Help in English and Spanish at *BigIdeasMath.com*

Write a system of linear inequalities represented by the graph.

6.



7.



Solving Real-Life Problems

EXAMPLE 6 **Modeling with Mathematics**

You have at most 8 hours to spend at the mall and at the beach. You want to spend at least 2 hours at the mall and more than 4 hours at the beach. Write and graph a system that represents the situation. How much time can you spend at each location?



SOLUTION

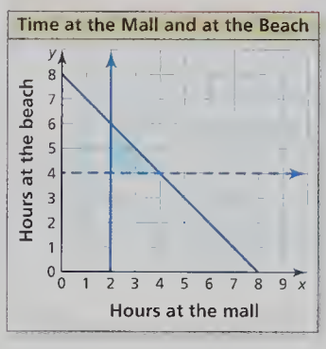
1. **Understand the Problem** You know the total amount of time you can spend at the mall and at the beach. You also know how much time you want to spend at each location. You are asked to write and graph a system that represents the situation and determine how much time you can spend at each location.
2. **Make a Plan** Use the given information to write a system of linear inequalities. Then graph the system and identify an ordered pair in the solution region.
3. **Solve the Problem** Let x be the number of hours at the mall and let y be the number of hours at the beach.

$x + y \leq 8$ at most 8 hours at the mall and at the beach

$x \geq 2$ at least 2 hours at the mall

$y > 4$ more than 4 hours at the beach

Graph the system.



One ordered pair in the solution region is (2.5, 5).

▶ So, you can spend 2.5 hours at the mall and 5 hours at the beach.

4. **Look Back** Check your solution by substituting it into the inequalities in the system, as shown.

Check

$x + y \leq 8$

$2.5 + 5 \stackrel{?}{\leq} 8$

$7.5 \leq 8$ ✓

$x \geq 2$

$2.5 \geq 2$ ✓

$y > 4$

$5 > 4$ ✓

Monitoring Progress

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8. Name another solution of Example 6.
9. **WHAT IF?** You want to spend at least 3 hours at the mall. How does this change the system? Is (2.5, 5) still a solution? Explain.

Extra Example 6

You have at most 7 hours to spend playing soccer and swimming. You want to spend at least 2 hours playing soccer and more than 2 hours swimming. Write a system that represents the situation. How much time can you spend on each activity?

Let x be the number of hours playing soccer and y be the number of hours swimming.

$x + y \leq 7$

$x \geq 2$

$y \geq 2$

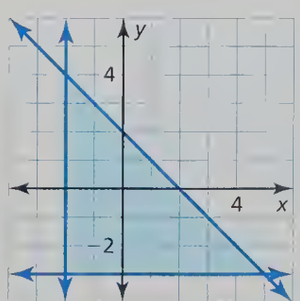
Sample answer: (3, 3). You can spend 3 hours playing soccer and 3 hours swimming.

MONITORING PROGRESS ANSWERS

8. *Sample answer:* 3 hours at the mall, 4.5 hours at the beach
9. $x \geq 2$ becomes $x \geq 3$; no; 2.5 is not greater than 3.

Closure

- Name an ordered pair that is a solution of the system of linear inequalities. *Sample answer:* (0, 0)
- Name an ordered pair that is not a solution of the system of linear inequalities. *Sample answer:* (3, 3)



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–29 odd, 35, 38, 40, 49–55

Average: 1, 2–40 even, 49–55

Advanced: 1, 2, 10, 18, 20, 24–40 even, 41–55

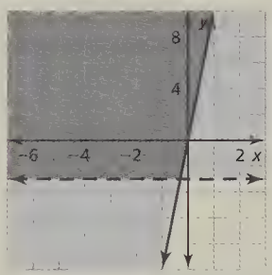
HOMEWORK CHECK

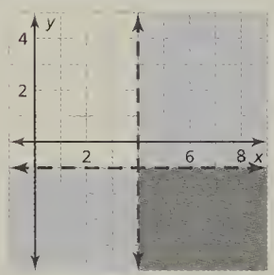
Basic: 7, 11, 21, 29

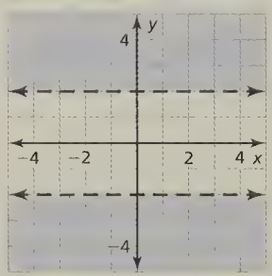
Average: 8, 14, 22, 30

Advanced: 10, 18, 24, 30

ANSWERS

1. Substitute the values into both inequalities and verify that both inequalities are true.
2. (1, −2); It is the only ordered pair that is not a solution of the system.
3. no
4. yes
5. no
6. no
7. yes
8. yes
9. no
10. yes
11. 

12. 

13. 

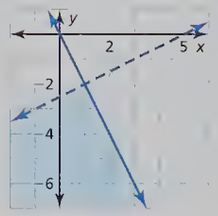
5.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** How can you verify that an ordered pair is a solution of a system of linear inequalities?

2. **WHICH ONE DOESN'T BELONG?** Use the graph shown. Which of the ordered pairs does *not* belong with the other three? Explain your reasoning.

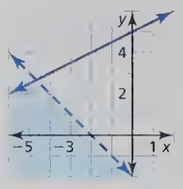


(1, −2) (0, −4) (−1, −6) (2, −4)

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, tell whether the ordered pair is a solution of the system of linear inequalities.

3. (−4, 3)
4. (−3, −1)
5. (−2, 5)
6. (1, 1)



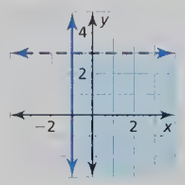
In Exercises 7–10, tell whether the ordered pair is a solution of the system of linear inequalities. *(See Example 1.)*

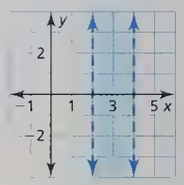
7. (−5, 2); $y < 4$, $y > x + 3$
8. (1, −1); $y > -2$, $y > x - 5$
9. (0, 0); $y \leq x + 7$, $y \geq 2x + 3$
10. (4, −3); $y \leq -x + 1$, $y \leq 5x - 2$

In Exercises 11–20, graph the system of linear inequalities. *(See Examples 2 and 3.)*

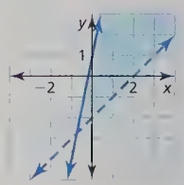
11. $y > -3$, $y \geq 5x$
12. $y < -1$, $x > 4$
13. $y < -2$, $y > 2$
14. $y < x - 1$, $y \geq x + 1$
15. $y \geq -5$, $y - 1 < 3x$
16. $x + y > 4$, $y \geq \frac{3}{2}x - 9$
17. $x + y > 1$, $-x - y < -3$
18. $2x + y \leq 5$, $y + 2 \geq -2x$
19. $x < 4$, $y > 1$, $y \geq -x + 1$
20. $x + y \leq 10$, $x - y \geq 2$, $y > 2$

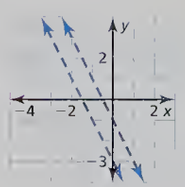
In Exercises 21–26, write a system of linear inequalities represented by the graph. *(See Examples 4 and 5.)*

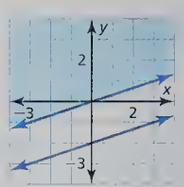
21. 

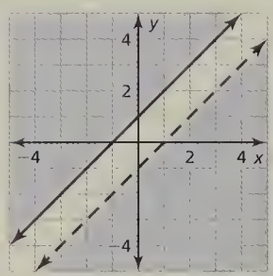
22. 

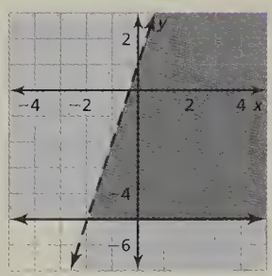
23. 

24. 

25. 

26. 

14. 

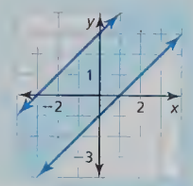
15. 

16–26. See Additional Answers.

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in graphing the system of linear inequalities.**

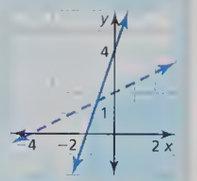
27.

$y \leq x - 1$
$y \geq x + 3$



28.

$y \leq 3x + 4$
$y > \frac{1}{2}x + 2$



29. **MODELING WITH MATHEMATICS** You can spend at most \$21 on fruit. Blueberries cost \$4 per pound, and strawberries cost \$3 per pound. You need at least 3 pounds of fruit to make muffins. *(See Example 6.)*

a. Write and graph a system of linear inequalities that represents the situation.

b. Identify and interpret a solution of the system.

c. Use the graph to determine whether you can buy 4 pounds of blueberries and 1 pound of strawberries.



30. **MODELING WITH MATHEMATICS** You earn \$10 per hour working as a manager at a grocery store. You are required to work at the grocery store at least 8 hours per week. You also teach music lessons for \$15 per hour. You need to earn at least \$120 per week, but you do not want to work more than 20 hours per week.

a. Write and graph a system of linear inequalities that represents the situation.

b. Identify and interpret a solution of the system.

c. Use the graph to determine whether you can work 8 hours at the grocery store and teach 1 hour of music lessons.

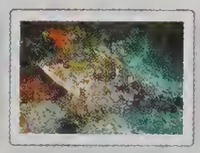
31. **MODELING WITH MATHEMATICS** You are fishing for surfperch and rockfish, which are species of bottomfish. Gaming laws allow you to catch no more than 15 surfperch per day, no more than 10 rockfish per day, and no more than 20 total bottomfish per day.

a. Write and graph a system of linear inequalities that represents the situation.

b. Use the graph to determine whether you can catch 11 surfperch and 9 rockfish in 1 day.



surfperch



rockfish

32. **REASONING** Describe the intersection of the half-planes of the system shown.

$$x - y \leq 4$$
$$x - y \geq 4$$

33. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded rectangle.

$(-1, 1), (6, 1), (6, -3), (-1, -3)$

a. Write a system of linear inequalities represented by the shaded rectangle.

b. Find the area of the rectangle.

34. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded triangle.

$(2, 5), (6, -3), (-2, -3)$

a. Write a system of linear inequalities represented by the shaded triangle.

b. Find the area of the triangle.

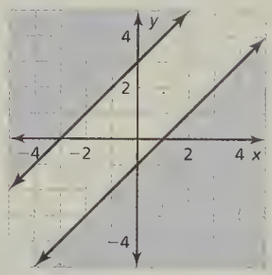
35. **PROBLEM SOLVING** You plan to spend less than half of your monthly \$2000 paycheck on housing and savings. You want to spend at least 10% of your paycheck on savings and at most 30% of it on housing. How much money can you spend on savings and housing?

36. **PROBLEM SOLVING** On a road trip with a friend, you drive about 70 miles per hour, and your friend drives about 60 miles per hour. The plan is to drive less than 15 hours and at least 600 miles each day. Your friend will drive more hours than you. How many hours can you and your friend each drive in 1 day?

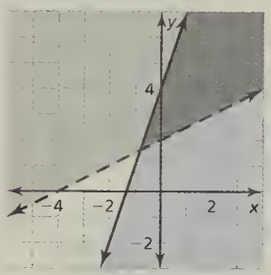
Dynamic Teaching Tools
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

ANSWERS

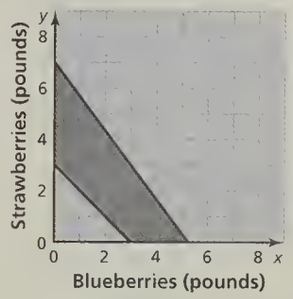
27. The wrong regions are shaded for both inequalities.



28. The wrong region is shaded for $y > \frac{1}{2}x + 2$.



29. a. $4x + 3y \leq 21$ and $x + y \geq 3$



b. *Sample answer:* (2, 4), You can buy 2 pounds of blueberries and 4 pounds of strawberries.

c. yes

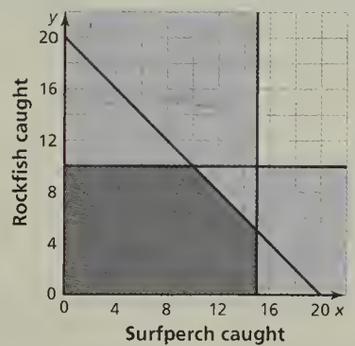
30. a. $x \geq 8$, $10x + 15y \geq 120$, and $x + y \leq 20$



b. *Sample answer:* (12, 2), You can work 12 hours at the grocery store and spend 2 hours teaching music lessons.

30. c. no

31. a. $x \leq 15$, $y \leq 10$, and $x + y \leq 20$



b. yes

32. the line $x - y = 4$

33. a. $x \geq -1$, $x \leq 6$, $y \geq -3$, and $y \leq 1$

b. 28 square units

34. a. $y \leq 2x + 1$, $y \leq -2x + 9$, and $y \geq -3$

b. 32 square units

35. *Sample answer:* \$300 on savings, \$500 on housing

36. *Sample answer:* you 6 hours, your friend 8 hours

ANSWERS

37. Both can be done by graphing; Solving a system of linear inequalities requires finding overlapping half-planes, solving a system of linear equations does not.

38. >; <; This puts point C in the overlapping half-planes and leaves the other points out.

39–55. See Additional Answers.

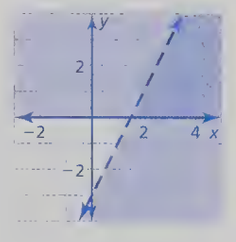
Mini-Assessment

1. Tell whether the ordered pair $(-4, 2)$ is a solution of the system of linear inequalities.

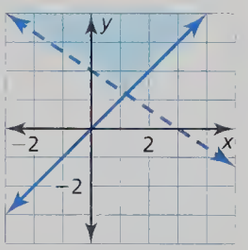
 $y < -4x, y \geq x - 4$ yes

2. Graph the system of linear inequalities.

 $y \geq 0, y < 2x - 3$



3. Write a system of linear inequalities represented by the graph.

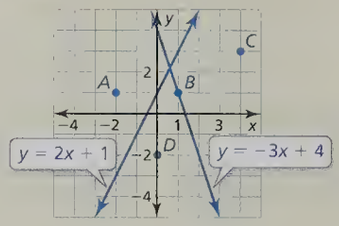


 $y \geq x, y > -\frac{2}{3}x + 2$

4. You can spend at most \$24 at the fair. Game tickets cost \$2 each, and ride tickets cost \$4 each. You want to buy at least 3 game tickets and more than 2 ride tickets. Write a system that represents the situation. How many of each ticket can you buy? Let x be the number of game tickets and y be the number of ride tickets: $2x + 4y \leq 24, x \geq 3, y > 2$. *Sample answer:* (4, 3). You can buy 4 game tickets and 3 ride tickets.

37. WRITING How are solving systems of linear inequalities and solving systems of linear equations similar? How are they different?

38. HOW DO YOU SEE IT? The graphs of two linear equations are shown.



Replace the equal signs with inequality symbols to create a system of linear inequalities that has point C as a solution, but not points A, B, and D. Explain your reasoning.

$y \quad -3x + 4$

$y \quad 2x + 1$

39. OPEN-ENDED Write a real-life problem that can be represented by a system of linear inequalities. Write the system of linear inequalities and graph the system.

40. MAKING AN ARGUMENT Your friend says that a system of linear inequalities in which the boundary lines are parallel must have no solution. Is your friend correct? Explain.

41. CRITICAL THINKING Is it possible for the solution set of a system of linear inequalities to be all real numbers? Explain your reasoning.

OPEN-ENDED In Exercises 42–44, write a system of linear inequalities with the given characteristic.

42. All solutions are in Quadrant I.

43. All solutions have one positive coordinate and one negative coordinate.

44. There are no solutions.

45. OPEN-ENDED One inequality in a system is $-4x + 2y > 6$. Write another inequality so the system has (a) no solution and (b) infinitely many solutions.

46. THOUGHT PROVOKING You receive a gift certificate for a clothing store and plan to use it to buy T-shirts and sweatshirts. Describe a situation in which you can buy 9 T-shirts and 1 sweatshirt, but you cannot buy 3 T-shirts and 8 sweatshirts. Write and graph a system of linear inequalities that represents the situation.

47. CRITICAL THINKING Write a system of linear inequalities that has exactly one solution.

48. MODELING WITH MATHEMATICS You make necklaces and key chains to sell at a craft fair. The table shows the amounts of time and money it takes to make a necklace and a key chain, and the amounts of time and money you have available for making them.

	Necklace	Key chain	Available
Time to make (hours)	0.5	0.25	20
Cost to make (dollars)	2	3	120

a. Write and graph a system of four linear inequalities that represents the number x of necklaces and the number y of key chains that you can make.

b. Find the vertices (corner points) of the graph of the system.

c. You sell each necklace for \$10 and each key chain for \$8. The revenue R is given by the equation $R = 10x + 8y$. Find the revenue corresponding to each ordered pair in part (b). Which vertex results in the maximum revenue?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the product using exponents. *(Skills Review Handbook)*

49. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **50.** $(-13) \cdot (-13) \cdot (-13)$ **51.** $x \cdot x \cdot x \cdot x \cdot x \cdot x$

Write an equation of the line with the given slope and y-intercept. *(Section 4.1)*

52. slope: 1, y-intercept: -6 **53.** slope: -3, y-intercept: 5 **54.** slope: $-\frac{1}{4}$, y-intercept: -1 **55.** slope: $\frac{4}{3}$, y-intercept: 0

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

5.5–5.7 What Did You Learn?

Core Vocabulary

linear inequality in two variables, *p. 252*
solution of a linear inequality in two variables, *p. 252*
graph of a linear inequality, *p. 252*
half-planes, *p. 252*
system of linear inequalities, *p. 260*
solution of a system of linear inequalities, *p. 260*
graph of a system of linear inequalities, *p. 261*

Core Concepts

Section 5.5

Solving Linear Equations by Graphing, *p. 246*

Section 5.6

Graphing a Linear Inequality in Two Variables, *p. 253*
Writing a Linear Inequality in Two Variables, *p. 254*

Section 5.7

Graphing a System of Linear Inequalities, *p. 261*
Writing a System of Linear Inequalities, *p. 262*

Mathematical Thinking

1. Describe how to solve Exercise 39 on page 250 algebraically.
2. Why is it important to be precise when answering part (a) of Exercise 48 on page 258?
3. Describe the overall step-by-step process you used to solve Exercise 35 on page 265.

Performance Task

Prize Patrol

You have been selected to drive a prize patrol cart and place prizes on the competing teams' predetermined paths. You know the teams' routes and you can only make one pass. Where will you place the prizes so that each team will have a chance to find a prize on their route?

To explore the answer to this question and more, go to *BigIdeasMath.com*.



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Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

ANSWERS

1. Solve $ax + b = cx + d$ for x. For each situation, determine whether the numerator and denominator are positive or negative. Then determine the sign of the quotient.
2. Exceeding the maximum weight could be unsafe.
3. Write the inequalities $h + s < \frac{1}{2}(2000)$, $s \geq 10\%(2000)$, and $h \leq 30\%(2000)$. Simplify the expressions on the right side of each inequality. Choose values for s and h that make the last two inequalities true and verify that they also make the first inequality true.

ANSWERS

1. $(2, -5)$
2. $(1, -1)$
3. $(4, -1)$
4. $(-2, -3)$
5. $(2, 1)$
6. $(2, 0)$
7. 8 brushes, 4 tubes of paint

5 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

5.1 Solving Systems of Linear Equations by Graphing *(pp. 219–224)*

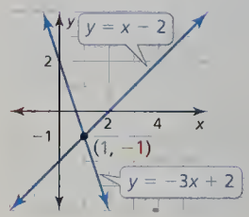
Solve the system by graphing. $y = x - 2$ Equation 1
$y = -3x + 2$ Equation 2

Step 1 Graph each equation.

Step 2 Estimate the point of intersection.
The graphs appear to intersect at $(1, -1)$.

Step 3 Check your point from Step 2.

Equation 1	Equation 2
$y = x - 2$	$y = -3x + 2$
$-1 \stackrel{?}{=} 1 - 2$	$-1 \stackrel{?}{=} -3(1) + 2$
$-1 = -1$ ✓	$-1 = -1$ ✓



▶ The solution is $(1, -1)$.

Solve the system of linear equations by graphing.

1. $y = -3x + 1$
 $y = x - 7$
2. $y = -4x + 3$
 $4x - 2y = 6$
3. $5x + 5y = 15$
 $2x - 2y = 10$

5.2 Solving Systems of Linear Equations by Substitution *(pp. 225–230)*

Solve the system by substitution. $-2x + y = -8$ Equation 1
$7x + y = 10$ Equation 2

Step 1 Solve for y in Equation 1.

$y = 2x - 8$ Revised Equation 1

Step 2 Substitute $2x - 8$ for y in Equation 2 and solve for x.

$7x + y = 10$	Equation 2
$7x + (2x - 8) = 10$	Substitute $2x - 8$ for y.
$9x - 8 = 10$	Combine like terms.
$9x = 18$	Add 8 to each side.
$x = 2$	Divide each side by 9.

Step 3 Substituting 2 for x in Equation 1 and solving for y gives $y = -4$.

▶ The solution is $(2, -4)$.

Solve the system of linear equations by substitution. Check your solution.

4. $3x + y = -9$
 $y = 5x + 7$
5. $x + 4y = 6$
 $x - y = 1$
6. $2x + 3y = 4$
 $y + 3x = 6$

7. You spend \$20 total on tubes of paint and disposable brushes for an art project. Tubes of paint cost \$4.00 each and paintbrushes cost \$0.50 each. You purchase twice as many brushes as tubes of paint. How many brushes and tubes of paint do you purchase?

5.3 Solving Systems of Linear Equations by Elimination *(pp. 231–236)*

Solve the system by elimination. $4x + 6y = -8$ Equation 1; $x - 2y = -2$ Equation 2

Step 1 Multiply Equation 2 by 3 so that the coefficients of the y-terms are opposites.

$4x + 6y = -8$ → $4x + 6y = -8$ Equation 1

$x - 2y = -2$ **Multiply by 3.** → $3x - 6y = -6$ Revised Equation 2

Step 2 Add the equations.

$4x + 6y = -8$ Equation 1

$3x - 6y = -6$ Revised Equation 2

$7x \quad = -14$ Add the equations.

Step 3 Solve for x.

$7x = -14$ Resulting equation from Step 2

$x = -2$ Divide each side by 7.

Step 4 Substitute -2 for x in one of the original equations and solve for y.

$4x + 6y = -8$ Equation 1

$4(-2) + 6y = -8$ Substitute -2 for x.

$-8 + 6y = -8$ Multiply.

$y = 0$ Solve for y.

Check

Equation 1: $4x + 6y = -8$; $4(-2) + 6(0) \stackrel{?}{=} -8$; $-8 = -8$ ✓

Equation 2: $x - 2y = -2$; $(-2) - 2(0) \stackrel{?}{=} -2$; $-2 = -2$ ✓

▶ The solution is $(-2, 0)$.

Solve the system of linear equations by elimination. Check your solution.

8. $9x - 2y = 34$; $5x + 2y = -6$

9. $x + 6y = 28$; $2x - 3y = -19$

10. $8x - 7y = -3$; $6x - 5y = -1$

5.4 Solving Special Systems of Linear Equations *(pp. 237–242)*

Solve the system. $4x + 2y = -14$ Equation 1; $y = -2x - 6$ Equation 2

Solve by substitution. Substitute $-2x - 6$ for y in Equation 1.

$4x + 2y = -14$ Equation 1

$4x + 2(-2x - 6) = -14$ Substitute $-2x - 6$ for y.

$4x - 4x - 12 = -14$ Distributive Property

$-12 = -14$ ✗ Combine like terms.

▶ The equation $-12 = -14$ is never true. So, the system has no solution.

Solve the system of linear equations.

11. $x = y + 2$; $-3x + 3y = 6$

12. $3x - 6y = -9$; $-5x + 10y = 10$

13. $-4x + 4y = 32$; $3x + 24 = 3y$

ANSWERS

8. $(2, -8)$

9. $(-2, 5)$

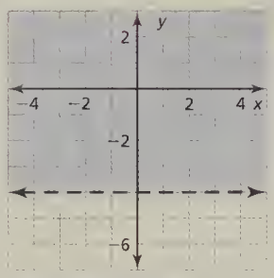
10. $(4, 5)$

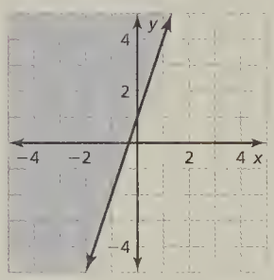
11. no solution

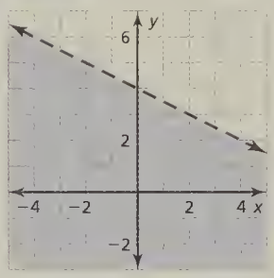
12. no solution

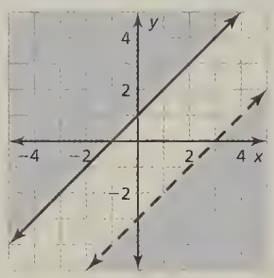
13. infinitely many solutions

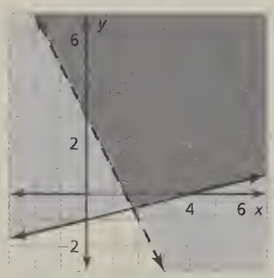
ANSWERS

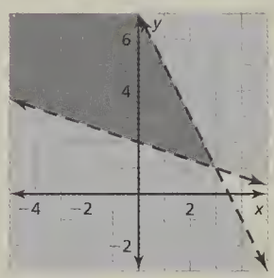
14. $x = -5$
15. $x = 13$
16. $x = -3$
17. 

18. 

19. 

20. 

21. 

22. 

5.5 Solving Equations by Graphing *(pp. 245–250)*

Solve $3x - 1 = -2x + 4$ by graphing. Check your solution.

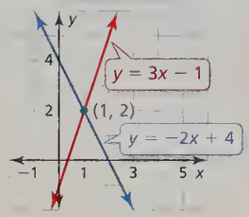
Step 1 Write a system of linear equations using each side of the original equation.

$y = 3x - 1$ → $3x - 1 = -2x + 4$ ← $y = -2x + 4$

Step 2 Graph the system.

$y = 3x - 1$ Equation 1
$y = -2x + 4$ Equation 2

The graphs intersect at $(1, 2)$.



▶ So, the solution of the equation is $x = 1$.

Check

$3x - 1 = -2x + 4$
$3(1) - 1 \stackrel{?}{=} -2(1) + 4$
$2 = 2$ ✓

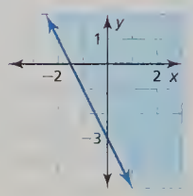
Solve the equation by graphing. Check your solution(s).

14. $x + 1 = -x - 9$ **15.** $2x - 8 = x + 5$ **16.** $\frac{1}{3}x + 5 = -2x - 2$

5.6 Linear Inequalities in Two Variables *(pp. 251–258)*

Graph $4x + 2y \geq -6$ in a coordinate plane.

Step 1 Graph $4x + 2y = -6$, or $y = -2x - 3$. Use a solid line because the inequality symbol is $\geq$.



Step 2 Test $(0, 0)$.

$4x + 2y \geq -6$ Write the inequality.
$4(0) + 2(0) \stackrel{?}{\geq} -6$ Substitute.
$0 \geq -6$ ✓ Simplify.

Step 3 Because $(0, 0)$ is a solution, shade the half-plane that contains $(0, 0)$.

Graph the inequality in a coordinate plane.

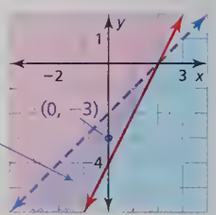
17. $y > -4$ **18.** $-9x + 3y \geq 3$ **19.** $5x + 10y < 40$

5.7 Systems of Linear Inequalities *(pp. 259–266)*

Graph the system. $y < x - 2$ Inequality 1
$y \geq 2x - 4$ Inequality 2

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. One solution is $(0, -3)$.



The solution is the purple-shaded region.

Graph the system of linear inequalities.

20. $y \leq x - 3$
$y \geq x + 1$

21. $y > -2x + 3$
$y \geq \frac{1}{4}x - 1$

22. $x + 3y > 6$
$2x + y < 7$

5 Chapter Test

Solve the system of linear equations using any method. Explain why you chose the method.

1. $8x + 3y = -9$
 $-8x + y = 29$

2. $\frac{1}{2}x + y = -6$
 $y = \frac{3}{5}x + 5$

3. $y = 4x + 4$
 $-8x + 2y = 8$

4. $x = y - 11$
 $x - 3y = 1$

5. $6x - 4y = 9$
 $9x - 6y = 15$

6. $y = 5x - 7$
 $-4x + y = -1$

7. Write a system of linear inequalities so the points $(1, 2)$ and $(4, -3)$ are solutions of the system, but the point $(-2, 8)$ is not a solution of the system.

Graph the system of linear inequalities.

8. $y > -1$
 $x < 3$

9. $y > \frac{1}{2}x + 4$
 $2y \leq x + 4$

10. $x + y < 1$
 $5x + y > 4$

11. $y \geq -\frac{2}{3}x + 1$
 $-3x + y > -2$

12. You pay \$45.50 for 10 gallons of gasoline and 2 quarts of oil at a gas station. Your friend pays \$22.75 for 5 gallons of the same gasoline and 1 quart of the same oil.

 a. Is there enough information to determine the cost of 1 gallon of gasoline and 1 quart of oil? Explain.

 b. The receipt shown is for buying the same gasoline and same oil. Is there now enough information to determine the cost of 1 gallon of gasoline and 1 quart of oil? Explain.

 c. Determine the cost of 1 gallon of gasoline and 1 quart of oil.

WELCOME
DATE 11/12/13 16:25
PUMP # 03
PRODUCT: REGUNL
GALLONS: 8.00
2 QUARTS OIL
TOTAL: \$38.40
THANK YOU
HAVE A NICE DAY

13. Describe the advantages and disadvantages of solving a system of linear equations by graphing.

14. You have at most \$60 to spend on trophies and medals to give as prizes for a contest.

 a. Write and graph an inequality that represents the numbers of trophies and medals you can buy. Identify and interpret a solution of the inequality.

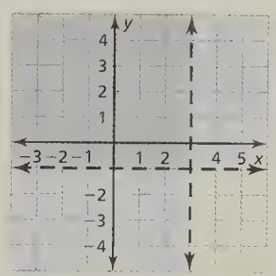
 b. You want to purchase at least 6 items. Write and graph a system that represents the situation. How many of each item can you buy?

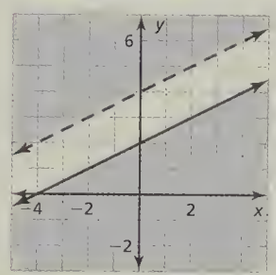


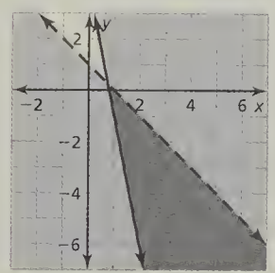
15. Compare the slopes and y-intercepts of the graphs of the equations in the linear system $8x + 4y = 12$ and $3y = -6x - 15$ to determine whether the system has one solution, no solution, or infinitely many solutions. Explain.

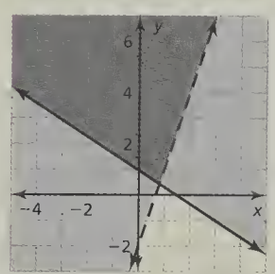
If students need help...	If students got it...
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

ANSWERS

1. $(-3, 5)$; *Sample answer:* elimination because one of the variables has opposite coefficients
2. $(-10, -1)$; *Sample answer:* substitution because one of the equations was solved for y
3. infinitely many solutions; *Sample answer:* substitution because one of the equations was solved for y
4. $(-17, -6)$; *Sample answer:* substitution because one of the equations was solved for x
5. no solution; *Sample answer:* elimination because no variable had a coefficient of 1 or –1
6. (6, 23); *Sample answer:* substitution because one of the equations was solved for y
7. *Sample answer:* $x > 0$ and $y < 6$
8. 

9. 

10. 

11. 

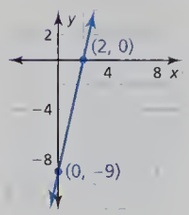
12–15. See Additional Answers.

ANSWERS

1. D
2. H
3. 3248
4. B
5. G

5 Standards Assessment

1. The graph of which equation is shown? *(TEKS A.2.B)*



 (A) $9x - 2y = -18$

 (B) $-9x - 2y = 18$

 (C) $9x + 2y = 18$

 (D) $-9x + 2y = -18$

2. A van rental company rents out 6-, 8-, 12-, and 16-passenger vans. The function $C(x) = 100 + 5x$ represents the cost C (in dollars) of renting an x-passenger van for a day. Which of the following numbers is in the range of the function? *(TEKS A.2.A)*

 (F) 6

 (G) 100

 (H) 130

 (J) 150

3. **GRIDDED ANSWER** The students in the graduating classes at three high schools in a school district have to pay for their cap-and-gown sets and extra tassels. At one high school, students pay \$3262 for 215 cap-and-gown sets and 72 extra tassels. At another high school, students pay \$3346 for 221 cap-and-gown sets and 72 extra tassels. How much (in dollars) do students at the third high school pay for 218 cap-and-gown sets and 56 extra tassels? *(TEKS A.2.I, TEKS A.5.C)*

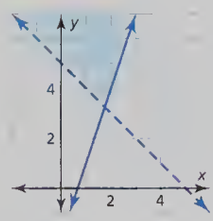
4. Which of the following points is *not* a solution of the system of linear inequalities represented by the graph? *(TEKS A.3.H)*

 (A) (4, 10)

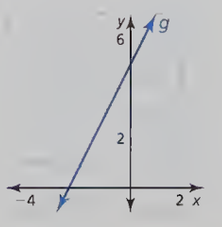
 (B) (5, 8)

 (C) (−2, 8)

 (D) (0, 10)



5. Consider the function $f(x) = 2x - 1$. Which of the following functions are represented by the graph? *(TEKS A.3.E)*



 I. $g(x) = f(x) + 6$

 II. $g(x) = f(x - 3)$

 III. $g(x) = f(x + 3)$

 IV. $g(x) = f(2x)$

 (F) I and II only

 (G) I and III only

 (H) I, II, and III only

 (J) I, II, III, and IV

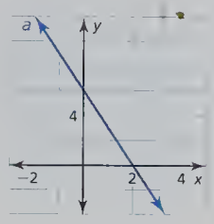
6. Line b is perpendicular to line a and passes through the point $(-9, 2)$. Which of the following is an equation for line b? *(TEKS A.2.F)*

Ⓐ $y = \frac{1}{3}x + 5$

Ⓑ $y = -\frac{1}{3}x - 1$

Ⓒ $y = \frac{1}{3}x - \frac{29}{3}$

Ⓓ $y = -\frac{1}{3}x + 1$



7. Which system of linear equations has no solution? *(TEKS A.5.C)*

Ⓕ $y - 2x = 5$
$4y - 10x = 20$

Ⓖ $y = -4x + 6$
$y = 8x - 12$

Ⓗ $2y = 6x + 4$
$y - \frac{1}{2} = 3x$

Ⓙ $y + 1 = 4x$
$y + 1 = \frac{1}{4}x$

8. A car dealership offers interest-free car loans for one day only. During this day, a salesperson at the dealership sells two cars. One of the clients decides to pay off a \$17,424 car in 36 monthly payments of \$484. The other client decides to pay off a \$15,840 car in 48 monthly payments of \$330. Which system of equations can you use to determine the number x of months after which both clients will have the same loan balance y (in dollars)? *(TEKS A.2.I)*

Ⓐ $y = -484x$
$y = -330x$

Ⓑ $y = -484x + 17{,}424$
$y = -330x + 15{,}840$

Ⓒ $y = -484x + 15{,}840$
$y = -330x + 17{,}424$

Ⓓ $y = 484x + 17{,}424$
$y = 330x + 15{,}840$

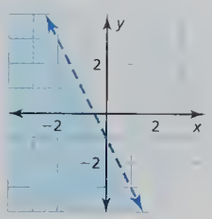
9. The graph of which linear inequality is shown? *(TEKS A.2.H)*

Ⓕ $y < -2x - 1$

Ⓖ $y \leq -2x - 1$

Ⓗ $y > -2x - 1$

Ⓙ $y \geq -2x - 1$



10. Simplify the expression $4(2x + 4) - 6(x + 1)$. *(TEKS A.10.D)*

Ⓐ $2x + 10$

Ⓑ $2x + 15$

Ⓒ $2x + 17$

Ⓓ $2x + 22$

ANSWERS

6. A
7. H
8. B
9. F
10. A

Chapter 6 Pacing Guide

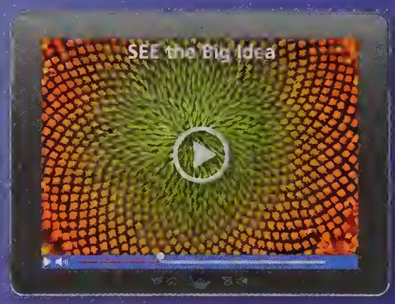
Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	3 Days
Quiz	1 Day
Section 5	2 Days
Section 6	2 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 6	17 Days
Year-to-Date	99 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
6.1	A.11.B
6.2	A.11.B
6.3	A.9.A, A.9.B, A.9.C, A.9.D, A.9.E
6.4	A.9.B, A.9.C, A.9.D
6.5	A.12.D
6.6	A.12.C

6 Exponential Functions and Sequences

6.1 Properties of Exponents
6.2 Radicals and Rational Exponents
6.3 Exponential Functions
6.4 Exponential Growth and Decay
6.5 Geometric Sequences
6.6 Recursively Defined Sequences



Fibonacci and Flowers *(p. 323)*


Soup Kitchen *(p. 318)*


Plutonium Decay *(p. 307)*


Coyote Population *(p. 297)*


Jellyfish *(p. 281)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

For an overview of this chapter, formative assessment tips, and teaching strategies, see Laurie's Notes at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

Using Order of Operations (6.7.A)

Example 1 Evaluate $10^2 \div (30 \div 3) - 4(3 - 9) + 5^1$.

First:	Parentheses	$10^2 \div (30 \div 3) - 4(3 - 9) + 5^1 = 10^2 \div 10 - 4(-6) + 5^1$
Second:	Exponents	$= 100 \div 10 - 4(-6) + 5$
Third:	Multiplication and Division (from left to right)	$= 10 + 24 + 5$
Fourth:	Addition and Subtraction (from left to right)	$= 39$

Evaluate the expression.

1. $12\left(\frac{14}{2}\right) - 3^3 + 15 - 9^2$
2. $5^2 \cdot 8 \div 2^2 + 20 \cdot 3 - 4$
3. $-7 + 16 \div 2^4 + (10 - 4^2)$

Finding Square Roots (A.11.A)

Example 2 Find $-\sqrt{81}$.

▶ $-\sqrt{81}$ represents the negative square root. Because $9^2 = 81$, $-\sqrt{81} = -\sqrt{9^2} = -9$.

Find the square root(s).

4. $\sqrt{64}$
5. $-\sqrt{4}$
6. $-\sqrt{25}$
7. $\pm\sqrt{121}$

Writing Equations for Arithmetic Sequences (A.12.D)

Example 3 Write an equation for the nth term of the arithmetic sequence 5, 15, 25, 35, ...

The first term is 5, and the common difference is 10.

$a_n = a_1 + (n - 1)d$	Equation for an arithmetic sequence
$a_n = 5 + (n - 1)(10)$	Substitute 5 for a_1 and 10 for d.
$a_n = 10n - 5$	Simplify.

Write an equation for the nth term of the arithmetic sequence.

8. 12, 14, 16, 18, . . .
9. 6, 3, 0, −3, . . .
10. 22, 15, 8, 1, . . .

11. **ABSTRACT REASONING** Recall that a perfect square is a number with integers as its square roots. Is the product of two perfect squares always a perfect square? Is the quotient of two perfect squares always a perfect square? Explain your reasoning.

Dynamic Solutions available at *BigIdeasMath.com*

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For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make Information Wheels for the following topics.

- Order of operations
- Square roots
- Arithmetic sequences

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

Texas Essential Knowledge and Skills

6.7.A The student is expected to generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization.

A.11.A The student is expected to simplify numerical radical expressions involving square roots.

A.12.D The student is expected to write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms.

ANSWERS

1. −9
2. 106
3. −12
4. 8
5. −2
6. −5
7. ±11
8. $a_n = 2n + 10$
9. $a_n = -3n + 9$
10. $a_n = -7n + 29$
11. yes; no; The product of two perfect squares can be represented by $m^2n^2 = (mm)(nn) = (mn)(mn) = (mn)^2$. If m and n are integers, their product is also an integer, so $(mn)^2$ is an integer. There are many counterexamples illustrating that the quotient of two perfect squares does not have to be a perfect square, such as $9 \div 4$.

MONITORING PROGRESS ANSWERS

1. 3435
2. 512

Mathematical Thinking

Mathematically proficient students use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. **(A.1.B)**

Problem-Solving Strategies

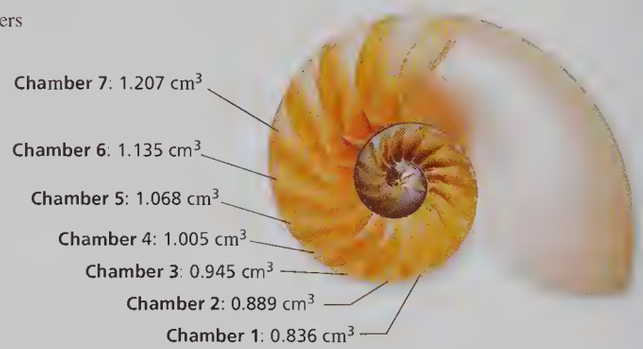
Core Concept

Finding a Pattern

When solving a real-life problem, look for a pattern in the data. The pattern could include repeating items, numbers, or events. After you find the pattern, describe it and use it to solve the problem.

EXAMPLE 1 **Using a Problem-Solving Strategy**

The volumes of seven chambers of a chambered nautilus are given. Find the volume of Chamber 10.



SOLUTION

To find a pattern, try dividing each volume by the volume of the previous chamber.

$$\frac{0.889}{0.836} \approx 1.063 \qquad \frac{0.945}{0.889} \approx 1.063 \qquad \frac{1.005}{0.945} \approx 1.063$$

$$\frac{1.068}{1.005} \approx 1.063 \qquad \frac{1.135}{1.068} \approx 1.063 \qquad \frac{1.207}{1.135} \approx 1.063$$

From this, you can see that the volume of each chamber is about 6.3% greater than the volume of the previous chamber. To find the volume of Chamber 10, multiply the volume of Chamber 7 by 1.063 three times.

$1.207(1.063) \approx \mathbf{1.283}$ ← volume of Chamber 8

$1.283(1.063) \approx \mathbf{1.364}$ ← volume of Chamber 9

$1.364(1.063) \approx \mathbf{1.450}$ ← volume of Chamber 10

▶ The volume of Chamber 10 is about 1.450 cubic centimeters.

Monitoring Progress

1. A rabbit population over 8 consecutive years is given by 50, 80, 128, 205, 328, 524, 839, 1342. Find the population in the tenth year.
2. The sums of the numbers in the first eight rows of Pascal's Triangle are 1, 2, 4, 8, 16, 32, 64, 128. Find the sum of the numbers in the tenth row.

For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

6.1 Properties of Exponents

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.11.B

Essential Question How can you write general rules involving properties of exponents?

EXPLORATION 1 **Writing Rules for Properties of Exponents**

Work with a partner.

ANALYZING MATHEMATICAL RELATIONSHIPS
To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results in writing general rules.

a. What happens when you multiply two powers with the same base? Write the product of the two powers as a single power. Then write a *general rule* for finding the product of two powers with the same base.

i. $(2^2)(2^3) =$ ii. $(4^1)(4^5) =$

iii. $(5^3)(5^5) =$ iv. $(x^2)(x^6) =$

b. What happens when you divide two powers with the same base? Write the quotient of the two powers as a single power. Then write a *general rule* for finding the quotient of two powers with the same base.

i. $\frac{4^3}{4^2} =$ ii. $\frac{2^5}{2^2} =$

iii. $\frac{x^6}{x^3} =$ iv. $\frac{3^4}{3^4} =$

c. What happens when you find a power of a power? Write the expression as a single power. Then write a *general rule* for finding a power of a power.

i. $(2^2)^4 =$ ii. $(7^3)^2 =$

iii. $(y^3)^3 =$ iv. $(x^4)^2 =$

d. What happens when you find a power of a product? Write the expression as the product of two powers. Then write a *general rule* for finding a power of a product.

i. $(2 \cdot 5)^2 =$ ii. $(5 \cdot 4)^3 =$

iii. $(6a)^2 =$ iv. $(3x)^2 =$

e. What happens when you find a power of a quotient? Write the expression as the quotient of two powers. Then write a *general rule* for finding a power of a quotient.

i. $\left(\frac{2}{3}\right)^2 =$ ii. $\left(\frac{4}{3}\right)^3 =$

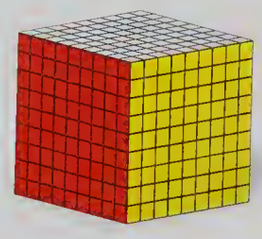
iii. $\left(\frac{x}{2}\right)^3 =$ iv. $\left(\frac{a}{b}\right)^4 =$

Communicate Your Answer

2. How can you write general rules involving properties of exponents?

3. There are 3^3 small cubes in the cube below. Write an expression for the number of small cubes in the large cube at the right.





For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students work in pairs to complete the exploration and Communicate Your Answer. Have them take turns reading aloud and discussing the instructions and questions with each other. Explain that to make certain they understand what their partner is saying, they should restate it in their own words and ask if that is what was meant.

Beginning Restate using simple phrases of two or three words.
Intermediate Restate using simple sentences.
Advanced/Advanced High Restate using multiple sentences.

ELPS 2.D.2 Seek clarification [of spoken language] as needed.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.11.B The student is expected to simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

ANSWERS

1. a. $a^m a^n = a^{m+n}$
- i. 2^5
- ii. 4^6
- iii. 5^8
- iv. x^8

b. $\frac{a^m}{a^n} = a^{m-n}$
- i. 4^1
- ii. 2^3
- iii. x^3
- iv. 3^0

c. $(a^m)^n = a^{mn}$
- i. 2^8
- ii. 7^6
- iii. y^9
- iv. x^8

d. $(ab)^m = a^m b^m$
- i. $2^2 \cdot 5^2$
- ii. $5^3 \cdot 4^3$
- iii. $6^2 a^2$
- iv. $3^2 x^2$

e. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- i. $\frac{2^2}{3^2}$
- ii. $\frac{4^3}{3^3}$
- iii. $\frac{x^3}{2^3}$
- iv. $\frac{a^4}{b^4}$

2. Try several examples to find a pattern, then express the pattern using variables.

3. 9^3

Extra Example 1

Evaluate each expression.

a. 2.5^0 1

b. $(-3)^{-2}$ $\frac{1}{9}$

Extra Example 2

Simplify the expression $\frac{2^0x^2}{y^{-2}}$. Write your answer using only positive exponents. x^2y^2

MONITORING PROGRESS ANSWERS

1. 1
2. $\frac{1}{27}$
3. -4
4. $\frac{1}{9x^5}$

6.1 Lesson

What You Will Learn

- Use zero and negative exponents.
- Use the properties of exponents.
- Solve real-life problems involving exponents.

Core Vocabulary

Previous

power
exponent
base
scientific notation

Using Zero and Negative Exponents

Core Concept

Zero Exponent

Words For any nonzero number a, $a^0 = 1$. The power 0^0 is undefined.

Numbers $4^0 = 1$ **Algebra** $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any nonzero number a, a^{-n} is the reciprocal of a^n.

Numbers $4^{-2} = \frac{1}{4^2}$ **Algebra** $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

EXAMPLE 1 Using Zero and Negative Exponents

Evaluate each expression.

a. 6.7^0 b. $(-2)^{-4}$

SOLUTION

a. $6.7^0 = 1$ Definition of zero exponent

b. $(-2)^{-4} = \frac{1}{(-2)^4}$ Definition of negative exponent

$= \frac{1}{16}$ Simplify.

EXAMPLE 2 Simplifying an Expression

Simplify the expression $\frac{4x^0}{y^{-3}}$. Write your answer using only positive exponents.

SOLUTION

$\frac{4x^0}{y^{-3}} = 4x^0y^3$ Definition of negative exponent

$= 4y^3$ Definition of zero exponent

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Evaluate the expression.

1. $(-9)^0$ 2. 3^{-3} 3. $\frac{-5^0}{2^{-2}}$

4. Simplify the expression $\frac{3^{-2}x^{-5}}{y^0}$. Write your answer using only positive exponents.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Inclusion

Challenge students to write expressions that are equivalent to a simplified expression. For example, give students the expression x^2y. Have them write two different expressions that can be simplified to x^2y. *Sample answer:* $\frac{x^3}{xy^{-1}}, \frac{x^{-1}y^2}{x^{-3}y}$

REMEMBER

The expression x^3 is called a *power*. The *base*, x, is used as a factor 3 times because the *exponent* is 3.

Core Concept

Product of Powers Property

Let a be a real number, and let m and n be integers.

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \cdot 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be integers.

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Let a be a real number, and let m and n be integers.

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

EXAMPLE 3 Using Properties of Exponents

Simplify each expression. Write your answer using only positive exponents.

a. $3^2 \cdot 3^6$ b. $\frac{(-4)^2}{(-4)^7}$ c. $(z^4)^{-3}$

SOLUTION

a. $3^2 \cdot 3^6 = 3^{2+6}$ Product of Powers Property

$= 3^8 = 6561$ Simplify.

b. $\frac{(-4)^2}{(-4)^7} = (-4)^{2-7}$ Quotient of Powers Property

$= (-4)^{-5}$ Simplify.

$= \frac{1}{(-4)^5} = -\frac{1}{1024}$ Definition of negative exponent

c. $(z^4)^{-3} = z^{4 \cdot (-3)}$ Power of a Power Property

$= z^{-12}$ Simplify.

$= \frac{1}{z^{12}}$ Definition of negative exponent

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Simplify the expression. Write your answer using only positive exponents.

5. $10^4 \cdot 10^{-6}$
6. $x^9 \cdot x^{-9}$
7. $\frac{-5^8}{-5^4}$
8. $\frac{y^6}{y^7}$
9. $(6^{-2})^{-1}$
10. $(w^{12})^5$

Extra Example 3

Simplify each expression. Write your answer using only positive exponents.

a. $2^3 \cdot 2^5$ $2^8 = 256$

b. $\frac{(-7)^2}{(-7)^4}$ $\frac{1}{(-7)^2} = \frac{1}{49}$

c. $(d^{-3})^5$ $d^{-15} = \frac{1}{d^{15}}$

MONITORING PROGRESS ANSWERS

5. $\frac{1}{100}$
6. 1
7. 625
8. $\frac{1}{y}$
9. 36
10. w^{60}

Extra Example 4

Simplify each expression. Write your answer using only positive exponents.

a. $(-3x)^3$ $-27x^3$

b. $\left(\frac{b}{-3}\right)^4$ $\frac{b^4}{81}$

c. $\left(\frac{2x}{5}\right)^3$ $\frac{8x^3}{125}$

d. $\left(\frac{3c}{4}\right)^{-2}$ $\frac{16}{9c^2}$

MONITORING PROGRESS ANSWERS

11. $\frac{1}{1000y^3}$

12. $\frac{-1024}{n^5}$

13. $\frac{1}{32k^{10}}$

14. $\frac{49}{36c^2}$

Core Concept

Power of a Product Property

Let a and b be real numbers, and let m be an integer.

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be an integer.

Words To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

EXAMPLE 4 Using Properties of Exponents

Simplify each expression. Write your answer using only positive exponents.

a. $(-1.5y)^2$ b. $\left(\frac{a}{-10}\right)^3$ c. $\left(\frac{3d}{2}\right)^4$ d. $\left(\frac{2x}{3}\right)^{-5}$

SOLUTION

a. $(-1.5y)^2 = (-1.5)^2 \cdot y^2$ Power of a Product Property

$= 2.25y^2$ Simplify.

b. $\left(\frac{a}{-10}\right)^3 = \frac{a^3}{(-10)^3}$ Power of a Quotient Property

$= -\frac{a^3}{1000}$ Simplify.

c. $\left(\frac{3d}{2}\right)^4 = \frac{(3d)^4}{2^4}$ Power of a Quotient Property

$= \frac{3^4 d^4}{2^4}$ Power of a Product Property

$= \frac{81d^4}{16}$ Simplify.

d. $\left(\frac{2x}{3}\right)^{-5} = \frac{(2x)^{-5}}{3^{-5}}$ Power of a Quotient Property

$= \frac{3^5}{(2x)^5}$ Definition of negative exponent

$= \frac{3^5}{2^5 x^5}$ Power of a Product Property

$= \frac{243}{32x^5}$ Simplify.

ANOTHER WAY

Because the exponent is negative, you could find the reciprocal of the base first. Then simplify.

$\left(\frac{2x}{3}\right)^{-5} = \left(\frac{3}{2x}\right)^5 = \frac{243}{32x^5}$

Monitoring Progress

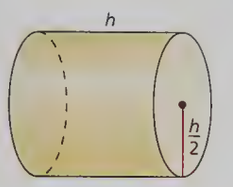
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Simplify the expression. Write your answer using only positive exponents.

11. $(10y)^{-3}$ 12. $\left(-\frac{4}{n}\right)^5$ 13. $\left(\frac{1}{2k^2}\right)^5$ 14. $\left(\frac{6c}{7}\right)^{-2}$

Solving Real-Life Problems

EXAMPLE 5 **Simplifying a Real-Life Expression**



Volume = ?

$2\pi r^3$ $\pi h^3 2^{-2}$ $\pi h 4^{-1}$

$\frac{\pi h^2}{4}$ $\frac{\pi h^3}{4}$ $\frac{\pi h^3}{2}$

Which of the expressions shown represent the volume of the cylinder, where r is the radius and h is the height?

SOLUTION

$V = \pi r^2 h$ Formula for the volume of a cylinder

$= \pi\left(\frac{h}{2}\right)^2 (h)$ Substitute $\frac{h}{2}$ for r.

$= \pi\left(\frac{h^2}{2^2}\right)(h)$ Power of a Quotient Property

$= \frac{\pi h^3}{4}$ Simplify.

Any expression equivalent to $\frac{\pi h^3}{4}$ represents the volume of the cylinder.

- You can use the properties of exponents to write $\pi h^3 2^{-2}$ as $\frac{\pi h^3}{4}$.
- Note $h = 2r$. When you substitute $2r$ for h in $\frac{\pi h^3}{4}$, you can write $\frac{\pi(2r)^3}{4}$ as $2\pi r^3$.
- None of the other expressions are equivalent to $\frac{\pi h^3}{4}$.

▶ The expressions $2\pi r^3$, $\pi h^3 2^{-2}$, and $\frac{\pi h^3}{4}$ represent the volume of the cylinder.

REMEMBER

A number is written in scientific notation when it is of the form $a \times 10^b$, where $1 \le a < 10$ and b is an integer.

EXAMPLE 6 **Solving a Real-Life Problem**

A jellyfish emits about 1.25×10^8 particles of light, or photons, in 6.25×10^{-4} second. How many photons does the jellyfish emit each second? Write your answer in scientific notation and in standard form.



SOLUTION

Divide to find the unit rate.

$\frac{1.25 \times 10^8}{6.25 \times 10^{-4}}$ ← photons, ← seconds — Write the rate.

$= \frac{1.25}{6.25} \times \frac{10^8}{10^{-4}}$ Rewrite.

$= 0.2 \times 10^{12}$ Simplify.

$= 2 \times 10^{11}$ Write in scientific notation.

▶ The jellyfish emits 2×10^{11}, or 200,000,000,000 photons per second.

Monitoring Progress

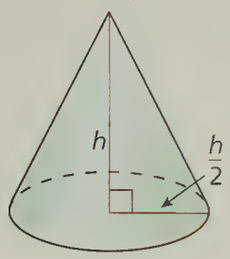
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15. Write two expressions that represent the area of a base of the cylinder in Example 5.

16. It takes the Sun about 2.3×10^8 years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^2 years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the center of the Milky Way? Write your answer in scientific notation.

Extra Example 5

Which of the expressions shown represent the volume of the cone, where r is the radius and h is the height?



$\frac{\pi h^3}{12}$ $\frac{\pi r^3}{3}$ $\frac{\pi h^2}{6}$

$\frac{\pi h^2}{12}$ $\frac{2\pi r^3}{3}$ $8\pi r^3$ $\frac{\pi h^3}{12}$, $\frac{2\pi r^3}{3}$

Extra Example 6

A DVD player uses approximately 1.35×10^{-3} kilowatt-hours of energy in 4.5×10^{-2} hours. How many kilowatt-hours of energy does the DVD player use per hour? Write your answer in scientific notation and in standard form.

3.0×10^{-2}, or 0.03 kilowatt-hour of energy per hour

MONITORING PROGRESS ANSWERS

15. πr^2; $\frac{\pi h^2}{4}$

16. 9.2×10^5

Closure

- **Response Logs:** Select from "What is confusing me the most is ..." or "I was successful in ...".

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–45 odd, 55, 64, 67, 70–75

Average: 1–3, 4–44 even, 45, 46–58 even, 64, 67, 70–75

Advanced: 1–4, 10–24 even, 32–44 even, 45, 46–64 even, 65–75

HOMEWORK CHECK

Basic: 5, 13, 23, 37, 55

Average: 8, 16, 24, 38, 56

Advanced: 10, 18, 32, 40, 56

ANSWERS

1. Product of Powers Property, Power of a Power Property, definition of negative exponents; Use the Product of Powers Property to simplify the expression inside the parentheses to 4^4. Then, use the Power of a Power Property to simplify the entire expression to 4^{-8}. Then, use the definition of negative exponents to produce the final answer, $\frac{1}{4^8} = \frac{1}{65{,}536}$.
2. The Power of a Product Property is used when finding a power of a product by finding the power of each factor and multiplying.
3. The Quotient of Powers Property is used when dividing powers that have the same base. The answer is the common base raised to the difference of the exponents of the numerator and denominator.
4. Simplify $3^{6 \cdot 3}$; 3^{18}; 3^9
5. 1
6. 1
7. $\frac{1}{625}$
8. $-\frac{1}{32}$
9. $\frac{1}{16}$
10. $-\frac{1}{5}$
11. $-\frac{4}{3}$
12. $\frac{81}{64}$
13. $\frac{1}{x^7}$
14. 1
15. $\frac{9}{y^3}$
16. $\frac{15}{c^8}$
17. $\frac{1}{4m^3}$
18. $\frac{s}{9r^{11}}$
19. $\frac{b^7}{64}$
20. $\frac{49q^9}{p^8}$
21. $\frac{32x^7}{y^6}$
22. $\frac{1625z^{10}}{x^5}$
23. 625
24. -216
25. 6561
26. 1
27. p^{24}
28. $\frac{1}{s^{15}}$
29. $\frac{1}{216}$
30. $-\frac{1}{343}$
31. x^2
32. z^5
33. 10^{-2} m
34. $14a^2b$ microns
35. The product has a base of 2, not $2 \cdot 2$; $2^4 \cdot 2^5 = 2^9$
36. The exponent of the quotient should be the difference of the exponents, not the quotient of the exponents; $\frac{x^8}{x^4} = x^4$

6.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Which definitions or properties would you use to simplify the expression $(4^8 \cdot 4^{-4})^{-2}$? Explain.
2. **WRITING** Explain when and how to use the Power of a Product Property.
3. **WRITING** Explain when and how to use the Quotient of Powers Property.
4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Simplify $3^3 \cdot 3^6$. | Simplify 3^{3+6}. | Simplify $3^{6 \cdot 3}$. | Simplify $3^6 \cdot 3^3$.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, evaluate the expression. *(See Example 1.)*

5. $(-7)^0$
6. 4^0
7. 5^{-4}
8. $(-2)^{-5}$
9. $\frac{2^{-4}}{4^0}$
10. $\frac{5^{-1}}{-9^0}$
11. $\frac{-3^{-3}}{6^{-2}}$
12. $\frac{(-8)^{-2}}{3^{-4}}$

In Exercises 13–22, simplify the expression. Write your answer using only positive exponents. *(See Example 2.)*

13. x^{-7}
14. y^0
15. $9x^0y^{-3}$
16. $15c^{-8}d^0$
17. $\frac{2^{-2}m^{-3}}{n^0}$
18. $\frac{10^0r^{-11}s}{3^2}$
19. $\frac{4^{-3}a^0}{b^{-7}}$
20. $\frac{p^{-8}}{7^{-2}q^{-9}}$
21. $\frac{2^2y^{-6}}{8^{-1}z^0x^{-7}}$
22. $\frac{13x^{-5}y^0}{5^{-3}z^{-10}}$

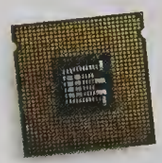
In Exercises 23–32, simplify the expression. Write your answer using only positive exponents. *(See Example 3.)*

23. $\frac{5^6}{5^2}$
24. $\frac{(-6)^8}{(-6)^5}$
25. $(-9)^2 \cdot (-9)^2$
26. $4^{-5} \cdot 4^5$
27. $(p^6)^4$
28. $(s^{-5})^3$
29. $6^{-8} \cdot 6^5$
30. $-7 \cdot (-7)^{-4}$
31. $\frac{x^5}{x^4} \cdot x$
32. $\frac{z^8 \cdot z^2}{z^5}$

33. **USING PROPERTIES** A microscope magnifies an object 10^5 times. The length of an object is 10^{-7} meter. What is its magnified length?



34. **USING PROPERTIES** The area of the rectangular computer chip is $112a^3b^2$ square microns. What is the length?



width = $8ab$ microns

ERROR ANALYSIS **In Exercises 35 and 36, describe and correct the error in simplifying the expression.**

35. ✗
$$2^4 \cdot 2^5 = (2 \cdot 2)^{4+5}$$
$$= 4^9$$

36. ✗
$$\frac{x^5 \cdot x^3}{x^4} = \frac{x^8}{x^4}$$
$$= x^{8/4}$$
$$= x^2$$

In Exercises 37–44, simplify the expression. Write your answer using only positive exponents. *(See Example 4.)*

37. $(-5z)^3$

38. $(4x)^{-4}$

39. $\left(\frac{6}{n}\right)^{-2}$

40. $\left(\frac{-t}{3}\right)^2$

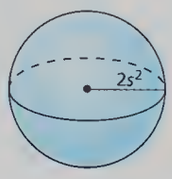
41. $(3s^8)^{-5}$

42. $(-5p^3)^3$

43. $\left(-\frac{w^3}{6}\right)^{-2}$

44. $\left(\frac{1}{2r^6}\right)^{-6}$

45. **USING PROPERTIES** Which of the expressions represent the volume of the sphere? Explain. *(See Example 5.)*



Ⓐ $\left(\frac{3s^2}{2^4\pi s^8}\right)^{-1}$

Ⓑ $(2^5\pi s^6)(3^{-1})$

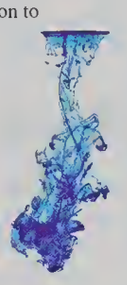
Ⓒ $\frac{32\pi s^6}{3}$

Ⓓ $(2s)^5 \cdot \frac{\pi s}{3}$

Ⓔ $\left(\frac{3\pi s^6}{32}\right)^{-1}$

Ⓕ $\frac{32}{3}\pi s^5$

46. **MODELING WITH MATHEMATICS** Diffusion is the movement of molecules from one location to another. The time t (in seconds) it takes molecules to diffuse a distance of x centimeters is given by $t = \frac{x^2}{2D}$, where D is the diffusion coefficient. The diffusion coefficient for a drop of ink in water is about 10^{-5} square centimeters per second. How long will it take the ink to diffuse 1 micrometer (10^{-4} centimeter)?



In Exercises 47–50, simplify the expression. Write your answer using only positive exponents.

47. $\left(\frac{2x^{-2}y^3}{3xy^{-4}}\right)^4$

48. $\left(\frac{4s^5t^{-7}}{-2s^{-2}t^4}\right)^3$

49. $\left(\frac{3m^{-5}n^2}{4m^{-2}n^0}\right)^2 \cdot \left(\frac{mn^4}{9n}\right)^2$

50. $\left(\frac{3x^3y^0}{x^{-2}}\right)^4 \cdot \left(\frac{y^2x^{-4}}{5xy^{-8}}\right)^3$

In Exercises 51–54, evaluate the expression. Write your answer in scientific notation and standard form.

51. $(3 \times 10^2)(1.5 \times 10^{-5})$

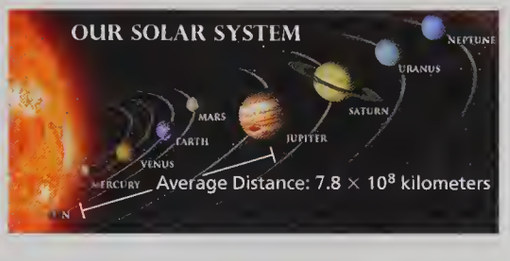
52. $(6.1 \times 10^{-3})(8 \times 10^9)$

53. $\frac{(6.4 \times 10^7)}{(1.6 \times 10^5)}$

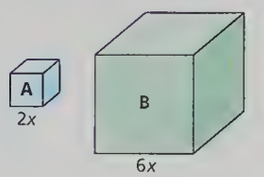
54. $\frac{(3.9 \times 10^{-5})}{(7.8 \times 10^{-8})}$

55. **PROBLEM SOLVING** In 2012, on average, about 9.46×10^{-1} pound of potatoes was produced for every 2.3×10^{-5} acre harvested. How many pounds of potatoes on average were produced for each acre harvested? Write your answer in scientific notation and in standard form. *(See Example 6.)*

56. **PROBLEM SOLVING** The speed of light is approximately 3×10^5 kilometers per second. How long does it take sunlight to reach Jupiter? Write your answer in scientific notation and in standard from.



57. **MATHEMATICAL CONNECTIONS** Consider Cube A and Cube B.



a. Which property of exponents should you use to simplify an expression for the volume of each cube?

b. How can you use the Power of a Quotient Property to find how many times greater the volume of Cube B is than the volume of Cube A?

58. **PROBLEM SOLVING** A byte is a unit used to measure a computer's memory. The table shows the numbers of bytes in several units of measure.

Unit	kilobyte	megabyte	gigabyte	terabyte
Number of bytes	2^{10}	2^{20}	2^{30}	2^{40}

a. How many kilobytes are in 1 terabyte?

b. How many megabytes are in 16 gigabytes?

c. Another unit used to measure a computer's memory is a bit. There are 8 bits in a byte. How can you convert the number of bytes in each unit of measure given in the table to bits? Can you still use a base of 2? Explain.

Dynamic Teaching Tools
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ANSWERS

37. $-125z^3$

38. $\frac{1}{256x^4}$

39. $\frac{n^2}{36}$

40. $\frac{t^2}{9}$

41. $\frac{1}{243s^{40}}$

42. $-125p^9$

43. $\frac{36}{w^6}$

44. $64r^{36}$

45. B, C, D; These expressions simplify to be the volume of the sphere, which is $\frac{32\pi s^6}{3}$.

46. 5×10^{-4} sec

47. $\frac{16y^{28}}{81x^{12}}$

48. $\frac{-8s^{21}}{t^{33}}$

49. $\frac{n^{10}}{144m^4}$

50. $\frac{81x^5y^{30}}{125}$

51. 4.5×10^{-3}; 0.0045

52. 4.88×10^7; 48,800,000

53. 4×10^2; 400

54. 5×10^2; 500

55. about 4.113×10^4 lb/acre; about 41,130 lb/acre

56. 2.6×10^3 sec; 2600 sec

57. a. Power of a Product Property

b. Express $\frac{(6x)^3}{(2x)^3}$ as $\left(\frac{6x}{2x}\right)^3$. Simplify the expression inside the parentheses to produce $(3)^3$, so the volume is 27 times greater.

58. a. 2^{30} kilobytes

b. 16,384 megabytes

c. Multiply each number in the table by 8; yes; The number 8 can be expressed as 2^3, so multiply each number in the table by 2^3. Because the values have a common base of 2, they can be simplified using the Product of Powers Property.

ANSWERS

59. $(2ab)^3$

60. $(4rs)^2$

61. $(2w^3z^2)^6$

62. $(9x^2y^4)^2$ or $(3xy^2)^4$

63. **a.** $\left(\frac{1}{6}\right)^n$

b. $\frac{1}{1296}$

c. $\frac{1}{32}$; The probability of flipping heads once is $\frac{1}{2}$, and $\left(\frac{1}{2}\right)^5$ is $\frac{1}{32}$.

64. **a.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$

b. 2^{-1}; 2^{-2}; 2^{-3}; 2^{-4}

65. $x = 8$, $y = -1$; Using the Quotient of Powers Property, you can conclude from the first equation that $x - y = 9$. Using the Product of Powers Property and the Quotient of Powers Property, you can conclude from the second equation that $x + 2 - 3y = 13$. Use these equations to solve a system of linear equations.

66. *Sample answer:* $r = x^2$, $h = 81x^4$

67. no; The mass of the seed from the double coconut palm is 10 kilograms.

68. **a.** 2^{13}

b. 2^{23}

c. They become 3^{13} and 3^{23}.

69–75. See Additional Answers.

REWRITING EXPRESSIONS **In Exercises 59–62, rewrite the expression as a power of a product.**

59. $8a^3b^3$

60. $16r^2s^2$

61. $64w^{18}z^{12}$

62. $81x^4y^8$

63. USING STRUCTURE The probability of rolling a 6 on a number cube is $\frac{1}{6}$. The probability of rolling a 6 twice in a row is $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$.

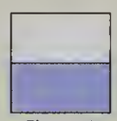


a. Write an expression that represents the probability of rolling a 6 n times in a row.

b. What is the probability of rolling a 6 four times in a row?

c. What is the probability of flipping heads on a coin five times in a row? Explain.

64. HOW DO YOU SEE IT? The shaded part of Figure n represents the portion of a piece of paper visible after folding the paper in half n times.


Figure 1

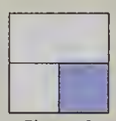

Figure 2


Figure 3

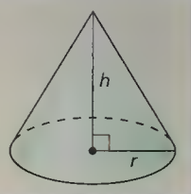

Figure 4

a. What fraction of the original piece of paper is each shaded part?

b. Rewrite each fraction from part (a) in the form 2^x.

65. REASONING Find x and y when $\frac{b^x}{b^y} = b^9$ and $\frac{b^x \cdot b^2}{b^{3y}} = b^{13}$. Explain how you found your answer.

66. THOUGHT PROVOKING Write expressions for r and h so that the volume of the cone can be represented by the expression $27\pi x^8$. Find r and h.



67. MAKING AN ARGUMENT One of the smallest plant seeds comes from an orchid, and one of the largest plant seeds comes from a double coconut palm. A seed from an orchid has a mass of 10^{-6} gram. The mass of a seed from a double coconut palm is 10^{10} times the mass of the seed from the orchid. Your friend says that the seed from the double coconut palm has a mass of about 1 kilogram. Is your friend correct? Explain.

68. CRITICAL THINKING Your school is conducting a survey. Students can answer the questions in either part with "agree" or "disagree."

Part 1: 13 questions
Part 2: 10 questions

Part 1: Classroom	Agree	Disagree
1. I come prepared for class.	○	○
2. I enjoy my assignments.	○	○

a. What power of 2 represents the number of different ways that a student can answer all the questions in Part 1?

b. What power of 2 represents the number of different ways that a student can answer all the questions on the entire survey?

c. The survey changes, and students can now answer "agree," "disagree," or "no opinion." How does this affect your answers in parts (a) and (b)?

69. ABSTRACT REASONING Compare the values of a^n and a^{-n} when $n < 0$, when $n = 0$, and when $n > 0$ for (a) $a > 1$ and (b) $0 < a < 1$. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the square root(s). *(Skills Review Handbook)*

70. $\sqrt{25}$

71. $-\sqrt{100}$

72. $\pm\sqrt{\frac{1}{64}}$

Classify the real number in as many ways as possible. *(Skills Review Handbook)*

73. 12

74. $\frac{65}{9}$

75. $\frac{\pi}{4}$

Mini-Assessment

1. Evaluate the expression $\frac{6^0}{7^{-2}}$. 49

Simplify the expression. Write your answer using only positive exponents.

2. $(x^{-4})^3 \; x^{-12} = \frac{1}{x^{12}}$

3. $\left(\frac{5n}{-3}\right)^{-3} \; -\frac{27}{125n^3}$

4. A laptop computer uses approximately 8.4×10^{-3} kilowatt-hours of energy in 3.5×10^{-1} hours. How many kilowatt-hours of energy does the laptop use per hour? Write your answer in scientific notation and in standard form. 2.4×10^{-2}, or 0.024 kilowatt-hours of energy per hour

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

6.2 Radicals and Rational Exponents

Texas Essential Knowledge and Skills
A.11.B

Essential Question How can you write and evaluate an nth root of a number?

Recall that you cube a number as follows.

3rd power

$2^3 = 2 \cdot 2 \cdot 2 = 8$ 2 cubed is 8.

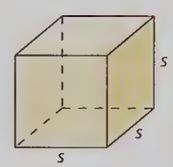
To "undo" cubing a number, take the cube root of the number.

Symbol for cube root is $\sqrt[3]{\ }$.

$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ The cube root of 8 is 2.

EXPLORATION 1 Finding Cube Roots

Work with a partner. Use a cube root symbol to write the side length of each cube. Then find the cube root. Check your answers by multiplying. Which cube is the largest? Which two cubes are the same size? Explain your reasoning.



a. Volume $= 27$ ft^3 **b.** Volume $= 125$ cm^3 **c.** Volume $= 3375$ in.^{3}

d. Volume $= 3.375$ m^3 **e.** Volume $= 1$ yd^3 **f.** Volume $= \frac{125}{8}$ mm^3

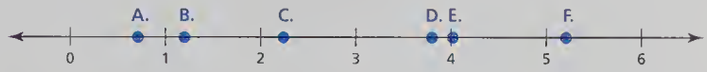
JUSTIFYING THE SOLUTION
To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 Estimating nth Roots

Work with a partner. Estimate each positive nth root. Then match each nth root with the point on the number line. Justify your answers.

a. $\sqrt[4]{25}$ **b.** $\sqrt{0.5}$ **c.** $\sqrt[5]{2.5}$

d. $\sqrt[3]{65}$ **e.** $\sqrt[3]{55}$ **f.** $\sqrt[6]{20{,}000}$



Communicate Your Answer

3. How can you write and evaluate an nth root of a number?
4. The body mass m (in kilograms) of a dinosaur that walked on two feet can be modeled by

$$m = (0.00016)C^{2.73}$$

where C is the circumference (in millimeters) of the dinosaur's femur. The mass of a *Tyrannosaurus rex* was 4000 kilograms. Use a calculator to approximate the circumference of its femur.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.11.B The student is expected to simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

ANSWERS

1. **a.** $\sqrt[3]{27}$ ft; 3 ft
 b. $\sqrt[3]{125}$ cm; 5 cm
 c. $\sqrt[3]{3375}$ in.; 15 in.
 d. $\sqrt[3]{3.375}$ m; 1.5 m
 e. $\sqrt[3]{1}$ yd; 1 yd
 f. $\sqrt[3]{\frac{125}{8}}$; 2.5 mm
 d; a and e; The cube in part (b) has the largest side length of 1.5 meters. The cubes in parts (a) and (e) have equal side lengths because 3 feet = 1 yard.
2. **a.** *Sample answer:* 2.2; C; 25 is between $2^4 = 16$ and $3^4 = 81$, and C is the only point on the graph between 2 and 3.
 b. *Sample answer:* 0.7; A; 0.5 is between $0^2 = 0$ and $1^2 = 1$, and A is the only point on the graph between 0 and 1.
 c. *Sample answer:* 1.2; B; 2.5 is between $1^5 = 1$ and $2^5 = 32$, and B is the only point on the graph between 1 and 2.

2d–4. See Additional Answers.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students complete the explorations in groups of mixed language abilities. Remind students that to obtain information from each other, they may want to use the question words *how, which, why, what, when, where,* or *who*.

Advanced/Advanced High Lead the explorations by reading the instructions. Ask others if they have questions. Question the teacher if clarification is needed.
Beginning Ask for basic information needed to do the calculations.
Intermediate Ask for clarification of the procedures needed.

ELPS 3.F.1 Ask [for] information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments.

Extra Example 1

Find the indicated real nth root(s) of a.

a. $n = 3, a = -64$ $\sqrt[3]{-64} = -4$, or $(-64)^{1/3} = -4$

b. $n = 2, a = 81$ $\pm\sqrt{81} = \pm 9$, or $\pm(81)^{1/2} = \pm 9$

MONITORING PROGRESS ANSWERS

1. -5
2. ± 2

6.2 Lesson

Core Vocabulary

*n*th root of a, *p. 286*
radical, *p. 286*
index of a radical, *p. 286*

Previous
square root

What You Will Learn

- Find nth roots.
- Evaluate expressions with rational exponents.
- Solve real-life problems involving rational exponents.

Finding nth Roots

You can extend the concept of a square root to other types of roots. For example, 2 is a cube root of 8 because $2^3 = 8$, and 3 is a fourth root of 81 because $3^4 = 81$. In general, for an integer n greater than 1, if $b^n = a$, then b is an **nth root of a**. An nth root of a is written as $\sqrt[n]{a}$, where the expression $\sqrt[n]{a}$ is called a **radical** and n is the **index** of the radical.

You can also write an nth root of a as a power of a. If you assume the Power of a Power Property applies to rational exponents, then the following is true.

$$(a^{1/2})^2 = a^{(1/2) \cdot 2} = a^1 = a$$
$$(a^{1/3})^3 = a^{(1/3) \cdot 3} = a^1 = a$$
$$(a^{1/4})^4 = a^{(1/4) \cdot 4} = a^1 = a$$

Because $a^{1/2}$ is a number whose square is a, you can write $\sqrt{a} = a^{1/2}$. Similarly, $\sqrt[3]{a} = a^{1/3}$ and $\sqrt[4]{a} = a^{1/4}$. In general, $\sqrt[n]{a} = a^{1/n}$ for any integer n greater than 1.

READING

$\pm\sqrt[n]{a}$ represents both the positive and negative nth roots of a.

Core Concept

Real nth Roots of a

Let n be an integer greater than 1, and let a be a real number.

- If n is odd, then a has one real nth root: $\sqrt[n]{a} = a^{1/n}$
- If n is even and $a > 0$, then a has two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If n is even and $a = 0$, then a has one real nth root: $\sqrt[n]{0} = 0$
- If n is even and $a < 0$, then a has no real nth roots.

The nth roots of a number may be real numbers or *imaginary numbers*. You will study imaginary numbers in a future course.

EXAMPLE 1 Finding nth Roots

Find the indicated real nth root(s) of a.

a. $n = 3, a = -27$ b. $n = 4, a = 16$

SOLUTION

a. The index $n = 3$ is odd, so -27 has one real cube root. Because $(-3)^3 = -27$, the cube root of -27 is $\sqrt[3]{-27} = -3$, or $(-27)^{1/3} = -3$.

b. The index $n = 4$ is even, and $a > 0$. So, 16 has two real fourth roots. Because $2^4 = 16$ and $(-2)^4 = 16$, the fourth roots of 16 are $\pm\sqrt[4]{16} = \pm 2$, or $\pm 16^{1/4} = \pm 2$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the indicated real nth root(s) of a.

1. $n = 3, a = -125$ 2. $n = 6, a = 64$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Evaluating Expressions with Rational Exponents

Recall that the radical $\sqrt{a}$ indicates the positive square root of a. Similarly, an nth root of a, $\sqrt[n]{a}$, with an *even* index indicates the positive nth root of a.

REMEMBER
The expression under the radical sign is the radicand.

EXAMPLE 2 Evaluating nth Root Expressions

Evaluate each expression.

a. $\sqrt[3]{-8}$ b. $-\sqrt[3]{8}$ c. $16^{1/4}$ d. $(-16)^{1/4}$

SOLUTION

a. $\sqrt[3]{-8} = \sqrt[3]{(-2) \cdot (-2) \cdot (-2)}$ Rewrite the expression showing factors.
$= -2$ Evaluate the cube root.

b. $-\sqrt[3]{8} = -(\sqrt[3]{2 \cdot 2 \cdot 2})$ Rewrite the expression showing factors.
$= -(2)$ Evaluate the cube root.
$= -2$ Simplify.

c. $16^{1/4} = \sqrt[4]{16}$ Rewrite the expression in radical form.
$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$ Rewrite the expression showing factors.
$= 2$ Evaluate the fourth root.

d. $(-16)^{1/4}$ is not a real number because there is no real number that can be multiplied by itself four times to produce -16.

A rational exponent does not have to be of the form $1/n$. Other rational numbers such as $3/2$ can also be used as exponents. You can use the properties of exponents to evaluate or simplify expressions involving rational exponents.

STUDY TIP
You can rewrite $27^{2/3}$ as $27^{(1/3) \cdot 2}$ and then use the Power of a Power Property to show that $27^{(1/3) \cdot 2} = (27^{1/3})^2$.

Core Concept

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

Algebra $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

Numbers $27^{2/3} = (27^{1/3})^2 = (\sqrt[3]{27})^2$

EXAMPLE 3 Evaluating Expressions with Rational Exponents

Evaluate (a) $16^{3/4}$ and (b) $27^{4/3}$.

SOLUTION

a. $16^{3/4} = (16^{1/4})^3$ Rational exponents
$= 2^3$ Evaluate the nth root.
$= 8$ Evaluate the power.

b. $27^{4/3} = (27^{1/3})^4$
$= 3^4$
$= 81$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Evaluate the expression.

3. $\sqrt[3]{-125}$ 4. $(-64)^{2/3}$ 5. $9^{5/2}$ 6. $256^{3/4}$

Extra Example 2

Evaluate each expression.

a. $\sqrt[4]{81}$ 3

b. $-\sqrt[4]{81}$ -3

c. $25^{1/2}$ 5

d. $(-25)^{1/2}$ $(-25)^{1/2}$ is not a real number because there is no real number that can be multiplied by itself to produce -25.

Extra Example 3

a. Evaluate $243^{2/5}$. 9

b. Evaluate $16^{3/2}$. 64

MONITORING PROGRESS ANSWERS

3. -5
4. 16
5. 243
6. 64

Differentiated Instruction

Kinesthetic

Some students may benefit by having access to a scientific or graphing calculator. After evaluating the expressions, students can enter the expressions in the calculator, find the values, and compare the calculated values to their answers.

Extra Example 4

Simplify the expression $8^{2/3} \cdot 8^{5/3}$. 128

Extra Example 5

The radius r of a sphere is given by the equation $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, where V is the volume of the sphere.

Volume = 463 cubic inches



Find the radius of the basketball to the nearest inch. Use 3.14 for π. The radius of the basketball is about 5 inches.

MONITORING PROGRESS ANSWERS

7. 144
8. $\frac{1}{4}$
9. $2y^{3/2}$
10. about 16 in.

EXAMPLE 4 Simplifying Expressions with Rational Exponents

a. $27^{4/3} \cdot 27^{1/3} = 27^{4/3 + 1/3}$ Product of Powers Property

$= 27^{5/3}$ Simplify.

$= (27^{1/3})^5$ Definition of rational exponent

$= 3^5$ Evaluate the nth root.

$= 243$ Evaluate the power.

b. $x^{1/2} \div x^{3/5} = x^{1/2 - 3/5}$ Quotient of Powers Property

$= x^{5/10 - 6/10}$ Rewrite fractions with common denominator.

$= x^{-1/10}$ Simplify.

$= \frac{1}{x^{1/10}}$ Definition of negative exponent

Solving Real-Life Problems

EXAMPLE 5 Solving a Real-Life Problem

Volume = 113 cubic feet



The radius r of a sphere is given by the equation $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, where V is the volume of the sphere. Find the radius of the beach ball to the nearest foot. Use 3.14 for π.

SOLUTION

1. **Understand the Problem** You know the equation that represents the radius of a sphere in terms of its volume. You are asked to find the radius for a given volume.
2. **Make a Plan** Substitute the given volume into the equation. Then evaluate to find the radius.
3. **Solve the Problem**

$r = \left(\frac{3V}{4\pi}\right)^{1/3}$ Write the equation.

$= \left(\frac{3(113)}{4(3.14)}\right)^{1/3}$ Substitute 113 for V and 3.14 for π.

$= \left(\frac{339}{12.56}\right)^{1/3}$ Multiply.

≈ 3 Use a calculator.

▶ The radius of the beach ball is about 3 feet.

4. **Look Back** To check that your answer is reasonable, compare the size of the ball to the size of the woman pushing the ball. The ball appears to be slightly taller than the woman. The average height of a woman is between 5 and 6 feet. So, a radius of 3 feet, or height of 6 feet, seems reasonable for the beach ball.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Simplify the expression. Write your answer using only positive exponents.

7. $12^{5/2} \div 12^{1/2}$
8. $16^{-3/4} \cdot 16^{1/4}$
9. $2y^{2/3} \cdot y^{5/6}$
10. **WHAT IF?** In Example 5, the volume of the beach ball is 17,000 cubic inches. Find the radius to the nearest inch. Use 3.14 for π.

Closure

- **Writing Prompt:** To evaluate the expression $64^{5/6}$... *Sample answer:* Rewrite $64^{5/6}$ as $(64^{1/6})^5$. Evaluate the 6th root of 64 to get 2 and then evaluate the power 2^5 to get 32.

6.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to evaluate $81^{1/4}$.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$(\sqrt[3]{27})^2$ $\quad 27^{2/3}$ $\quad 3^2$ $\quad (\sqrt[2]{27})^3$

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, rewrite the expression in rational exponent form.

3. $\sqrt{10}$ $\quad$ 4. $\sqrt[5]{34}$

In Exercises 5 and 6, rewrite the expression in radical form.

5. $15^{1/3}$ $\quad$ 6. $140^{1/8}$

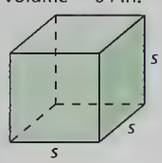
In Exercises 7–10, find the indicated real nth root(s) of a. *(See Example 1.)*

7. $n = 2, a = 36$ $\quad$ 8. $n = 4, a = 81$

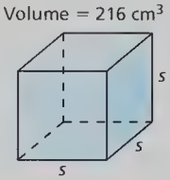
9. $n = 3, a = 1000$ $\quad$ 10. $n = 9, a = -512$

MATHEMATICAL CONNECTIONS **In Exercises 11 and 12, find the dimensions of the cube. Check your answer.**

11. Volume = 64 in.^{3}



12. Volume = 216 cm^3



In Exercises 13–18, evaluate the expression. *(See Example 2.)*

13. $\sqrt[4]{256}$ $\quad$ 14. $\sqrt[3]{-216}$

15. $\sqrt[3]{-343}$ $\quad$ 16. $-\sqrt[5]{1024}$

17. $128^{1/7}$ $\quad$ 18. $(-64)^{1/2}$

In Exercises 19 and 20, rewrite the expression in rational exponent form.

19. $(\sqrt[5]{8})^4$ $\quad$ 20. $(\sqrt[5]{-21})^6$

In Exercises 21 and 22, rewrite the expression in radical form.

21. $(-4)^{2/7}$ $\quad$ 22. $9^{5/2}$

In Exercises 23–28, evaluate the expression. *(See Example 3.)*

23. $32^{3/5}$ $\quad$ 24. $125^{2/3}$

25. $(-36)^{3/2}$ $\quad$ 26. $(-243)^{2/5}$

27. $(-128)^{5/7}$ $\quad$ 28. $343^{4/3}$

29. **ERROR ANALYSIS** Describe and correct the error in rewriting the expression in rational exponent form.

✗ $(\sqrt[3]{2})^4 = 2^{3/4}$

30. **ERROR ANALYSIS** Describe and correct the error in evaluating the expression.

✗ $(-81)^{3/4} = [(-81)^{1/4}]^3$
$= (-3)^3$
$= -27$

In Exercises 31–34, evaluate the expression.

31. $\left(\frac{1}{1000}\right)^{1/3}$ $\quad$ 32. $\left(\frac{1}{64}\right)^{1/6}$

33. $(27)^{-2/3}$ $\quad$ 34. $(9)^{-5/2}$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–29 odd, 35–45 odd, 48, 50, 63–66

Average: 1, 2–50 even, 55, 63–66

Advanced: 1, 2, 8–12 even, 16–46 even, 47–66

HOMEWORK CHECK

Basic: 7, 13, 23, 37, 45

Average: 8, 14, 26, 38, 46

Advanced: 10, 16, 26, 40, 46

ANSWERS

1. Find the fourth root of 81, or what real number multiplied by itself four times produces 81.
2. $(\sqrt[2]{27})^3$; It is the only one that is not equivalent to 9.
3. $10^{1/2}$
4. $34^{1/5}$
5. $\sqrt[3]{15}$
6. $\sqrt[8]{140}$
7. ± 6
8. ± 3
9. 10
10. -2
11. $s = 4$ in.
12. $s = 6$ cm
13. 4
14. -6
15. -7
16. -4
17. 2
18. not a real number
19. $8^{4/5}$
20. $-21^{6/5}$
21. $(\sqrt[7]{-4})^2$
22. $(\sqrt{9})^5$
23. 8
24. 25
25. not a real number
26. 9
27. -32
28. 2401
29. The numerator and denominator are reversed; $(\sqrt[3]{2})^4 = 2^{4/3}$
30. The number -81 does not have a real fourth root; not a real number
31. $\frac{1}{10}$ $\quad$ 32. $\frac{1}{2}$
33. $\frac{1}{9}$ $\quad$ 34. $\frac{1}{243}$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

35. 6 ft^2
36. 243 mm
37. 25
38. 8
39. 5
40. $\frac{1}{32}$
41. $d^{7/12}$
42. $\frac{1}{z^{11/15}}$
43. $4n^{13/8}$
44. $\frac{1}{7a^{2/3}}$
45. about 1 in.
46. about 44 m^3
47. Write the radicand, a, as the base and write the exponent as a fraction with the power, m, as the numerator and the index, n, as the denominator.
48. $x^{1/2}$ in.
49. -1, 0, and 1
50. no; If n is odd and a is negative, $\sqrt[n]{a}$ will be negative, and $-\sqrt[n]{a}$ will be positive.
51. $(xy)^{1/2}$
52. y^2
53. $2xy^2$
54. x^3y^5
55. about 1.38 ft
56. *Sample answer:* period of a pendulum: $T = 2\pi\left(\frac{\ell}{g}\right)^{1/2}$
57. always; Power of a Power Property
58. sometimes; true when $x = 1$, otherwise false
59. always; definition of rational exponent

60–66. See Additional Answers.

Mini-Assessment

1. Find the indicated real nth roots of a.
 $n = 4$, $a = 625$ $\pm\sqrt[4]{625} = \pm 5$, or $\pm(625)^{1/4} = \pm 5$

Evaluate the expression.

2. $-\sqrt[4]{1296}$ -6
3. $64^{4/3}$ 256

35. **PROBLEM SOLVING** A math club is having a bake sale. Find the area of the bake sale sign.



$4^{1/2}$ ft

$\sqrt[6]{729}$ ft

36. **PROBLEM SOLVING** The volume of a cube-shaped box is 27^5 cubic millimeters. Find the length of one side of the box.

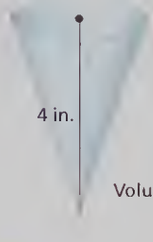
In Exercises 37–44, simplify the expression. Write your answer using only positive exponents. *(See Example 4.)*

37. $5^{-3/2} \cdot 5^{7/2}$
38. $2^{5/3} \div 2^{-4/3}$
39. $125^3 \div 125^{8/3}$
40. $\left(\frac{1}{16}\right)^{1/2} \cdot \left(\frac{1}{16}\right)^{3/4}$
41. $d^{3/4} \div d^{1/6}$
42. $z^{-7/5} \cdot z^{2/3}$
43. $4n^{1/8} \cdot n^{3/2}$
44. $a^{-3/4} \div 7a^{-1/12}$

45. **MODELING WITH MATHEMATICS** The radius r of the base of a cone is given by the equation

$$r = \left(\frac{3V}{\pi h}\right)^{1/2}$$

where V is the volume of the cone and h is the height of the cone. Find the radius of the paper cup to the nearest inch. Use 3.14 for π. *(See Example 5.)*

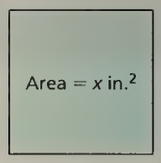


Volume = 5 in.^3

46. **MODELING WITH MATHEMATICS** The volume of a sphere is given by the equation $V = \frac{1}{6\sqrt{\pi}}S^{3/2}$, where S is the surface area of the sphere. Find the volume of a sphere, to the nearest cubic meter, that has a surface area of 60 square meters. Use 3.14 for π.

47. **WRITING** Explain how to write $(\sqrt[n]{a})^m$ in rational exponent form.

48. **HOW DO YOU SEE IT?** Write an expression in rational exponent form that represents the side length of the square.



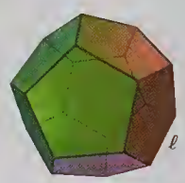
49. **REASONING** For what values of x is $x = x^{1/5}$?

50. **MAKING AN ARGUMENT** Your friend says that for a real number a and a positive integer n, the value of $\sqrt[n]{a}$ is always positive and the value of $-\sqrt[n]{a}$ is always negative. Is your friend correct? Explain.

In Exercises 51–54, simplify the expression.

51. $(y^{1/6})^3 \cdot \sqrt{x}$
52. $(y^n \cdot y^{1/3})^{3/2}$
53. $x \cdot \sqrt[3]{y^6} + y^2 \cdot \sqrt[3]{x^3}$
54. $(x^{1/3} \cdot y^{1/2})^9 \cdot \sqrt{y}$

55. **PROBLEM SOLVING** The formula for the volume of a regular dodecahedron is $V \approx 7.66\ell^3$, where ℓ is the length of an edge. The volume of the dodecahedron is 20 cubic feet. Estimate the edge length.



56. **THOUGHT PROVOKING** Find a formula (for instance, from geometry or physics) that contains a radical. Rewrite the formula using rational exponents.

ABSTRACT REASONING **In Exercises 57–62, let x be a nonnegative real number. Determine whether the statement is *always*, *sometimes*, or *never* true. Justify your answer.**

57. $(x^{1/3})^3 = x$
58. $x^{1/3} = x^{-3}$
59. $x^{1/3} = \sqrt[3]{x}$
60. $x^{1/3} = x^3$
61. $\frac{x^{2/3}}{x^{1/3}} = \sqrt[3]{x}$
62. $x = x^{1/3} \cdot x^3$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the function when $x = -3$, 0, and 8. *(Section 3.3)*

63. $f(x) = 2x - 10$
64. $w(x) = -5x - 1$
65. $h(x) = 13 - x$
66. $g(x) = 8x + 16$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

6.3 Exponential Functions

Texas Essential Knowledge and Skills
A.9.A
A.9.B
A.9.C
A.9.D
A.9.E

Essential Question What are some of the characteristics of the graph of an exponential function?

EXPLORATION 1 Exploring an Exponential Function

Work with a partner. Copy and complete each table for the *exponential function* $y = 16(2)^x$. In each table, what do you notice about the values of x? What do you notice about the values of y?

x	$y = 16(2)^x$
0	
1	
2	
3	
4	
5	

x	$y = 16(2)^x$
0	
2	
4	
6	
8	
10	

JUSTIFYING THE SOLUTION
To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 Exploring an Exponential Function

Work with a partner. Repeat Exploration 1 for the exponential function $y = 16\left(\frac{1}{2}\right)^x$. Do you think the statement below is true for *any* exponential function? Justify your answer.

> *"As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor."*

EXPLORATION 3 Graphing Exponential Functions

Work with a partner. Sketch the graphs of the functions given in Explorations 1 and 2. How are the graphs similar? How are they different?

Communicate Your Answer

4. What are some of the characteristics of the graph of an exponential function?
5. Sketch the graph of each exponential function. Does each graph have the characteristics you described in Question 4? Explain your reasoning.
 a. $y = 2^x$ b. $y = 2(3)^x$ c. $y = 3(1.5)^x$
 d. $y = \left(\frac{1}{2}\right)^x$ e. $y = 3\left(\frac{1}{2}\right)^x$ f. $y = 2\left(\frac{3}{4}\right)^x$

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.9.A The student is expected to determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

A.9.B The student is expected to interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems.

A.9.C The student is expected to write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

A.9.D The student is expected to graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

A.9.E The student is expected to write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

ANSWERS

1. 16, 32, 64, 128, 256, 512; 16, 64, 256, 1024, 4096, 16,384; Each value of x increases by the same amount; Each value of y is multiplied by the same factor.

2–5. See Additional Answers.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students complete the explorations and Communicate Your Answer in groups of mixed language abilities. Explain that these are the tasks they will perform as they work through the explorations: *copy, complete, repeat, justify, sketch,* and *explain*. Review the meanings of these words, if needed.

Advanced/Advanced High Lead the work in the explorations by reading the instructions and questions, and paraphrasing what needs to be done.

Beginning Use simple phrases to provide information.
Intermediate Use complete sentences to provide information.

ELPS 3.F.2 Give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments.

Extra Example 1

Does each table represent a *linear* or an *exponential* function? Explain.

a.

x	0	2	4	6
y	2	4	6	8

The function is linear. As x increases by 2, y increases by 2. The rate of change is constant.

b.

x	0	1	2	3
y	1	3	9	27

The function is exponential. As x increases by 1, y is multiplied by 3.

Extra Example 2

Evaluate each function for the given value of x.

a. $y = -3(4)^x$; $x = 2$ -48

b. $y = 2(0.25)^x$; $x = -3$ 128

MONITORING PROGRESS ANSWERS

1. exponential; As x increases by 1, y is multiplied by $\frac{1}{2}$.
2. linear; As x increases by 4, y decreases by 1. The rate of change is constant.
3. $\frac{2}{81}$; 2; 6
4. 0.375; 1.5; about 2.12

6.3 Lesson

Core Vocabulary

exponential function, *p. 292*
asymptote, *p. 293*

Previous
independent variable
dependent variable
parent function

What You Will Learn

- Identify and evaluate exponential functions.
- Graph exponential functions.
- Solve real-life problems involving exponential functions.

Identifying and Evaluating Exponential Functions

An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$. As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor, which means consecutive y-values form a constant ratio.

EXAMPLE 1 Identifying Functions

Does each table represent a *linear* or an *exponential* function? Explain.

a.

x	0	1	2	3
y	2	4	6	8

b.

x	0	1	2	3
y	4	8	16	32

SOLUTION

a.

	+1	+1	+1	
x	0	1	2	3
y	2	4	6	8
	+2	+2	+2	

▶ As x increases by 1, y increases by 2. The rate of change is constant. So, the function is linear.

b.

	+1	+1	+1	
x	0	1	2	3
y	4	8	16	32
	×2	×2	×2	

▶ As x increases by 1, y is multiplied by 2. So, the function is exponential.

STUDY TIP

In Example 1b, consecutive y-values form a constant ratio.

$\frac{8}{4} = 2, \frac{16}{8} = 2, \frac{32}{16} = 2$

EXAMPLE 2 Evaluating Exponential Functions

Evaluate each function for the given value of x.

a. $y = -2(5)^x$; $x = 3$

b. $y = 3(0.5)^x$; $x = -2$

SOLUTION

a.

$y = -2(5)^x$		Write the function.
$= -2(5)^3$		Substitute for x.
$= -2(125)$		Evaluate the power.
$= -250$		Multiply.

b.

$y = 3(0.5)^x$	
$= 3(0.5)^{-2}$	
$= 3(4)$	
$= 12$	

Monitoring Progress

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Does the table represent a *linear* or an *exponential* function? Explain.

1.

x	0	1	2	3
y	8	4	2	1

2.

x	−4	0	4	8
y	1	0	−1	−2

Evaluate the function when $x = -2$, 0, and $\frac{1}{2}$.

3. $y = 2(9)^x$

4. $y = 1.5(2)^x$

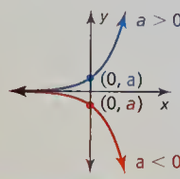
For classroom suggestions on teaching this lesson, see Laurie's Notes at *BigIdeasMath.com*.

Graphing Exponential Functions

The graph of a function $y = ab^x$ is a vertical stretch or shrink by a factor of $|a|$ of the graph of the parent function $y = b^x$. When $a < 0$, the graph is also reflected in the x-axis. The y-intercept of the graph of $y = ab^x$ is a.

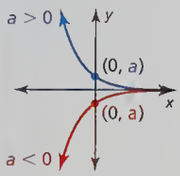
Core Concept

Graphing $y = ab^x$ When $b > 1$



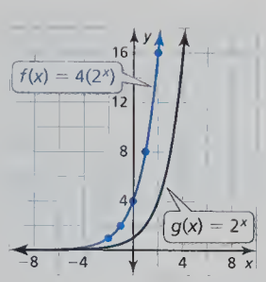
The x-axis is an asymptote of the graph of $y = ab^x$. An asymptote is a line that a graph approaches but never intersects.

Graphing $y = ab^x$ When $0 < b < 1$



EXAMPLE 3 Graphing $y = ab^x$ When $b > 1$

Graph $f(x) = 4(2)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.



SOLUTION

Step 1 Make a table of values.

Step 2 Plot the ordered pairs.

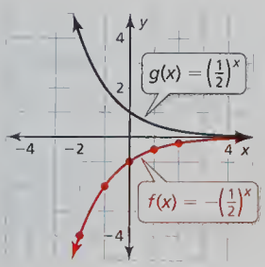
Step 3 Draw a smooth curve through the points.

x	−2	−1	0	1	2
$f(x)$	1	2	4	8	16

▶ The parent function is $g(x) = 2^x$. The graph of f is a vertical stretch by a factor of 4 of the graph of g. The y-intercept of the graph of f, 4, is above the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g. From the graph of f, you can see that the domain is all real numbers and the range is $y > 0$.

EXAMPLE 4 Graphing $y = ab^x$ When $0 < b < 1$

Graph $f(x) = -\left(\frac{1}{2}\right)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.



SOLUTION

Step 1 Make a table of values.

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

x	−2	−1	0	1	2
$f(x)$	−4	−2	−1	$-\frac{1}{2}$	$-\frac{1}{4}$

▶ The parent function is $g(x) = \left(\frac{1}{2}\right)^x$. The graph of f is a reflection in the x-axis of the graph of g. The y-intercept of the graph of f, −1, is below the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g. From the graph of f, you can see that the domain is all real numbers and the range is $y < 0$.

Monitoring Progress

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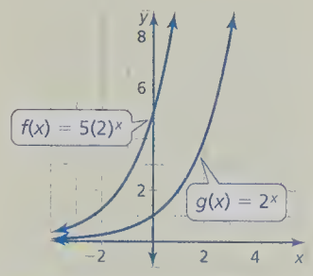
Graph the function. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.

5. $f(x) = -2(4)^x$

6. $f(x) = 2\left(\frac{1}{4}\right)^x$

Extra Example 3

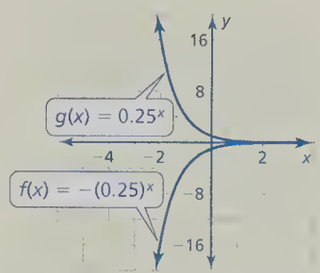
Graph $f(x) = 5(2)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.



The parent function is $g(x) = 2^x$. The graph of f is a vertical stretch by a factor of 5 of the graph of g. The y-intercept of the graph of f, 5, is above the y-intercept of the graph of g, 1. The x-axis is an asymptote of both graphs. The domain of f is all real numbers and the range is $y > 0$.

Extra Example 4

Graph $f(x) = -(0.25)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.



The parent function is $g(x) = (0.25)^x$. The graph of f is a reflection in the x-axis of the graph of g. The y-intercept of the graph of f, −1, is below the y-intercept of the graph of g, 1. The x-axis is an asymptote of both graphs. The domain of f is all real numbers and the range is $y < 0$.

English Language Learners

Pair Activity

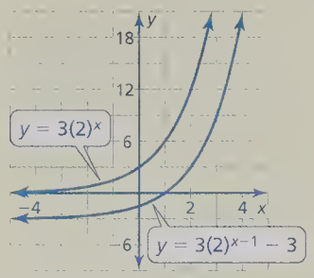
Pair each English learner with an English speaker. Ask one student to make the table of values and the other student to plot the ordered pairs and draw a smooth curve through the points. Have the students work together to compare the graph to the graph of the parent function, identify the y-intercepts and asymptotes, and describe the domain and range.

MONITORING PROGRESS ANSWERS

5–6. See Additional Answers.

Extra Example 5

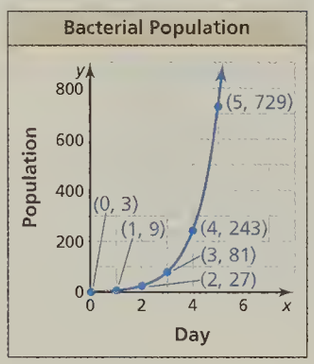
Graph $y = 3(2)^{x-1} - 3$. Identify the asymptote. Describe the domain and range.



The graph has an asymptote at $y = -3$. The domain is all real numbers and the range is $y > -3$.

Extra Example 6

The graph represents a bacterial population y after x days.



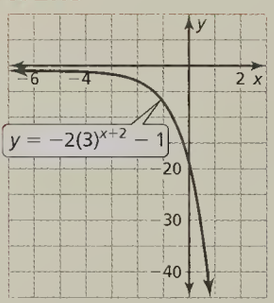
a. Write an exponential function that represents the population. Identify and interpret the y-intercept. $y = 3(3)^x$; The y-intercept of 3 means that the initial bacterial population is 3.

b. Describe the domain and range of the function. The domain is $x \geq 0$ and the range is $y \geq 3$.

c. Find the population after 6 days. 2187 bacteria

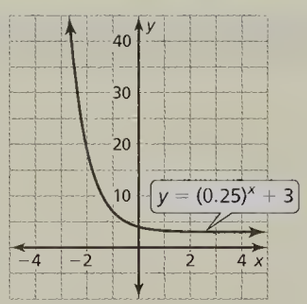
MONITORING PROGRESS ANSWERS

7.



asymptote: $y = -1$; domain: all real numbers, range: $y < -1$

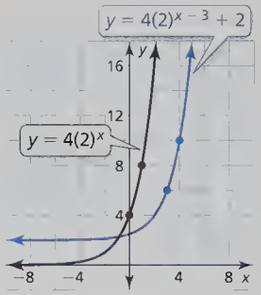
8.



asymptote: $y = 3$; domain: all real numbers, range: $y > 3$

To graph a function of the form $y = ab^{x-h} + k$, begin by graphing $y = ab^x$. Then translate the graph horizontally h units and vertically k units.

EXAMPLE 5 Graphing $y = ab^{x-h} + k$



Graph $y = 4(2)^{x-3} + 2$. Identify the asymptote. Describe the domain and range.

SOLUTION

Step 1 Graph $y = 4(2)^x$. This is the same function that is in Example 3, which passes through (0, 4) and (1, 8).

Step 2 Translate the graph 3 units right and 2 units up. The graph passes through (3, 6) and (4, 10).

Notice that the graph approaches the line $y = 2$ but does not intersect it.

▶ So, the graph has an asymptote at $y = 2$. From the graph, you can see that the domain is all real numbers and the range is $y > 2$.

Monitoring Progress

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Graph the function. Identify the asymptote. Describe the domain and range.

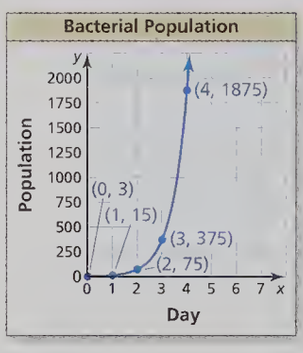
7. $y = -2(3)^{x+2} - 1$

8. $f(x) = (0.25)^x + 3$

Solving Real-Life Problems

For an exponential function of the form $y = ab^x$, the y-values change by a factor of b as x increases by 1. You can use this fact to write an exponential function when you know the y-intercept, a.

EXAMPLE 6 Modeling with Mathematics



The graph represents a bacterial population y after x days.

a. Write an exponential function that represents the population. Identify and interpret the y-intercept.

b. Describe the domain and range of the function.

c. Find the population after 5 days.

SOLUTION

a. Use the graph to make a table of values. The y-intercept is 3. The y-values increase by a factor of 5 as x increases by 1.

		+1	+1	+1	+1
x	0	1	2	3	4
y	3	15	75	375	1875
		×5	×5	×5	×5

▶ So, the population can be modeled by $y = 3(5)^x$. The y-intercept of 3 means that the initial bacterial population is 3.

b. From the graph, you can see that the domain is $x \geq 0$ and the range is $y \geq 3$. Note that $x \geq 0$ because the number of days cannot be negative.

c. Substitute 5 for x in the function $y = 3(5)^x$.

▶ After 5 days, there are $y = 3(5)^5 = 3(3125) = 9375$ bacteria.

You learned to use the *linear regression* feature of a graphing calculator to find an equation of the line of best fit in Section 4.6. Similarly, you can use exponential regression to find an exponential function that fits a data set.

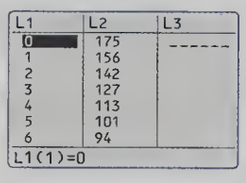
EXAMPLE 7 **Writing an Exponential Function Using Technology**

Time, x	Temperature, y
0	175
1	156
2	142
3	127
4	113
5	101
6	94
7	84
8	75

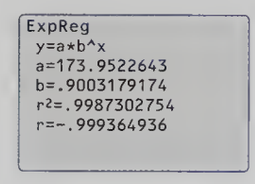
The table shows the time x (in minutes) since a cup of hot coffee was poured and the temperature y (in degrees Fahrenheit) of the coffee. (a) Use a graphing calculator to find an exponential function that fits the data. Then plot the data and graph the function in the same viewing window. (b) Predict the temperature of the coffee 10 minutes after it is poured.

SOLUTION

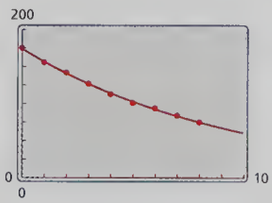
a. Step 1 Enter the data from the table into two lists.



Step 2 Use the *exponential regression* feature. The values in the equation can be rounded to obtain $y = 174(0.9)^x$.



Step 3 Enter the equation $y = 174(0.9)^x$ into the calculator. Then plot the data and graph the equation in the same viewing window.



b. After 10 minutes, the temperature of the coffee will be about $y = 174(0.9)^{10} \approx 60.7$ degrees Fahrenheit.

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9. A bacterial population y after x days can be represented by an exponential function whose graph passes through (0, 100) and (1, 200). (a) Write a function that represents the population. Identify and interpret the y-intercept. (b) Find the population after 6 days. (c) Does this bacterial population grow faster than the bacterial population in Example 6? Explain.

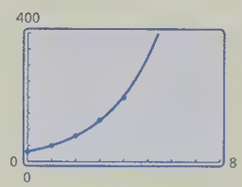
10. In Example 7, (a) identify and interpret the correlation coefficient and (b) predict the temperature of the coffee 15 minutes after it is poured.

Extra Example 7

The table shows the fish population in a lake after t years.

Year, t	Population, y
0	31
1	50
2	80
3	130
4	200

a. Use a graphing calculator to find an exponential function that fits the data. Then plot the data and graph the function. $y = 31.2(1.6)^x$



b. Predict the fish population after 5 years. about 327 fish

MONITORING PROGRESS ANSWERS

9. a. $y = 100(2)^x$; $y = 100$; The initial bacterial population is 100.
 b. 6400 cells
 c. no; This population doubles each day, but the population in Example 6 increases by a factor of 5 each day.

10. a. $r = -0.999$; The relationship between the time and the temperature has a strong, negative correlation and the equation closely models the data.
 b. 35.8°F

Closure

- **Point of Most Significance:** Ask students to identify, aloud or on a paper to be collected, the most significant point (or part) in the lesson that aided their learning.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–35 odd, 41–47 odd, 50, 54, 60–63

Average: 1–4, 6–50 even, 53, 54, 58, 60–63

Advanced: 1–4, 10–14 even, 20–28 even, 36–50 even, 51–63

HOMEWORK CHECK

Basic: 11, 15, 25, 31, 41

Average: 12, 16, 26, 32, 42

Advanced: 14, 20, 28, 36, 42

6.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **OPEN-ENDED** Sketch an increasing exponential function whose graph has a y-intercept of 2.
2. **REASONING** Why is a the y-intercept of the graph of the function $y = ab^x$?
3. **WRITING** Compare the graph of $y = 2(5)^x$ with the graph of $y = 5^x$.
4. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$y = 3^x$ $\quad f(x) = 2(4)^x$ $\quad f(x) = (-3)^x$ $\quad y = 5(3)^x$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, determine whether the equation represents an exponential function. Explain.

5. $y = 4(7)^x$
6. $y = -6x$
7. $y = 2x^3$
8. $y = -3^x$
9. $y = 9(-5)^x$
10. $y = \frac{1}{2}(1)^x$

In Exercises 11–14, determine whether the table represents a *linear* or an *exponential* function. Explain. *(See Example 1.)*

11.

x	y
1	−2
2	0
3	2
4	4

12.

x	y
1	6
2	12
3	24
4	48

13.

x	−1	0	1	2	3
y	0.25	1	4	16	64

14.

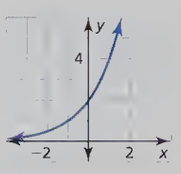
x	−3	0	3	6	9
y	10	1	−8	−17	−26

In Exercises 15–20, evaluate the function for the given value of x. *(See Example 2.)*

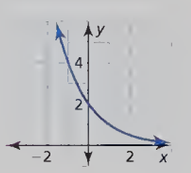
15. $y = 3^x$; $x = 2$
16. $f(x) = 3(2)^x$; $x = -1$
17. $y = -4(5)^x$; $x = 2$
18. $f(x) = 0.5^x$; $x = -3$
19. $f(x) = \frac{1}{3}(6)^x$; $x = 3$
20. $y = \frac{1}{4}(4)^x$; $x = \frac{3}{2}$

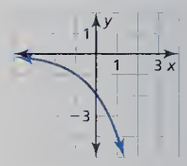
USING STRUCTURE In Exercises 21–24, match the function with its graph.

21. $f(x) = 2(0.5)^x$
22. $y = -2(0.5)^x$
23. $y = 2(2)^x$
24. $f(x) = -2(2)^x$

A. 

B. 

C. 

D. 

In Exercises 25–30, graph the function. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f. *(See Examples 3 and 4.)*

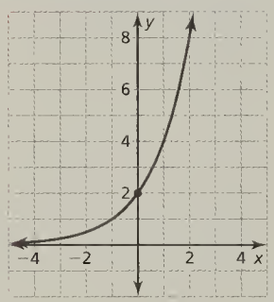
25. $f(x) = 3(0.5)^x$
26. $f(x) = -4^x$
27. $f(x) = -2(7)^x$
28. $f(x) = 6\left(\frac{1}{3}\right)^x$
29. $f(x) = \frac{1}{2}(8)^x$
30. $f(x) = \frac{3}{2}(0.25)^x$

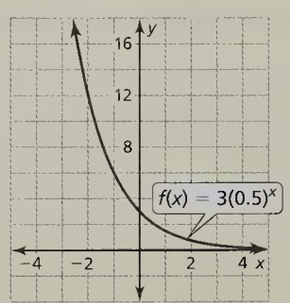
In Exercises 31–36, graph the function. Identify the asymptote. Describe the domain and range. *(See Example 5.)*

31. $f(x) = 3^x - 1$
32. $f(x) = 4^{x+3}$

ANSWERS

1. *Sample answer:*



2. The y-intercept is the point where $x = 0$. At the y-intercept $y = ab^0 = a \cdot 1 = a$.
3. The graph of $y = 2(5)^x$ is a vertical stretch by a factor of 2 of the graph of $y = 5^x$. The y-intercept of $y = 2(5)^x$, 2, is above the y-intercept of $y = 5^x$, 1.
4. $f(x) = (-3)^x$; It is the only one that is not an exponential function.
5. yes; It fits the pattern $y = ab^x$.
6. no; It is a linear function.
7. no; The exponent is a constant.
8. yes: It fits the pattern $y = ab^x$.
9. no; Although it fits the pattern $y = ab^x$, b cannot be negative.
10. no; Although it fits the pattern $y = ab^x$, b cannot be 1.
11. linear; As x increases by 1, y increases by 2. The rate of change is constant.
12. exponential; As x increases by 1, y is multiplied by 2.
13. exponential; As x increases by 1, y is multiplied by 4.
14. linear; As x increases by 3, y decreases by 9. The rate of change is constant.
15. 9
16. $\frac{3}{2}$
17. -100
18. 8
19. 72
20. 2
21. C
22. B
23. A
24. D
25. 

The graph of f is a vertical stretch by a factor of 3 of the graph of $g(x) = 0.5^x$. The y-intercept of the graph of f, 3, is above the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y > 0$

26–32. See Additional Answers.

33. $y = 5^{x-2} + 7$

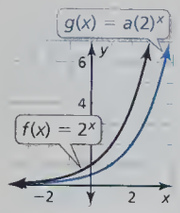
34. $y = -\left(\frac{1}{2}\right)^{x+1} - 3$

35. $y = -8(0.75)^{x+2} - 2$

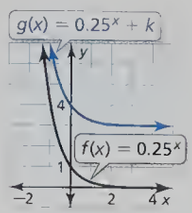
36. $f(x) = 3(6)^{x-1} - 5$

In Exercises 37–40, compare the graphs. Find the value of h, k, or a.

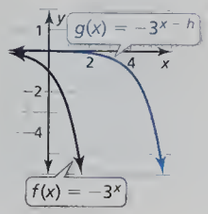
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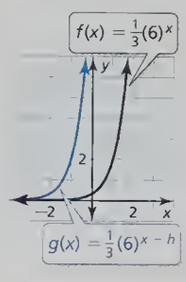
38.



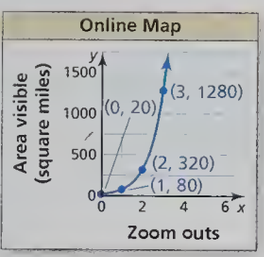
39.



40.



41. **MODELING WITH MATHEMATICS** The graph represents the amount of area visible y on an online map after you zoom out x times. *(See Example 6.)*

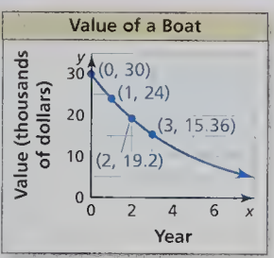


a. Write an exponential function that represents the area. Identify and interpret the y-intercept.

b. Describe the domain and range of the function.

c. Find the area visible after you zoom out 10 times.

42. **MODELING WITH MATHEMATICS** The graph represents the value y of a boat after x years.



a. Write an exponential function that represents the value. Identify and interpret the y-intercept.

b. Describe the domain and range of the function.

c. Find the value of the boat after 8 years.

43. **ERROR ANALYSIS** Describe and correct the error in evaluating the function.

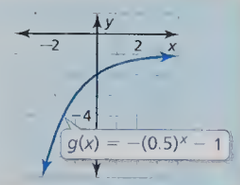
✗

$$g(x) = 6(0.5)^x;\ x = -2$$
$$g(-2) = 6(0.5)^{-2}$$
$$= 3^{-2}$$
$$= \frac{1}{9}$$

44. **ERROR ANALYSIS** Describe and correct the error in finding the domain and range of the function.

✗

The domain is all real numbers, and the range is $y < 0$.



45. **MODELING WITH MATHEMATICS** The table shows the number y of views an online video receives after being online for x days. *(See Example 7.)*

Day, x	0	1	2	3	4	5
Views, y	12	68	613	3996	27,810	205,017

a. Use a graphing calculator to find an exponential function that fits the data. Then plot the data and graph the function in the same viewing window.

b. Predict the number of views the video receives after being online for 7 days.

46. **MODELING WITH MATHEMATICS** The table shows the coyote population y in a national park after t decades.

Decade, t	0	1	2	3	4
Population, y	15	26	41	72	123

a. Use a graphing calculator to find an exponential function that fits the data. Then plot the data and graph the function in the same viewing window.

b. Predict the coyote population after 60 years.



Dynamic Teaching Tools

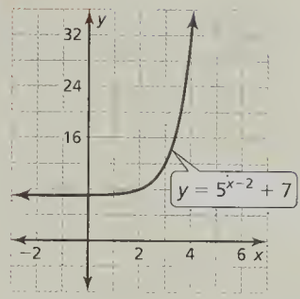
Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

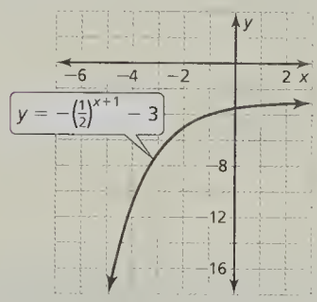
ANSWERS

33.



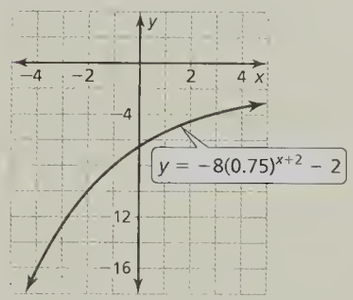
asymptote: $y = 7$; domain: all real numbers, range: $y > 7$

34.



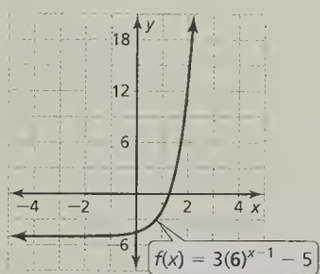
asymptote: $y = -3$; domain: all real numbers, range: $y < -3$

35.



asymptote: $y = -2$; domain: all real numbers, range: $y < -2$

36.



asymptote: $y = -5$; domain: all real numbers, range: $y > -5$

37. The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ of the graph of f; $a = \frac{1}{2}$

38. The graph of g is a vertical translation 3 units up of the graph of f; $k = 3$

39. The graph of g is a horizontal translation 4 units right of the graph of f; $h = 4$

40. The graph of g is a horizontal translation 2 units left of the graph of f; $h = -2$

41. a. $y = 20(4)^x$; $y = 20$; The initial area visible is 20 square miles.

b. domain: all whole numbers, range: $y \geq 20$

c. 20,971,520 mi^2

42. a. $y = 30(0.8)^x$; $y = 30$; The initial value of the boat is \$30,000.

b. domain: $x \geq 0$, range: $y \leq 30$

c. about \$5000

43. need to simplify the power before multiplying; $6(0.5)^{-2} = 6(4) = 24$

44. The graph approaches the line $y = -1$, not $y = 0$; The range is $y < -1$.

45–46. See Additional Answers.

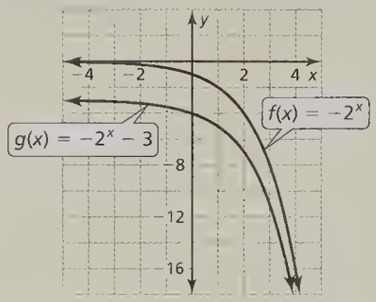
ANSWERS

47. **a.** $y = 40\left(\frac{3}{2}\right)^x$

b. about 304 visitors

48. about 4681 grills; $y = 3300(1.06)^x$

49.



The y-intercept of g is 3 units below the y-intercept of f. The domain of both functions is all real numbers. The range of g is $y < -3$ and the range of f is $y < 0$.

50. no; The values of x are not increasing by a constant amount.

51–63. See Additional Answers.

Mini-Assessment

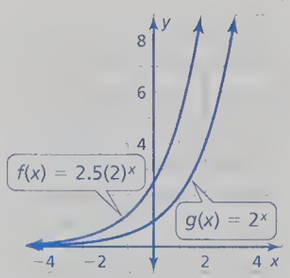
1. Does the table represent a *linear* or an *exponential* function? Explain.

x	0	1	2	3
y	−2	−6	−18	−54

The function is exponential. As x increases by 1, y is multiplied by 3.

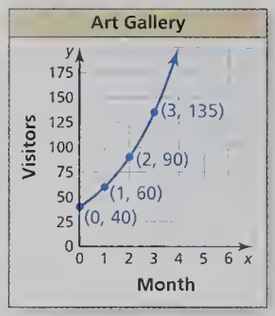
2. Evaluate the function $y = 5(3)^x$ for $x = 2$. 45

3. Graph $f(x) = 2.5(2)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.



The graph of f is a vertical stretch by a factor of 2.5 of the graph of g; y-intercept of the graph of f: 2.5, y-intercept of the graph of g: 1; asymptote: x-axis; domain: all real numbers, range: $y > 0$.

47. MODELING WITH MATHEMATICS The graph represents the number y of visitors to a new art gallery after x months.



a. Write an exponential function that represents this situation.

b. Approximate the number of visitors after 5 months.

48. PROBLEM SOLVING A sales report shows that 3300 gas grills were purchased from a chain of hardware stores last year. The store expects grill sales to increase 6% each year. About how many grills does the store expect to sell in Year 6? Use an equation to justify your answer.

49. WRITING Graph the function $f(x) = -2^x$. Then graph $g(x) = -2^x - 3$. How are the y-intercept, domain, and range affected by the translation?

50. MAKING AN ARGUMENT Your friend says that the table represents an exponential function because y is multiplied by a constant factor. Is your friend correct? Explain.

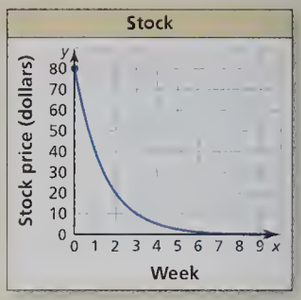
x	0	1	3	6
y	2	10	50	250

51. WRITING Describe the effect of a on the graph of $y = a \cdot 2^x$ when a is positive and when a is negative.

52. OPEN-ENDED Write a function whose graph is a horizontal translation of the graph of $h(x) = 4^x$.

53. USING STRUCTURE The graph of g is a translation 4 units up and 3 units right of the graph of $f(x) = 5^x$. Write an equation for g.

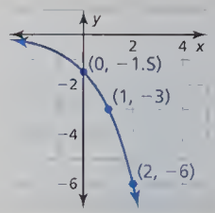
54. HOW DO YOU SEE IT? The exponential function $y = V(x)$ represents the projected value of a stock x weeks after a corporation loses an important legal battle. The graph of the function is shown.



a. After how many weeks will the stock be worth \$20?

b. Describe the change in the stock price from Week 1 to Week 3.

55. USING GRAPHS The graph represents the exponential function f. Find $f(7)$.



56. THOUGHT PROVOKING Write a function of the form $y = ab^x$ that represents a real-life population. Explain the meaning of each of the constants a and b in the real-life context.

57. REASONING Let $f(x) = ab^x$. Show that when x is increased by a constant k, the quotient $\frac{f(x+k)}{f(x)}$ is always the same regardless of the value of x.

58. PROBLEM SOLVING A function g models a relationship in which the dependent variable is multiplied by 4 for every 2 units the independent variable increases. The value of the function at 0 is 5. Write an equation that represents the function.

59. PROBLEM SOLVING Write an exponential function f so that the slope from the point $(0, f(0))$ to the point $(2, f(2))$ is equal to 12.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the percent as a decimal. *(Skills Review Handbook)*

60. 4% **61.** 35% **62.** 128% **63.** 250%

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

6.4 Exponential Growth and Decay

Texas Essential Knowledge and Skills
A.9.B
A.9.C
A.9.D

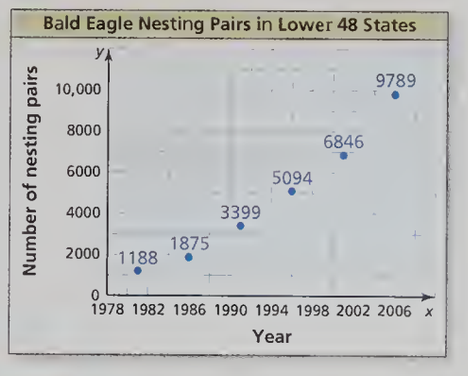
Essential Question What are some of the characteristics of exponential growth and exponential decay functions?

EXPLORATION 1 Predicting a Future Event

Work with a partner. It is estimated, that in 1782, there were about 100,000 nesting pairs of bald eagles in the United States. By the 1960s, this number had dropped to about 500 nesting pairs. In 1967, the bald eagle was declared an endangered species in the United States. With protection, the nesting pair population began to increase. Finally, in 2007, the bald eagle was removed from the list of endangered and threatened species.

APPLYING MATHEMATICS
To be proficient in math, you need to apply the mathematics you know to solve problems arising in everyday life.

Describe the pattern shown in the graph. Is it exponential growth? Assume the pattern continues. When will the population return to that of the late 1700s? Explain your reasoning.



EXPLORATION 2 Describing a Decay Pattern

Work with a partner. A forensic pathologist was called to estimate the time of death of a person. At midnight, the body temperature was 80.5°F and the room temperature was a constant 60°F. One hour later, the body temperature was 78.5°F.

a. By what percent did the difference between the body temperature and the room temperature drop during the hour?

b. Assume that the original body temperature was 98.6°F. Use the percent decrease found in part (a) to make a table showing the decreases in body temperature. Use the table to estimate the time of death.

Communicate Your Answer

3. What are some of the characteristics of exponential growth and exponential decay functions?

4. Use the Internet or some other reference to find an example of each type of function. Your examples should be different than those given in Explorations 1 and 2.

 a. exponential growth **b.** exponential decay

SUPPORTING English Language Learners

As students work in pairs, circulate through the classroom and listen to them read aloud portions of the explorations. Ask them to employ the following reading strategies.

Beginning/Advanced Beginning students read the headings, graph, and single sentences while asking Advanced students to clarify things that they don't understand. Advanced students read longer paragraphs and request clarification from teachers or classroom aides as needed.

Intermediate/Advanced High Advanced High students read the problem setups for the explorations, and request clarification from teachers or classroom aides as needed. Intermediate students read questions and information from the graph and ask Advanced High students to clarify things that they don't understand.

ELPS 4.F.7 Use support from peers and teachers to enhance and confirm understanding.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.9.B The student is expected to interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems.

A.9.C The student is expected to write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

A.9.D The student is expected to graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

ANSWERS

1. As x increases by 5, y is multiplied by about 1.5; yes; 2036; The function can be approximately modeled by $y = 1188(1.5)^x$, where x represents the number of 5-year intervals since 1981. Setting y equal to 100,000 and solving for x gives 11 intervals, or 55 years.
2. **a.** $2/20.5 \approx 9.8\%$
 b. See Additional Answers.
3. As the independent variable changes by a constant amount, the dependent variable is multiplied by a constant factor.
4. **a.** *Sample answer:* the value of a CD each year that earns 2.5% interest compounded annually
 b. *Sample answer:* the worth of a farm tractor that depreciates at a rate of 10% per year

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Extra Example 1

The inaugural attendance of an annual art festival is 80,000. The attendance y increases by 6% each year.

a. Write an exponential growth function that represents the attendance after t years. The festival attendance can be represented by $y = 80{,}000(1.06)^t$.

b. How many people will attend the festival in the fifth year? Round your answer to the nearest thousand. About 107,000 people will attend the festival in the fifth year.

MONITORING PROGRESS ANSWER

1. a. $y = 500{,}000(1.15)^t$
 b. about 1,160,000

6.4 Lesson

What You Will Learn

- Use and identify exponential growth and decay functions.
- Interpret and rewrite exponential growth and decay functions.
- Solve real-life problems involving exponential growth and decay.

Core Vocabulary

exponential growth, *p. 300*
exponential growth function, *p. 300*
exponential decay, *p. 301*
exponential decay function, *p. 301*
compound interest, *p. 303*

Exponential Growth and Decay Functions

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time.

Core Concept

Exponential Growth Functions

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.

initial amount; rate of growth (in decimal form); final amount $= y = a(1 + r)^t$; time; growth factor

STUDY TIP

Notice that an exponential growth function is of the form $y = ab^x$, where b is replaced by $1 + r$ and x is replaced by t.

EXAMPLE 1 Using an Exponential Growth Function

The inaugural attendance of an annual music festival is 150,000. The attendance y increases by 8% each year.

a. Write an exponential growth function that represents the attendance after t years.

b. How many people will attend the festival in the fifth year? Round your answer to the nearest thousand.



SOLUTION

a. The initial amount is 150,000, and the rate of growth is 8%, or 0.08.

$y = a(1 + r)^t$ — Write the exponential growth function.
$= 150{,}000(1 + 0.08)^t$ — Substitute 150,000 for a and 0.08 for r.
$= 150{,}000(1.08)^t$ — Add.

▶ The festival attendance can be represented by $y = 150{,}000(1.08)^t$.

b. The value $t = 4$ represents the fifth year because $t = 0$ represents the first year.

$y = 150{,}000(1.08)^t$ — Write the exponential growth function.
$= 150{,}000(1.08)^4$ — Substitute 4 for t.
$\approx 204{,}073$ — Use a calculator.

▶ About 204,000 people will attend the festival in the fifth year.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. A website has 500,000 members in 2010. The number y of members increases by 15% each year. (a) Write an exponential growth function that represents the website membership t years after 2010. (b) How many members will there be in 2016? Round your answer to the nearest ten thousand.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Notebook Development

Have students create an exponential function page in their notebooks. Include the definition of exponential function from Section 6.3 along with the definitions of exponential growth and decay functions from this section. Have students include an example with each definition.

Exponential decay occurs when a quantity decreases by the same factor over equal intervals of time.

Core Concept

Exponential Decay Functions

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an **exponential decay function**.

initial amount; rate of decay (in decimal form); final amount; $y = a(1 - r)^t$; time; decay factor

STUDY TIP

Notice that an exponential decay function is of the form $y = ab^x$, where b is replaced by $1 - r$ and x is replaced by t.

For exponential growth, the value inside the parentheses is greater than 1 because r is added to 1. For exponential decay, the value inside the parentheses is less than 1 because r is subtracted from 1.

EXAMPLE 2 Identifying Exponential Growth and Decay

Determine whether each table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

a.

x	y
0	270
1	90
2	30
3	10

b.

x	0	1	2	3
y	5	10	20	40

SOLUTION

a.

x	y	
0	270	
1	90	$\times \frac{1}{3}$
2	30	$\times \frac{1}{3}$
3	10	$\times \frac{1}{3}$

(+1, +1, +1)

▶ As x increases by 1, y is multiplied by $\frac{1}{3}$. So, the table represents an exponential decay function.

b.

	+1	+1	+1	
x	0	1	2	3
y	5	10	20	40
	×2	×2	×2	

▶ As x increases by 1, y is multiplied by 2. So, the table represents an exponential growth function.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

2.

x	0	1	2	3
y	64	16	4	1

3.

x	1	3	5	7
y	4	11	18	25

Extra Example 2

Determine whether each table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

a.

x	y
0	24
1	12
2	6
3	3

The table represents an exponential decay function, because as x increases by 1, y is multiplied by $\frac{1}{2}$.

b.

x	0	1	2	3
y	4	40	400	4000

The table represents an exponential growth function, because as x increases by 1, y is multiplied by 10.

MONITORING PROGRESS ANSWERS

2. exponential decay; As x increases by 1, y is multiplied by $\frac{1}{4}$.
3. neither; As x increases by 2, y increases by 7.

Extra Example 3

Determine whether each function represents *exponential growth* or *exponential decay*. Identify the initial amount and interpret the growth factor or decay factor.

a. The function $y = 4(1.09)^t$ represents the value y (in dollars) of a comic book t years after it is issued. The function represents exponential growth. The initial amount is \$4, and the growth factor of 1.09 means that the value of the comic book increases by 9% each year.

b. The function $y = 24(0.50)^x$ represents the number y of teams left in a baseball tournament after x rounds. The function represents exponential decay. The initial amount is 24 teams, and the decay factor of 0.5 means that 50% of the teams are left after each round.

Extra Example 4

Rewrite each function in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*.

a. $f(x) = 20(0.81)^{x/2}$ $f(x) = 20(0.9)^x$; The function represents exponential decay.

b. $f(x) = (2.5)^{x-2}$ $f(x) = 0.16(2.5)^x$; The function represents exponential growth.

MONITORING PROGRESS ANSWERS

4. exponential growth; 10,000 people; The attendance increases by 12% each season.
5. $f(x) = 3(1.22)^x$; exponential growth
6. $f(x) = 0.9(0.95)^x$; exponential decay

Interpreting and Rewriting Exponential Functions

EXAMPLE 3 Interpreting Exponential Functions

Determine whether each function represents *exponential growth* or *exponential decay*. Identify the initial amount and interpret the growth factor or decay factor.

a. The function $y = 0.5(1.07)^t$ represents the value y (in dollars) of a baseball card t years after it is issued.

b. The function $y = 128(0.5)^x$ represents the number y of players left in a video game tournament after x rounds.

SOLUTION

a. The function is of the form $y = a(1 + r)^t$, where $1 + r > 1$, so it represents exponential growth. The initial amount is \$0.50, and the growth factor of 1.07 means that the value of the baseball card increases by 7% each year.

b. The function is of the form $y = a(1 - r)^t$, where $1 - r < 1$, so it represents exponential decay. The initial amount is 128 players, and the decay factor of 0.5 means that 50% of the players are left after each round.

STUDY TIP

You can rewrite exponential expressions and functions using the properties of exponents. Changing the form of an exponential function can reveal important attributes of the function.

EXAMPLE 4 Rewriting Exponential Functions

Rewrite each function in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*.

a. $f(x) = 100(0.96)^{x/4}$ **b.** $f(x) = (1.1)^{x-3}$

SOLUTION

a. $f(x) = 100(0.96)^{x/4}$ Write the function.

$= 100(0.96^{1/4})^x$ Power of a Power Property

$\approx 100(0.99)^x$ Evaluate the power.

▶ So, the function represents exponential decay.

b. $f(x) = (1.1)^{x-3}$ Write the function.

$= \frac{(1.1)^x}{(1.1)^3}$ Quotient of Powers Property

$\approx 0.75(1.1)^x$ Evaluate the power and simplify.

▶ So, the function represents exponential growth.

Monitoring Progress

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4. The function $y = 10(1.12)^n$ represents the average game attendance y (in thousands of people) of a professional baseball team in its nth season. Determine whether the function represents *exponential growth* or *exponential decay*. Identify the initial amount and interpret the growth factor or decay factor.

Rewrite the function in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*.

5. $f(x) = 3(1.02)^{10x}$ 6. $f(x) = (0.95)^{x+2}$

Solving Real-Life Problems

Exponential growth functions are used in real-life situations involving *compound interest*. Although interest earned is expressed as an *annual* rate, the interest is usually compounded more frequently than once per year. So, the formula $y = a(1 + r)^t$ must be modified for compound interest problems.

Core Concept

Compound Interest

Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P = principal (initial amount)
r = annual interest rate (in decimal form)
t = time (in years)
n = number of times interest is compounded per year

STUDY TIP

For interest compounded yearly, you can substitute 1 for n in the formula to get $y = P(1 + r)^t$.

EXAMPLE 5 Writing a Function

You deposit \$100 in a savings account that earns 6% annual interest compounded monthly. Write a function that represents the balance after t years.

SOLUTION

$$y = P\left(1 + \frac{r}{n}\right)^{nt}$$ Write the compound interest formula.

$$= 100\left(1 + \frac{0.06}{12}\right)^{12t}$$ Substitute 100 for P, 0.06 for r, and 12 for n.

$$= 100(1.005)^{12t}$$ Simplify.

EXAMPLE 6 Solving a Real-Life Problem

The table shows the balance of a money market account over time.

Year, t	Balance
0	\$100
1	\$110
2	\$121
3	\$133.10
4	\$146.41
5	\$161.05

a. Write a function that represents the balance after t years.

b. Graph the functions from part (a) and from Example 5 in the same coordinate plane. Compare the account balances.

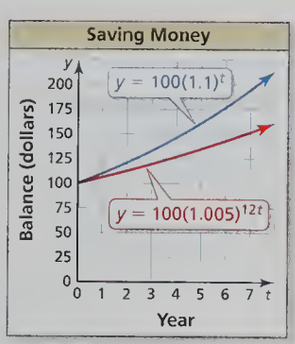
SOLUTION

a. From the table, you know the initial balance is \$100, and it increases 10% each year. So, $P = 100$ and $r = 0.1$.

$$y = P(1 + r)^t$$ Write the compound interest formula when $n = 1$.

$$= 100(1 + 0.1)^t$$ Substitute 100 for P and 0.1 for r.

$$= 100(1.1)^t$$ Add.



b. The money market account earns 10% interest each year, and the savings account earns 6% interest each year. So, the balance of the money market account increases faster.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. You deposit \$500 in a savings account that earns 9% annual interest compounded monthly. Write and graph a function that represents the balance y (in dollars) after t years.

Extra Example 5

You deposit \$300 in a savings account that earns 3% annual interest compounded monthly. Write a function that represents the balance after t years. $y = 300(1.0025)^{12t}$

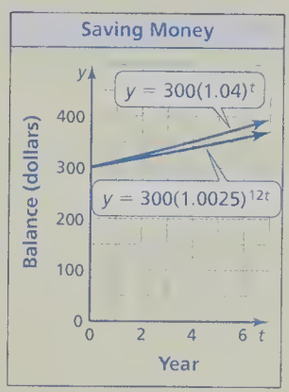
Extra Example 6

The table shows the balance of a money market account over time.

Year, t	Balance
0	\$300
1	\$312
2	\$324.48
3	\$337.46
4	\$350.96
5	\$365

a. Write a function that represents the balance after t years. $y = 300(1.04)^t$

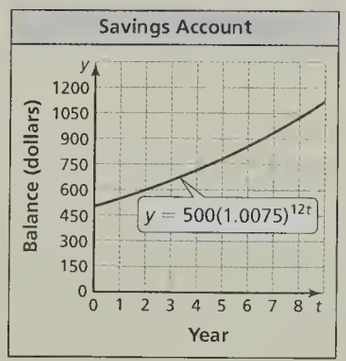
b. Graph the functions from part (a) and from Extra Example 5 in the same coordinate plane. Compare the account balances.



The money market account earns 4% interest each year, and the savings account earns 3% interest each year. So, the balance of the money market account increases faster.

MONITORING PROGRESS ANSWER

7. $y = 500(1.0075)^{12t}$



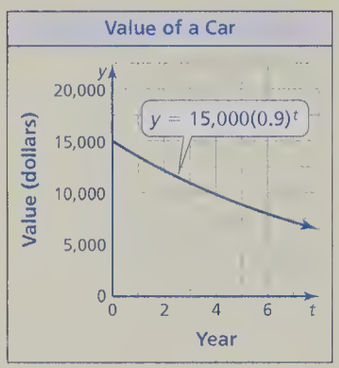
Extra Example 7

The value of a car is \$15,000. It loses 10% of its value every year.

a. Write a function that represents the value y (in dollars) of the car after t years. $y = 15{,}000(0.9)^t$

b. Find the approximate monthly percent decrease in value. about 1%

c. Graph the function from part (a).



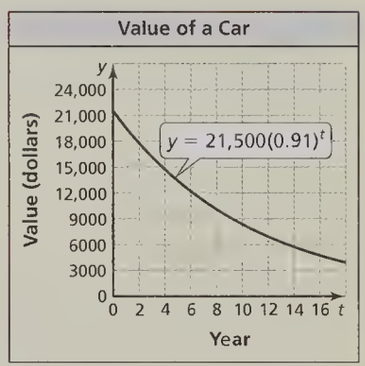
d. Estimate the value of the car after 6 years. about \$8,000

MONITORING PROGRESS ANSWERS

8. a. $y = 21{,}500(0.91)^t$

b. about 0.8%

c.



about \$7000



EXAMPLE 7 Solving a Real-Life Problem

The value of a car is \$21,500. It loses 12% of its value every year. (a) Write a function that represents the value y (in dollars) of the car after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Identify and interpret any asymptotes of the graph. (d) Estimate the value of the car after 6 years.

SOLUTION

1. **Understand the Problem** You know the value of the car and its annual percent decrease in value. You are asked to write a function that represents the value of the car over time and approximate the monthly percent decrease in value. Then graph the function and use the graph to estimate the value of the car in the future.

STUDY TIP
In real life, the percent decrease in value of an asset is called the *depreciation rate*.

2. **Make a Plan** Use the initial amount and the annual percent decrease in value to write an exponential decay function. Rewrite the function using the properties of exponents to approximate the monthly percent decrease (rate of decay). Then graph the original function and use the graph to estimate the y-value when the t-value is 6.

3. **Solve the Problem**

a. The initial value is \$21,500, and the rate of decay is 12%, or 0.12.

$y = a(1 - r)^t$ Write the exponential decay function.

$= 21{,}500(1 - 0.12)^t$ Substitute 21,500 for a and 0.12 for r.

$= 21{,}500(0.88)^t$ Subtract.

▶ The value of the car can be represented by $y = 21{,}500(0.88)^t$.

b. Use the fact that $t = \frac{1}{12}(12t)$ and the properties of exponents to rewrite the function in a form that reveals the monthly rate of decay.

$y = 21{,}500(0.88)^t$ Write the original function.

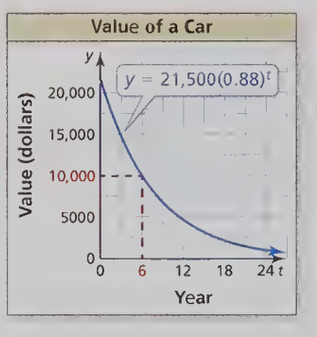
$= 21{,}500(0.88)^{(1/12)(12t)}$ Rewrite the exponent.

$= 21{,}500(0.88^{1/12})^{12t}$ Power of a Power Property

$\approx 21{,}500(0.989)^{12t}$ Evaluate the power.

Use the decay factor $1 - r \approx 0.989$ to find the rate of decay $r \approx 0.011$.

▶ So, the monthly percent decrease is about 1.1%.



c. You can see that the graph approaches, but never intersects, the t-axis.

▶ So, the graph has an asymptote at $t = 0$. This makes sense because the car will never have a value of \$0.

d. From the graph, you can see that the y-value is about 10,000 when $t = 6$.

▶ So, the value of the car is about \$10,000 after 6 years.

4. **Look Back** To check that the monthly percent decrease is reasonable, multiply it by 12 to see if it is close in value to the annual percent decrease of 12%.

$1.1\% \times 12 = 13.2\%$ 13.2% is close to 12%, so 1.1% is reasonable.

When you evaluate $y = 21{,}500(0.88)^t$ for $t = 6$, you get about \$9985. So, \$10,000 is a reasonable estimation.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

8. **WHAT IF?** The car loses 9% of its value every year. (a) Write a function that represents the value y (in dollars) of the car after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Estimate the value of the car after 12 years. Round your answer to the nearest thousand.

Closure

- **Exit Ticket:** Write an exponential growth function. *Sample answer:* $y = 100(1.06)^x$
 Write an exponential decay function. *Sample answer:* $y = 250(0.8)^x$

6.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** In the exponential growth function $y = a(1 + r)^t$, the quantity r is called the _____.
2. **VOCABULARY** What is the decay factor in the exponential decay function $y = a(1 - r)^t$?
3. **VOCABULARY** Compare exponential growth and exponential decay.
4. **WRITING** When does the function $y = ab^x$ represent exponential growth? exponential decay?

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, identify the initial amount a and the rate of growth r (as a percent) of the exponential function. Evaluate the function when $t = 5$. Round your answer to the nearest tenth.

5. $y = 350(1 + 0.75)^t$
6. $y = 10(1 + 0.4)^t$
7. $y = 25(1.2)^t$
8. $y = 12(1.05)^t$
9. $f(t) = 1500(1.074)^t$
10. $h(t) = 175(1.028)^t$
11. $g(t) = 6.72(2)^t$
12. $p(t) = 1.8^t$

In Exercises 13–16, write a function that represents the situation.

13. Sales of \$10,000 increase by 65% each year.
14. Your starting annual salary of \$35,000 increases by 4% each year.
15. A population of 210,000 increases by 12.5% each year.
16. An item costs \$4.50, and its price increases by 3.5% each year.
17. **MODELING WITH MATHEMATICS** The population of a city has been increasing by 2% annually. The sign shown is from the year 2000. *(See Example 1.)*



 a. Write an exponential growth function that represents the population t years after 2000.
 b. What will the population be in 2020? Round your answer to the nearest thousand.
18. **MODELING WITH MATHEMATICS** A young channel catfish weighs about 0.1 pound. During the next 8 weeks, its weight increases by about 23% each week.
 a. Write an exponential growth function that represents the weight of the catfish after t weeks during the 8-week period.
 b. About how much will the catfish weigh after 4 weeks? Round your answer to the nearest thousandth.



In Exercises 19–26, identify the initial amount a and the rate of decay r (as a percent) of the exponential function. Evaluate the function when $t = 3$. Round your answer to the nearest tenth.

19. $y = 575(1 - 0.6)^t$
20. $y = 8(1 - 0.15)^t$
21. $g(t) = 240(0.75)^t$
22. $f(t) = 475(0.5)^t$
23. $w(t) = 700(0.995)^t$
24. $h(t) = 1250(0.865)^t$
25. $y = \left(\frac{7}{8}\right)^t$
26. $y = 0.5\left(\frac{3}{4}\right)^t$

In Exercises 27–30, write a function that represents the situation.

27. A population of 100,000 decreases by 2% each year.
28. A \$900 sound system decreases in value by 9% each year.
29. A stock valued at \$100 decreases in value by 9.5% each year.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–21 odd, 31–59 odd, 64, 67, 69–75

Average: 1–4, 6–64 even, 67, 69–75

Advanced: 1–4, 12, 16, 18, 24–44 even, 50–62 even, 63–75

HOMEWORK CHECK

Basic: 17, 33, 41, 53, 57

Average: 16, 34, 42, 50, 58

Advanced: 18, 34, 42, 54, 60

ANSWERS

1. rate of growth
2. $1 - r$
3. Exponential growth occurs when a quantity increases by the same factor over equal intervals of time. Exponential decay occurs when a quantity decreases by the same factor over equal intervals of time.
4. when $b > 1$ and x represents time; when $0 < b < 1$ and x represents time
5. $a = 350$, $r = 75\%$; about 5744.6
6. $a = 10$, $r = 40\%$; about 53.8
7. $a = 25$, $r = 20\%$; about 62.2
8. $a = 12$, $r = 55\%$; about 15.3
9. $a = 1500$, $r = 7.4\%$; about 2143.4
10. $a = 175$, $r = 2.8\%$; about 200.9
11. $a = 6.72$, $r = 100\%$; about 215.0
12. $a = 1$, $r = 80\%$; about 18.9
13. $y = 10{,}000(1.65)^t$
14. $y = 35{,}000(1.04)^t$
15. $y = 210{,}000(1.125)^t$
16. $y = 4.50(1.035)^t$
17. **a.** $y = 315{,}000(1.02)^t$
 b. about 468,000
18. **a.** $y = 0.1(1.23)^t$
 b. about 0.229 lb
19. $a = 575$, $r = 60\%$; about 36.8
20. $a = 8$, $r = 15\%$; about 4.9
21. $a = 240$, $r = 25\%$; about 101.3
22. $a = 475$, $r = 50\%$; about 59.4
23. $a = 700$, $r = 0.5\%$; about 689.6
24. $a = 1250$, $r = 13.5\%$; about 809.0
25. $a = 1$, $r = 12.5\%$; about 0.7
26. $a = 0.5$, $r = 25\%$; about 0.2
27. $y = 100{,}000(0.98)^t$
28. $y = 900(0.91)^t$
29. $y = 100(0.905)^t$

Dynamic Teaching Tools

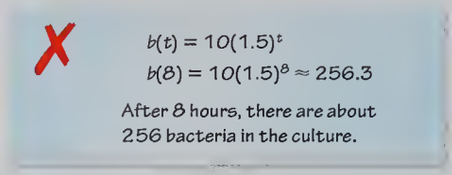
- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

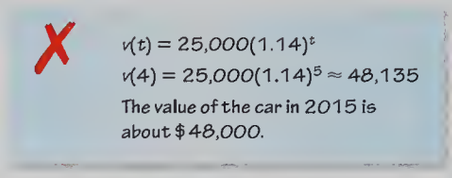
30. $y = 20{,}000(0.866)^t$
31. The growth rate is 1 + 1.5, not just 1.5; $b(t) = 10(2.5)^t$; $b(8) = 10(2.5)^8 \approx 15{,}258.8$; After 8 hours, there are about 15,259 bacteria in the culture.
32. The decay rate is 1 − 0.14, not 1 + 0.14; $v(t) = 25{,}000(0.86)^5 \approx 11{,}761$; The value of the car in 2015 is about \$12,000.
33. exponential decay; As x increases by 1, y is multiplied by $\frac{1}{5}$.
34. neither; As x increases by 1, y decreases by 4.
35. neither; As x increases by 1, y decreases by 6.
36. exponential growth; As x increases by 1, y is multiplied by 3.
37. exponential growth; As x increases by 5, y is multiplied by 4.
38. exponential decay; As x increases by 2, y is multiplied by $\frac{1}{6}$.
39. a. exponential decay
 b. about \$15,155
40. a. exponential growth
 b. about 17,716 people
41. exponential decay; 22,000 people; The population decreases by 6% each year.
42. exponential growth; 13 customers; The number of customers increases by 20% each day.
43. exponential growth; 1 sprint; The number of sprints increases by 100% each day.
44. exponential decay; 900 students; The number of healthy students decreases by 15% each day.
45. $f(x) = 1.52(0.9)^x$; exponential decay
46. $f(x) = 14.8(1.4)^x$; exponential growth
47. $f(x) = 2(1.69)^x$; exponential growth
48. $f(x) = 5(0.96)^x$; exponential decay
49. $f(x) = (1.20)^x$; exponential growth
50. $f(x) = 0.34(1.16)^x$; exponential growth
51. $f(x) = 0.67(0.55)^x$; exponential decay
52. $f(x) = (0.60)^x$; exponential decay

30. A company profit of \$20,000 decreases by 13.4% each year.

31. **ERROR ANALYSIS** The growth rate of a bacterial culture is 150% each hour. Initially, there are 10 bacteria. Describe and correct the error in finding the number of bacteria in the culture after 8 hours.



32. **ERROR ANALYSIS** You purchase a car in 2010 for \$25,000. The value of the car decreases by 14% annually. Describe and correct the error in finding the value of the car in 2015.



In Exercises 33–38, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain. *(See Example 2.)*

33.

x	y
−1	50
0	10
1	2
2	0.4

34.

x	y
0	32
1	28
2	24
3	20

35.

x	y
0	35
1	29
2	23
3	17

36.

x	y
1	17
2	51
3	153
4	459

37.

x	y
5	2
10	8
15	32
20	128

38.

x	y
3	432
5	72
7	12
9	2

39. **ANALYZING RELATIONSHIPS** The table shows the value of a camper t years after it is purchased.

t	Value
1	\$37,000
2	\$29,600
3	\$23,680
4	\$18,944

a. Determine whether the table represents an exponential growth function, an exponential decay function, or neither.

b. What is the value of the camper after 5 years?

40. **ANALYZING RELATIONSHIPS** The table shows the total numbers of visitors to a website t days after it is online.

t	42	43	44	45
Visitors	11,000	12,100	13,310	14,641

a. Determine whether the table represents an exponential growth function, an exponential decay function, or neither.

b. How many people will have visited the website after it is online 47 days?

In Exercises 41–44, determine whether the function represents *exponential growth* or *exponential decay*. Identify the initial amount and interpret the growth factor or decay factor. *(See Example 3.)*

41. The function $y = 22(0.94)^t$ represents the population y (in thousands of people) of a town after t years.

42. The function $y = 13(1.2)^x$ represents the number y of customers at a store x days after the store first opens.

43. The function $y = 2^n$ represents the number y of sprints required n days since the start of a training routine.

44. The function $y = 900(0.85)^d$ represents the number y of students who are healthy d days after a flu outbreak.

In Exercises 45–52, rewrite the function in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*. *(See Example 4.)*

45. $f(x) = (0.9)^{x-4}$
46. $f(x) = (1.4)^{x+8}$
47. $f(x) = 2(1.06)^{9x}$
48. $f(x) = 5(0.82)^{x/5}$
49. $f(x) = (1.45)^{x/2}$
50. $f(x) = 0.4(1.16)^{x-1}$
51. $f(x) = 4(0.55)^{x+3}$
52. $f(x) = (0.88)^{4x}$

In Exercises 53–56, write a function that represents the balance after t years. *(See Example 5.)*

53. $2000 deposit that earns 5% annual interest compounded quarterly

54. $1400 deposit that earns 10% annual interest compounded semiannually

55. $6200 deposit that earns 8.4% annual interest compounded monthly

56. $3500 deposit that earns 9.2% annual interest compounded quarterly

57. PROBLEM SOLVING The cross-sectional area of a tree 4.5 feet from the ground is called its *basal area*. The table shows the basal areas (in square inches) of Tree A over time. *(See Example 6.)*

Year, t	0	1	2	3	4
Basal area, A	120	132	145.2	159.7	175.7



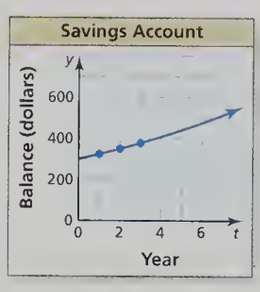
a. Write functions that represent the basal areas of the trees after t years.

b. Graph the functions from part (a) in the same coordinate plane. Compare the basal areas.

58. PROBLEM SOLVING You deposit $300 into an investment account that earns 12% annual interest compounded quarterly. The graph shows the balance of a savings account over time.

a. Write functions that represent the balances of the accounts after t years.

b. Graph the functions from part (a) in the same coordinate plane. Compare the account balances.



59. PROBLEM SOLVING A city has a population of 25,000. The population is expected to increase by 5.5% annually for the next decade. *(See Example 7.)*

a. Write a function that represents the population y after t years.

b. Find the approximate monthly percent increase in population.

c. Graph the function from part (a). Estimate the population after 4 years.



60. PROBLEM SOLVING Plutonium-238 is a material that generates steady heat due to decay and is used in power systems for some spacecraft. The function $y = a(0.5)^{t/x}$ represents the amount y of a substance remaining after t years, where a is the initial amount and x is the length of the half-life (in years).



a. A scientist is studying a 3-gram sample. Write a function that represents the amount y of plutonium-238 after t years.

b. What is the yearly percent decrease of plutonium-238?

c. Graph the function from part (a). Identify and interpret any asymptotes of the graph.

d. Estimate the amount remaining after 12 years.

61. COMPARING FUNCTIONS The three given functions describe the amount y of ibuprofen (in milligrams) in a person's bloodstream t hours after taking the dosage.

$y \approx 800(0.71)^t$

$y \approx 800(0.9943)^{60t}$

$y \approx 800(0.843)^{2t}$

a. Show that these expressions are approximately equivalent.

b. Describe the information given by each of the functions.

ANSWERS

53. $y = 2000(1.0125)^{4t}$

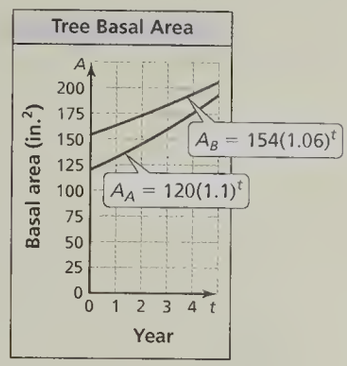
54. $y = 1400(1.05)^{2t}$

55. $y = 6200(1.007)^{12t}$

56. $y = 3500(1.023)^{4t}$

57. a. $A_A = 120(1.1)^t$; $A_B = 154(1.06)^t$

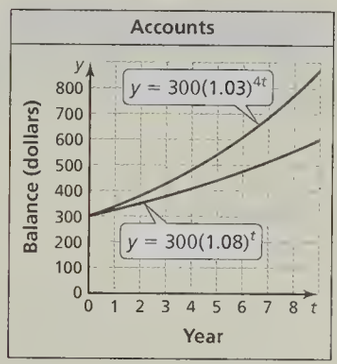
b.



The basal area of Tree B is larger than the basal area of Tree A, but the difference between the basal areas is decreasing.

58. a. investment account: $y = 300(1.03)^{4t}$; savings account: $y = 300(1.08)^t$

b.

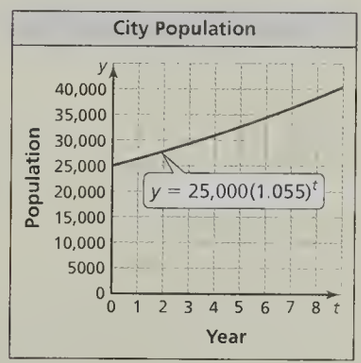


Both accounts start with the same balance. The investment account balance is increasing at a faster rate, so it is greater than the savings account balance after the start.

59. a. $y = 25{,}000(1.055)^t$

b. about 0.45%

c.

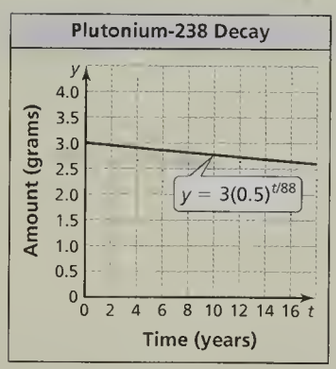


about 30,971 people

60. a. $y = 3(0.5)^{t/88}$

b. about 0.8%

60. c.



$y = 0$, The amount of plutonium−238 approaches but never equals 0.

d. about 2.7 g

61. a. $y \approx 800(0.9943)^{60t} \approx 800(0.9943^{60})^t \approx 800(0.7097)^t$; $y \approx 800(0.843)^{2t} \approx 800(0.843^2)^t \approx 800(0.7106)^t$

b. See Additional Answers.

ANSWERS

62. $C(t) = 9000(1.003)^{12t} + 480t$; The total amount of money saved both in the savings account and at home.

63. 200%; The growth factor is 3, which is also $r + 1$, so r, the growth rate, is 2, or 200%.

64. **a.** D; The graph is increasing, and the value from $x = 0$ to $x = 2$ increases from 40 to 160, which is the expected value.

b. B; The graph is decreasing, and the value from $x = 0$ to $x = 2$ decreases from 500 to about 350, which is the expected value.

c. A; The graph is increasing, and the value from $x = 0$ to $x = 2$ increases from 40 to about 50, which is the expected value.

d. C; The graph is decreasing, and the value from $x = 0$ to $x = 2$ decreases from 500 to about 450, which is the expected value.

65–75. See Additional Answers.

Mini-Assessment

1. The inaugural attendance of an annual music festival is 20,000. The attendance y increases by 5% each year.
 a. Write an exponential growth function that represents the attendance after t years.
 $y = 20{,}000(1.05)^t$
 b. How many people will attend the festival in the fifth year? Round your answer to the nearest thousand. about 26,000 people
2. Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

x	0	1	2	3
y	7	35	175	875

an exponential growth function

3. Rewrite the function $f(x) = 300(0.7)^{x/4}$ in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*.
$f(x) = 300(0.91)^x$; exponential decay

62. COMBINING FUNCTIONS You deposit \$9000 in a savings account that earns 3.6% annual interest compounded monthly. You also save \$40 per month in a safe at home. Write a function $C(t) = b(t) + h(t)$, where $b(t)$ represents the balance of your savings account and $h(t)$ represents the amount in your safe after t years. What does $C(t)$ represent?

63. NUMBER SENSE During a flu epidemic, the number of sick people triples every week. What is the growth rate as a percent? Explain your reasoning.

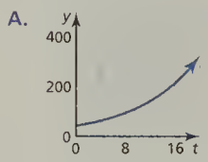
64. HOW DO YOU SEE IT? Match each situation with its graph. Explain your reasoning.

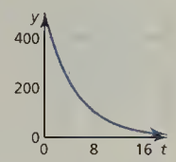
a. A bacterial population doubles each hour.

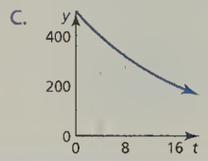
b. The value of a computer decreases by 18% each year.

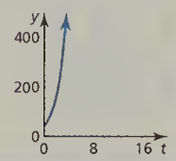
c. A deposit earns 11% annual interest compounded yearly.

d. A radioactive element decays 5.5% each year.

A.


B.


C.


D.


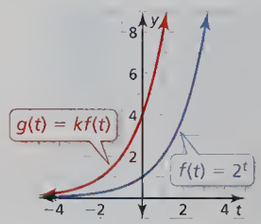
65. WRITING Give an example of an equation in the form $y = ab^x$ that does not represent an exponential growth function or an exponential decay function. Explain your reasoning.

66. THOUGHT PROVOKING Describe two account options into which you can deposit \$1000 and earn compound interest. Write a function that represents the balance of each account after t years. Which account would you rather use? Explain your reasoning.

67. MAKING AN ARGUMENT A store is having a sale on sweaters. On the first day, the prices of the sweaters are reduced by 20%. The prices will be reduced another 20% each day until the sweaters are sold. Your friend says the sweaters will be free on the fifth day. Is your friend correct? Explain.



68. COMPARING FUNCTIONS The graphs of f and g are shown.



a. Explain why f is an exponential growth function. Identify the rate of growth.

b. Describe the transformation from the graph of f to the graph of g. Determine the value of k.

c. The graph of g is the same as the graph of $h(t) = f(t + r)$. Use properties of exponents to find the value of r.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.3)*

69. $8x + 12 = 4x$ **70.** $5 - t = 7t + 21$ **71.** $6(r - 2) = 2r + 8$

Determine whether the sequence is arithmetic. If so, find the common difference. *(Section 4.7)*

72. $-20, -26, -32, -38, \ldots$ **73.** $9, 18, 36, 72, \ldots$

74. $-5, -8, -12, -17, \ldots$ **75.** $10, 20, 30, 40, \ldots$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

6.1–6.4 What Did You Learn?

Core Vocabulary

*n*th root of *a*, *p. 286*
radical, *p. 286*
index of a radical, *p. 286*
exponential function, *p. 292*
asymptote, *p. 293*
exponential growth, *p. 300*
exponential growth function, *p. 300*
exponential decay, *p. 301*
exponential decay function, *p. 301*
compound interest, *p. 303*

Core Concepts

Section 6.1

Zero Exponent, *p. 278*
Negative Exponents, *p. 278*
Product of Powers Property, *p. 279*
Quotient of Powers Property, *p. 279*
Power of a Power Property, *p. 279*
Power of a Product Property, *p. 280*
Power of a Quotient Property, *p. 280*

Section 6.2

Real *n*th Roots of *a*, *p. 286*
Rational Exponents, *p. 287*

Section 6.3

Graphing $y = ab^x$ When $b > 1$, *p. 293*
Graphing $y = ab^x$ When $0 < b < 1$, *p. 293*

Section 6.4

Exponential Growth Functions, *p. 300*
Exponential Decay Functions, *p. 301*
Compound Interest, *p. 303*

Mathematical Thinking

1. How did you apply what you know to simplify the complicated situation in Exercise 56 on page 283?
2. How can you use previously established results to construct an argument in Exercise 50 on page 290?
3. How is the form of the function you wrote in Exercise 62 on page 308 related to the forms of other types of functions you have learned about in this course?

Study Skills

Analyzing Your Errors

Misreading Directions

- **What Happens:** You incorrectly read or do not understand directions.
- **How to Avoid This Error:** Read the instructions for exercises at least twice and make sure you understand what they mean. Make this a habit and use it when taking tests.



309

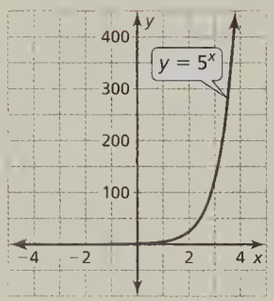
Dynamic Teaching Tools

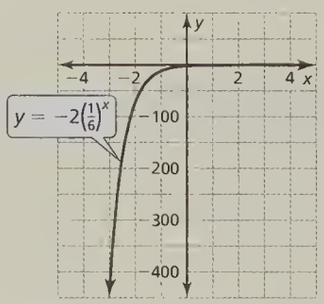
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

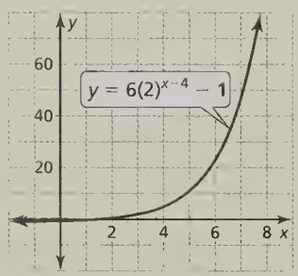
ANSWERS

1. *Sample answer:* Apply the distance formula ($d = rt$) to find an equation solved for time, and apply the laws of exponents to simplify the powers of 10.
2. *Sample answer:* Apply what you previously established about the sign of a radical with an odd index, which is the same as the sign of the radicand.
3. *Sample answer:* The first term, $b(t)$, is in the form of an exponential function, and the second term, $h(t)$, is in the form of a linear function.

ANSWERS

1. 3^6
2. $\frac{1}{k^{12}}$
3. $\frac{64r^6}{27s^{15}}$
4. $\frac{x^4}{4y^8}$
5. 3
6. $\frac{1}{2}$
7. 64
8. 32
9. 

asymptote: $y = 0$; domain: all real numbers, range: $y > 0$
10. 

asymptote: $y = 0$; domain: all real numbers, range: $y < 0$
11. 

asymptote: $y = -1$; domain: all real numbers, range: $y > -1$
12. exponential growth; As x increases by 1, y is multiplied by 3.
13. exponential decay; As x increases by 1, y is multiplied by $\frac{1}{11}$.
14. $f(x) = 1.88(1.6)^x$; exponential growth
15. $f(x) = 0.28(0.96)^x$; exponential decay
16. $f(x) = 71.55\left(\frac{4}{5}\right)^x$; exponential decay
17. a. 10^{12}
 b. 10^5
 c. 1000 dg; 10,000 mg = 10 g, 1000 dg = 100 g
18. 48 ft^3

6.1–6.4 Quiz

Simplify the expression. Write your answer using only positive exponents. *(Section 6.1)*

1. $3^2 \cdot 3^4$
2. $(k^4)^{-3}$
3. $\left(\frac{4r^2}{3s^5}\right)^3$
4. $\left(\frac{2x^0}{4x^{-2}y^4}\right)^2$

Evaluate the expression. *(Section 6.2)*

5. $\sqrt[3]{27}$
6. $\left(\frac{1}{16}\right)^{1/4}$
7. $512^{2/3}$
8. $(\sqrt{4})^5$

Graph the function. Identify the asymptote. Describe the domain and range. *(Section 6.3)*

9. $y = 5^x$
10. $y = -2\left(\frac{1}{6}\right)^x$
11. $y = 6(2)^{x-4} - 1$

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain. *(Section 6.4)*

12.

x	0	1	2	3
y	7	21	63	189

13.

x	1	2	3	4
y	14,641	1331	121	11

Rewrite the function in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*. *(Section 6.4)*

14. $f(x) = 3(1.6)^{x-1}$
15. $f(x) = \frac{1}{3}(0.96)^{4x}$
16. $f(x) = 80\left(\frac{4}{5}\right)^{x/2}$

17. The table shows several units of mass. *(Section 6.1)*

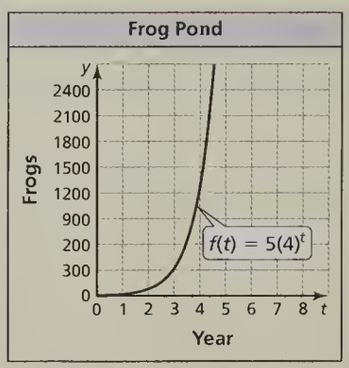
Unit of mass	kilogram	hectogram	dekagram	decigram	centigram	milligram	microgram	nanogram
Mass (in grams)	10^3	10^2	10^1	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}

 a. How many times larger is a kilogram than a nanogram? Write your answer using only positive exponents.
 b. How many times smaller is a milligram than a hectogram? Write your answer using only positive exponents.
 c. Which is greater, 10,000 milligrams or 1000 decigrams? Explain your reasoning.

18. You store blankets in a cedar chest. What is the volume of the cedar chest? *(Section 6.2)*



19. The function $f(t) = 5(4)^t$ represents the number of frogs in a pond after t years. *(Section 6.3 and Section 6.4)*
 a. Does the function represent *exponential growth* or *exponential decay*? Explain.
 b. Identify the initial amount and interpret the growth factor or decay factor.
 c. Graph the function. Describe the domain and range.
 d. What is the approximate monthly percent change?
 e. How many frogs are in the pond after 4 years?

19. a. exponential growth; As t increases by 1, y is multiplied by 4.
 b. 5 frogs; The number of frogs in the pond increases by 300% each year.
 c. 

 domain: $x \geq 0$, range: $y > 0$
 d. about 12%
 e. 1280 frogs

6.5 Geometric Sequences

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.12.D

Essential Question How can you use a geometric sequence to describe a pattern?

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**.

EXPLORATION 1 Describing Calculator Patterns

Work with a partner. Enter the keystrokes on a calculator and record the results in the table. Describe the pattern.

a. Step 1: 2 =
Step 2: × 2 =
Step 3: × 2 =
Step 4: × 2 =
Step 5: × 2 =

b. Step 1: 6 4 =
Step 2: × . 5 =
Step 3: × . 5 =
Step 4: × . 5 =
Step 5: × . 5 =

Step	1	2	3	4	5
Calculator display					

Step	1	2	3	4	5
Calculator display					

c. Use a calculator to make your own sequence. Start with any number and multiply by 3 each time. Record your results in the table.

Step	1	2	3	4	5
Calculator display					

d. Part (a) involves a geometric sequence with a common ratio of 2. What is the common ratio in part (b)? part (c)?

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to notice when calculations are repeated and look both for general methods and for shortcuts.

EXPLORATION 2 Folding a Sheet of Paper

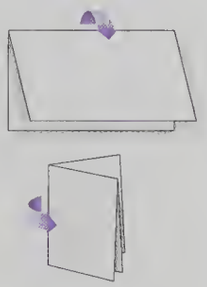
Work with a partner. A sheet of paper is about 0.1 millimeter thick.

a. How thick will it be when you fold it in half once? twice? three times?

b. What is the greatest number of times you can fold a piece of paper in half? How thick is the result?

c. Do you agree with the statement below? Explain your reasoning.

"If it were possible to fold the paper in half 15 times, it would be taller than you."



Communicate Your Answer

3. How can you use a geometric sequence to describe a pattern?
4. Give an example of a geometric sequence from real life other than paper folding.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.12.D The student is expected to write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms.

ANSWERS

1. a. 2, 4, 8, 16, 32; Each number is twice the preceding number.
 b. 64; 32; 16; 8; 4; Each number is half of the preceding number.
 c. *Sample answer:* 10, 30, 90, 270, 810
 d. $\frac{1}{2}$; 3
2. a. 0.2 mm; 0.4 mm; 0.8 mm
 b. *Sample answer:* 6 times; 6.4 mm
 c. yes; The thickness is a geometric sequence that doubles with each fold. After 15 folds, the thickness would be 3276.8 millimeters, or 3.2768 meters, which is taller than a human being.
3. Indicate the common ratio between each consecutive pair of terms.
4. *Sample answer:* If you drop a tennis ball, the height after each bounce will be a common fraction of the height of the preceding bounce.

Extra Example 1

Decide whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

a. 8, 24, 72, 216, ...

The sequence is geometric. The ratios are the same. The common ratio is $\frac{3}{1}$, or 3.

b. 8, 3, −2, −7, ...

The sequence is arithmetic. The differences are the same. The common difference is −5.

MONITORING PROGRESS ANSWERS

1. arithmetic; There is a common difference of −4.
2. geometric; There is a common ratio of $\frac{1}{8}$.
3. neither; There is neither a common difference nor a common ratio.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

6.5 Lesson

Core Vocabulary

geometric sequence, *p. 312*
common ratio, *p. 312*

Previous
arithmetic sequence
common difference

What You Will Learn

- Identify geometric sequences.
- Extend and graph geometric sequences.
- Write geometric sequences as functions.

Identifying Geometric Sequences

Core Concept

Geometric Sequence

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**. Each term is found by multiplying the previous term by the common ratio.

1, 5, 25, 125, . . . Terms of a geometric sequence

×5 ×5 ×5 ← common ratio

EXAMPLE 1 **Identifying Geometric Sequences**

Decide whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

a. 120, 60, 30, 15, . . .

b. 2, 6, 11, 17, . . .

SOLUTION

a. Find the ratio between each pair of consecutive terms.

120 60 30 15

$\frac{60}{120} = \frac{1}{2}$ $\frac{30}{60} = \frac{1}{2}$ $\frac{15}{30} = \frac{1}{2}$

The ratios are the same. The common ratio is $\frac{1}{2}$.

▶ So, the sequence is geometric.

b. Find the ratio between each pair of consecutive terms.

2 6 11 17

$\frac{6}{2} = 3$ $\frac{11}{6} = 1\frac{5}{6}$ $\frac{17}{11} = 1\frac{6}{11}$

There is no common ratio, so the sequence is *not* geometric.

Find the difference between each pair of consecutive terms.

2 6 11 17

6 − 2 = 4 11 − 6 = 5 17 − 11 = 6

There is no common difference, so the sequence is *not* arithmetic.

▶ So, the sequence is *neither* geometric nor arithmetic.

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Decide whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

1. 5, 1, −3, −7, . . .

2. 1024, 128, 16, 2, . . .

3. 2, 6, 10, 16, . . .

SUPPORTING English Language Learners

Have students practice reading comprehension by reading aloud portions of page 312 in groups of mixed language abilities. Explain that they should ask for help from the group, other classmates, or the teacher if there is something they don't understand.

Beginning Read the Core Vocabulary and headings.
Intermediate Read What You Will Learn, the problem statement in Example 1, and Monitoring Progress.
Advanced/Advanced High Read the Core Concept and steps of the Solution in Example 1.

ELPS 4.F.8 Use support from peers and teachers to develop vocabulary needed to comprehend increasingly challenging language.

Extending and Graphing Geometric Sequences

EXAMPLE 2 Extending Geometric Sequences

Write the next three terms of each geometric sequence.

a. 3, 6, 12, 24, . . . **b.** 64, −16, 4, −1, . . .

SOLUTION

Use tables to organize the terms and extend each sequence.

a.

Position	1	2	3	4	5	6	7
Term	3	6	12	24	48	96	192

×2 ×2 ×2 ×2 ×2 ×2

Each term is twice the previous term. So, the common ratio is 2.

Multiply a term by 2 to find the next term.

▶ The next three terms are 48, 96, and 192.

b.

Position	1	2	3	4	5	6	7
Term	64	−16	4	−1	$\frac{1}{4}$	$-\frac{1}{16}$	$\frac{1}{64}$

$\times\left(-\frac{1}{4}\right)$ $\times\left(-\frac{1}{4}\right)$ $\times\left(-\frac{1}{4}\right)$ $\times\left(-\frac{1}{4}\right)$ $\times\left(-\frac{1}{4}\right)$ $\times\left(-\frac{1}{4}\right)$

Multiply a term by $-\frac{1}{4}$ to find the next term.

▶ The next three terms are $\frac{1}{4}$, $-\frac{1}{16}$, and $\frac{1}{64}$.

ANALYZING MATHEMATICAL RELATIONSHIPS

When the terms of a geometric sequence alternate between positive and negative terms, or vice versa, the common ratio is negative.

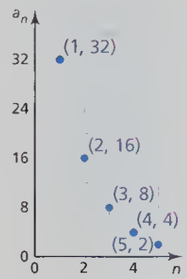
EXAMPLE 3 Graphing a Geometric Sequence

Graph the geometric sequence 32, 16, 8, 4, 2, ... What do you notice?

SOLUTION

Make a table. Then plot the ordered pairs (n, a_n).

Position, n	1	2	3	4	5
Term, a_n	32	16	8	4	2



▶ The points appear to lie on an exponential curve.

STUDY TIP

The points of any geometric sequence with a *positive* common ratio lie on an exponential curve.

Monitoring Progress

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Write the next three terms of the geometric sequence. Then graph the sequence.

4. 1, 3, 9, 27, . . . **5.** 2500, 500, 100, 20, . . .

6. 80, −40, 20, −10, . . . **7.** −2, 4, −8, 16, . . .

Differentiated Instruction

Auditory

Students may benefit from explaining the process for extending a geometric sequence and how to use the common ratio in that process. Have students take turns working at the board with one of several geometric sequences. Students should calculate the common ratio and then use it to find the next 3 terms, explaining each step in the process.

Extra Example 2

Write the next three terms of each geometric sequence.

a. 972, 324, 108, 36, ...

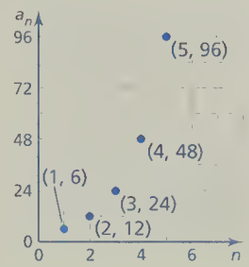
12, 4, $\frac{4}{3}$

b. 7, −14, 28, −56, ...

112, −224, 448

Extra Example 3

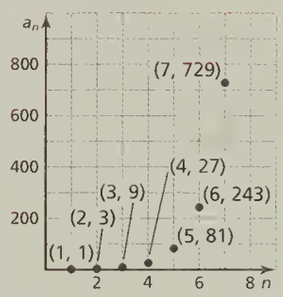
Graph the geometric sequence 6, 12, 24, 48, 96, ... What do you notice?



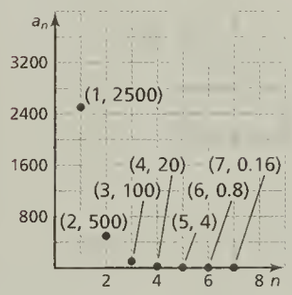
The points appear to lie on an exponential curve.

MONITORING PROGRESS ANSWERS

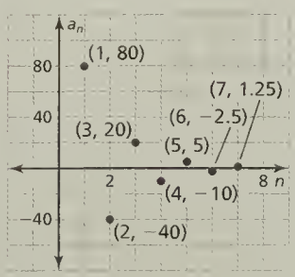
4. 81, 243, 729



5. 4, 0.8, 0.16



6. 5, −2.5, 1.25



7. See Additional Answers.

Extra Example 4

Write an equation for the nth term of the geometric sequence 3, 12, 48, 192, ... Then find a_{10}.

$a_n = 3(4)^{n-1}$; The 10th term of the geometric sequence is 786,432.

MONITORING PROGRESS ANSWERS

8. $a_n = (-5)^{n-1}$; 15,625
9. $a_n = 13(2)^{n-1}$; 832
10. $a_n = 432\left(\frac{1}{6}\right)^{n-1}$; $\frac{1}{108}$
11. $a_n = 4(2.5)^{n-1}$; 976.5625

Writing Geometric Sequences as Functions

Because consecutive terms of a geometric sequence have a common ratio, you can use the first term a_1 and the common ratio r to write an exponential function that describes a geometric sequence. Let $a_1 = 1$ and $r = 5$.

Position, n	Term, a_n	Written using a_1 and r	Numbers
1	first term, a_1	a_1	1
2	second term, a_2	$a_1 r$	$1 \cdot 5 = 5$
3	third term, a_3	$a_1 r^2$	$1 \cdot 5^2 = 25$
4	fourth term, a_4	$a_1 r^3$	$1 \cdot 5^3 = 125$
⋮	⋮	⋮	⋮
n	nth term, a_n	$a_1 r^{n-1}$	$1 \cdot 5^{n-1}$

STUDY TIP

Notice that the equation $a_n = a_1 r^{n-1}$ is of the form $y = ab^x$.

Core Concept

Equation for a Geometric Sequence

Let a_n be the nth term of a geometric sequence with first term a_1 and common ratio r. The nth term is given by

$$a_n = a_1 r^{n-1}.$$

EXAMPLE 4 Finding the nth Term of a Geometric Sequence

Write an equation for the nth term of the geometric sequence 2, 12, 72, 432, ... Then find a_{10}.

SOLUTION

The first term is 2, and the common ratio is 6.

$a_n = a_1 r^{n-1}$ Equation for a geometric sequence

$a_n = 2(6)^{n-1}$ Substitute 2 for a_1 and 6 for r.

Use the equation to find the 10th term.

$a_n = 2(6)^{n-1}$ Write the equation.

$a_{10} = 2(6)^{10-1}$ Substitute 10 for n.

$= 20{,}155{,}392$ Simplify.

▶ The 10th term of the geometric sequence is 20,155,392.

Monitoring Progress

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Write an equation for the nth term of the geometric sequence. Then find a_7.

8. 1, −5, 25, −125, . . .
9. 13, 26, 52, 104, . . .
10. 432, 72, 12, 2, . . .
11. 4, 10, 25, 62.5, . . .

You can rewrite the equation for a geometric sequence with first term a_1 and common ratio r in function notation by replacing a_n with $f(n)$.

$$f(n) = a_1 r^{n-1}$$

The domain of the function is the set of positive integers.

EXAMPLE 5 **Modeling with Mathematics**



Clicking the *zoom-out* button on a mapping website doubles the side length of the square map. After how many clicks on the *zoom-out* button is the side length of the map 640 miles?

Zoom-out clicks	1	2	3
Map side length (miles)	5	10	20

SOLUTION

1. **Understand the Problem** You know that the side length of the square map doubles after each click on the *zoom-out* button. So, the side lengths of the map represent the terms of a geometric sequence. You need to find the number of clicks it takes for the side length of the map to be 640 miles.
2. **Make a Plan** Begin by writing a function f for the nth term of the geometric sequence. Then find the value of n for which $f(n) = 640$.
3. **Solve the Problem** The first term is 5, and the common ratio is 2.

 $f(n) = a_1 r^{n-1}$ Function for a geometric sequence

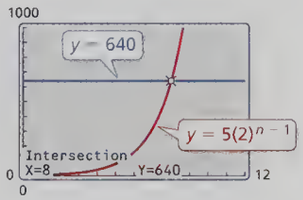
 $f(n) = 5(2)^{n-1}$ Substitute 5 for a_1 and 2 for r.

 The function $f(n) = 5(2)^{n-1}$ represents the geometric sequence. Use this function to find the value of n for which $f(n) = 640$. So, use the equation $640 = 5(2)^{n-1}$ to write a system of equations.

 $y = 5(2)^{n-1}$ Equation 1

 $y = 640$ Equation 2

 Then use a graphing calculator to graph the equations and find the point of intersection. The point of intersection is (8, 640).



 ▶ So, after eight clicks, the side length of the map is 640 miles.

4. **Look Back** You can use the *table* feature of a graphing calculator to find the value of n for which $f(n) = 640$.

X	Y1	Y2
3	20	640
4	40	640
5	80	640
6	160	640
7	320	640
8	640	640
9	1280	640

X=8

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12. **WHAT IF?** After how many clicks on the *zoom-out* button is the side length of the map 2560 miles?

Extra Example 5

Use the information in Example 5. After how many clicks on the *zoom-out* button is the side length of the map 5120 miles?

After 11 clicks, the side length of the map is 5120 miles.

MONITORING PROGRESS ANSWER

12. 10 clicks

Closure

- **Writing:** Describe in words how to find the 10th term of a geometric sequence. *Sample answer:* Find the common ratio of the sequence. In the equation $a_n = a_1 r^{n-1}$, substitute the first term for a_1, the common ratio for r, and 10 for n. Then simplify the equation.
- **Exit Ticket:** Find the 10th term of the geometric sequence 3, 15, 75, 375, ... 5,859,375

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–37 odd, 44, 46, 55, 56

Average: 1, 2–50 even, 55, 56

Advanced: 1, 2, 8, 12–24 even, 30–44 even, 45–56

HOMEWORK CHECK

Basic: 9, 19, 25, 37

Average: 12, 22, 26, 38

Advanced: 12, 22, 30, 38

ANSWERS

1. The first sequence is an arithmetic sequence with a common difference of 2. The second sequence is a geometric sequence with a common ratio of 2.
2. A negative number raised to an odd exponent is negative, and raised to an even exponent is positive. So, if the common ratio is negative, the terms will alternate signs.
3. 3
4. $\frac{1}{6}$
5. -8
6. 10
7. $\frac{3}{4}$
8. $-\frac{1}{3}$
9. arithmetic; There is a common difference of 8.
10. neither; There is neither a common difference nor a common ratio.
11. neither; There is neither a common difference nor a common ratio.
12. geometric; There is a common ratio of 7.
13. geometric; There is a common ratio of $\frac{1}{8}$.
14. arithmetic; There is a common difference of 7.
15. geometric; There is a common ratio of 5.
16. neither; There is neither a common difference nor a common ratio.
17. neither; There is neither a common difference nor a common ratio.
18. arithmetic; There is a common difference of 9.

6.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Compare the two sequences.

 2, 4, 6, 8, 10, . . . 2, 4, 8, 16, 32, . . .

2. **CRITICAL THINKING** Why do the points of a geometric sequence lie on an exponential curve only when the common ratio is positive?

Monitoring Progress and Modeling with Mathematics

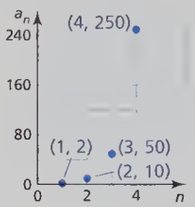
In Exercises 3–8, find the common ratio of the geometric sequence.

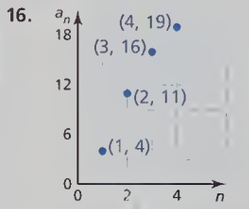
3. 4, 12, 36, 108, . . .
4. 36, 6, 1, $\frac{1}{6}$, . . .
5. $\frac{3}{8}$, -3, 24, -192, . . .
6. 0.1, 1, 10, 100, . . .
7. 128, 96, 72, 54, . . .
8. -162, 54, -18, 6, . . .

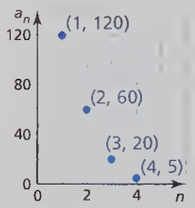
In Exercises 9–14, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning. *(See Example 1.)*

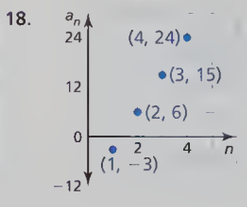
9. -8, 0, 8, 16, . . .
10. -1, 4, -7, 10, . . .
11. 9, 14, 20, 27, . . .
12. $\frac{3}{49}$, $\frac{3}{7}$, 3, 21, . . .
13. 192, 24, 3, $\frac{3}{8}$, . . .
14. -25, -18, -11, -4, . . .

In Exercises 15–18, determine whether the graph represents an *arithmetic sequence*, a *geometric sequence*, or *neither*. Explain your reasoning.

15. 

16. 

17. 

18. 

In Exercises 19–24, write the next three terms of the geometric sequence. Then graph the sequence. *(See Examples 2 and 3.)*

19. 5, 20, 80, 320, . . .
20. -3, 12, -48, 192, . . .
21. 81, -27, 9, -3, . . .
22. -375, -75, -15, -3, . . .
23. 32, 8, 2, $\frac{1}{2}$, . . .
24. $\frac{16}{9}$, $\frac{8}{3}$, 4, 6, . . .

In Exercises 25–32, write an equation for the nth term of the geometric sequence. Then find a_6. *(See Example 4.)*

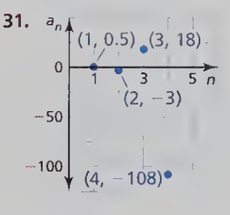
25. 2, 8, 32, 128, . . .
26. 0.6, -3, 15, -75, . . .
27. $-\frac{1}{8}$, $-\frac{1}{4}$, $-\frac{1}{2}$, -1, . . .
28. 0.1, 0.9, 8.1, 72.9, . . .

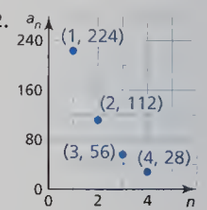
29.

n	1	2	3	4
a_n	7640	764	76.4	7.64

30.

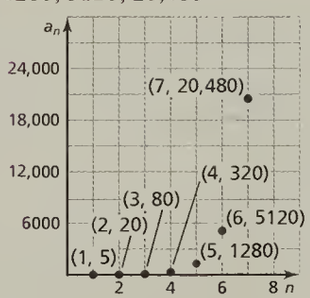
n	1	2	3	4
a_n	-192	48	-12	3

31. 

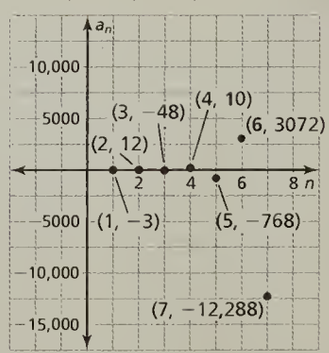
32. 

33. **PROBLEM SOLVING** A badminton tournament begins with 128 teams. After the first round, 64 teams remain. After the second round, 32 teams remain. How many teams remain after the third, fourth, and fifth rounds?

19. 1280; 5120; 20,480

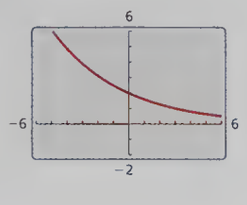


20. -768; 3072; $-12,288$



21–33. See Additional Answers.

34. **PROBLEM SOLVING** A graphing calculator screen displays an area of 96 square units. After you zoom out once, the area is 384 square units. After you zoom out a second time, the area is 1536 square units. What is the screen area after you zoom out four times?



35. **ERROR ANALYSIS** Describe and correct the error in writing the next three terms of the geometric sequence.

✗ $-8, 4, -2, 1, \ldots$

$\times(-2)$ $\times(-2)$ $\times(-2)$

The next three terms are -2, 4, and -8.

36. **ERROR ANALYSIS** Describe and correct the error in writing an equation for the nth term of the geometric sequence.

✗ $-2, -12, -72, -432, \ldots$

The first term is -2, and the common ratio is -6.

$a_n = a_1 r^{n-1}$

$a_n = -2(-6)^{n-1}$

37. **MODELING WITH MATHEMATICS** The distance (in millimeters) traveled by a swinging pendulum decreases after each swing, as shown in the table. *(See Example 5.)*

Swing	1	2	3
Distance (in millimeters)	625	500	400



a. Write a function that represents the distance the pendulum swings on its nth swing.

b. After how many swings is the distance 256 millimeters?

38. **MODELING WITH MATHEMATICS** You start a chain email and send it to six friends. The next day, each of your friends forwards the email to six people. The process continues for a few days.

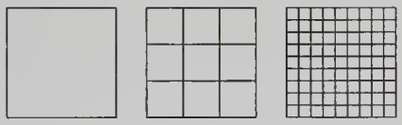


a. Write a function that represents the number of people who have received the email after n days.

b. After how many days will 1296 people have received the email?

MATHEMATICAL CONNECTIONS In Exercises 39 and 40, (a) write a function that represents the sequence of figures and (b) describe the 10th figure in the sequence.

39.



40.



41. **REASONING** Write a sequence that represents the number of teams that have been eliminated after n rounds of the badminton tournament in Exercise 33. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

42. **REASONING** Write a sequence that represents the perimeter of the graphing calculator screen in Exercise 34 after you zoom out n times. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

43. **WRITING** Compare the graphs of arithmetic sequences to the graphs of gcometric sequences.

44. **MAKING AN ARGUMENT** You are given two consecutive terms of a sequence.

$\ldots, -8, 0, \ldots$

Your friend says that the sequence is not geometric. A classmate says that is impossible to know given only two terms. Who is correct? Explain.

Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

ANSWERS

34. 24,576 square units

35. The common factor is $-\frac{1}{2}$, not -2;

$-8, \overset{\times(-\frac{1}{2})}{\longrightarrow} 4, \overset{\times(-\frac{1}{2})}{\longrightarrow} -2, \overset{\times(-\frac{1}{2})}{\longrightarrow} 1, \ldots$

The next three terms are $-\frac{1}{2}$, $\frac{1}{4}$, and $-\frac{1}{8}$.

36. The common ratio is 6, not -6; The first term is -2 and the common ratio is 6; $a_n = a_1 r^{n-1}$; $a_n = -2(6)^{n-1}$

37. a. $a_n = 625\left(\frac{4}{5}\right)^{n-1}$

b. 5 swings

38. a. $a_n = 6^n$

b. 4 days

39. a. $a_n = 9^{n-1}$

b. a large square containing 387,420,489 small squares

40. a. $a_n = 4^{n-1}$

b. a large triangle containing 262,144 small triangles

41. $a_n = 64\left(\frac{1}{2}\right)^{n-1}$; geometric; There is a common ratio of $\frac{1}{2}$.

42. $a_n = 80(2)^{n-1}$; geometric; There is a common ratio of 2.

43. Graphs of arithmetic sequences form a linear pattern. Graphs of geometric sequences form an exponential pattern when the common ratio is positive, and a pattern of points alternating between Quadrants I and IV when the common ratio is negative.

44. your friend; The only way to get 0 as a term would be with a common ratio of 0. If the common ratio is 0, then there is no possible previous term that would result in -8 as the next term.

ANSWERS

45. yes; yes; It is an arithmetic sequence with a common difference of 0, and a geometric sequence with a common ratio of 1.

46. a. B

 b. A

 When the common ratio is greater than 1, the values in the sequence increase. When the common ratio is between 0 and 1, the values in the sequence decrease.

47. 59,049

48. *Sample answer:* 1, 1, 2, 3, 5, 8, 13, ...; After the first two terms, each term is the sum of the preceding two terms.

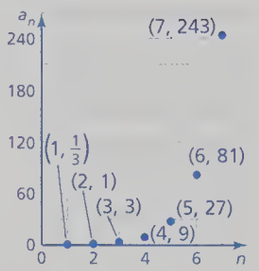
49. dependent; Each term is calculated from the preceding term.

50–56. See Additional Answers.

Mini-Assessment

1. Decide whether the sequence 13, 23, 33, 43, ... is *arithmetic*, *geometric*, or *neither*. Explain your reasoning. arithmetic; The differences are the same. The common difference is 10.

2. Write the next three terms of the geometric sequence $\frac{1}{3}$, 1, 3, 9, ... Then graph the sequence.
 27, 81, 243



4. Write an equation for the nth term of the geometric sequence 11, −33, 99, −297, ... Then find a_8.
 $a_n = 11(-3)^{n-1}$; $a_8 = -24{,}057$

5. The number of bacteria in a culture doubles each hour. After how many hours is the number of bacteria 7680?

Hours	1	2	3
Number of bacteria	120	240	480

 after 7 hours

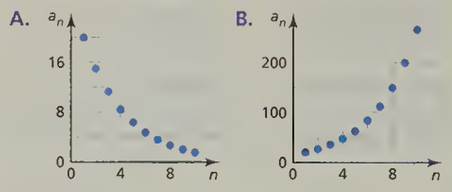
45. **CRITICAL THINKING** Is the sequence shown an *arithmetic* sequence? a *geometric* sequence? Explain your reasoning.

 3, 3, 3, 3, . . .

46. **HOW DO YOU SEE IT?** Without performing any calculations, match each equation with its graph. Explain your reasoning.

 a. $a_n = 20\left(\frac{4}{3}\right)^{n-1}$

 b. $a_n = 20\left(\frac{3}{4}\right)^{n-1}$



47. **REASONING** What is the 9th term of the geometric sequence where $a_3 = 81$ and $r = 3$?

48. **OPEN-ENDED** Write a sequence that has a pattern but is not arithmetic or geometric. Describe the pattern.

49. **ATTENDING TO PRECISION** Are the terms of a geometric sequence independent or dependent? Explain your reasoning.

50. **DRAWING CONCLUSIONS** A college student makes a deal with her parents to live at home instead of living on campus. She will pay her parents $0.01 for the first day of the month, $0.02 for the second day, $0.04 for the third day, and so on.

 a. Write an equation that represents the nth term of the geometric sequence.

 b. What will she pay on the 25th day?

 c. Did the student make a good choice or should she have chosen to live on campus? Explain.

51. **REPEATED REASONING** A soup kitchen makes 16 gallons of soup. Each day, a quarter of the soup is served and the rest is saved for the next day.



 a. Write the first five terms of the sequence of the number of fluid ounces of soup left each day.

 b. Write an equation that represents the nth term of the sequence.

 c. When is all the soup gone? Explain.

52. **THOUGHT PROVOKING** Find the sum of the terms of the geometric sequence.

 $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2^{n-1}}, \ldots$

 Explain your reasoning. Write a different infinite geometric sequence that has the same sum.

53. **OPEN-ENDED** Write a geometric sequence in which $a_2 < a_1 < a_3$.

54. **NUMBER SENSE** Write an equation that represents the nth term of each geometric sequence shown.

n	1	2	3	4
a_n	2	6	18	54

n	1	2	3	4
b_n	1	5	25	125

 a. Do the terms $a_1 - b_1, a_2 - b_2, a_3 - b_3, \ldots$ form a geometric sequence? If so, how does the common ratio relate to the common ratios of the sequences above?

 b. Do the terms $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \ldots$ form a geometric sequence? If so, how does the common ratio relate to the common ratios of the sequences above?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use residuals to determine whether the model is a good fit for the data in the table. Explain. *(Section 4.6)*

55. $y = 3x - 8$

x	0	1	2	3	4	5	6
y	−10	−2	−1	2	1	7	10

56. $y = -5x + 1$

x	−3	−2	−1	0	1	2	3
y	6	4	6	1	2	−4	−3

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

6.6 Recursively Defined Sequences

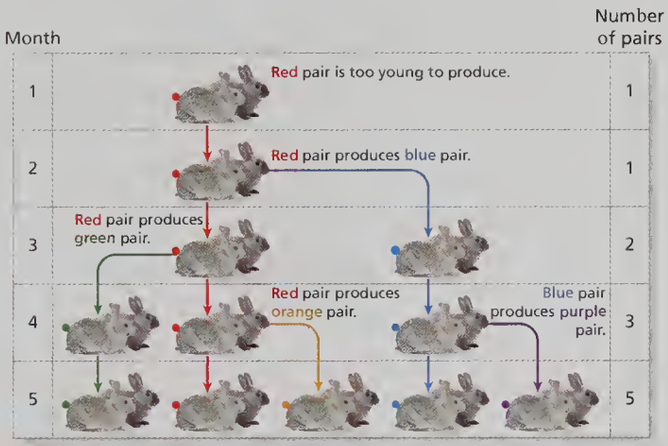
Texas Essential Knowledge and Skills
A.12.C

Essential Question
How can you define a sequence recursively?

A **recursive rule** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms.

EXPLORATION 1 Describing a Pattern

Work with a partner. Consider a hypothetical population of rabbits. Start with one breeding pair. After each month, each breeding pair produces another breeding pair. The total number of rabbits each month follows the exponential pattern 2, 4, 8, 16, 32, ... Now suppose that in the first month after each pair is born, the pair is too young to reproduce. Each pair produces another pair after it is 2 months old. Find the total number of pairs in months 6, 7, and 8.



ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to look closely to discern a pattern or structure.

EXPLORATION 2 Using a Recursive Equation

Work with a partner. Consider the following recursive equation.

$$a_n = a_{n-1} + a_{n-2}$$

Each term in the sequence is the sum of the two preceding terms.

Copy and complete the table. Compare the results with the sequence of the number of pairs in Exploration 1.

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1						

Communicate Your Answer

3. How can you define a sequence recursively?
4. Use the Internet or some other reference to determine the mathematician who first described the sequences in Explorations 1 and 2.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.12.C The student is expected to identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

ANSWERS

1. 8 pairs, 13 pairs, 21 pairs
2. 2, 3, 5, 8, 13, 21; They are the same.
3. Give the beginning term(s) of a sequence and a recursive equation that tells how a_n is related to one or more preceding terms.
4. Leonardo Fibonacci

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students work in pairs to complete the explorations and Communicate Your Answer. Have them take turns reading aloud and discussing the instructions and questions with each other. Explain that to make certain they understand what their partner is saying, they should restate it in their own words, and ask if that is what was meant.

Beginning Restate using simple phrases of two or three words.
Intermediate Restate using simple sentences.
Advanced/Advanced High Restate using multiple sentences.

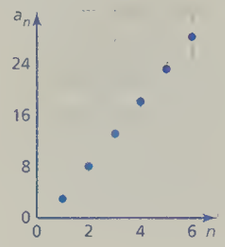
ELPS 2.D.2 Seek clarification [of spoken language] as needed.

Extra Example 1

Write the first six terms of each sequence. Then graph each sequence.

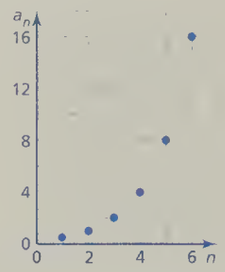
a. $a_1 = 3, a_n = a_{n-1} + 5$

3, 8, 13, 18, 23, 28



b. $a_1 = 0.5, a_n = 2a_{n-1}$

0.5, 1, 2, 4, 8, 16



6.6 Lesson

What You Will Learn

- Write terms of recursively defined sequences.
- Write recursive rules for sequences.
- Translate between recursive rules and explicit rules.
- Write recursive rules for special sequences.

Core Vocabulary

explicit rule, *p. 320*
recursive rule, *p. 320*

Previous
arithmetic sequence
geometric sequence

Writing Terms of Recursively Defined Sequences

So far in this book, you have defined arithmetic and geometric sequences *explicitly*. An **explicit rule** gives a_n as a function of the term's position number n in the sequence. For example, an explicit rule for the arithmetic sequence 3, 5, 7, 9, . . . is $a_n = 3 + 2(n - 1)$, or $a_n = 2n + 1$.

Now, you will define arithmetic and geometric sequences *recursively*. A **recursive rule** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms.

Core Concept

Recursive Equation for an Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference

Recursive Equation for a Geometric Sequence

$a_n = r \cdot a_{n-1}$, where r is the common ratio

EXAMPLE 1 Writing Terms of Recursively Defined Sequences

Write the first six terms of each sequence. Then graph each sequence.

a. $a_1 = 2, a_n = a_{n-1} + 3$ **b.** $a_1 = 1, a_n = 3a_{n-1}$

SOLUTION

You are given the first term. Use the recursive equation to find the next five terms.

a. $a_1 = 2$

$a_2 = a_1 + 3 = 2 + 3 = 5$

$a_3 = a_2 + 3 = 5 + 3 = 8$

$a_4 = a_3 + 3 = 8 + 3 = 11$

$a_5 = a_4 + 3 = 11 + 3 = 14$

$a_6 = a_5 + 3 = 14 + 3 = 17$

b. $a_1 = 1$

$a_2 = 3a_1 = 3(1) = 3$

$a_3 = 3a_2 = 3(3) = 9$

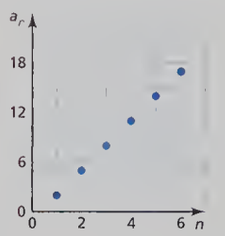
$a_4 = 3a_3 = 3(9) = 27$

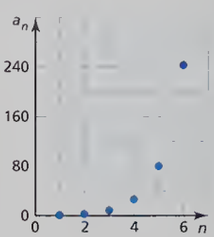
$a_5 = 3a_4 = 3(27) = 81$

$a_6 = 3a_5 = 3(81) = 243$

STUDY TIP

A sequence is a discrete function. So, the points on the graph are not connected.





For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Inclusion

Some students may have difficulty understanding the difference between explicit and recursive equations for a sequence. Point out that an explicit equation allows you to find *any* term in a sequence. For example, given the first term and the explicit equation, you can find the second term, the 8th term, the 15th term, or any other term. A recursive equation allows you to find only the *next* term in a sequence. For example, given the first term and the recursive equation, you can only find the second term. Then you must use the second term and the recursive equation to find the third term, and so on.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write the first six terms of the sequence. Then graph the sequence.

1. $a_1 = 0, a_n = a_{n-1} - 8$
2. $a_1 = -7.5, a_n = a_{n-1} + 2.5$
3. $a_1 = -36, a_n = \frac{1}{2}a_{n-1}$
4. $a_1 = 0.7, a_n = 10a_{n-1}$

Writing Recursive Rules

EXAMPLE 2 **Writing Recursive Rules**

Write a recursive rule for each sequence.

a. $-30, -18, -6, 6, 18, \ldots$ **b.** $500, 100, 20, 4, 0.8, \ldots$

SOLUTION

Use a table to organize the terms and find the pattern.

a.

Position, n	1	2	3	4	5
Term, a_n	-30	-18	-6	6	18

$+12$ $+12$ $+12$ $+12$

The sequence is arithmetic, with first term $a_1 = -30$ and common difference $d = 12$.

$a_n = a_{n-1} + d$ Recursive equation for an arithmetic sequence

$a_n = a_{n-1} + 12$ Substitute 12 for d.

▶ So, a recursive rule for the sequence is $a_1 = -30, a_n = a_{n-1} + 12$.

COMMON ERROR

When writing a recursive rule for a sequence, you need to write both the beginning term(s) and the recursive equation.

b.

Position, n	1	2	3	4	5
Term, a_n	500	100	20	4	0.8

$\times\frac{1}{5}$ $\times\frac{1}{5}$ $\times\frac{1}{5}$ $\times\frac{1}{5}$

The sequence is geometric, with first term $a_1 = 500$ and common ratio $r = \frac{1}{5}$.

$a_n = r \cdot a_{n-1}$ Recursive equation for a geometric sequence

$a_n = \frac{1}{5}a_{n-1}$ Substitute $\frac{1}{5}$ for r.

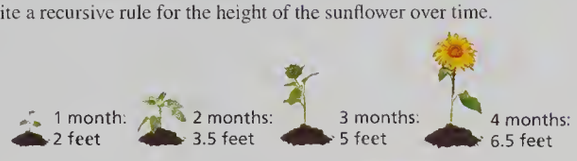
▶ So, a recursive rule for the sequence is $a_1 = 500, a_n = \frac{1}{5}a_{n-1}$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write a recursive rule for the sequence.

5. $8, 3, -2, -7, -12, \ldots$
6. $1.3, 2.6, 3.9, 5.2, 6.5, \ldots$
7. $4, 20, 100, 500, 2500, \ldots$
8. $128, -32, 8, -2, 0.5, \ldots$
9. Write a recursive rule for the height of the sunflower over time.



Extra Example 2

Write a recursive rule for each sequence.

a. $-8, -1, 6, 13, 20, \ldots$

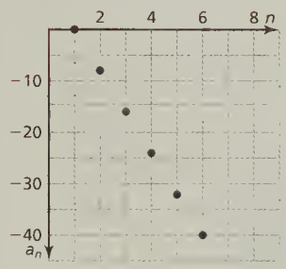
$a_1 = -8; a_n = a_{n-1} + 7$

b. $405, 135, 45, 15, 5, \ldots$

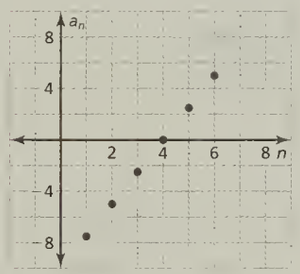
$a_1 = 405, a_n = \frac{1}{3}a_{n-1}$

MONITORING PROGRESS ANSWERS

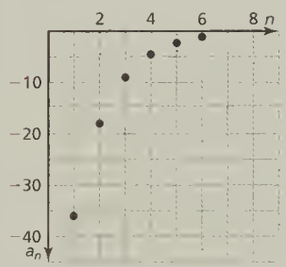
1. $0, -8, -16, -24, -32, -40$



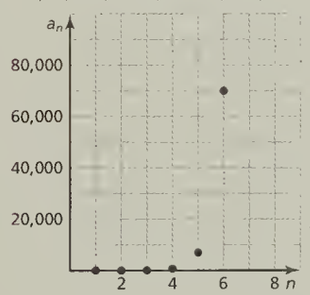
2. $-7.5, -5, -2.5, 0, 2.5, 5$



3. $-36, -18, -9, -\frac{9}{2}, -\frac{9}{4}, -\frac{9}{8}$



4. $0.7; 7; 70; 700; 7000; 70{,}000$



5. $a_1 = 8, a_n = a_{n-1} - 5$
6. $a_1 = 1.3, a_n = a_{n-1} + 1.3$
7. $a_1 = 4, a_n = 5a_{n-1}$
8. $a_1 = 128, a_n = -\frac{1}{4}a_{n-1}$
9. $a_1 = 2$ ft, $a_n = (a_{n-1} + 1.5)$ ft

Extra Example 3

Write an explicit rule for each recursive rule.

a. $a_1 = 9, a_n = a_{n-1} - 4$

$a_n = -4n + 13$

b. $a_1 = 4.5, a_n = 3a_{n-1}$

$a_n = 4.5(3)^{n-1}$

Extra Example 4

Write a recursive rule for each explicit rule.

a. $a_n = 6n - 5$

$a_1 = 1, a_n = a_{n-1} + 6$

b. $a_n = 4(3)^{n-1}$

$a_1 = 4, a_n = 3a_{n-1}$

MONITORING PROGRESS ANSWERS

10. $a_n = 20n - 65$

11. $a_n = 13(-3)^{n-1}$

12. $a_1 = 0, a_n = a_{n-1} - 1$

13. $a_1 = -2.5, a_n = 4a_{n-1}$

Translating between Recursive and Explicit Rules

EXAMPLE 3 Translating from Recursive Rules to Explicit Rules

Write an explicit rule for each recursive rule.

a. $a_1 = 25, a_n = a_{n-1} - 10$ **b.** $a_1 = 19.6, a_n = -0.5a_{n-1}$

SOLUTION

a. The recursive rule represents an arithmetic sequence, with first term $a_1 = 25$ and common difference $d = -10$.

$a_n = a_1 + (n - 1)d$ Explicit rule for an arithmetic sequence

$a_n = 25 + (n - 1)(-10)$ Substitute 25 for a_1 and -10 for d.

$a_n = -10n + 35$ Simplify.

▶ An explicit rule for the sequence is $a_n = -10n + 35$.

b. The recursive rule represents a geometric sequence, with first term $a_1 = 19.6$ and common ratio $r = -0.5$.

$a_n = a_1 r^{n-1}$ Explicit rule for a geometric sequence

$a_n = 19.6(-0.5)^{n-1}$ Substitute 19.6 for a_1 and -0.5 for r.

▶ An explicit rule for the sequence is $a_n = 19.6(-0.5)^{n-1}$.

EXAMPLE 4 Translating from Explicit Rules to Recursive Rules

Write a recursive rule for each explicit rule.

a. $a_n = -2n + 3$ **b.** $a_n = -3(2)^{n-1}$

SOLUTION

a. The explicit rule represents an arithmetic sequence, with first term $a_1 = -2(1) + 3 = 1$ and common difference $d = -2$.

$a_n = a_{n-1} + d$ Recursive equation for an arithmetic sequence

$a_n = a_{n-1} + (-2)$ Substitute -2 for d.

▶ So, a recursive rule for the sequence is $a_1 = 1, a_n = a_{n-1} - 2$.

b. The explicit rule represents a geometric sequence, with first term $a_1 = -3$ and common ratio $r = 2$.

$a_n = r \cdot a_{n-1}$ Recursive equation for a geometric sequence

$a_n = 2a_{n-1}$ Substitute 2 for r.

▶ So, a recursive rule for the sequence is $a_1 = -3, a_n = 2a_{n-1}$.

Monitoring Progress



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Write an explicit rule for the recursive rule.

10. $a_1 = -45, a_n = a_{n-1} + 20$ **11.** $a_1 = 13, a_n = -3a_{n-1}$

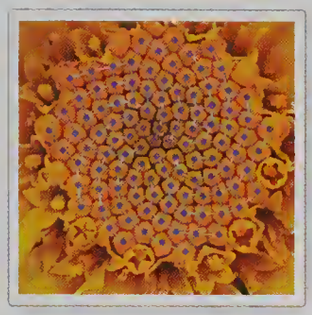
Write a recursive rule for the explicit rule.

12. $a_n = -n + 1$ **13.** $a_n = -2.5(4)^{n-1}$

Writing Recursive Rules for Special Sequences

You can write recursive rules for sequences that are neither arithmetic nor geometric. One way is to look for patterns in the sums of consecutive terms.

EXAMPLE 5 **Writing Recursive Rules for Other Sequences**



The sequence in Example 5 is called the *Fibonacci sequence*. This pattern is naturally occurring in many objects, such as flowers.

Use the sequence shown.

$$1, 1, 2, 3, 5, 8, \ldots$$

a. Write a recursive rule for the sequence.

b. Write the next three terms of the sequence.

SOLUTION

a. Find the difference and ratio between each pair of consecutive terms.

1 1 2 3

$1 - 1 = 0 \quad 2 - 1 = 1 \quad 3 - 2 = 1$

There is no common difference, so the sequence is *not* arithmetic.

1 1 2 3

$\frac{1}{1} = 1 \quad \frac{2}{1} = 2 \quad \frac{3}{2} = 1\frac{1}{2}$

There is no common ratio, so the sequence is *not* geometric.

Find the sum of each pair of consecutive terms.

$a_1 + a_2 = 1 + 1 = 2$ — 2 is the third term.

$a_2 + a_3 = 1 + 2 = 3$ — 3 is the fourth term.

$a_3 + a_4 = 2 + 3 = 5$ — 5 is the fifth term.

$a_4 + a_5 = 3 + 5 = 8$ — 8 is the sixth term.

Beginning with the third term, each term is the sum of the two previous terms. A recursive equation for the sequence is $a_n = a_{n-2} + a_{n-1}$.

▶ So, a recursive rule for the sequence is $a_1 = 1, a_2 = 1, a_n = a_{n-2} + a_{n-1}$.

b. Use the recursive equation $a_n = a_{n-2} + a_{n-1}$ to find the next three terms.

$a_7 = a_5 + a_6 = 5 + 8 = 13$

$a_8 = a_6 + a_7 = 8 + 13 = 21$

$a_9 = a_7 + a_8 = 13 + 21 = 34$

▶ The next three terms are 13, 21, and 34.

Monitoring Progress

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Write a recursive rule for the sequence. Then write the next three terms of the sequence.

14. 5, 6, 11, 17, 28, . . .

15. −3, −4, −7, −11, −18, . . .

16. 1, 1, 0, −1, −1, 0, 1, 1, . . .

17. 4, 3, 1, 2, −1, 3, −4, . . .

Extra Example 5

Use the sequence shown.

−4, −5, −9, −14, −23, −37, . . .

a. Write a recursive rule for the sequence.

$a_1 = -4, a_2 = -5,$

$a_n = a_{n-2} + a_{n-1}$

b. Write the next three terms of the sequence. −60, −97, −157

MONITORING PROGRESS ANSWERS

14. $a_1 = 5, a_2 = 6, a_n = a_{n-2} + a_{n-1}$; 45, 73, 118

15. $a_1 = -3, a_2 = -4,$ $a_n = a_{n-2} + a_{n-1}$; −29, −47, −76

16. $a_1 = 1, a_2 = 1, a_n = a_{n-1} - a_{n-2}$; 0, −1, −1

17. $a_1 = 4, a_2 = 3, a_n = a_{n-2} - a_{n-1}$; 7, −11, 18

Closure

- **Look Back:** Have students *Look Back* over Sections 6.5 and 6.6. Have them summarize what they understand about sequences and what helped them learn it.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–45 odd, 54, 58, 60–67

Average: 1, 2–54 even, 58, 60–67

Advanced: 1, 2, 6, 8, 12, 16–22 even, 26–34 even, 38–48 even, 52–67

HOMEWORK CHECK

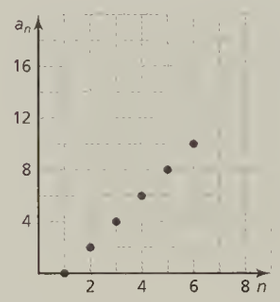
Basic: 7, 15, 23, 29, 39

Average: 8, 14, 26, 30, 40

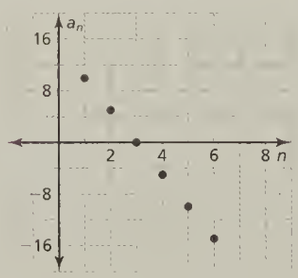
Advanced: 12, 16, 26, 30, 40

ANSWERS

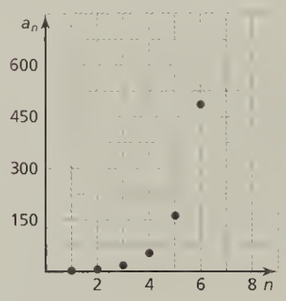
1. recursive equation
2. $a_n = 6n - 2$; It is the only explicit rule.
3. geometric
4. arithmetic
5. arithmetic
6. geometric
7. 0, 2, 4, 6, 8, 10



8. 10, 5, 0, −5, −10, −15



9. 2, 6, 18, 54, 162, 486



6.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A recursive rule gives the beginning term(s) of a sequence and a(n) _____ that tells how a_n is related to one or more preceding terms.

2. **WHICH ONE DOESN'T BELONG?** Which rule does *not* belong with the other three? Explain your reasoning.

$a_1 = -1, a_n = 5a_{n-1}$ $\quad a_n = 6n - 2$ $\quad a_1 = -3, a_n = a_{n-1} + 1$ $\quad a_1 = 9, a_n = 4a_{n-1}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, determine whether the recursive rule represents an *arithmetic sequence* or a *geometric sequence*.

3. $a_1 = 2, a_n = 7a_{n-1}$
4. $a_1 = 18, a_n = a_{n-1} + 1$
5. $a_1 = 5, a_n = a_{n-1} - 4$
6. $a_1 = 3, a_n = -6a_{n-1}$

In Exercises 7–12, write the first six terms of the sequence. Then graph the sequence. *(See Example 1.)*

7. $a_1 = 0, a_n = a_{n-1} + 2$
8. $a_1 = 10, a_n = a_{n-1} - 5$
9. $a_1 = 2, a_n = 3a_{n-1}$
10. $a_1 = 8, a_n = 1.5a_{n-1}$
11. $a_1 = 80, a_n = -\frac{1}{2}a_{n-1}$
12. $a_1 = -7, a_n = -4a_{n-1}$

In Exercises 13–20, write a recursive rule for the sequence. *(See Example 2.)*

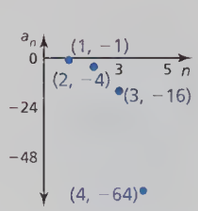
13.

n	1	2	3	4
a_n	7	16	25	34

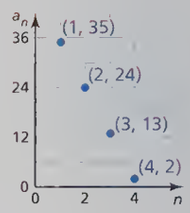
14.

n	1	2	3	4
a_n	8	24	72	216

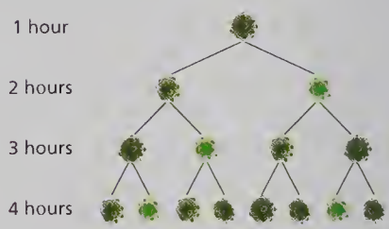
15. 243, 81, 27, 9, 3, . . .
16. 3, 11, 19, 27, 35, . . .
17. 0, −3, −6, −9, −12, . . .
18. 5, −20, 80, −320, 1280, . . .
19.



20.



21. **MODELING WITH MATHEMATICS** Write a recursive rule for the number of bacterial cells over time.

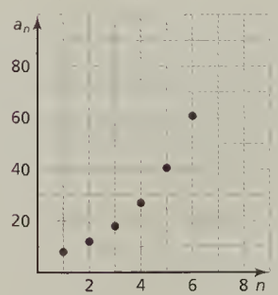


22. **MODELING WITH MATHEMATICS** Write a recursive rule for the length of the deer antler over time.

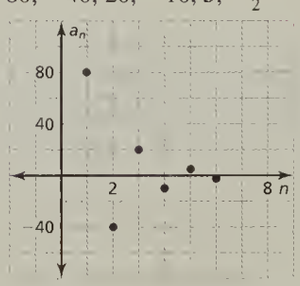


1 day: $4\frac{1}{2}$ in. 2 days: $4\frac{3}{4}$ in. 3 days: 5 in. 4 days: $5\frac{1}{4}$ in.

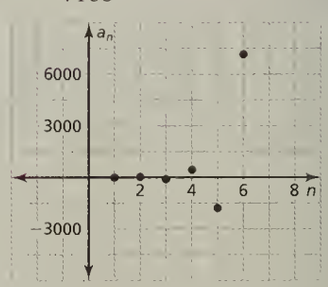
10. 8, 12, 18, 27, 40.5, 60.75



11. 80, −40, 20, −10, 5, $-\frac{5}{2}$



12. −7, 28, −112, 448, −1792, 7168



13–22. See Additional Answers.

In Exercises 23–28, write an explicit rule for the recursive rule. *(See Example 3.)*

23. $a_1 = -3, a_n = a_{n-1} + 3$

24. $a_1 = 8, a_n = a_{n-1} - 12$

25. $a_1 = 16, a_n = 0.5a_{n-1}$

26. $a_1 = -2, a_n = 9a_{n-1}$

27. $a_1 = 4, a_n = a_{n-1} + 17$

28. $a_1 = 5, a_n = -5a_{n-1}$

In Exercises 29–34, write a recursive rule for the explicit rule. *(See Example 4.)*

29. $a_n = 7(3)^{n-1}$

30. $a_n = -4n + 2$

31. $a_n = 1.5n + 3$

32. $a_n = 6n - 20$

33. $a_n = (-5)^{n-1}$

34. $a_n = -81\left(\frac{2}{3}\right)^{n-1}$

In Exercises 35–38, graph the first four terms of the sequence with the given description. Write a recursive rule and an explicit rule for the sequence.

35. The first term of a sequence is 5. Each term of the sequence is 15 more than the preceding term.

36. The first term of a sequence is 16. Each term of the sequence is half the preceding term.

37. The first term of a sequence is -1. Each term of the sequence is -3 times the preceding term.

38. The first term of a sequence is 19. Each term of the sequence is 13 less than the preceding term.

In Exercises 39–44, write a recursive rule for the sequence. Then write the next two terms of the sequence. *(See Example 5.)*

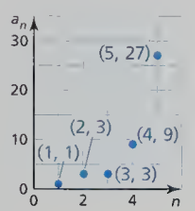
39. 1, 3, 4, 7, 11, . . .

40. 10, 9, 1, 8, -7, 15, . . .

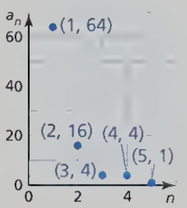
41. 2, 4, 2, -2, -4, -2, . . .

42. 6, 1, 7, 8, 15, 23, . . .

43.



44.



45. ERROR ANALYSIS Describe and correct the error in writing an explicit rule for the recursive rule $a_1 = 6$, $a_n = a_{n-1} - 12$.

✗

$a_n = a_1 + (n - 1)d$
$a_n = 6 + (n - 1)(12)$
$a_n = 6 + 12n - 12$
$a_n = -6 + 12n$

46. ERROR ANALYSIS Describe and correct the error in writing a recursive rule for the sequence 2, 4, 6, 10, 16, ...

✗

2, 4, 6, . . .
+2 +2

The sequence is arithmetic, with first term $a_1 = 2$ and common difference $d = 2$.

$a_n = a_{n-1} + d$
$a_1 = 2, a_n = a_{n-1} + 2$

In Exercises 47–51, the function f represents a sequence. Find the 2nd, 5th, and 10th terms of the sequence.

47. $f(1) = 3, f(n) = f(n - 1) + 7$

48. $f(1) = -1, f(n) = 6f(n - 1)$

49. $f(1) = 8, f(n) = -f(n - 1)$

50. $f(1) = 4, f(2) = 5, f(n) = f(n - 2) + f(n - 1)$

51. $f(1) = 10, f(2) = 15, f(n) = f(n - 1) - f(n - 2)$

52. MODELING WITH MATHEMATICS The X-ray shows the lengths (in centimeters) of bones in a human hand.



a. Write a recursive rule for the lengths of the bones.

b. Measure the lengths of different sections of your hand. Can the lengths be represented by a recursively defined sequence? Explain.

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ANSWERS

23. $a_n = 3n - 6$

24. $a_n = -12n + 20$

25. $a_n = 16(0.5)^{n-1}$

26. $a_n = -2(9)^{n-1}$

27. $a_n = 17n - 13$

28. $a_n = 5(-5)^{n-1}$

29. $a_1 = 7, a_n = 3a_{n-1}$

30. $a_1 = -2, a_n = a_{n-1} - 4$

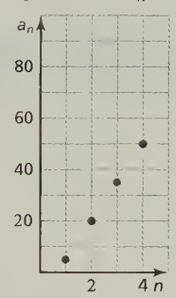
31. $a_1 = 4.5, a_n = a_{n-1} + 1.5$

32. $a_1 = -14, a_n = a_{n-1} + 6$

33. $a_1 = 1, a_n = -5a_{n-1}$

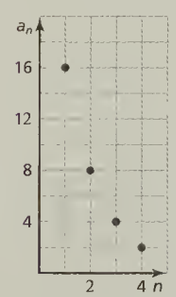
34. $a_1 = -81, a_n = \frac{2}{3}a_{n-1}$

35.



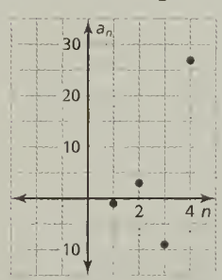
$a_1 = 5, a_n = a_{n-1} + 15$; $a_n = 15n - 10$

36.



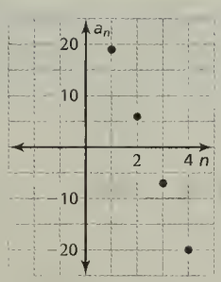
$a_1 = 16, a_n = \frac{1}{2}a_{n-1}$; $a_n = 16\left(\frac{1}{2}\right)^{n-1}$

37.



$a_1 = -1, a_n = -3a_{n-1}$; $a_n = -(-3)^{n-1}$

38.



$a_1 = 19, a_n = a_{n-1} - 13$; $a_n = -13n + 32$

39. $a_1 = 1, a_2 = 3, a_n = a_{n-2} + a_{n-1}$; 18, 29

40. $a_1 = 10, a_2 = 9, a_n = a_{n-2} - a_{n-1}$; -22, 37

41. $a_1 = 2, a_2 = 4, a_n = a_{n-1} - a_{n-2}$; 2, 4

42. $a_1 = 6, a_2 = 1, a_n = a_{n-2} + a_{n-1}$; 38, 61

43. $a_1 = 1, a_2 = 3, a_n = (a_n - 1)(a_{n-2})$; 243, 6561

44. $a_1 = 64, a_2 = 16, a_n = \frac{a_{n-2}}{a_{n-1}}$; 4, $\frac{1}{4}$

45. The common difference is -12, not 12; $a_n = 6 + (n - 1)(-12)$; $a_n = 6 - 12n + 12$; $a_n = 18 - 12n$

46. The sequence is not arithmetic; Each term in the sequence is the sum of the preceding two terms; $a_1 = 2, a_2 = 4$, $a_n = a_{n-2} + a_{n-1}$

47–52. See Additional Answers.

ANSWERS

53. **a.** 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

b. 3; 12; 48; 192; 768; 3072; 12,288; 49,152; 196,608; 786,432

c. 4, 7, 3, −4, −7, −3, 4, 7, 3, −4

54. **a.** 1, 1, 2, 3, 5, 8, or $a_{mn} = a_{(m-1)(n-1)} + a_{m(n-1)}$

b. Fibonacci sequence; 13

c. Check students' work. The diagram from the book should be drawn, along with an extended 13 by 13 grid at the bottom. The curve should continue into this grid and end in the bottom right corner. The grid should be labeled with a "7."

55. 5, 19, 61, 187, 565; neither; There is no common difference or common ratio.

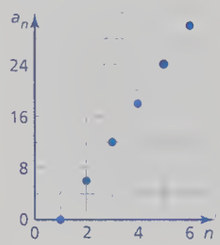
56–67. See Additional Answers.

Mini-Assessment

1. Write the first six terms of the sequence. Then graph the sequence.

 $a_1 = 0, a_n = a_{n-1} + 6$

 0, 6, 12, 18, 24, 30



2. Write a recursive rule for the sequence 0.5, 4, 32, 256, 2048, ...

 $a_1 = 0.5, a_n = 8a_{n-1}$

3. Write an explicit rule for the recursive rule.

 $a_1 = -3, a_n = a_{n-1} + 5$

 $a_n = 5n - 8$

4. Write a recursive rule for the explicit rule.

 $a_n = -1.5(2)^{n-1}$

 $a_1 = -1.5, a_n = 2a_{n-1}$

5. Use the sequence shown.

 7, 9, 16, 25, 41, 66, ...

 a. Write a recursive rule for the sequence.

 $a_1 = 7, a_2 = 9,$

 $a_n = a_{n-2} + a_{n-1}$

 b. Write the next three terms of the sequence. 107, 173, 280

53. **USING TOOLS** You can use a spreadsheet to generate the terms of a sequence.

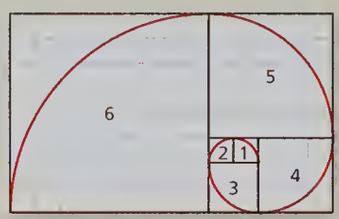
A2	=A1+2		
	A	B	C
1	3		
2	5		
3			
4			

a. To generate the terms of the sequence $a_1 = 3$, $a_n = a_{n-1} + 2$, enter the value of a_1, 3, into cell A1. Then enter "=A1+2" into cell A2, as shown. Use the *fill down* feature to generate the first 10 terms of the sequence.

b. Use a spreadsheet to generate the first 10 terms of the sequence $a_1 = 3$, $a_n = 4a_{n-1}$. (*Hint:* Enter "=4*A1" into cell A2.)

c. Use a spreadsheet to generate the first 10 terms of the sequence $a_1 = 4$, $a_2 = 7$, $a_n = a_{n-1} - a_{n-2}$. (*Hint:* Enter "=A2-A1" into cell A3.)

54. **HOW DO YOU SEE IT?** Consider Squares 1–6 in the diagram.



a. Write a sequence in which each term a_n is the side length of square n.

b. What is the name of this sequence? What is the next term of this sequence?

c. Use the term in part (b) to add another square to the diagram and extend the spiral.

55. **REASONING** Write the first 5 terms of the sequence $a_1 = 5$, $a_n = 3a_{n-1} + 4$. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

56. **THOUGHT PROVOKING** Describe the pattern for the numbers in Pascal's Triangle, shown below. Write a recursive rule that gives the mth number in the nth row.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
```

57. **REASONING** The explicit rule $a_n = a_1 + (n-1)d$ defines an arithmetic sequence.

a. Explain why $a_{n-1} = a_1 + [(n-1) - 1]d$.

b. Justify each step in showing that a recursive equation for the sequence is $a_n = a_{n-1} + d$.

$a_n = a_1 + (n-1)d$

$= a_1 + [(n-1) + 0]d$

$= a_1 + [(n-1) - 1 + 1]d$

$= a_1 + [((n-1) - 1) + 1]d$

$= a_1 + [(n-1) - 1]d + d$

$= a_{n-1} + d$

58. **MAKING AN ARGUMENT** Your friend claims that the sequence

$-5, 5, -5, 5, -5, \ldots$

cannot be represented by a recursive rule. Is your friend correct? Explain.

59. **PROBLEM SOLVING** Write a recursive rule for the sequence.

3, 7, 15, 31, 63, ...

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

60. $5x + 12x$ **61.** $9 - 6y - 14$ **62.** $2d - 7 - 8d$ **63.** $3 - 3m + 11m$

Write a linear function f with the given values. *(Section 4.2)*

64. $f(2) = 6, f(-1) = -3$ **65.** $f(-2) = 0, f(6) = -4$

66. $f(-3) = 5, f(-1) = 5$ **67.** $f(3) = -1, f(-4) = -15$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

6.5–6.6 What Did You Learn?

Core Vocabulary

geometric sequence, *p. 312*
common ratio, *p. 312*
explicit rule, *p. 320*
recursive rule, *p. 320*

Core Concepts

Section 6.5

Geometric Sequence, *p. 312*
Equation for a Geometric Sequence, *p. 314*

Section 6.6

Recursive Equation for an Arithmetic Sequence, *p. 320*
Recursive Equation for a Geometric Sequence, *p. 320*

Mathematical Thinking

1. Explain how writing a function in Exercise 39 part (a) on page 317 created a shortcut for answering part (b).
2. How did you choose an appropriate tool in Exercise 52 part (b) on page 325?

Performance Task

The New Car

There is so much more to buying a new car than the purchase price. Interest rates, depreciation, and inflation are all factors. So, what is the real cost of your new car?

To explore the answer to this question and more, go to ***BigIdeasMath.com***.



Dynamic Teaching Tools

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- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

1. *Sample answer:* Use the function from part (a) to calculate the number of squares for the tenth figure in part (b) without determining the number of squares in the fourth through ninth figures.
2. *Sample answer:* Parts of a hand can be conveniently measured in centimeters, so a 30-centimeter ruler is an appropriate tool to use.

ANSWERS

1. $\frac{1}{y^2}$
2. $\frac{1}{x^3}$
3. y^6
4. $\frac{25y^8}{4x^4}$
5. 2
6. -3
7. 125
8. not a real number

6 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

6.1 Properties of Exponents *(pp. 277–284)*

Simplify $\left(\frac{x}{4}\right)^{-4}$. Write your answer using only positive exponents.

$$\left(\frac{x}{4}\right)^{-4} = \frac{x^{-4}}{4^{-4}} \quad \text{Power of a Quotient Property}$$
$$= \frac{4^4}{x^4} \quad \text{Definition of negative exponent}$$
$$= \frac{256}{x^4} \quad \text{Simplify.}$$

Simplify the expression. Write your answer using only positive exponents.

1. $y^3 \cdot y^{-5}$
2. $\frac{x^4}{x^7}$
3. $(x^0y^2)^3$
4. $\left(\frac{2x^2}{5y^4}\right)^{-2}$

6.2 Radicals and Rational Exponents *(pp. 285–290)*

Evaluate $512^{1/3}$.

$$512^{1/3} = \sqrt[3]{512} \quad \text{Rewrite the expression in radical form.}$$
$$= \sqrt[3]{8 \cdot 8 \cdot 8} \quad \text{Rewrite the expression showing factors.}$$
$$= 8 \quad \text{Evaluate the cube root.}$$

Evaluate the expression.

5. $\sqrt[3]{8}$
6. $\sqrt[5]{-243}$
7. $625^{3/4}$
8. $(-25)^{1/2}$

6.3 Exponential Functions *(pp. 291–298)*

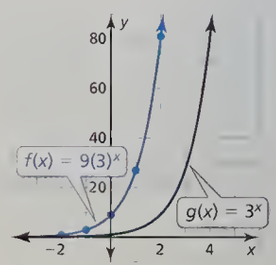
Graph $f(x) = 9(3)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.

Step 1 Make a table of values.

x	-2	-1	0	1	2
$f(x)$	1	3	9	27	81

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



▶ The parent function is $g(x) = 3^x$. The graph of f is a vertical stretch by a factor of 9 of the graph of g. The y-intercept of the graph of f, 9, is above the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g. From the graph of f, you can see that the domain is all real numbers and the range is $y > 0$.

Graph the function. Compare the graph to the graph of the parent function. Identify the *y*-intercepts and asymptotes of the graphs. Describe the domain and range of *f*.

9. $f(x) = -4\left(\frac{1}{4}\right)^x$

10. $f(x) = 2(3)^x$

11. Graph $f(x) = 2^{x-4} - 3$. Identify the asymptote. Describe the domain and range.

12. Use the table shown. (a) Use a graphing calculator to find an exponential function that fits the data. Then plot the data and graph the function in the same viewing window. (b) Identify and interpret the correlation coefficient.

x	0	1	2	3	4
y	200	96	57	25	10

6.4 Exponential Growth and Decay *(pp. 299–308)*

The function $f(x) = 75(0.5)^{x/10}$ represents the amount (in milligrams) of a radioactive substance remaining after x days. Rewrite the function in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*. Identify the initial amount and interpret the growth factor or decay factor.

$f(x) = 75(0.5)^{x/10}$ Write the function.

$= 75(0.5^{1/10})^x$ Power of a Power Property

$\approx 75(0.93)^x$ Evaluate the power.

The function is of the form $y = a(1 - r)^t$, where $1 - r < 1$, so it represents exponential decay. The initial amount is 75 milligrams, and the decay factor of 0.93 means that about 93% of the radioactive substance remains after each day.

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

13.

x	0	1	2	3
y	3	6	12	24

14.

x	1	2	3	4
y	162	108	72	48

15. The function $y = 1.75(1.02)^t$ represents the value y (in dollars) of a share of stock after t days. Determine whether the function represents *exponential growth* or *exponential decay*. Identify the initial amount and interpret the growth factor or decay factor.

Rewrite the function in the form $f(x) = ab^x$ to determine whether it represents *exponential growth* or *exponential decay*.

16. $f(x) = 4(1.25)^{x+3}$

17. $f(x) = (1.06)^{8x}$

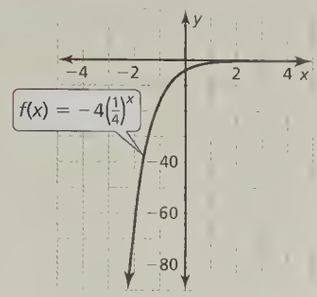
18. $f(x) = 6(0.84)^{x-4}$

19. You deposit \$750 in a savings account that earns 5% annual interest compounded quarterly. (a) Write a function that represents the balance after t years. (b) What is the balance of the account after 4 years?

20. The value of a TV is \$1500. Its value decreases by 14% each year. (a) Write a function that represents the value y (in dollars) of the TV after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Identify and interpret any asymptotes of the graph. (d) Estimate the value of the TV after 3 years.

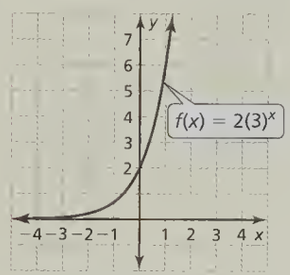
ANSWERS

9.



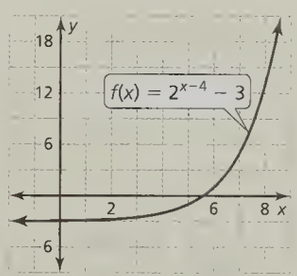
The graph of f is a vertical stretch by a factor of 3 and a reflection in the x-axis of the graph of $g(x) = \left(\frac{1}{4}\right)^x$. The y-intercept of the graph of f, -4, is below the y-intercept of the graph of g, 1. The x-axis is the asymptote of both the graphs of f and g; domain: all real numbers, range: $y < 0$

10.



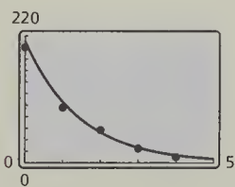
The graph of f is a vertical stretch by a factor of 2 of the graph of $g(x) = 3^x$. The y-intercept of the graph of f, 2, is above the y-intercept of the graph of g, 1. The x-axis is the asymptote of both the graphs of f and g; domain: all real numbers, range: $y > 0$

11.



asymptote: $y = -3$; domain: all real numbers, range: $y > -3$

12. a. $y = 211(0.5)^x$



b. $r = -0.996$; The relationship between x and y has a strong negative correlation and the equation closely models the data.

13. exponential growth; As x increases by 1, y is multiplied by 2.

14. exponential decay; As x increases by 1, y is multiplied by $\frac{2}{3}$.

15. exponential growth; \$1.75; The value of the stock increases by 2% each day.

16. $f(x) = 7.81(1.25)^x$, exponential growth; 25%

17. $f(x) = 1.59^x$, exponential growth; 59%

18. $f(x) = 12.05(0.84)^x$, exponential decay; 16%

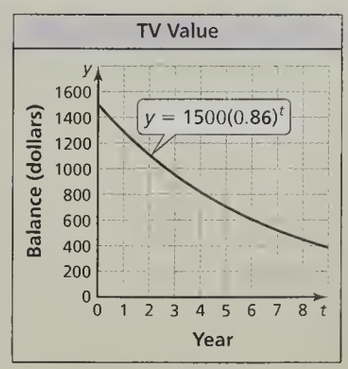
19. a. $y = 750(1.0125)^{4t}$

b. \$914.92

20. a. $y = 1500(0.86)^t$

b. about 1.2%

c.

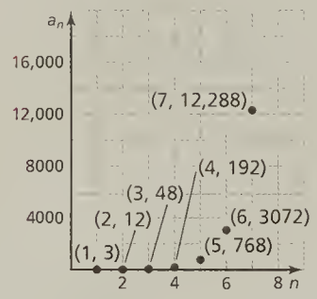


asymptote: $y = 0$; The value of the TV approaches but never equals \$0.

d. about \$950

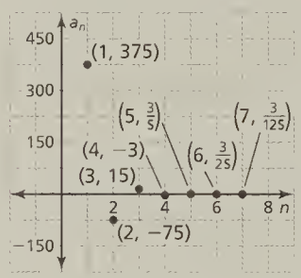
ANSWERS

21. geometric; There is a common ratio of 4; 768, 3072, 12,288



22. neither; There is neither a common ratio nor a common difference.

23. geometric; There is a common ratio of $-\frac{1}{5}, \frac{3}{5}, -\frac{3}{25}, \frac{3}{125}$

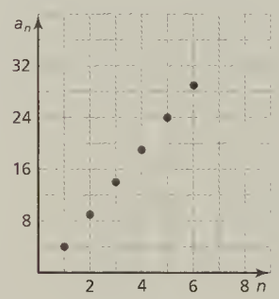


24. $a_n = 4^{n-1}$; 65,536

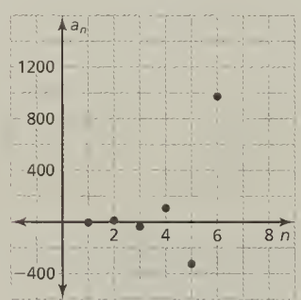
25. $a_n = 5(-2)^{n-1}$; 1280

26. $a_n = 486\left(\frac{1}{3}\right)^{n-1}$; $\frac{2}{27}$

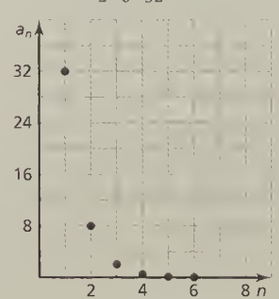
27. 4, 9, 14, 19, 24, 29



28. −4, 12, −36, 108, −324, 972



29. 32, 8, 2, $\frac{1}{2}, \frac{1}{8}, \frac{1}{32}$



6.5 Geometric Sequences *(pp. 311–318)*

Write the next three terms of the geometric sequence 2, 6, 18, 54, ...

Use a table to organize the terms and extend the sequence.

Position	1	2	3	4	5	6	7
Term	2	6	18	54	162	486	1458
		×3	×3	×3	×3	×3	×3

Each term is 3 times the previous term. So, the common ratio is 3.

Multiply a term by 3 to find the next term.

▶ The next three terms are 162, 486, and 1458.

Decide whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning. If the sequence is geometric, write the next three terms and graph the sequence.

21. 3, 12, 48, 192, . . . **22.** 9, −18, 27, −36, . . . **23.** 375, −75, 15, −3, . . .

Write an equation for the *n*th term of the geometric sequence. Then find a_9.

24. 1, 4, 16, 64, . . . **25.** 5, −10, 20, −40, . . . **26.** 486, 162, 54, 18, . . .

6.6 Recursively Defined Sequences *(pp. 319–326)*

Write a recursive rule for the sequence 5, 12, 19, 26, 33, ...

Use a table to organize the terms and find the pattern.

Position, n	1	2	3	4	5
Term, a_n	5	12	19	26	33
		+7	+7	+7	+7

The sequence is arithmetic, with first term $a_1 = 5$ and common difference $d = 7$.

$a_n = a_{n-1} + d$ Recursive equation for an arithmetic sequence

$a_n = a_{n-1} + 7$ Substitute 7 for d.

▶ So, a recursive rule for the sequence is $a_1 = 5$, $a_n = a_{n-1} + 7$.

Write the first six terms of the sequence. Then graph the sequence.

27. $a_1 = 4, a_n = a_{n-1} + 5$ **28.** $a_1 = -4, a_n = -3a_{n-1}$ **29.** $a_1 = 32, a_n = \frac{1}{4}a_{n-1}$

Write a recursive rule for the sequence.

30. 3, 8, 13, 18, 23, . . . **31.** 3, 6, 12, 24, 48, . . . **32.** 7, 6, 13, 19, 32, . . .

33. The first term of a sequence is 8. Each term of the sequence is 5 times the preceding term. Graph the first four terms of the sequence. Write a recursive rule and an explicit rule for the sequence.

30. $a_1 = 3, a_n = a_{n-1} + 5$

31. $a_1 = 3, a_n = 2a_{n-1}$

32. $a_1 = 7, a_2 = 6, a_n = a_{n-2} + a_{n-1}$

33.



$a_1 = 8, a_n = 5a_{n-1}; a_n = 8(5)^{n-1}$

6 Chapter Test

Evaluate the expression.

1. $-\sqrt[4]{16}$
2. $729^{1/6}$
3. $(-32)^{7/5}$

Simplify the expression. Write your answer using only positive exponents.

4. $z^{-1/4} \cdot z^{3/5}$
5. $\dfrac{b^{-5}}{a^0 b^{-8}}$
6. $\left(\dfrac{2c^4}{5}\right)^{-3}$

Write and graph a function that represents the situation.

7. Your starting annual salary of \$42,500 increases by 3% each year.
8. You deposit \$500 in an account that earns 6.5% annual interest compounded yearly.

Write an explicit rule and a recursive rule for the sequence.

9.

n	1	2	3	4
a_n	−6	8	22	36

10.

n	1	2	3	4
a_n	400	100	25	6.25

11. Graph $f(x) = 2(6)^x$. Compare the graph to the graph of $g(x) = 6^x$. Identify the y-intercepts and asymptotes of the graphs. Describe the domain and range of f.

12. The function $y = ab^t$ represents the population y (in millions of people) of a country after t years. Choose values of a and b so that the population is initially 13 million people, but is below 10 million people after 3 years.

Use the equation to complete the statement "a ☐ b" with the symbol <, >, or =. Do not attempt to solve the equation.

13. $\dfrac{5^a}{5^b} = 5^{-3}$
14. $9^a \cdot 9^{-b} = 1$

15. The first two terms of a sequence are $a_1 = 3$ and $a_2 = -12$. Let a_3 be the third term when the sequence is arithmetic and let b_3 be the third term when the sequence is geometric. Find $a_3 - b_3$.

16. At sea level, Earth's atmosphere exerts a pressure of 1 atmosphere. Atmospheric pressure P (in atmospheres) decreases with altitude. It can be modeled by $P = (0.99988)^a$, where a is the altitude (in meters).
 a. Identify the initial amount, decay factor, and decay rate.
 b. Use a graphing calculator to graph the function. Use the graph to estimate the atmospheric pressure at an altitude of 5000 feet.

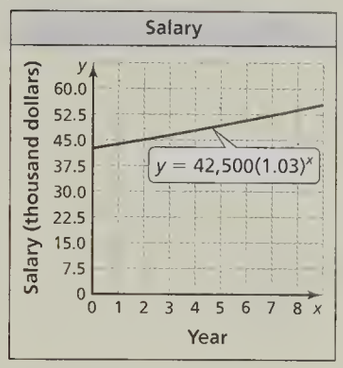
17. You follow the training schedule from your coach.
 a. Write an explicit rule and a recursive rule for the geometric sequence.
 b. On what day do you run approximately 3 kilometers?

> **Training On Your Own**
> **Day 1:** Run 1 km.
> **Each day after Day 1:** Run 20% farther than the previous day.

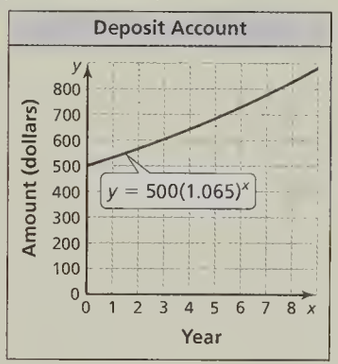
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Skills Review Handbook	Performance Task
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ANSWERS

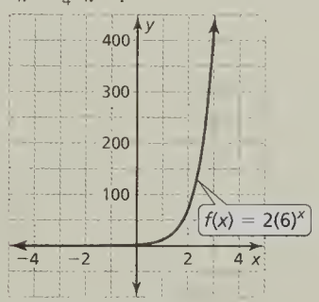
1. -2
2. 3
3. -128
4. $z^{7/20}$
5. b^3
6. $\dfrac{125}{8c^{12}}$
7. $y = 42{,}500(1.03)^x$



8. $y = 500(1.065)^x$



9. $a_n = 14n - 20$; $a_1 = -6$, $a_n = a_{n-1} + 14$
10. $a_n = 400\left(\frac{1}{4}\right)^{n-1}$; $a_1 = 400$, $a_n = \frac{1}{4}a_{n-1}$
11.



The graph of f is a vertical stretch by a factor of 2 of the graph of $g(x) = 6^x$. The y-intercept of the graph of f, 2, is above the y-intercept of the graph of g, 1. The x-axis is the asymptote of both the graphs f and g; domain: all real numbers, range: $y > 0$

12. *Sample answer:* $a = 13$, $b = 0.9$
13. <
14. =

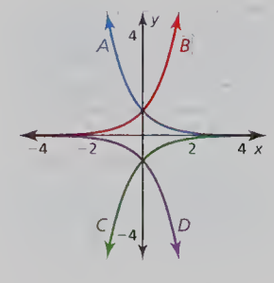
15–17. See Additional Answers.

ANSWERS

1. D
2. 3
3. F
4. B
5. G

6 Standards Assessment

1. Which graph represents the function $f(x) = -3^x$? *(TEKS A.9.D)*



Ⓐ A Ⓑ B

Ⓒ C Ⓓ D

2. **GRIDDED ANSWER** What number should you use to complete the expression so that the statement is true? *(TEKS A.11.B)*

$$\frac{x^{5/3} \cdot x^{-1} \cdot \sqrt[3]{x}}{x^{-2} \cdot x^{0}} = x^{\square}$$

3. Which symbol should you use to complete Inequality 2 so that the system of linear inequalities has no solution? *(TEKS A.3.H)*

Inequality 1 $y - 2x \leq 4$

Inequality 2 $6x - 3y \square -12$

Ⓕ $<$ Ⓖ $>$

Ⓗ $\leq$ Ⓙ $\geq$

4. The second term of a sequence is 7. Each term of the sequence is 10 more than the preceding term. Which equation gives the nth term of the sequence? *(TEKS A.12.D)*

Ⓐ $a_n = -3n + 13$ Ⓑ $a_n = 10n - 13$

Ⓒ $a_n = 10n - 3$ Ⓓ $a_n = 10n + 7$

5. Which statement is true about the function $f(x) = -2(9)^x$? *(TEKS A.9.A)*

Ⓕ The domain is $x < 0$. The range is $y < 0$.

Ⓖ The domain is all real numbers. The range is $y < 0$.

Ⓗ The domain is all real numbers. The range is $y > 0$.

Ⓙ The domain is all real numbers. The range is all real numbers.

ANSWERS

6. C
7. J
8. A
9. F

6. For 10 years, the population of a city increases in a pattern that is approximately exponential. Using exponential regression, you fit the function $p(t) = 15(1.064)^t$ to the population data, where p is the population (in thousands) of the city in year t. What is a reasonable prediction for the future population of the city? *(TEKS A.9.E)*

 (A) The population will be under 7000 in Year 12.

 (B) The population will be over 40,000 in Year 15.

 (C) The population will be under 50,000 in Year 18.

 (D) The population will be over 300,000 by Year 20.

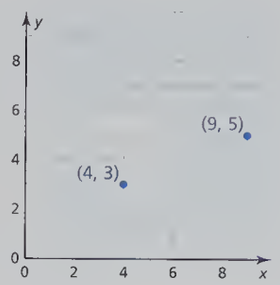
7. Which equation represents the line that passes through the two points shown? *(TEKS A.2.B)*

 (F) $y = \frac{5}{2}x - 13$

 (G) $y = \frac{2}{5}x - \frac{23}{5}$

 (H) $-5x + 2y = -14$

 (J) $-2x + 5y = 7$



8. Which of the relations is *not* a function? *(TEKS A.12.A)*

 (A)

x	-2	0	0	2
y	1	-3	3	5

 (B)

x	0	2	4	6
y	-5	-5	-5	-5

 (C)

x	1	2	3	4
y	-1	6	13	21

 (D)

x	-1	0	1	2
y	7	6	7	10

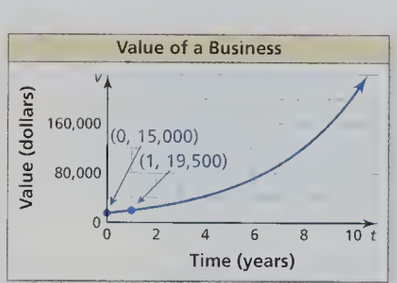
9. The graph shows the value of a business over time. Which equation models the value v (in dollars) of the business over time t (in years)? *(TEKS A.9.C)*

 (F) $v = 15{,}000(1.30)^t$

 (G) $v = 15{,}000(0.70)^t$

 (H) $v = 15{,}000(0.50)^t$

 (J) $v = 15{,}000(0.30)^t$



Chapter 7 Pacing Guide

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	2 Days
Section 5	2 Days
Quiz	1 Day
Section 6	2 Days
Section 7	2 Days
Section 8	2 Days
Section 9	2 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 7	22 Days
Year-to-Date	121 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
7.1	A.10.A
7.2	A.10.B, A.10.D
7.3	A.10.B
7.4	A.10.C
7.5	A.8.A
7.6	A.8.A, A.10.E
7.7	A.8.A, A.10.D, A.10.E
7.8	A.8.A, A.10.E, A.10.F
7.9	A.10.D, A.10.E

7 Polynomial Equations and Factoring

7.1 Adding and Subtracting Polynomials
7.2 Multiplying Polynomials
7.3 Special Products of Polynomials
7.4 Dividing Polynomials
7.5 Solving Polynomial Equations in Factored Form
7.6 Factoring $x^2 + bx + c$
7.7 Factoring $ax^2 + bx + c$
7.8 Factoring Special Products
7.9 Factoring Polynomials Completely



Height of a Falling Object *(p. 386)*

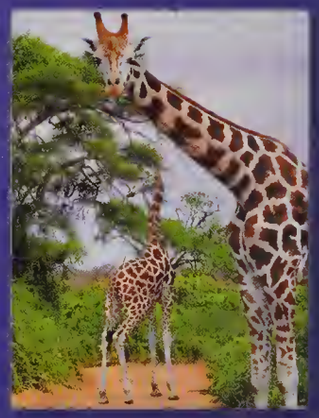

Game Reserve *(p. 380)*


Photo Cropping *(p. 376)*


Framing a Photo *(p. 350)*


Gateway Arch *(p. 368)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

For an overview of this chapter, formative assessment tips, and teaching strategies, see **Laurie's Notes** at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

Simplifying Algebraic Expressions (6.7.D)

Example 1 Simplify $6x + 5 - 3x - 4$.

$$6x + 5 - 3x - 4 = 6x - 3x + 5 - 4 \quad \text{Commutative Property of Addition}$$
$$= (6 - 3)x + 5 - 4 \quad \text{Distributive Property}$$
$$= 3x + 1 \quad \text{Simplify.}$$

Example 2 Simplify $-8(y - 3) + 2y$.

$$-8(y - 3) + 2y = -8(y) - (-8)(3) + 2y \quad \text{Distributive Property}$$
$$= -8y + 24 + 2y \quad \text{Multiply.}$$
$$= -8y + 2y + 24 \quad \text{Commutative Property of Addition}$$
$$= (-8 + 2)y + 24 \quad \text{Distributive Property}$$
$$= -6y + 24 \quad \text{Simplify.}$$

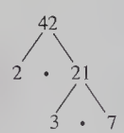
Simplify the expression.

1. $3x - 7 + 2x$
2. $4r + 6 - 9r - 1$
3. $-5t + 3 - t - 4 + 8t$
4. $3(s - 1) + 5$
5. $2m - 7(3 - m)$
6. $4(h + 6) - (h - 2)$

Writing Prime Factorizations (6.7.A)

Example 3 Write the prime factorization of 42.

Make a factor tree.



▶ The prime factorization of 42 is $2 \cdot 3 \cdot 7$.

Write the prime factorization of the number.

7. 36
8. 63
9. 54
10. 72
11. 28
12. 30

13. **ABSTRACT REASONING** Is it possible for two integers to have the same prime factorization? Explain your reasoning.

Dynamic Solutions available at *BigIdeasMath.com*

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For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make Concept Circles for the following topics.

- Using Properties to Simplify Algebraic Expressions
- Writing Prime Factorizations of Numbers

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

Texas Essential Knowledge and Skills

6.7.D The student is expected to generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties.

6.7.A The student is expected to generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization.

ANSWERS

1. $5x - 7$
2. $-5r + 5$
3. $2t - 1$
4. $3s + 2$
5. $9m - 21$
6. $3h + 26$
7. $2^2 \cdot 3^2$
8. $3^2 \cdot 7$
9. $2 \cdot 3^3$
10. $2^3 \cdot 3^2$
11. $2^2 \cdot 7$
12. $2 \cdot 3 \cdot 5$
13. no; The product of any factorization is a unique number.

MONITORING PROGRESS ANSWERS

1. $3x^2 - x + 1$
2. $-x^2 + x$
3. $-x^2 - 2x$
4. $x^2 + x - 1$
5. $x^2 + 2$
6. $x - 6$
7. $-x^2 + 2x$
8. 9
9. $2x^2$

Mathematical Thinking

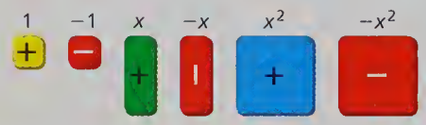
Mathematically proficient students select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems. **(A.1.C)**

Using Models

Core Concept

Using Algebra Tiles

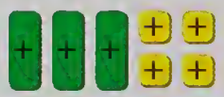
When solving a problem, it can be helpful to use a model. For instance, you can use algebra tiles to model algebraic expressions and operations with algebraic expressions.

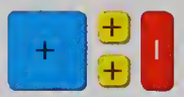


EXAMPLE 1 **Writing Expressions Modeled by Algebra Tiles**

Write the algebraic expression modeled by the algebra tiles.

a. 

b. 

c. 

SOLUTION

a. The algebraic expression is x^2.

b. The algebraic expression is $3x + 4$.

c. The algebraic expression is $x^2 - x + 2$.

Monitoring Progress

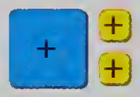
Write the algebraic expression modeled by the algebra tiles.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

7.1 Adding and Subtracting Polynomials

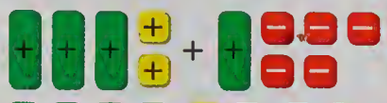
TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.10.A

Essential Question
How can you add and subtract polynomials?

EXPLORATION 1 Adding Polynomials

Work with a partner. Write the expression modeled by the algebra tiles in each step.

Step 1



$(3x + 2) + (x - 5)$

Step 2



Step 3



Step 4



EXPLORATION 2 Subtracting Polynomials

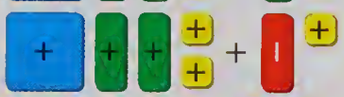
Work with a partner. Write the expression modeled by the algebra tiles in each step.

Step 1



$(x^2 + 2x + 2) - (x - 1)$

Step 2



Step 3



Step 4



Step 5



REASONING
To be proficient in math, you need to represent a given situation using symbols.

Communicate Your Answer

3. How can you add and subtract polynomials?
4. Use your methods in Question 3 to find each sum or difference.
 a. $(x^2 + 2x - 1) + (2x^2 - 2x + 1)$
 b. $(4x + 3) + (x - 2)$
 c. $(x^2 + 2) - (3x^2 + 2x + 5)$
 d. $(2x - 3x) - (x^2 - 2x + 4)$

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.10.A The student is expected to add and subtract polynomials of degree one and degree two.

ANSWERS

1. $4x + 2 - 5$; $4x + (2 - 2) - 3$; $4x - 3$
2. $(x^2 + 2x + 2) + (-x + 1)$; $x^2 + 2x - x + 3$; $x^2 + x + (x - x) + 3$; $x^2 + x + 3$
3. Add or subtract like terms.
4. a. $3x^2$
 b. $5x + 1$
 c. $-2x^2 - 2x - 3$
 d. $-x^2 + x - 4$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Extra Example 1

Find the degree of each monomial.

a. $2x$

The degree of the monomial is 1.

b. $\frac{3}{4}x^2y^2$

The degree of the monomial is 4.

c. $5x^2y$

The degree of the monomial is 3.

d. -15

The degree of the monomial is 0.

MONITORING PROGRESS ANSWERS

1. 4
2. 5
3. 1
4. 0

7.1 Lesson

Core Vocabulary

monomial, *p. 338*
degree of a monomial, *p. 338*
polynomial, *p. 339*
binomial, *p. 339*
trinomial, *p. 339*
degree of a polynomial, *p. 339*
standard form, *p. 339*
leading coefficient, *p. 339*
closed, *p. 340*

What You Will Learn

- Find the degrees of monomials.
- Classify polynomials.
- Add and subtract polynomials.
- Solve real-life problems.

Finding the Degrees of Monomials

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents.

The **degree of a monomial** is the sum of the exponents of the variables in the monomial. The degree of a nonzero constant term is 0. The constant 0 does not have a degree.

Monomial	Degree
10	0
$3x$	1
$\frac{1}{2}ab^2$	$1 + 2 = 3$
$-1.8m^5$	5

Not a monomial	Reason
$5 + x$	A sum is not a monomial.
$\frac{2}{n}$	A monomial cannot have a variable in the denominator.
4^a	A monomial cannot have a variable exponent.
x^{-1}	The variable must have a whole number exponent.

EXAMPLE 1 **Finding the Degrees of Monomials**

Find the degree of each monomial.

a. $5x^2$ **b.** $-\frac{1}{2}xy^3$ **c.** $8x^3y^3$ **d.** -3

SOLUTION

a. The exponent of x is 2.

▶ So, the degree of the monomial is 2.

b. The exponent of x is 1, and the exponent of y is 3.

▶ So, the degree of the monomial is $1 + 3$, or 4.

c. The exponent of x is 3, and the exponent of y is 3.

▶ So, the degree of the monomial is $3 + 3$, or 6.

d. You can rewrite -3 as $-3x^0$.

▶ So, the degree of the monomial is 0.

Monitoring Progress

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Find the degree of the monomial.

1. $-3x^4$ **2.** $7c^3d^2$ **3.** $\frac{5}{3}y$ **4.** -20.5

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the information about Finding the Degrees of Monomials and Example 1 orally. Have students work in pairs and collaborate with one another to complete Monitoring Progress. Remind students to refer to Example 1 as a guide for their work.

Beginning/Advanced Advanced students clarify information for beginners as they work through the Monitoring Progress questions together.

Intermediate/Advanced High Students clarify information for one another as needed.

ELPS 2.I.4 Demonstrate listening comprehension of increasingly complex spoken English by collaborating with peers commensurate with content and grade-level needs.

Classifying Polynomials

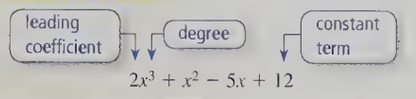
Core Concept

Polynomials

A **polynomial** is a monomial or a sum of monomials. Each monomial is called a *term* of the polynomial. A polynomial with two terms is a **binomial**. A polynomial with three terms is a **trinomial**.

Binomial	Trinomial
$5x + 2$	$x^2 + 5x + 2$

The **degree of a polynomial** is the greatest degree of its terms. A polynomial in one variable is in **standard form** when the exponents of the terms decrease from left to right. When you write a polynomial in standard form, the coefficient of the first term is the **leading coefficient**.

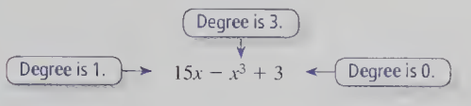


EXAMPLE 2 Writing a Polynomial in Standard Form

Write $15x - x^3 + 3$ in standard form. Identify the degree and leading coefficient of the polynomial.

SOLUTION

Consider the degree of each term of the polynomial.



▶ You can write the polynomial in standard form as $-x^3 + 15x + 3$. The greatest degree is 3, so the degree of the polynomial is 3, and the leading coefficient is -1.

EXAMPLE 3 Classifying Polynomials

Write each polynomial in standard form. Identify the degree and classify each polynomial by the number of terms.

a. $-3z^4$ **b.** $4 + 5x^2 - x$ **c.** $8q + q^5$

SOLUTION

Polynomial	Standard Form	Degree	Type of Polynomial
a. $-3z^4$	$-3z^4$	4	monomial
b. $4 + 5x^2 - x$	$5x^2 - x + 4$	2	trinomial
c. $8q + q^5$	$q^5 + 8q$	5	binomial

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

5. $4 - 9z$ **6.** $t^2 - t^3 - 10t$ **7.** $2.8x + x^3$

Extra Example 2

Write $-8 + 9x^2 - 2x$ in standard form. Identify the degree and leading coefficient of the polynomial.

standard form: $9x^2 - 2x - 8$; degree of polynomial: 2; leading coefficient: 9

Extra Example 3

Write each polynomial in standard form. Identify the degree and classify each polynomial by the number of terms.

a. $3 + 7a^2$ standard form: $7a^2 + 3$; degree: 2; type: binomial

b. $-b^2 + 2b^4 + 6b$ standard form: $2b^4 - b^2 + 6b$; degree: 4; type: trinomial

c. $12c^5$ standard form: $12c^5$; degree: 5; type: monomial

MONITORING PROGRESS ANSWERS

5. $-9z + 4$; 1; -9; binomial

6. $-t^3 + t^2 - 10t$; 3; -1; trinomial

7. $x^3 + 2.8x$; 3; 1; binomial

English Language Learners

Notebook Development

Have students create in their notebooks a polynomial page that includes all the vocabulary words on pages 338 and 339. For each word, have students include the definition and an example.

Extra Example 4

Find the sum.

a. $(3x + 4) + (2x + 1)$

$5x + 5$

b. $(-5a^2 + a + 2) + (2a^2 - a - 9)$

$-3a^2 - 7$

Extra Example 5

Find the difference.

a. $(3y + 8) - (6y - 2)$

$-3y + 10$

b. $(12t^2 + 5t - 7) - (-4t^2 + 3t - 1)$

$16t^2 + 2t - 6$

Adding and Subtracting Polynomials

A set of numbers is **closed** under an operation when the operation performed on any two numbers in the set results in a number that is also in the set. For example, the set of integers is closed under addition, subtraction, and multiplication. This means that if a and b are two integers, then $a + b$, $a - b$, and ab are also integers.

The set of polynomials is closed under addition and subtraction. So, the sum or difference of any two polynomials is also a polynomial.

To add polynomials, add like terms. You can use a vertical or a horizontal format.

EXAMPLE 4 Adding Polynomials

Find the sum.

a. $(6x + 15) + (8x - 3)$ **b.** $(3x^2 + x - 6) + (x^2 + 4x + 10)$

SOLUTION

a. Vertical format: Align like terms vertically and add.

$$\begin{array}{r} 6x + 15 \\ +\ 8x - 3 \\ \hline 14x + 12 \end{array}$$

▶ The sum is $14x + 12$.

b. Horizontal format: Group like terms and simplify.

$$\begin{aligned}(3x^2 + x - 6) + (x^2 + 4x + 10) &= (3x^2 + x^2) + (x + 4x) + (-6 + 10) \\ &= 4x^2 + 5x + 4\end{aligned}$$

▶ The sum is $4x^2 + 5x + 4$.

To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by -1.

EXAMPLE 5 Subtracting Polynomials

Find the difference.

a. $(4n + 5) - (-2n + 4)$ **b.** $(4x^2 - 3x + 5) - (3x^2 - x - 8)$

COMMON ERROR

Remember to multiply *each* term of the polynomial by -1 when you write the subtraction as addition.

SOLUTION

a. Vertical format: Align like terms vertically and subtract.

$$\begin{array}{r} 4n + 5 \\ -\ (-2n + 4) \end{array} \quad \Rightarrow \quad \begin{array}{r} 4n + 5 \\ +\ 2n - 4 \\ \hline 6n + 1 \end{array}$$

▶ The difference is $6n + 1$.

b. Horizontal format: Group like terms and simplify.

$$\begin{aligned}(4x^2 - 3x + 5) - (3x^2 - x - 8) &= 4x^2 - 3x + 5 - 3x^2 + x + 8 \\ &= (4x^2 - 3x^2) + (-3x + x) + (5 + 8) \\ &= x^2 - 2x + 13\end{aligned}$$

▶ The difference is $x^2 - 2x + 13$.

Monitoring Progress

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Find the sum or difference.

8. $(b - 10) + (4b - 3)$

9. $(x^2 - x - 2) + (7x^2 - x + 8)$

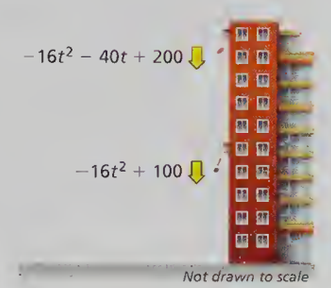
10. $(k + 5) - (3k - 6)$

11. $(p^2 + p + 3) - (-4p^2 - p + 3)$

Solving Real-Life Problems

EXAMPLE 6 **Solving a Real-Life Problem**

A penny is thrown straight down from a height of 200 feet. At the same time, a paintbrush is dropped from a height of 100 feet. The polynomials represent the heights (in feet) of the objects after t seconds.



a. Write a polynomial that represents the distance between the penny and the paintbrush after t seconds.

b. Interpret the coefficients of the polynomial in part (a).

STUDY TIP

When a power of the variable appears in one polynomial but not the other, leave a space in that column, or write the term with a coefficient of 0.

SOLUTION

a. To find the distance between the objects after t seconds, subtract the polynomials.

Penny	$-16t^2 - 40t + 200$	$-16t^2 - 40t + 200$
Paintbrush	$-(-16t^2 + 100)$	$+ 16t^2 - 100$
		$-40t + 100$

▶ The polynomial $-40t + 100$ represents the distance between the objects after t seconds.

b. When $t = 0$, the distance between the objects is $-40(0) + 100 = 100$ feet. So, the constant term 100 represents the distance between the penny and the paintbrush when both objects begin to fall.

As the value of t increases by 1, the value of $-40t + 100$ decreases by 40. This means that the objects become 40 feet closer to each other each second. So, -40 represents the amount that the distance between the objects changes each second.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

12. WHAT IF? The polynomial $-16t^2 - 25t + 200$ represents the height of the penny after t seconds.

a. Write a polynomial that represents the distance between the penny and the paintbrush after t seconds.

b. Interpret the coefficients of the polynomial in part (a).

Extra Example 6

Use the information in Example 6. Suppose the polynomial $-16t^2 + 75$ represents the height (in feet) of the paintbrush after t seconds.

a. Write a polynomial that represents the distance between the penny and the paintbrush after t seconds.
$-40t + 125$

b. Interpret the coefficients of the polynomial in part (a).
The constant term 125 represents the distance between the penny and the paintbrush when both objects begin to fall. The coefficient -40 represents the amount that the distance between the objects changes each second.

MONITORING PROGRESS ANSWERS

8. $5b - 13$

9. $8x^2 - 2x + 6$

10. $-2k + 11$

11. $5p^2 + 2p$

12. a. $-25t + 100$

b. The constant term 100 indicates the distance between the penny and the paintbrush is 100 feet when they begin to fall. The coefficient of the linear term -25 indicates that the penny and the paintbrush become 25 feet closer to each other each second.

Closure

- **Exit Ticket:** Write an example of the sum of a trinomial and a binomial that is a binomial.
Sample answer: $(3x^2 - 2x - 4) + (2x - 8) = 3x^2 - 12$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–41 odd, 53, 56, 57, 62–64

Average: 1–4, 6–52 even, 53–57, 62–64

Advanced: 1–4, 8, 12, 18–22 even, 26–52 even, 53–64

HOMEWORK CHECK

Basic: 5, 13, 23, 33, 53

Average: 10, 16, 24, 34, 53

Advanced: 12, 18, 28, 34, 53

ANSWERS

1. when the exponents of the terms decrease from left to right
2. *Sample answer:* $2x^5 - 3x + 7$
3. Determine if performing the operation on any two numbers in the set always results in a number that is also in the set.
4. $x^2 - 8^x$; It is the only one that is not a polynomial.
5. 1
6. 4
7. 2
8. 0
9. 9
10. 6
11. 11
12. 11
13. $2c^4 + 6c^2 - c$; 4; 2; trinomial
14. $-w^{12} + 4w^{11}$; 12; -1; binomial
15. $3p^2 + 7$; 2; 3; binomial
16. $-4d^3 + 8d - 2$; 3; -4; trinomial
17. $3t^8$; 8; 3; monomial
18. $3z^4 + 2z^3 + 5z$; 4; 3; trinomial
19. $-\frac{5}{7}r^8 + 2r^5 + \pi r^2$; 8; $-\frac{5}{7}$; trinomial
20. $\sqrt{7}n^4$; 4; $\sqrt{7}$; monomial
21. It is the product of a number, $\frac{4}{3}\pi$, and a variable with a whole number exponent, r^3; 3
22. binomial; 8
23. $3y + 10$
24. $x - 8$
25. $n^2 - 8n + 5$
26. $-4p^2 - 3p - 17$
27. $6g^2 - 9g + 4$
28. $12r^2 + r - 7$
29. $a^3 - 5a^2 + 4a + 5$
30. $-5s^3 + 2s^2 - s - 9$
31. $-2d - 8$
32. $-x + 8$
33. $-2y^2 + 2y + 18$
34. $7m^2 - 11m - 2$
35. $k^3 - k^2 - 7k + 14$
36. $4r^3 - r^2 - 8r - 10$

7.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** When is a polynomial in one variable in standard form?
2. **OPEN-ENDED** Write a trinomial in one variable of degree 5 in standard form.
3. **VOCABULARY** How can you determine whether a set of numbers is closed under an operation?
4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$a^3 + 4a$ $\quad x^2 - 8^x$ $\quad b - 2^{-1}$ $\quad -\frac{\pi}{3} + 6y^8z$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, find the degree of the monomial. *(See Example 1.)*

5. $4g$
6. $23x^4$
7. $-1.75k^2$
8. $-\frac{4}{9}$
9. s^8t
10. $8m^2n^4$
11. $9xy^3z^7$
12. $-3q^4rs^6$

In Exercises 13–20, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms. *(See Examples 2 and 3.)*

13. $6c^2 + 2c^4 - c$
14. $4w^{11} - w^{12}$
15. $7 + 3p^2$
16. $8d - 2 - 4d^3$
17. $3t^8$
18. $5z + 2z^3 + 3z^4$
19. $\pi r^2 - \frac{5}{7}r^8 + 2r^5$
20. $\sqrt{7}n^4$
21. **MODELING WITH MATHEMATICS** The expression $\frac{4}{3}\pi r^3$ represents the volume of a sphere with radius r. Why is this expression a monomial? What is its degree?



22. **MODELING WITH MATHEMATICS** The amount of money you have after investing \$400 for 8 years and \$600 for 6 years at the same interest rate is represented by $400x^8 + 600x^6$, where x is the growth factor. Classify the polynomial by the number of terms. What is its degree?

In Exercises 23–30, find the sum. *(See Example 4.)*

23. $(5y + 4) + (-2y + 6)$
24. $(-8x - 12) + (9x + 4)$
25. $(2n^2 - 5n - 6) + (-n^2 - 3n + 11)$
26. $(-3p^2 + 5p - 2) + (-p^2 - 8p - 15)$
27. $(3g^2 - g) + (3g^2 - 8g + 4)$
28. $(9r^2 + 4r - 7) + (3r^2 - 3r)$
29. $(4a - a^3 - 3) + (2a^3 - 5a^2 + 8)$
30. $(s^3 - 2s - 9) + (2s^2 - 6s^3 + s)$

In Exercises 31–38, find the difference. *(See Example 5.)*

31. $(d - 9) - (3d - 1)$
32. $(6x + 9) - (7x + 1)$
33. $(y^2 - 4y + 9) - (3y^2 - 6y - 9)$
34. $(4m^2 - m + 2) - (-3m^2 + 10m + 4)$
35. $(k^3 - 7k + 2) - (k^2 - 12)$
36. $(-r - 10) - (-4r^3 + r^2 + 7r)$

37. $(t^4 - t^2 + t) - (12 - 9t^2 - 7t)$

38. $(4d - 6d^3 + 3d^2) - (10d^3 + 7d - 2)$

ERROR ANALYSIS **In Exercises 39 and 40, describe and correct the error in finding the sum or difference.**

39.

$$\begin{aligned}(x^2 + x) - (2x^2 - 3x) &= x^2 + x - 2x^2 - 3x \\ &= (x^2 - 2x^2) + (x - 3x) \\ &= -x^2 - 2x\end{aligned}$$

40.

$$\begin{array}{r} x^3 - 4x^2 + 3 \\ +\ -3x^3 + 8x - 2 \\ \hline -2x^3 + 4x^2 + 1 \end{array}$$

41. **MODELING WITH MATHEMATICS** The cost (in dollars) of making b bracelets is represented by $4 + 5b$. The cost (in dollars) of making b necklaces is represented by $8b + 6$. Write a polynomial that represents how much more it costs to make b necklaces than b bracelets.



42. **MODELING WITH MATHEMATICS** The number of individual memberships at a fitness center in m months is represented by $142 + 12m$. The number of family memberships at the fitness center in m months is represented by $52 + 6m$. Write a polynomial that represents the total number of memberships at the fitness center.

In Exercises 43–46, find the sum or difference.

43. $(2s^2 - 5st - t^2) - (s^2 + 7st - t^2)$

44. $(a^2 - 3ab + 2b^2) + (-4a^2 + 5ab - b^2)$

45. $(c^2 - 6d^2) + (c^2 - 2cd + 2d^2)$

46. $(-x^2 + 9xy) - (x^2 + 6xy - 8y^2)$

REASONING **In Exercises 47–50, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

47. The terms of a polynomial are _____ monomials.

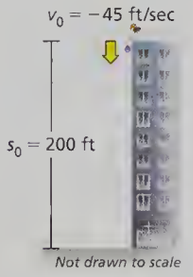
48. The difference of two trinomials is _____ a trinomial.

49. A binomial is _____ a polynomial of degree 2.

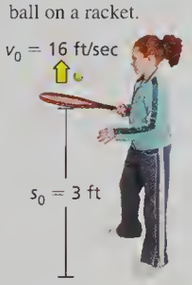
50. The sum of two polynomials is _____ a polynomial.

MODELING WITH MATHEMATICS **The polynomial $-16t^2 + v_0t + s_0$ represents the height (in feet) of an object, where v_0 is the initial vertical velocity (in feet per second), s_0 is the initial height of the object (in feet), and t is the time (in seconds). In Exercises 51 and 52, write a polynomial that represents the height of the object. Then find the height of the object after 1 second.**

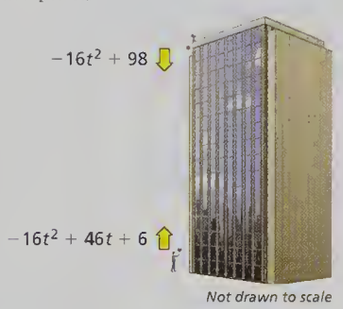
51. You throw a water balloon from a building.



52. You bounce a tennis ball on a racket.



53. **MODELING WITH MATHEMATICS** You drop a ball from a height of 98 feet. At the same time, your friend throws a ball upward. The polynomials represent the heights (in feet) of the balls after t seconds. *(See Example 6.)*



a. Write a polynomial that represents the distance between your ball and your friend's ball after t seconds.

b. Interpret the coefficients of the polynomial in part (a).

Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

ANSWERS

37. $t^4 + 8t^2 + 8t - 12$

38. $-16d^3 + 3d^2 - 3d + 2$

39. When writing the subtraction as addition, the last term of the polynomial was not multiplied by -1;
$= (x^2 + x) + (-2x^2 + 3x)$
$= (x^2 - 2x^2) + (x + 3x) = -x^2 + 4x$

40. $-4x^2$ and $8x$ are not like terms, so they cannot be added;

$$\begin{array}{r} x^3 - 4x^2 + 3 \\ +\ -3x^3 + 8x - 2 \\ \hline -2x^3 - 4x^2 + 8x + 1 \end{array}$$

41. $3b + 2$

42. $194 + 18m$

43. $s^2 - 12st$

44. $-3a^2 + 2ab + b^2$

45. $2c^2 - 2cd - 4d^2$

46. $-2x^2 + 3xy + 8y^2$

47. always; A polynomial is a monomial or a sum of monomials, and each monomial is a term of the polynomial.

48. sometimes; If like terms have the same coefficient, they will cancel when subtracted so the difference will have fewer than 3 terms. Or, if the terms in the trinomial are not all of the same degree, then the difference could have more than 3 terms.

49. sometimes; The two terms in the binomial can be of any degree.

50. always; Polynomials are closed under addition.

51. $-16t^2 - 45t + 200$; 139 ft

52. $-16t^2 + 16t + 3$; 3 ft

53. **a.** $-46t + 92$

b. The constant term 92 indicates that the distance between the two balls is 92 feet when they begin. The coefficient of the linear term -46 indicates that the two balls become 46 feet closer to each other each second.

ANSWERS

54. **a.** $-0.028t^3 - 0.32t^2 + 1.6t + 59$

b. \$55.5 million

55. $12x - 3$

56. **a.** Subtract the polynomials to find the difference.

b. The lines intersect, so the vertical distance is 0; The difference of the polynomials evaluated at $x = 1$ is 0.

57. yes; Addition is commutative and associative, so you can add in any order.

58. $\frac{1}{2}x^2 + \frac{1}{2}, \frac{1}{2}x^2 - \frac{1}{2}$

59–64. See Additional Answers.

Mini-Assessment

1. Find the degree of the monomial $15d^3$. 3
2. Write the polynomial $-8 - x^2 + 2x$ in standard form. Identify the degree and classify the polynomial by the number of terms.
 standard form: $-x^2 + 2x - 8$; degree: 2; type: trinomial

Find the sum or difference.

3. $(6n^2 + 2n + 3) + (-5n^2 - 4n - 7)$
 $n^2 - 2n - 4$
4. $(-4x + 7) - (4x - 7)$
 $-8x + 14$
5. A ball is thrown straight down from a height of 300 feet. At the same time, a quarter is dropped from a height of 150 feet. The polynomials represent the heights (in feet) of the objects after t seconds.
 Ball: $-16t^2 - 50t + 300$
 Quarter: $-16t^2 + 150$
 a. Write a polynomial that represents the distance between the ball and the quarter after t seconds. $-50t + 150$
 b. Interpret the coefficients of the polynomial in part (a). The constant term 150 represents the distance between the ball and the quarter when both objects begin to fall. The coefficient -50 represents the amount that the distance between the objects changes each second.

54. MODELING WITH MATHEMATICS During a 7-year period, the amounts (in millions of dollars) spent each year on buying new vehicles N and used vehicles U by United States residents are modeled by the equations

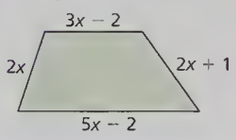
$$N = -0.028t^3 + 0.06t^2 + 0.1t + 17$$
$$U = -0.38t^2 + 1.5t + 42$$

where $t = 1$ represents the first year in the 7-year period.

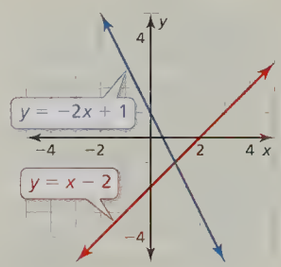
a. Write a polynomial that represents the total amount spent each year on buying new and used vehicles in the 7-year period.

b. How much is spent on buying new and used vehicles in the fifth year?

55. MATHEMATICAL CONNECTIONS Write the polynomial in standard form that represents the perimeter of the quadrilateral.



56. HOW DO YOU SEE IT? The right side of the equation of each line is a polynomial.



a. Explain how you can use the polynomials to find the vertical distance between points on the lines with the same x-value.

b. What do you know about the vertical distance when $x = 1$? How does this relate to your answer in part (a)?

57. MAKING AN ARGUMENT Your friend says that when adding polynomials, the order in which you add does not matter. Is your friend correct? Explain.

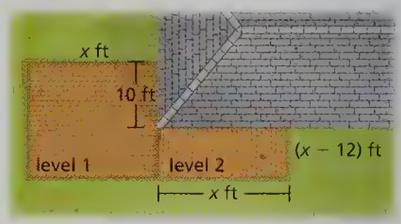
58. THOUGHT PROVOKING Write two polynomials whose sum is x^2 and whose difference is 1.

59. REASONING Determine whether the set is closed under the given operation. Explain.

a. the set of negative integers; multiplication

b. the set of whole numbers; addition

60. PROBLEM SOLVING You are building a multi-level deck.

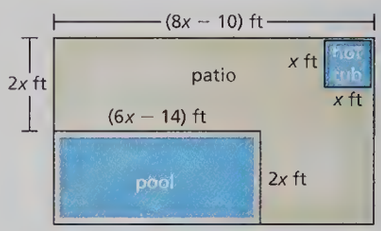


a. For each level, write a polynomial in standard form that represents the area of that level. Then write the polynomial in standard form that represents the total area of the deck.

b. What is the total area of the deck when $x = 20$?

c. A gallon of deck sealant covers 400 square feet. How many gallons of sealant do you need to cover the deck in part (b) once? Explain.

61. PROBLEM SOLVING A hotel installs a new swimming pool and a new hot tub.



a. Write the polynomial in standard form that represents the area of the patio.

b. The patio will cost \$10 per square foot. Determine the cost of the patio when $x = 9$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

62. $2(x - 1) + 3(x + 2)$ **63.** $8(4y - 3) + 2(y - 5)$ **64.** $5(2r + 1) - 3(-4r + 2)$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.2 Multiplying Polynomials

Essential Question How can you multiply two polynomials?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.10.B
A.10.D

REASONING
To be proficient in math, you need to reason abstractly and quantitatively. You need to pause as needed to recall the meanings of the symbols, operations, and quantities involved.

EXPLORATION 1 Multiplying Monomials Using Algebra Tiles

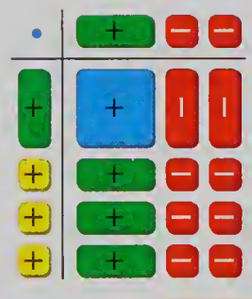
Work with a partner. Write each product. Explain your reasoning.

a. (+) • (+) =
b. (+) • (−) =
c. (−) • (−) =
d. (+x) • (+) =
e. (+x) • (−) =
f. (−x) • (+) =
g. (−x) • (−) =
h. (+x) • (+x) =
i. (+x) • (−x) =
j. (−x) • (−x) =

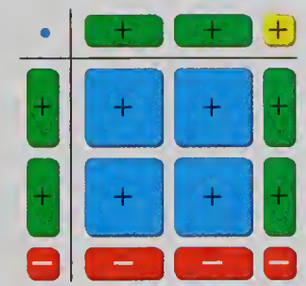
EXPLORATION 2 Multiplying Binomials Using Algebra Tiles

Work with a partner. Write the product of two binomials modeled by each rectangular array of algebra tiles. In parts (c) and (d), first draw the rectangular array of algebra tiles that models each product.

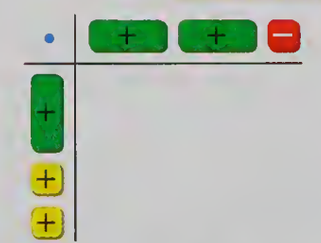
a. $(x + 3)(x - 2) =$



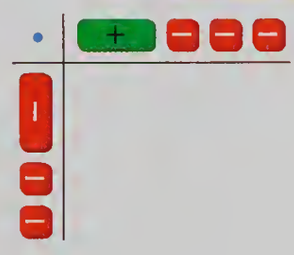
b. $(2x - 1)(2x + 1) =$



c. $(x + 2)(2x - 1) =$



d. $(-x - 2)(x - 3) =$



Communicate Your Answer

3. How can you multiply two polynomials?
4. Give another example of multiplying two binomials using algebra tiles that is similar to those in Exploration 2.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Have students work in pairs to complete the explorations and Communicate Your Answer. When they have finished, ask them to explain aloud their reasoning for each part in Exploration 1. Then ask aloud the Communicate Your Answer questions.

Beginning Use simple phrases to answer and demonstrate visually when possible.
Intermediate Use simple sentences to answer.
Advanced/Advanced High Explain answers using multiple complex sentences.

ELPS 2.I.3 Demonstrate listening comprehension of increasingly complex spoken English by responding to questions and requests commensurate with content and grade-level needs.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.10.B The student is expected to multiply polynomials of degree one and degree two.

A.10.D The student is expected to rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

ANSWERS

1. a. 1; The product of 1 and 1 is 1.
 b. -1; The product of 1 and -1 is -1.
 c. 1; The product of -1 and -1 is 1.
 d. x; The product of any number and 1 is that number.
 e. $-x$; The product of any number and -1 is the opposite of that number.
 f. $-x$; The product of any number and 1 is that number.
 g. x; The product of the opposite of a number and -1 is the number.
 h. x^2; The product of any number multiplied by itself is the number squared.
 i. $-x^2$; The product of any number and its opposite is the opposite of the number squared.
 j. x^2; The product of any number multiplied by itself is the number squared.
2. a. $x^2 + x - 6$
 b. $4x^2 - 1$
 c.



$2x^2 + 3x - 2$

 d.



$-x^2 + x + 6$

3–4. See Additional Answers.

Extra Example 1

a. Find $(x + 8)(x + 2)$.
$x^2 + 10x + 16$

b. Find $(x - 5)(x + 1)$.
$x^2 - 4x - 5$

Extra Example 2

Find $(2x + 3)(x - 3)$.
$2x^2 - 3x - 9$

MONITORING PROGRESS ANSWERS

1. $y^2 + 5y + 4$
2. $z^2 + 4z - 12$
3. $p^2 - 5p - 24$
4. $2r^2 - 11r + 5$

7.2 Lesson

Core Vocabulary

FOIL Method, *p. 347*

Previous
polynomial
closed
binomial
trinomial

What You Will Learn

- Multiply binomials.
- Use the FOIL Method.
- Multiply binomials and trinomials.

Multiplying Binomials

The product of two polynomials is always a polynomial. So, like the set of integers, the set of polynomials is closed under multiplication. You can use the Distributive Property to multiply two binomials.

EXAMPLE 1 **Multiplying Binomials Using the Distributive Property**

Find (a) $(x + 2)(x + 5)$ and (b) $(x + 3)(x - 4)$.

SOLUTION

a. Use the horizontal method.

$(x + 2)(x + 5) = x(x + 5) + 2(x + 5)$ — Distribute $(x + 5)$ to each term of $(x + 2)$.

$= x(x) + x(5) + 2(x) + 2(5)$ — Distributive Property

$= x^2 + 5x + 2x + 10$ — Multiply.

$= x^2 + 7x + 10$ — Combine like terms.

▶ The product is $x^2 + 7x + 10$.

b. Use the vertical method.

$$\begin{array}{rl} & x + 3 \\ \times & x - 4 \\ \hline & -4x - 12 \\ & x^2 + 3x \\ \hline & x^2 - x - 12 \end{array}$$

Align like terms vertically.
Multiply $-4(x + 3)$. — Distributive Property
Multiply $x(x + 3)$. — Distributive Property
Combine like terms.

▶ The product is $x^2 - x - 12$.

EXAMPLE 2 **Multiplying Binomials Using a Table**

Find $(2x - 3)(x + 5)$.

SOLUTION

Step 1 Write each binomial as a sum of terms.

$(2x - 3)(x + 5) = [2x + (-3)](x + 5)$

Step 2 Make a table of products.

	$2x$	-3
x	$2x^2$	$-3x$
5	$10x$	-15

▶ The product is $2x^2 - 3x + 10x - 15$, or $2x^2 + 7x - 15$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use the Distributive Property to find the product.

1. $(y + 4)(y + 1)$
2. $(z - 2)(z + 6)$

Use a table to find the product.

3. $(p + 3)(p - 8)$
4. $(r - 5)(2r - 1)$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Using the FOIL Method

The **FOIL Method** is a shortcut for multiplying two binomials.

Core Concept

FOIL Method

To multiply two binomials using the FOIL Method, find the sum of the products of the

First terms, $(x + 1)(x + 2) \rightarrow x(x) = x^2$

Outer terms, $(x + 1)(x + 2) \rightarrow x(2) = 2x$

Inner terms, and $(x + 1)(x + 2) \rightarrow 1(x) = x$

Last terms. $(x + 1)(x + 2) \rightarrow 1(2) = 2$

$$(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

EXAMPLE 3 Multiplying Binomials Using the FOIL Method

Find each product.

a. $(x - 3)(x - 6)$ **b.** $(2x^2 + 1)(3x^2 - 5)$

SOLUTION

a. Use the FOIL Method.

$$\begin{aligned} (x - 3)(x - 6) &= x(x) + x(-6) + (-3)(x) + (-3)(-6) && \text{FOIL Method} \\ &= x^2 + (-6x) + (-3x) + 18 && \text{Multiply.} \\ &= x^2 - 9x + 18 && \text{Combine like terms.} \end{aligned}$$

▶ The product is $x^2 - 9x + 18$.

b. Use the FOIL Method.

$$\begin{aligned} (2x^2 + 1)(3x^2 - 5) &= 2x^2(3x^2) + 2x^2(-5) + 1(3x^2) + 1(-5) && \text{FOIL Method} \\ &= 6x^4 + (-10x^2) + 3x^2 + (-5) && \text{Multiply.} \\ &= 6x^4 - 7x^2 - 5 && \text{Combine like terms.} \end{aligned}$$

▶ The product is $6x^4 - 7x^2 - 5$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use the FOIL Method to find the product.

5. $(m - 3)(m - 7)$
6. $(x - 4)(x + 2)$
7. $(n^2 + 2)(n^2 + 3)$
8. $\left(2u^2 + \frac{1}{2}\right)\left(4u^2 - \frac{3}{2}\right)$

Extra Example 3

Find each product.

a. $(x + 3)(x - 7)$
$x^2 - 4x - 21$

b. $(2x^2 - 1)(5x^2 - 4)$
$10x^4 - 13x^2 + 4$

MONITORING PROGRESS ANSWERS

5. $m^2 - 10m + 21$
6. $x^2 - 2x - 8$
7. $n^4 + 5n^2 + 6$
8. $8u^4 - u^2 - \frac{3}{4}$

Differentiated Instruction

Auditory/Visual

Assign each group one method for multiplying binominals: using the Distributive Property, a table, or FOIL. Have each student solve a different problem using that method and share the solution within the group. Then have the group solve the same problems again using the other two methods, and compare solutions.

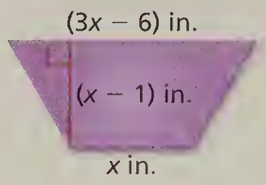
Extra Example 4

Find $(x - 2)(x^2 - 3x + 1)$.

$x^2 - 5x^2 + 7x - 2$

Extra Example 5

The purple piece of stained glass has a trapezoidal shape.



a. Write a polynomial that represents the area of the piece of glass.
$2x^2 - 5x + 3$

b. Find the area of the piece of glass when the shorter base is 7 inches.
66 square inches

MONITORING PROGRESS ANSWERS

9. $x^3 + 6x^2 + 13x + 8$
10. $n^3 - 5n^2 + 10n - 12$
11. It becomes $x^2 - \frac{3}{2}x - \frac{77}{2}$; The longer base becomes $x + 11$. Substituting this value in the formula for the area of a trapezoid along with the other unchanged values changes the linear and constant terms in the polynomial.

Multiplying Binomials and Trinomials

EXAMPLE 4 Multiplying a Binomial and a Trinomial

Find $(x + 5)(x^2 - 3x - 2)$.

SOLUTION

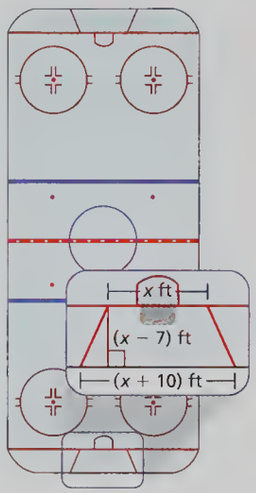
Multiply $5(x^2 - 3x - 2)$.

Multiply $x(x^2 - 3x - 2)$.

$$\begin{array}{rll} & x^2 - 3x - 2 & \text{Align like terms vertically.} \\ \times & \quad x + 5 & \\ \hline & 5x^2 - 15x - 10 & \text{Distributive Property} \\ & x^3 - 3x^2 - 2x & \text{Distributive Property} \\ \hline & x^3 + 2x^2 - 17x - 10 & \text{Combine like terms.} \end{array}$$

▶ The product is $x^3 + 2x^2 - 17x - 10$.

EXAMPLE 5 Solving a Real-Life Problem



In hockey, a goalie behind the goal line can only play a puck in the trapezoidal region.

a. Write a polynomial that represents the area of the trapezoidal region.

b. Find the area of the trapezoidal region when the shorter base is 18 feet.

SOLUTION

a. $\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(x - 7)[x + (x + 10)]$ — Substitute.

$= \frac{1}{2}(x - 7)(2x + 10)$ — Combine like terms.

F O I L

$= \frac{1}{2}[2x^2 + 10x + (-14x) + (-70)]$ — FOIL Method

$= \frac{1}{2}(2x^2 - 4x - 70)$ — Combine like terms.

$= x^2 - 2x - 35$ — Distributive Property

▶ A polynomial that represents the area of the trapezoidal region is $x^2 - 2x - 35$.

b. Find the value of $x^2 - 2x - 35$ when $x = 18$.

$x^2 - 2x - 35 = 18^2 - 2(18) - 35$ — Substitute 18 for x.

$= 324 - 36 - 35$ — Simplify.

$= 253$ — Subtract.

▶ The area of the trapezoidal region is 253 square feet.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the product.

9. $(x + 1)(x^2 + 5x + 8)$
10. $(n - 3)(n^2 - 2n + 4)$
11. **WHAT IF?** In Example 5(a), how does the polynomial change when the longer base is extended by 1 foot? Explain.

Closure

- **Point of Most Significance:** Ask students to identify, aloud or on a paper to be collected, the most significant point (or part) in the lesson that aided their learning.

7.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Describe two ways to find the product of two binomials.
2. **WRITING** Explain how the letters of the word FOIL can help you to remember how to multiply two binomials.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, use the Distributive Property to find the product. *(See Example 1.)*

3. $(x + 1)(x + 3)$
4. $(y + 6)(y + 4)$
5. $(z - 5)(z + 3)$
6. $(a + 8)(a - 3)$
7. $(g - 7)(g - 2)$
8. $(n - 6)(n - 4)$
9. $(3m + 1)(m + 9)$
10. $(5s + 6)(s - 2)$

In Exercises 11–18, use a table to find the product. *(See Example 2.)*

11. $(x + 3)(x + 2)$
12. $(y + 10)(y - 5)$
13. $(h - 8)(h - 9)$
14. $(c - 6)(c - 5)$
15. $(3k - 1)(4k + 9)$
16. $(5g + 3)(g + 8)$
17. $(-3 + 2j)(4j - 7)$
18. $(5d - 12)(-7 + 3d)$

ERROR ANALYSIS **In Exercises 19 and 20, describe and correct the error in finding the product of the binomials.**

19. ✗
$$(t-2)(t+5) = t - 2(t+5)$$
$$= t - 2t - 10$$
$$= -t - 10$$

20. ✗
$(x-5)(3x+1)$

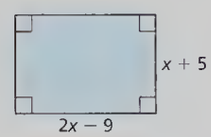
	$3x$	1
x	$3x^2$	x
5	$15x$	5

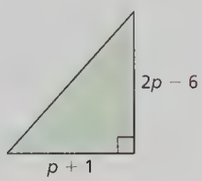
$(x-5)(3x+1) = 3x^2 + 16x + 5$

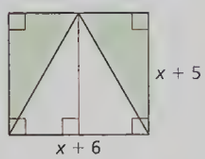
In Exercises 21–30, use the FOIL Method to find the product. *(See Example 3.)*

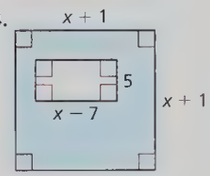
21. $(b + 3)(b + 7)$
22. $(w + 9)(w + 6)$
23. $\left(q - \frac{3}{4}\right)\left(q + \frac{1}{4}\right)$
24. $\left(z - \frac{5}{3}\right)\left(z - \frac{2}{3}\right)$
25. $(9 - r)(2 - 3r)$
26. $(8 - 4x)(2x + 6)$
27. $(6s^2 + 1)(2s^2 - 9)$
28. $(2x^2 - 4)(5x^2 + 3)$
29. $(w + 5)(w^2 + 3w)$
30. $(v - 3)(v^2 + 8v)$

MATHEMATICAL CONNECTIONS **In Exercises 31–34, write a polynomial that represents the area of the shaded region.**

31.


32.


33.


34.


In Exercises 35–42, find the product. *(See Example 4.)*

35. $(x + 4)(x^2 + 3x + 2)$
36. $(f + 1)(f^2 + 4f + 8)$
37. $(y + 3)(y^2 + 8y - 2)$
38. $(t - 2)(t^2 - 5t + 1)$
39. $(4 - b)(5b^2 + 5b - 4)$
40. $(d + 6)(2d^2 - d + 7)$
41. $(3e^2 - 5e + 7)(6e^2 + 1)$
42. $(6v^2 + 2v - 9)(4 - 5v^2)$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–43 odd, 53, 54, 58–61

Average: 1, 2–48 even, 53, 54, 58–61

Advanced: 1, 2, 8, 10, 16–20 even, 28–34 even, 40–48 even, 50–61

HOMEWORK CHECK

Basic: 3, 13, 21, 35, 43

Average: 4, 14, 22, 38, 44

Advanced: 8, 16, 28, 40, 44

ANSWERS

1. *Sample answer:* Distribute one of the binomials over each term in the other binomial and simplify; Write each binomial as a sum of terms and make a table of products.
2. The letters stand for the sets of terms to multiply: first, outer, inner, and last.
3. $x^2 + 4x + 3$
4. $y^2 + 10y + 24$
5. $z^2 - 2z - 15$
6. $a^2 + 5a - 24$
7. $g^2 - 9g + 14$
8. $n^2 - 10n + 24$
9. $3m^2 + 28m + 9$
10. $5s^2 - 4s - 12$
11. $x^2 + 5x + 6$
12. $y^2 + 5y - 50$
13. $h^2 - 17h + 72$
14. $c^2 - 11c + 30$
15. $12k^2 + 23k - 9$
16. $5g^2 + 43g + 24$
17. $8j^2 - 26j + 21$
18. $15d^2 - 71d + 84$
19. t also should be multiplied by $t + 5$;
$= t(t + 5) - 2(t + 5)$
$= t^2 + 5t - 2t - 10 = t^2 + 3t - 10$
20. The 5 in the left column should be -5;

	$3x$	1
x	$3x^2$	x
-5	$-15x$	-5

$(x - 5)(3x + 1) = 3x^2 - 14x - 5$

21. $b^2 + 10b + 21$
22. $w^2 + 15w + 54$
23. $q^2 - \frac{1}{2}q - \frac{3}{16}$
24. $z^2 - \frac{7}{3}z + \frac{10}{9}$
25. $3r^2 - 29r + 18$
26. $-8x^2 - 8x + 48$
27. $12s^4 - 52s^2 - 9$
28. $10x^4 - 14x^2 - 12$
29. $w^3 + 8w^2 + 15w$
30. $v^3 + 5v^2 - 24v$
31. $2x^2 + x - 45$
32. $p^2 - 2p - 3$
33. $\frac{1}{2}x^2 + \frac{11}{2}x + 15$
34. $x^2 - 3x + 36$
35. $x^3 + 7x^2 + 14x + 8$
36. $f^3 + 5f^2 + 12f + 8$
37. $y^3 + 11y^2 + 22y - 6$
38. $t^3 - 7t^2 + 11t - 2$
39. $-5b^3 + 15b^2 + 24b - 16$
40. $2d^3 + 11d^2 + d + 42$
41. $18e^4 - 30e^3 + 45e^2 - 5e + 7$
42. $-30v^4 - 10v^3 + 69v^2 + 8v - 36$

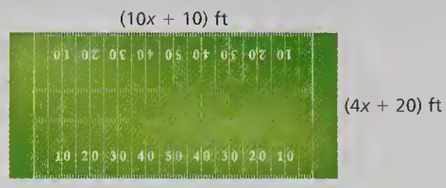
Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

43. **a.** $(40x^2 + 240x + 200)$ ft^2
 b. 57,600 ft^2
44. **a.** $(4x^2 + 84x + 440)$ in.^{2}
 b. 840 in.^{2}
45. $y^5 - y^4 - 6y^3$
46. $s^5 + 9s^4 + 20s^3$
47. $x^5 + 4x^4 - x^3 - 8x^2 - 2x$
48. $h^5 - 2h^4 + 4h^3 - 6h^2 + 3h$
49. $r^4 + 3r^3 - r - 3$
50. $t^4 + t^3 - 17t^2 + 6t + 2$
51. The degree of the product is the sum of the degrees of each binomial.
52. *Sample answer:* $(2x - 6)(x^2 + 3x + 4)$
53. no; FOIL would leave out the products that include the middle terms of the two trinomials.
54. **a.** $(-8x - 9)(-4x + 3)$
 b. a and c are positive, b and d are negative.
55. yes; You are both multiplying the same binomials, and neither the order in which you multiply nor the method used will make a difference.
56. $(4x^3 + 9x^2 - x - 6)$ ft^3
57. **a.** They have the same signs.
 b. They have opposite signs.
58. slope: -6; y-intercept: 7
59. slope: $\frac{1}{4}$; y-intercept: 7
60. slope: 2; y-intercept: -4
61. slope: $-\frac{1}{2}$; y-intercept: 4

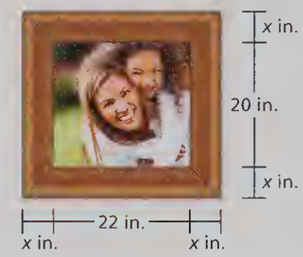
43. MODELING WITH MATHEMATICS The football field is rectangular. *(See Example 5.)*



a. Write a polynomial that represents the area of the football field.

b. Find the area of the football field when the width is 160 feet.

44. MODELING WITH MATHEMATICS You design a frame to surround a rectangular photo. The width of the frame is the same on every side, as shown.



a. Write a polynomial that represents the combined area of the photo and the frame.

b. Find the combined area of the photo and the frame when the width of the frame is 4 inches.

In Exercises 45–50, find the product.

45. $(y^3 + 2y^2)(y^2 - 3y)$
46. $(s^2 + 5s)(s^3 + 4s^2)$
47. $(x^2 - 2)(x^3 + 4x^2 + x)$
48. $(h^2 + 3)(h^3 - 2h^2 + h)$
49. $(r^2 + r + 1)(r^2 + 2r - 3)$
50. $(t^2 - 4t + 2)(t^2 + 5t + 1)$

51. **WRITING** When multiplying two binomials, explain how the degree of the product is related to the degree of each binomial.

52. **THOUGHT PROVOKING** Write two polynomials that are not monomials whose product is a trinomial of degree 3.

53. **MAKING AN ARGUMENT** Your friend says the FOIL Method can be used to multiply two trinomials. Is your friend correct? Explain your reasoning.

54. **HOW DO YOU SEE IT?** The table shows one method of finding the product of two binomials.

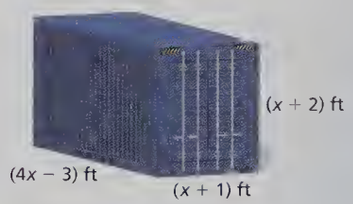
	$-4x$	3
$-8x$	a	b
-9	c	d

a. Write the two binomials being multiplied.

b. Determine whether a, b, c, and d will be positive or negative when $x > 0$.

55. **COMPARING METHODS** You use the Distributive Property to multiply $(x + 3)(x - 5)$. Your friend uses the FOIL Method to multiply $(x - 5)(x + 3)$. Should your answers be equivalent? Justify your answer.

56. **USING STRUCTURE** The shipping container is a rectangular prism. Write a polynomial that represents the volume of the container.



57. **ABSTRACT REASONING** The product of $(x + m)(x + n)$ is $x^2 + bx + c$.

a. What do you know about m and n when $c > 0$?

b. What do you know about m and n when $c < 0$?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the slope and the y-intercept of the graph of the linear equation. *(Section 3.5)*

58. $y = -6x + 7$
59. $y = \frac{1}{4}x + 7$
60. $3y = 6x - 12$
61. $2y + x = 8$

Mini-Assessment

1. Use the Distributive Property to find $(x - 7)(x - 5)$.
 $x^2 - 12x + 35$
2. Use a table to find $(2a + 1)(a + 4)$.
 $2a^2 + 9a + 4$
3. Use the FOIL method to find $(3x^2 - 1)(x^2 + 5)$. $3x^4 + 14x^2 - 5$
4. Find $(n - 3)(n^2 - 4n - 4)$.
 $n^3 - 7n^2 + 8n + 12$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.3 Special Products of Polynomials

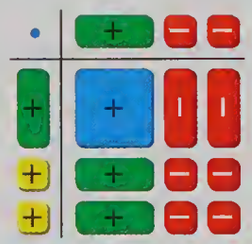
Texas Essential Knowledge and Skills
A.10.B

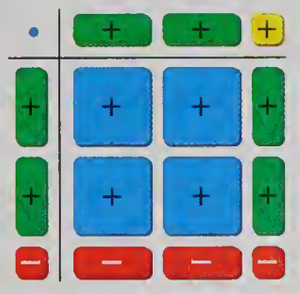
Essential Question What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$?

EXPLORATION 1 **Finding a Sum and Difference Pattern**

Work with a partner. Write the product of two binomials modeled by each rectangular array of algebra tiles.

a. $(x + 2)(x - 2) =$ **b.** $(2x - 1)(2x + 1) =$

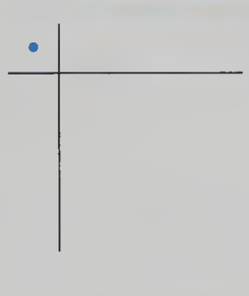


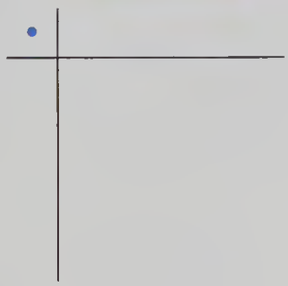


EXPLORATION 2 **Finding the Square of a Binomial Pattern**

Work with a partner. Draw the rectangular array of algebra tiles that models each product of two binomials. Write the product.

a. $(x + 2)^2 =$ **b.** $(2x - 1)^2 =$





ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$?

4. Use the appropriate special product pattern to find each product. Check your answers using algebra tiles.

a. $(x + 3)(x - 3)$ **b.** $(x - 4)(x + 4)$ **c.** $(3x + 1)(3x - 1)$

d. $(x + 3)^2$ **e.** $(x - 2)^2$ **f.** $(3x + 1)^2$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.10.B The student is expected to multiply polynomials of degree one and degree two.

ANSWERS

1. **a.** $x^2 - 4$
 b. $4x^2 - 1$
2. **a.**



$x^2 + 4x + 4$

b.



$4x^2 - 4x + 1$

3. $(a + b)(a - b) = a^2 - b^2$;
 $(a + b)^2 = a^2 + 2ab + b^2$;
 $(a - b)^2 = a^2 - 2ab + b^2$
4. **a.** $x^2 - 9$
 b. $x^2 - 16$
 c. $9x^2 - 1$
 d. $x^2 + 6x + 9$
 e. $x^2 - 4x + 4$
 f. $9x^2 + 6x + 1$

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Extra Example 1

Find each product.

a. $(2x + 6)^2$

$4x^2 + 24x + 36$

b. $(3x - y)^2$

$9x^2 - 6xy + y^2$

MONITORING PROGRESS ANSWERS

1. $x^2 + 14x + 49$
2. $49x^2 - 42x + 9$
3. $16x^2 - 8xy + y^2$
4. $9m^2 + 6mn + n^2$

7.3 Lesson

Core Vocabulary

Previous
binomial

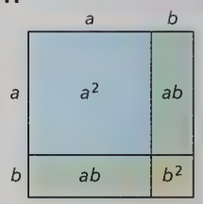
What You Will Learn

- Use the square of a binomial pattern.
- Use the sum and difference pattern.
- Use special product patterns to solve real-life problems.

Using the Square of a Binomial Pattern

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$



This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$ Replace b with $-b$ in the pattern above.

$(a - b)^2 = a^2 - 2ab + b^2$ Simplify.

Core Concept

Square of a Binomial Pattern

Algebra	Example
$(a + b)^2 = a^2 + 2ab + b^2$	$(x + 5)^2 = (x)^2 + 2(x)(5) + (5)^2$ $= x^2 + 10x + 25$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + (3)^2$ $= 4x^2 - 12x + 9$

ANALYZING MATHEMATICAL RELATIONSHIPS

When you use special product patterns, remember that a and b can be numbers, variables, or variable expressions.

EXAMPLE 1 Using the Square of a Binomial Pattern

Find each product.

a. $(3x + 4)^2$ b. $(5x - 2y)^2$

SOLUTION

a. $(3x + 4)^2 = (3x)^2 + 2(3x)(4) + 4^2$ Square of a binomial pattern

$= 9x^2 + 24x + 16$ Simplify.

▶ The product is $9x^2 + 24x + 16$.

b. $(5x - 2y)^2 = (5x)^2 - 2(5x)(2y) + (2y)^2$ Square of a binomial pattern

$= 25x^2 - 20xy + 4y^2$ Simplify.

▶ The product is $25x^2 - 20xy + 4y^2$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the product.

1. $(x + 7)^2$
2. $(7x - 3)^2$
3. $(4x - y)^2$
4. $(3m + n)^2$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Using the Sum and Difference Pattern

To find the product $(x + 2)(x - 2)$, you can multiply the two binomials using the FOIL Method.

$$(x + 2)(x - 2) = x^2 - 2x + 2x - 4 \qquad \text{FOIL Method}$$
$$= x^2 - 4 \qquad \text{Combine like terms.}$$

This suggests a pattern for the product of the sum and difference of two terms.

Core Concept

Sum and Difference Pattern

Algebra	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

EXAMPLE 2 Using the Sum and Difference Pattern

Find each product.

a. $(t + 5)(t - 5)$ b. $(3x + y)(3x - y)$

SOLUTION

a. $(t + 5)(t - 5) = t^2 - 5^2$ Sum and difference pattern
$= t^2 - 25$ Simplify.

▶ The product is $t^2 - 25$.

b. $(3x + y)(3x - y) = (3x)^2 - y^2$ Sum and difference pattern
$= 9x^2 - y^2$ Simplify.

▶ The product is $9x^2 - y^2$.

The special product patterns can help you use mental math to find certain products of numbers.

EXAMPLE 3 Using Special Product Patterns and Mental Math

Use special product patterns to find the product $26 \cdot 34$.

SOLUTION

Notice that 26 is 4 less than 30, while 34 is 4 more than 30.

$26 \cdot 34 = (30 - 4)(30 + 4)$ Write as product of difference and sum.
$= 30^2 - 4^2$ Sum and difference pattern
$= 900 - 16$ Evaluate powers.
$= 884$ Simplify.

▶ The product is 884.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the product.

5. $(x + 10)(x - 10)$
6. $(2x + 1)(2x - 1)$
7. $(x + 3y)(x - 3y)$
8. Describe how to use special product patterns to find 21^2.

Extra Example 2

Find each product.

a. $(a + 8)(a - 8)$
$a^2 - 64$

b. $(4x + y)(4x - y)$
$16x^2 - y^2$

Extra Example 3

Use special product patterns to find the product $48 \cdot 52$.
$(50 - 2)(50 + 2) = 2500 - 4 = 2496$

MONITORING PROGRESS ANSWERS

5. $x^2 - 100$
6. $4x^2 - 1$
7. $x^2 - 9y^2$
8. Rewrite 21^2 as $(20 + 1)^2$, then use the square of a binomial pattern $(20 + 1)^2 = 20^2 + 2 \cdot 20 \cdot 1 + 1^2 = 400 + 40 + 1 = 441$.

Differentiated Instruction

Organization

Have students make a table to show the patterns of special products. Have them include their own examples in the table.

Pattern	Algebra	Example
Square of a binomial (sum)	$(a + b)^2 = a^2 + 2ab + b^2$	
Square of a binomial (difference)	$(a - b)^2 = a^2 - 2ab + b^2$	
Sum and difference	$(a + b)(a - b) = a^2 - b^2$	

Extra Example 4

Each of two cats has one black patch gene (B) and one tan patch gene (t). Any gene combination with a B results in black patches in an offspring's fur. The Punnett square shows the possible gene combinations of the offspring and the resulting patch colors.

	B	t
B	BB	Bt
t	Bt	tt

a. What percent of the possible gene combinations result in tan patches?
25%

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.
$0.25B^2 + 0.5Bt + 0.25t^2$; The coefficient of t^2 shows that 25% of the possible gene combinations result in tan patches.

MONITORING PROGRESS ANSWERS

9. a. 25%

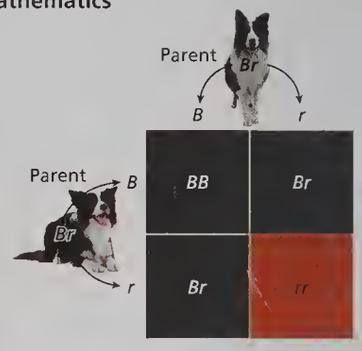
b. $(0.5B + 0.5W)^2 = 0.25B^2 + 0.5BW + 0.25W^2$; The coefficients show 25% black, 50% gray, and 25% white.

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

A combination of two genes determines the color of the dark patches of a border collie's coat. An offspring inherits one patch color gene from each parent. Each parent has two color genes, and the offspring has an equal chance of inheriting either one.

The gene B is for black patches, and the gene r is for red patches. Any gene combination with a B results in black patches. Suppose each parent has the same gene combination Br. The Punnett square shows the possible gene combinations of the offspring and the resulting patch colors.



a. What percent of the possible gene combinations result in black patches?

b. Show how you could use a polynomial to model the possible gene combinations.

SOLUTION

a. Notice that the Punnett square shows four possible gene combinations of the offspring. Of these combinations, three result in black patches.

▶ So, 75% of the possible gene combinations result in black patches.

b. Model the gene from each parent with $0.5B + 0.5r$. There is an equal chance that the offspring inherits a black or a red gene from each parent.

You can model the possible gene combinations of the offspring with $(0.5B + 0.5r)^2$. Notice that this product also represents the area of the Punnett square.

Expand the product to find the possible patch colors of the offspring.

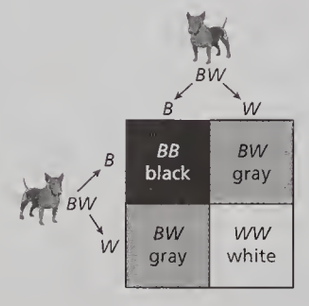
$$(0.5B + 0.5r)^2 = (0.5B)^2 + 2(0.5B)(0.5r) + (0.5r)^2$$
$$= 0.25B^2 + 0.5Br + 0.25r^2$$

Consider the coefficients in the polynomial.

$$0.25B^2 + 0.5Br + 0.25r^2$$

- $0.25B^2$: 25% BB, black patches
- $0.5Br$: 50% Br, black patches
- $0.25r^2$: 25% rr, red patches

The coefficients show that 25% + 50% = 75% of the possible gene combinations result in black patches.



Monitoring Progress

Help in English and Spanish at BigIdeasMath.com

9. Each of two dogs has one black gene (B) and one white gene (W). The Punnett square shows the possible gene combinations of an offspring and the resulting colors.

a. What percent of the possible gene combinations result in black?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.

SUPPORTING English Language Learners

Ask students to share what they know about genes and physical characteristics. Explain that a Punnett square demonstrates how genes might be inherited. Have students practice reading comprehension by reading aloud Example 4 in groups of mixed language abilities. Students should consult their group or the teacher when there is a concept they do not understand.

Beginning Read the headings.
Intermediate Read the first paragraph.
Advanced Read the second paragraph and parts (a) and (b).
Advanced High Read the solution.

ELPS 4.F.10 Use support from peers and teachers to develop background knowledge needed to comprehend increasingly challenging language.

Closure

- **Give Me Five:** "How did today's lesson help you better understand multiplication of binomials?"

7.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to use the square of a binomial pattern.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

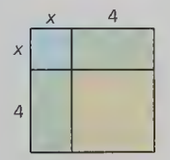
$(x + 1)(x - 1)$ $(3x + 2)(3x - 2)$ $(x + 2)(x - 3)$ $(2x + 5)(2x - 5)$

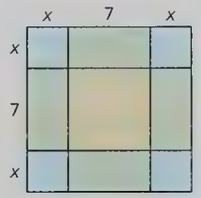
Monitoring Progress and Modeling with Mathematics

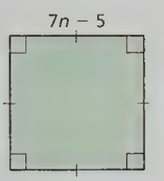
In Exercises 3–10, find the product. *(See Example 1.)*

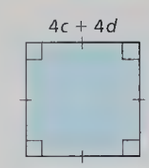
3. $(x + 8)^2$
4. $(a - 6)^2$
5. $(2f - 1)^2$
6. $(5p + 2)^2$
7. $(-7t + 4)^2$
8. $(-12 - n)^2$
9. $(2a + b)^2$
10. $(6x - 3y)^2$

MATHEMATICAL CONNECTIONS **In Exercises 11–14, write a polynomial that represents the area of the square.**

11. 

12. 

13. 

14. 

In Exercises 15–24, find the product. *(See Example 2.)*

15. $(t - 7)(t + 7)$
16. $(m + 6)(m - 6)$
17. $(4x + 1)(4x - 1)$
18. $(2k - 4)(2k + 4)$
19. $(8 + 3a)(8 - 3a)$
20. $\left(\frac{1}{2} - c\right)\left(\frac{1}{2} + c\right)$
21. $(p - 10q)(p + 10q)$
22. $(7m + 8n)(7m - 8n)$
23. $(-y + 4)(-y - 4)$
24. $(-5g - 2h)(-5g + 2h)$

In Exercises 25–30, use special product patterns to find the product. *(See Example 3.)*

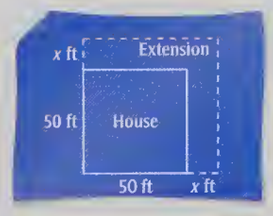
25. $16 \cdot 24$
26. $33 \cdot 27$
27. 42^2
28. 29^2
29. 30.5^2
30. $10\frac{1}{3} \cdot 9\frac{2}{3}$

ERROR ANALYSIS **In Exercises 31 and 32, describe and correct the error in finding the product.**

31. ✗ $(k + 4)^2 = k^2 + 4^2$
$= k^2 + 16$

32. ✗ $(s + 5)(s - 5) = s^2 + 2(s)(5) - 5^2$
$= s^2 + 10s - 25$

33. **MODELING WITH MATHEMATICS** A contractor extends a house on two sides.



a. The area of the house after the renovation is represented by $(x + 50)^2$. Find this product.

b. Use the polynomial in part (a) to find the area when $x = 15$. What is the area of the extension?

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–35 odd, 38, 43, 48–51

Average: 1, 2–34 even, 35–39, 43, 48–51

Advanced: 1, 2, 8–14 even, 22–34 even, 35–37, 38–40 even, 42–51

HOMEWORK CHECK

Basic: 3, 15, 17, 25, 35

Average: 4, 16, 18, 26, 35

Advanced: 10, 22, 24, 28, 35

ANSWERS

1. Substitute the first term of the binomial for a and the second term of the binomial for b in the square of a binomial pattern, then simplify.
2. $(x + 2)(x - 3)$; It is the only one that cannot be simplified using the sum and difference pattern.
3. $x^2 + 16x + 64$
4. $a^2 - 12a + 36$
5. $4f^2 - 4f + 1$
6. $25p^2 + 20p + 4$
7. $49t^2 - 56t + 16$
8. $n^2 + 24n + 144$
9. $4a^2 + 4ab + b^2$
10. $36x^2 - 36xy + 9y^2$
11. $x^2 + 8x + 16$
12. $4x^2 + 28x + 49$
13. $49n^2 - 70n + 25$
14. $16c^2 + 32cd + 16d^2$
15. $t^2 - 49$
16. $m^2 - 36$
17. $16x^2 - 1$
18. $4k^2 - 16$
19. $64 - 9a^2$
20. $\frac{1}{4} - c^2$
21. $p^2 - 100q^2$
22. $49m^2 - 64n^2$
23. $y^2 - 16$
24. $25g^2 - 4h^2$
25. 384
26. 891
27. 1764
28. 841
29. 930.25
30. $99\frac{8}{9}$
31. The middle term in the square of a binomial pattern was not included; $= k^2 + 2(k)(4) + 4^2 = k^2 + 8k + 16$
32. There is no middle term in the sum and difference pattern; $= s^2 - 5^2 = s^2 - 25$
33. a. $(x^2 + 100x + 2500)$ ft^2
 b. 4225 ft^2; 1725 ft^2

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

34. a. $(10{,}000 - x^2)$ ft^2

b. decrease; The original area is 10,000 ft^2, so the new area is the original area decreased by x^2.

c. 9559 ft^2

35. a. 25%

b. $(0.5N + 0.5a)^2 = 0.25N^2 + 0.5Na + 0.25a^2$

36. a. $(\pi x^2 - 12\pi x + 36\pi)$ mm^2

b. 4 times greater; The area increases from 4π square millimeters to 16π square millimeters, which is 4 times greater.

37. $(x + 11)(x - 11)$; $x^2 - 121$ fits the product side of the sum and difference pattern, so working backwards, a and b are the square roots of a^2 and b^2.

38. Use the Punnett Square which shows that $\frac{3}{4} = 75\%$ of the combinations result in green pods; Because both the G^2 and the Gy combinations result in green pods, add the coefficients of these terms in the polynomial, $0.25 + 0.5 = 0.75 = 75\%$.

39. $x^4 - 1$

40. $y^6 + 8y^3 + 16$

41. $4m^4 - 20m^2n^2 + 25n^4$

42. $r^6 - 36t^8$

43. no; $\left(4\frac{1}{3}\right)^2$ can be written as $\left(4 + \frac{1}{3}\right)^2$, however using the square of a binomial pattern results in $16 + \frac{8}{3} + \frac{1}{9}$, which is $18\frac{7}{9}$, not $16\frac{1}{9}$.

44–51. See Additional Answers.

34. MODELING WITH MATHEMATICS A square-shaped parking lot with 100-foot sides is reduced by x feet on one side and extended by x feet on an adjacent side.

a. The area of the new parking lot is represented by $(100 - x)(100 + x)$. Find this product.

b. Does the area of the parking lot increase, decrease, or stay the same? Explain.

c. Use the polynomial in part (a) to find the area of the new parking lot when $x = 21$.

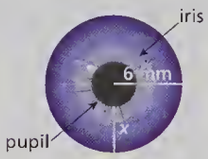
35. MODELING WITH MATHEMATICS In deer, the gene N is for normal coloring and the gene a is for no coloring, or albino. Any gene combination with an N results in normal coloring. The Punnett square shows the possible gene combinations of an offspring and the resulting colors from parents that both have the gene combination Na. *(See Example 4.)*

a. What percent of the possible gene combinations result in albino coloring?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.

Parent B \ Parent A	N	a
N	NN normal	Na normal
Na	Na normal	aa albino

36. MODELING WITH MATHEMATICS Your iris controls the amount of light that enters your eye by changing the size of your pupil.

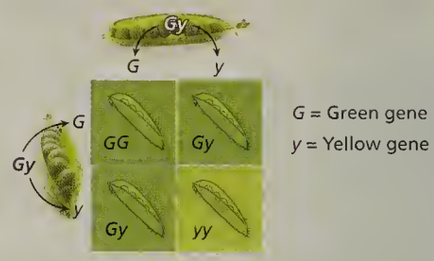


a. Write a polynomial that represents the area of your pupil. Write your answer in terms of π.

b. The width x of your iris decreases from 4 millimeters to 2 millimeters when you enter a dark room. How many times greater is the area of your pupil after entering the room than before entering the room? Explain.

37. CRITICAL THINKING Write two binomials that have the product $x^2 - 121$. Explain.

38. HOW DO YOU SEE IT? In pea plants, any gene combination with a green gene (G) results in a green pod. The Punnett square shows the possible gene combinations of the offspring of two Gy pea plants and the resulting pod colors.



A polynomial that models the possible gene combinations of the offspring is

$$(0.5G + 0.5y)^2 = 0.25G^2 + 0.5Gy + 0.25y^2.$$

Describe two ways to determine the percent of possible gene combinations that result in green pods.

In Exercises 39–42, find the product.

39. $(x^2 + 1)(x^2 - 1)$

40. $(y^3 + 4)^2$

41. $(2m^2 - 5n^2)^2$

42. $(r^3 - 6t^4)(r^3 + 6t^4)$

43. MAKING AN ARGUMENT Your friend claims to be able to use a special product pattern to determine that $\left(4\frac{1}{3}\right)^2$ is equal to $16\frac{1}{9}$. Is your friend correct? Explain.

44. THOUGHT PROVOKING The area (in square meters) of the surface of an artificial lake is represented by x^2. Describe three ways to modify the dimensions of the lake so that the new area can be represented by the three types of special product patterns discussed in this section.

45. REASONING Find k so that $9x^2 - 48x + k$ is the square of a binomial.

46. REPEATED REASONING Find $(x + 1)^3$ and $(x + 2)^3$. Find a pattern in the terms and use it to write a pattern for the cube of a binomial $(a + b)^3$.

47. PROBLEM SOLVING Find two numbers a and b such that $(a + b)(a - b) < (a - b)^2 < (a + b)^2$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. Write your answer using only positive exponents. *(Skills Review Handbook)*

48. $\frac{10^2}{10^5}$

49. $\frac{x^5 \cdot x}{x^8}$

50. $(3z^6)^{-3}$

51. $\left(\frac{2y^4}{y^3}\right)^{-2}$

Mini-Assessment

Find the product.

1. $(4x + 1)^2$ $16x^2 + 8x + 1$
2. $(3a - 5b)^2$ $9a^2 - 30ab + 25b^2$
3. $(2d + 3)(2d - 3)$ $4d^2 - 9$
4. Use special product patterns to find the product $43 \cdot 37$.
 $(40 + 3)(40 - 3) = 1600 - 9 = 1591$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.4 Dividing Polynomials

Texas Essential Knowledge and Skills
A.10.C

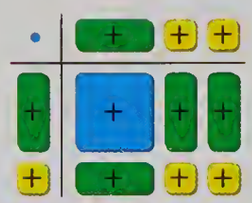
Essential Question How can you use algebra tiles to divide two polynomials?

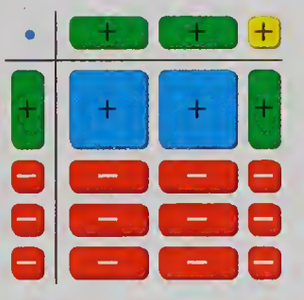
EXPLORATION 1 **Dividing Polynomials Using Algebra Tiles**

Work with a partner. Use algebra tiles to write each quotient.

a. $\frac{(x^2 + 3x + 2)}{(x + 1)} =$ [redacted]

b. $\frac{(2x^2 - 5x - 3)}{(x - 3)} =$ [redacted]



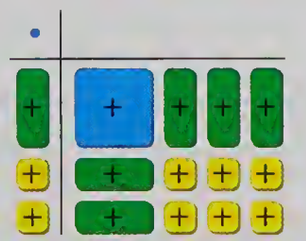


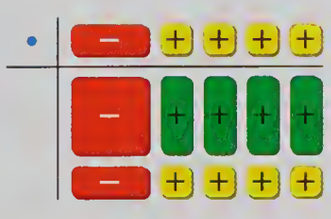
EXPLORATION 2 **Dividing Polynomials Using Algebra Tiles**

Work with a partner. Use algebra tiles to write each quotient.

a. $\frac{(x^2 + 5x + 6)}{(x + 2)} =$ [redacted]

b. $\frac{(-x^2 + 3x + 4)}{(-x + 4)} =$ [redacted]





c. $\frac{(x^2 + 4x - 5)}{(x - 1)} =$ [redacted]

d. $\frac{(2x^2 - 6x + 4)}{(2x - 2)} =$ [redacted]

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. How can you use algebra tiles to divide two polynomials?
4. Give another example of dividing two polynomials using algebra tiles that is similar to those in Explorations 1 and 2.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

4. *Sample answer:* $\frac{x^2 - x - 6}{x + 2} = x - 3$



Dynamic Teaching Tools

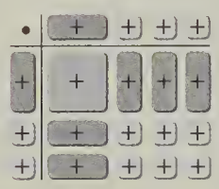
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.10.C The student is expected to determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend.

ANSWERS

1. a. $x + 2$
 b. $2x + 1$
2. a. $x + 3$



b. $x + 1$



c. $x + 5$



d. $x - 2$



3. Arrange algebra tiles that model the dividend into a rectangular array. Arrange algebra tiles that model the divisor along one of the dimensions of the array. Then use additional algebra tiles to model the missing dimension of the rectangle, the polynomial represented by these tiles is the quotient.

Extra Example 1
Find $(10x^2 + 2x) \div (2x)$. $5x + 1$

MONITORING PROGRESS ANSWERS
1. $2x - 6$
2. $-4x - 1$

7.4 Lesson

What You Will Learn

- Divide polynomials by monomials.
- Use polynomial long division to divide polynomials by other polynomials.
- Use synthetic division to divide polynomials by binomials of the form $x - k$.

Core Vocabulary

polynomial long division, *p. 358*
synthetic division, *p. 360*

Previous
polynomial
degree of a polynomial

Dividing Polynomials by Monomials

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

EXAMPLE 1 **Dividing a Polynomial by a Monomial**

Find $(6x^2 + 12x) \div (3x)$.

SOLUTION

$$\frac{6x^2 + 12x}{3x} = \frac{6x^2}{3x} + \frac{12x}{3x}$$
$$= 2x + 4$$

▶ $(6x^2 + 12x) \div (3x) = 2x + 4$

REMEMBER
When dividing terms with exponents, use the Quotient of Powers Property: $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Divide.

1. $(8x - 24) \div 4$
2. $(20x^2 + 5x) \div (-5x)$

Long Division of Polynomials

When you divide a polynomial $f(x)$ by a nonzero polynomial divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

dividend → $f(x)$, divisor → $d(x)$, remainder ← $r(x)$, quotient ← $q(x)$

The degree of the remainder must be less than the degree of the divisor. When the remainder is 0, the divisor *divides evenly* into the dividend. Also, the degree of the divisor is less than or equal to the degree of the dividend $f(x)$. One way to divide polynomials is called **polynomial long division**.

Core Concept

Long Division of Polynomials

Step 1 Write the dividend and divisor in standard form.

Step 2 Insert placeholders with zero coefficients for missing powers of the variable.

Step 3 Perform the long division of the polynomials as you would with integers. At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor. Line up the terms in the quotient with the like term in the dividend.

Step 4 Continue until the degree of the remainder is less than the degree of the divisor.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

EXAMPLE 2 Using Polynomial Long Division

Find $(4x + 5) \div (x + 1)$. Determine whether the divisor evenly divides into the dividend.

SOLUTION

$$\begin{array}{r} 4 \\ x + 1\overline{)4x + 5} \\ \underline{4x + 4} \\ 1 \end{array}$$

Multiply divisor by $\frac{4x}{x} = 4$.

Subtract.

▶ $(4x + 5) \div (x + 1) = 4 + \frac{1}{x + 1}$. Because the remainder does not equal 0, the divisor does not evenly divide into the dividend.

COMMON ERROR

The expression added to the quotient in the result of a long division problem is $\frac{r(x)}{d(x)}$, not $r(x)$.

Check

You can check the result of a division problem by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\begin{aligned}(4)(x + 1) + 1 &= (4x + 4) + 1 \\ &= 4x + 5 \checkmark\end{aligned}$$

EXAMPLE 3 Using Polynomial Long Division

Find $(6x^2 + 17x + 20) \div (3x + 4)$.

SOLUTION

$$\begin{array}{r} 2x + 3 \\ 3x + 4\overline{)6x^2 + 17x + 20} \\ \underline{6x^2 + 8x} \\ 9x + 20 \\ \underline{9x + 12} \\ 8 \end{array}$$

Multiply divisor by $\frac{6x^2}{3x} = 2x$.

Subtract. Bring down next term.

Multiply divisor by $\frac{9x}{3x} = 3$.

Subtract.

▶ $(6x^2 + 17x + 20) \div (3x + 4) = 2x + 3 + \frac{8}{3x + 4}$

Check

$$\begin{aligned}&(2x + 3)(3x + 4) + 8 \\ &= (6x^2 + 17x + 12) + 8 \\ &= 6x^2 + 17x + 20 \checkmark\end{aligned}$$

EXAMPLE 4 Using Polynomial Long Division

Find $(3x^2 + 2x + 15) \div (1 + x^2)$.

SOLUTION

$$\begin{array}{r} 3 \\ x^2 + 0x + 1\overline{)3x^2 + 2x + 15} \\ \underline{3x^2 + 0x + 3} \\ 2x + 12 \end{array}$$

Multiply divisor by $\frac{3x^2}{x^2} = 3$.

Subtract.

▶ $(3x^2 + 2x + 15) \div (1 + x^2) = 3 + \frac{2x + 12}{x^2 + 1}$

COMMON ERROR

Be sure to write $1 + x^2$ in standard form and insert $0x$ as a placeholder.

Extra Example 2

Find $(3x + 8) \div (x + 2)$. Determine whether the divisor evenly divides into the dividend.

$3 + \frac{2}{x + 2}$; Because the remainder does not equal 0, the divisor does not evenly divide into the dividend.

Extra Example 3

Find $(2x^2 + 11x + 18) \div (2x + 5)$.

$x + 3 + \frac{3}{2x + 5}$

Extra Example 4

Find $(4x^2 + 2x + 17) \div (2 + x^2)$.

$4 + \frac{2x + 9}{x^2 + 2}$

English Language Learners

Build on Past Knowledge

Remind students from their study of whole-number long division that the steps for dividing are divide, multiply, subtract, and bring down. These are the same steps they will use for polynomial long division.

Extra Example 5

Find $(2x^2 - 15x + 35) \div (x - 5)$ using synthetic division.

$2x - 5 + \frac{10}{x - 5}$

MONITORING PROGRESS ANSWERS

3. $5 + \frac{-19}{x + 3}$

4. $-2 + \frac{3}{-4x - 2}$

5. $7x + 18 + \frac{47}{x - 2}$

6. $-2x + 2 + \frac{2}{-5x + 1}$

7. $-6 + \frac{2x + 48}{x^2 + 5}$

8. $-2 + \frac{8x + 24}{-2x^2 + x + 3}$

9. $2x + 7 + \frac{24}{x - 3}$

10. $-x + 9 + \frac{-38}{x + 4}$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use polynomial long division to divide.

3. $(5x - 4) \div (x + 3)$
4. $(8x + 7) \div (-4x - 2)$
5. $(7x^2 + 4x + 11) \div (x - 2)$
6. $(10x^2 - 12x + 4) \div (-5x + 1)$
7. $(-6x^2 + 2x + 18) \div (x^2 + 5)$
8. $(4x^2 + 18 + 6x) \div (-2x^2 + x + 3)$

Synthetic Division

There is a shortcut for dividing polynomials by binomials of the form $x - k$. This shortcut is called **synthetic division**. The method is shown in the next example.

EXAMPLE 5 Using Synthetic Division

Find $(3x^2 - 10x + 20) \div (x - 2)$.

SOLUTION

Step 1 Write the coefficients of the dividend in order of descending exponents. Include a "0" for any missing powers of the variable. Because the divisor is $x - 2$, use $k = 2$. Write the k-value to the left of the vertical bar.

k-value $\rightarrow$ 2 | 3 −10 20 $\leftarrow$ coefficients of $3x^2 - 10x + 20$

Step 2 Bring down the leading coefficient. Multiply the leading coefficient by the k-value. Write the product under the second coefficient. Add.

2	3	−10	20
		6	
	3	−4	

Step 3 Multiply the previous sum by the k-value. Write the product under the third coefficient. Add. The first two numbers in the bottom row are the coefficients of the quotient, and the last number is the remainder.

2	3	−10	20
		6	−8
coefficients of quotient $\rightarrow$	3	−4	12 $\leftarrow$ remainder

▶ $(3x^2 - 10x + 20) \div (x - 2) = 3x - 4 + \frac{12}{x - 2}$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use synthetic division to divide.

9. $(2x^2 + x + 3) \div (x - 3)$
10. $(-x^2 + 5x - 2) \div (x + 4)$

Closure

- **Exit Ticket:** Use polynomial long division and synthetic division to divide: $(6x^2 - 21x - 5) \div (x - 4)$. $6x + 3 + \frac{7}{x - 4}$

7.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Describe how you can check the result of a polynomial division problem. Give an example.
2. **VOCABULARY** Can you use synthetic division to find $(3x^2 + x - 2) \div (x + 1)$? Explain your reasoning.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, divide. *(See Example 1.)*

3. $(36x - 42) \div 6$
4. $(45x + 20) \div (-5)$
5. $(16x^2 + 8x) \div (-2x)$
6. $(-4x^2 - 28x) \div (4x)$

In Exercises 7–12, use polynomial long division to divide. Determine whether the divisor evenly divides into the dividend. *(See Example 2.)*

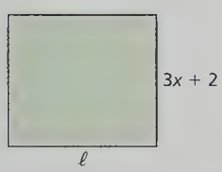
7. $(7x + 3) \div (x + 2)$
8. $(5x + 11) \div (x + 3)$
9. $(6x - 18) \div (2x - 6)$
10. $(8x - 5) \div (2x + 1)$
11. $(-8x - 10) \div (4x + 5)$
12. $(18x - 24) \div (-3x + 4)$

In Exercises 13–18, use polynomial long division to divide. *(See Example 3.)*

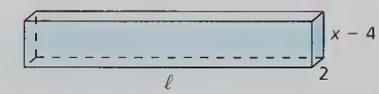
13. $(14x^2 - 23x + 6) \div (7x - 1)$
14. $(28x^2 + 36x - 1) \div (2x + 3)$
15. $(9x^2 + 24x + 15) \div (-3x - 2)$
16. $(8x^2 + 30x + 35) \div (4x + 3)$
17. $(15x + 14x^2 - 40) \div (-2x - 5)$
18. $(11x - 6x^2 - 8) \div (3x - 1)$

MATHEMATICAL CONNECTIONS **In Exercises 19 and 20, write an expression that represents the length ℓ of the figure.**

19. Area $= 9x^2 + 15x + 6$



20. Volume $= 4x^2 + 2x - 72$



In Exercises 21–28, use polynomial long division to divide. *(See Example 4.)*

21. $(4x^2 + 7x + 12) \div (x^2 + 2x + 1)$
22. $(3x^2 + 2x - 20) \div (x^2 - 8x + 12)$
23. $(16x^2 - 25 - 13x) \div (2x^2 + x - 10)$
24. $(7 - 8x^2 - 3x) \div (4x^2 + 5x - 3)$
25. $(5x^2 - 8x + 2) \div (x^2 - 3)$
26. $(9x^2 + 3x - 8) \div (3x^2 + 2)$
27. $(6x - 20x^2) \div (5x^2 - 3x + 2)$
28. $(-17 + 4x^2) \div (-2x^2 + 3x)$

In Exercises 29–36, use synthetic division to divide. *(See Example 5.)*

29. $(8x^2 + 4x - 19) \div (x - 1)$
30. $(2x^2 - 15x + 8) \div (x - 4)$
31. $(3x^2 + 17x + 15) \div (x + 5)$
32. $(9x^2 + 25x - 4) \div (x + 3)$
33. $(15x^2 - 48) \div (x - 2)$
34. $(2x^2 - 5x) \div (x - 3)$
35. $(3 - x^2 - x) \div (x + 4)$
36. $(12 - 7x^2) \div (x + 1)$

SUPPORTING English Language Learners

Have students share with the class their answers for the Vocabulary and Core Concept Check.

Beginning Use simple phrases to share answers.
Intermediate Use complete sentences to share information.
Advanced Use multiple complete sentences arranged appropriately.
Advanced High Explain answers using multiple complex sentences.

ELPS 3.G.1 Express opinions ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–37 odd, 41, 43, 44, 48–55

Average: 1, 2–38 even, 41–44, 48–55

Advanced: 1, 2, 6, 10, 12, 16–20 even, 24–38 even, 40–55

HOMEWORK CHECK

Basic: 3, 7, 13, 21, 29

Average: 4, 8, 14, 22, 30

Advanced: 6, 10, 16, 24, 32

ANSWERS

1. See Additional Answers.
2. yes; $3x^2 + x - 2$ is a polynomial and the binomial $x + 1$ can be rewritten as $x - (-1)$, so it is of the form $x - k$, where $k = -1$.
3. $6x - 7$
4. $-9x - 4$
5. $-8x - 4$
6. $-x - 7$
7. $7 + \frac{-11}{x + 2}$; no
8. $5 + \frac{-4}{x + 3}$; no
9. 3; yes
10. $4 + \frac{-9}{2x + 1}$; no
11. -2; yes
12. -6; yes
13. $2x - 3 + \frac{3}{7x - 1}$
14. $14x - 3 + \frac{8}{2x + 3}$
15. $-3x - 6 + \frac{3}{-3x - 2}$
16. $2x + 6 + \frac{17}{4x + 3}$
17. $-7x + 10 + \frac{10}{-2x - 5}$
18. $-2x + 3 + \frac{-5}{3x - 1}$
19. $3x + 3$
20. $2x + 9$
21. $4 + \frac{-x + 8}{x^2 + 2x + 1}$
22. $3 + \frac{26x - 56}{x^2 - 8x + 12}$
23. $8 + \frac{-21x + 55}{2x^2 + x - 10}$
24. $-2 + \frac{7x + 1}{4x^2 + 5x - 3}$

25–36. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

37. The remainder should be divided by the divisor;
$(3x^2 + 10x + 6) \div (x + 3) = 3x + 1 + \frac{3}{x + 3}$

38. The quotient should be one degree less than the dividend;
$(2x^2 - 6x - 3) \div (x - 4) = 2x + 2 + \frac{5}{x - 4}$

39. When dividing the dividend by the divisor, the remainder is 0.

40. $x + 4$; $3x^2 + 14x$; The divisor is the expression that the remainder is divided by, and the dividend is the result of multiplying the quotient by the divisor and adding the remainder.

41. $\frac{6x + 4}{x + 3}$; $6 + \frac{-14}{x + 3}$

42. 10

43. no; The divisor $x^2 + 1$ is not of the form $x - k$.

44. divisor: $x + 4$;
dividend: $5x^2 + 18x + 6$;
quotient: $5x - 2$; remainder: 14

45–55. See Additional Answers.

Mini-Assessment

1. Find $(18x^2 - 6x) \div (-3x)$.
 $-6x + 2$
2. Find $(15x - 20) \div (3x - 4)$. Determine whether the divisor evenly divides into the dividend.
 5; The divisor evenly divides into the dividend.
3. Find $(10x^2 + 24x + 13) \div (2x + 4)$ using polynomial long division.
 $5x + 2 + \frac{5}{2x + 4}$
4. Find $(5x^2 + 3x - 8) \div (x^2 - 2)$ using polynomial long division.
 $5 + \frac{3x + 2}{x^2 - 2}$

37. ERROR ANALYSIS Describe and correct the error in dividing $3x^2 + 10x + 6$ by $x + 3$.

$$\begin{array}{r} 3x + 1 \\ x + 3 \overline{)\,3x^2 + 10x + 6} \\ \underline{3x^2 + 9x} \\ x + 6 \\ \underline{x + 3} \\ 3 \end{array}$$

$(3x^2 + 10x + 6) \div (x + 3) = 3x + 1 + 3$

38. ERROR ANALYSIS Describe and correct the error in using synthetic division to divide $2x^2 - 6x - 3$ by $x - 4$.

$$\begin{array}{r|rrr} 4 & 2 & -6 & -3 \\ & & 8 & 8 \\ \hline & 2 & 2 & 5 \end{array}$$

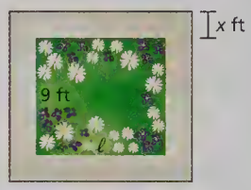
$(2x^2 - 6x - 3) \div (x - 4) = 2x^2 + 2x + 5$

39. WRITING Explain what it means for a divisor to divide evenly into a dividend.

40. USING STRUCTURE You divide two polynomials and obtain the result $3x + 2 - \frac{8}{x + 4}$. What is the divisor? What is the dividend? Explain how you found your answer.

41. MODELING WITH MATHEMATICS Your class is participating in a food drive. The total number of cans donated is represented by $6x + 4$, where x represents the number of students who participate. The total number of students in the class is represented by $x + 3$. Write an expression that represents the average number of cans donated by each student in the class. Use polynomial long division to rewrite the expression.

42. MODELING WITH MATHEMATICS A rectangular garden is bordered by a walkway. The width of the walkway is the same on every side. The total area (in square feet) of the garden and the walkway is represented by $4x^2 + 38x + 90$. Determine the length ℓ (in feet) of the garden without the walkway.



43. MAKING AN ARGUMENT Your friend says that you can use synthetic division to divide $3x^2 + x - 2$ by $x^2 + 1$ because the divisor is a binomial. Is your friend correct? Explain.

44. HOW DO YOU SEE IT? Write the divisor, dividend, quotient, and remainder represented by the synthetic division shown.

$$\begin{array}{r|rrr} -4 & 5 & 18 & 6 \\ & & -20 & 8 \\ \hline & 5 & -2 & 14 \end{array}$$

45. COMPARING METHODS Divide $6x^2 + 18x + 2$ by $3x + 6$ using polynomial long division. Then divide $6x^2 + 18x + 2$ by $x + 2$ using synthetic division. What do you notice about the results of the two division problems? Why do you think this is so?

46. THOUGHT PROVOKING Use polynomial long division to divide $3x^3 + 6x^2 - 12x + 13$ by $x + 4$. Can you also use synthetic division to divide the polynomials? Explain your reasoning.

47. CRITICAL THINKING Find a value for k so that $x + 8$ divides evenly into $x^2 + kx + 8$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the expression using the GCF. *(Skills Review Handbook)*

48. $12y - 18$ **49.** $9r + 27$ **50.** $49s + 35t$ **51.** $15x - 10y$

Solve the equation. Check your solution. *(Section 1.1)*

52. $p - 9 = 0$ **53.** $z + 12 = -5$ **54.** $6 = \frac{c}{-7}$ **55.** $4k = 0$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.5 Solving Polynomial Equations in Factored Form

Texas Essential Knowledge and Skills
A.8.A

Essential Question
How can you solve a polynomial equation?

EXPLORATION 1 Matching Equivalent Forms of an Equation

Work with a partner. An equation is considered to be in *factored form* when the product of the factors is equal to 0. Match each factored form of the equation with its equivalent standard form and nonstandard form.

	Factored Form		Standard Form		Nonstandard Form
a.	$(x - 1)(x - 3) = 0$	A.	$x^2 - x - 2 = 0$	1.	$x^2 - 5x = -6$
b.	$(x - 2)(x - 3) = 0$	B.	$x^2 + x - 2 = 0$	2.	$(x - 1)^2 = 4$
c.	$(x + 1)(x - 2) = 0$	C.	$x^2 - 4x + 3 = 0$	3.	$x^2 - x = 2$
d.	$(x - 1)(x + 2) = 0$	D.	$x^2 - 5x + 6 = 0$	4.	$x(x + 1) = 2$
e.	$(x + 1)(x - 3) = 0$	E.	$x^2 - 2x - 3 = 0$	5.	$x^2 - 4x = -3$

SELECTING TOOLS
To be proficient in math, you need to consider using tools such as a table or a spreadsheet to organize your results.

EXPLORATION 2 Writing a Conjecture

Work with a partner. Substitute 1, 2, 3, 4, 5, and 6 for x in each equation and determine whether the equation is true. Organize your results in a table. Write a conjecture describing what you discovered.

a. $(x - 1)(x - 2) = 0$
b. $(x - 2)(x - 3) = 0$
c. $(x - 3)(x - 4) = 0$
d. $(x - 4)(x - 5) = 0$
e. $(x - 5)(x - 6) = 0$
f. $(x - 6)(x - 1) = 0$

EXPLORATION 3 Special Properties of 0 and 1

Work with a partner. The numbers 0 and 1 have special properties that are shared by no other numbers. For each of the following, decide whether the property is true for 0, 1, both, or neither. Explain your reasoning.

a. When you add [redacted] to a number n, you get n.
b. If the product of two numbers is [redacted], then at least one of the numbers is 0.
c. The square of [redacted] is equal to itself.
d. When you multiply a number n by [redacted], you get n.
e. When you multiply a number n by [redacted], you get 0.
f. The opposite of [redacted] is equal to itself.

Communicate Your Answer
4. How can you solve a polynomial equation?
5. One of the properties in Exploration 3 is called the Zero-Product Property. It is one of the most important properties in all of algebra. Which property is it? Why do you think it is called the Zero-Product Property? Explain how it is used in algebra and why it is so important.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.8.A The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

ANSWERS
1. a. C; 5
 b. D; 1
 c. A; 3
 d. B; 4
 e. E; 2
2. See Additional Answers.
3. a. 0; Adding 0 does not change a value, but adding 1 increases the value.
 b. 0; The product of 0 and any number is always 0, never 1.
 c. both; $0^2 = 0$ and $1^2 = 1$
 d. 1; The product of any number and 0 is 0, the product of any number and 1 is the number.
 e. 0; The product of any number and 0 is 0, the product of any number and 1 is the number.
 f. 0; Zero is neither positive nor negative, so it is its own opposite.
4. Set each polynomial factor equal to 0 and solve.
5. See Additional Answers.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the Essential Question. Have students work in pairs to complete the explorations and Communicate Your Answer.

Beginning/Advanced Beginning students read the headings, equations, and parts (a)–(f) of Exploration 3 while asking Advanced students for help as needed. Advanced students read instructions and Communicate Your Answer questions, requesting help from teachers or classroom aides as needed.

Intermediate/Advanced High Intermediate and Advanced High students alternate reading the explorations and Communicate Your Answer questions, and request clarification from each other, teachers, or classroom aides as needed.

ELPS 4.F.9 Use support from peers and teachers to develop grasp of language structures needed to comprehend increasingly challenging language.

Extra Example 1

Solve each equation.

a. $3x(x - 6) = 0$

The roots are $x = 0$ and $x = 6$.

b. $(x + 5)(x - 4) = 0$

The roots are $x = -5$ and $x = 4$.

MONITORING PROGRESS ANSWERS

1. $x = 0, x = 1$
2. $t = 0, t = -2$
3. $z = 4, z = 6$

7.5 Lesson

Core Vocabulary

factored form, *p. 364*
Zero-Product Property, *p. 364*
roots, *p. 364*
repeated roots, *p. 365*

Previous

polynomial
standard form
greatest common factor (GCF)
monomial

What You Will Learn

- Use the Zero-Product Property.
- Factor polynomials using the GCF.
- Use the Zero-Product Property to solve real-life problems.

Using the Zero-Product Property

A polynomial is in **factored form** when it is written as a product of factors.

Standard form	Factored form
$x^2 + 2x$	$x(x + 2)$
$x^2 + 5x - 24$	$(x - 3)(x + 8)$

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of a polynomial equation are also called **roots**.

Core Concept

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 Solving Polynomial Equations

Solve each equation.

a. $2x(x - 4) = 0$ **b.** $(x - 3)(x - 9) = 0$

SOLUTION

a. $2x(x - 4) = 0$ Write equation.

$2x = 0$ *or* $x - 4 = 0$ Zero-Product Property

$x = 0$ *or* $x = 4$ Solve for x.

▶ The roots are $x = 0$ and $x = 4$.

b. $(x - 3)(x - 9) = 0$ Write equation.

$x - 3 = 0$ *or* $x - 9 = 0$ Zero-Product Property

$x = 3$ *or* $x = 9$ Solve for x.

▶ The roots are $x = 3$ and $x = 9$.

Check

To check the solutions of Example 1(a), substitute each solution in the original equation.

$2(0)(0 - 4) \stackrel{?}{=} 0$

$0(-4) \stackrel{?}{=} 0$

$0 = 0$ ✓

$2(4)(4 - 4) \stackrel{?}{=} 0$

$8(0) \stackrel{?}{=} 0$

$0 = 0$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Check your solutions.

1. $x(x - 1) = 0$
2. $3t(t + 2) = 0$
3. $(z - 4)(z - 6) = 0$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

When two or more roots of an equation are the same number, the equation has **repeated roots**.

EXAMPLE 2 Solving Polynomial Equations

Solve each equation.

a. $(2x + 7)(2x - 7) = 0$ **b.** $(x - 1)^2 = 0$ **c.** $(x + 1)(x - 3)(x - 2) = 0$

SOLUTION

a. $(2x + 7)(2x - 7) = 0$ Write equation.

$2x + 7 = 0$ *or* $2x - 7 = 0$ Zero-Product Property

$x = -\frac{7}{2}$ *or* $x = \frac{7}{2}$ Solve for x.

▶ The roots are $x = -\frac{7}{2}$ and $x = \frac{7}{2}$.

b. $(x - 1)^2 = 0$ Write equation.

$(x - 1)(x - 1) = 0$ Expand equation.

$x - 1 = 0$ *or* $x - 1 = 0$ Zero-Product Property

$x = 1$ *or* $x = 1$ Solve for x.

▶ The equation has repeated roots of $x = 1$.

c. $(x + 1)(x - 3)(x - 2) = 0$ Write equation.

$x + 1 = 0$ *or* $x - 3 = 0$ *or* $x - 2 = 0$ Zero-Product Property

$x = -1$ *or* $x = 3$ *or* $x = 2$ Solve for x.

▶ The roots are $x = -1$, $x = 3$, and $x = 2$.

STUDY TIP

You can extend the Zero-Product Property to products of more than two real numbers.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Check your solutions.

4. $(3s + 5)(5s + 8) = 0$ **5.** $(b + 7)^2 = 0$ **6.** $(d - 2)(d + 6)(d + 8) = 0$

Factoring Polynomials Using the GCF

To solve a polynomial equation using the Zero-Product Property, you may need to *factor* the polynomial, or write it as a product of other polynomials. Look for the *greatest common factor* (GCF) of the terms of the polynomial. This is a monomial that divides evenly into each term.

EXAMPLE 3 Finding the Greatest Common Monomial Factor

Factor out the greatest common monomial factor from $4x^4 + 24x^3$.

SOLUTION

The GCF of 4 and 24 is 4. The GCF of x^4 and x^3 is x^3. So, the greatest common monomial factor of the terms is $4x^3$.

▶ So, $4x^4 + 24x^3 = 4x^3(x + 6)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. Factor out the greatest common monomial factor from $8y^2 - 24y$.

Extra Example 2

Solve each equation.

a. $(4x + 5)(4x - 5) = 0$

The roots are $x = -\frac{5}{4}$ and $x = \frac{5}{4}$.

b. $(c + 6)^2 = 0$

The equation has repeated roots of $c = -6$.

c. $(a + 5)(a - 2)(a - 7) = 0$

The roots are $a = -5$, $a = 2$, and $a = 7$.

Extra Example 3

Factor out the greatest common monomial factor from $12x^3 + 3x^2$. $3x^2(4x + 1)$

MONITORING PROGRESS ANSWERS

4. $s = -\frac{5}{3}, s = -\frac{8}{5}$

5. $b = -7$

6. $d = 2, d = -6, d = -8$

7. $8y(y - 3)$

Differentiated Instruction

Visual

When using the GCF to factor, some students may benefit from seeing the GCF in each term of the polynomial. For example, write $4x^4 + 24x^3$ in Example 3 as $4x^3(x) + 4x^3(6)$. This may help students see that $4x^3$ is a factor of each term and that $4x^4 + 24x^3 = 4x^3(x + 6)$.

Extra Example 4

a. Solve $4x^2 + 12x = 0$.

The roots are $x = 0$ and $x = -3$.

b. Solve $-10a^2 = 8a$.

The roots are $a = 0$ and $a = -\frac{4}{5}$.

Extra Example 5

You can model the arch of a building entrance by using the equation $y = -\frac{1}{3}(x + 6)(x - 6)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the arch at the ground. 12 feet

MONITORING PROGRESS ANSWERS

8. $a = 0, a = -5$

9. $s = 0, s = 3$

10. $x = 0, x = \frac{1}{2}$

11. 8 ft

EXAMPLE 4 Solving Equations by Factoring

Solve (a) $2x^2 + 8x = 0$ and (b) $6n^2 = 15n$.

SOLUTION

a.

$2x^2 + 8x = 0$ — Write equation.

$2x(x + 4) = 0$ — Factor left side.

$2x = 0$ *or* $x + 4 = 0$ — Zero-Product Property

$x = 0$ *or* $x = -4$ — Solve for x.

▶ The roots are $x = 0$ and $x = -4$.

b.

$6n^2 = 15n$ — Write equation.

$6n^2 - 15n = 0$ — Subtract $15n$ from each side.

$3n(2n - 5) = 0$ — Factor left side.

$3n = 0$ *or* $2n - 5 = 0$ — Zero-Product Property

$n = 0$ *or* $n = \frac{5}{2}$ — Solve for n.

▶ The roots are $n = 0$ and $n = \frac{5}{2}$.

Monitoring Progress

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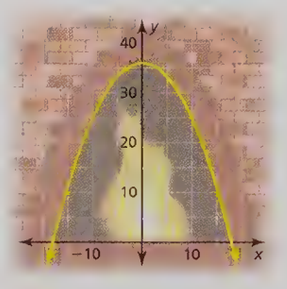
Solve the equation. Check your solutions.

8. $a^2 + 5a = 0$ **9.** $3s^2 - 9s = 0$ **10.** $4x^2 = 2x$

Solving Real-Life Problems

EXAMPLE 5 Modeling with Mathematics

You can model the arch of a fireplace using the equation $y = -\frac{1}{9}(x + 18)(x - 18)$, where x and y are measured in inches. The x-axis represents the floor. Find the width of the arch at floor level.



SOLUTION

Use the x-coordinates of the points where the arch meets the floor to find the width. At floor level, $y = 0$. So, substitute 0 for y and solve for x.

$y = -\frac{1}{9}(x + 18)(x - 18)$ — Write equation.

$0 = -\frac{1}{9}(x + 18)(x - 18)$ — Substitute 0 for y.

$0 = (x + 18)(x - 18)$ — Multiply each side by -9.

$x + 18 = 0$ *or* $x - 18 = 0$ — Zero-Product Property

$x = -18$ *or* $x = 18$ — Solve for x.

The width is the distance between the x-coordinates, -18 and 18.

▶ So, the width of the arch at floor level is $|-18 - 18| = 36$ inches.

Monitoring Progress

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11. You can model the entrance to a mine shaft using the equation $y = -\frac{1}{2}(x + 4)(x - 4)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the entrance at ground level.

Closure

- **Exit Ticket:** Write a polynomial equation that has -6 and 4 as solutions. *Sample answer:* $(x + 6)(x - 4) = 0$

7.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to use the Zero-Product Property to find the solutions of the equation $3x(x - 6) = 0$.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find *both* answers.

Solve the equation $(2k + 4)(k - 3) = 0$.

Find the values of k for which $2k + 4 = 0$ *or* $k - 3 = 0$.

Find the value of k for which $(2k + 4) + (k - 3) = 0$.

Find the roots of the equation $(2k + 4)(k - 3) = 0$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, solve the equation. *(See Example 1.)*

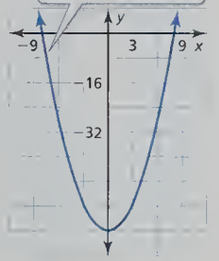
3. $x(x + 7) = 0$
4. $r(r - 10) = 0$
5. $12t(t - 5) = 0$
6. $-2v(v + 1) = 0$
7. $(s - 9)(s - 1) = 0$
8. $(y + 2)(y - 6) = 0$

In Exercises 9–20, solve the equation. *(See Example 2.)*

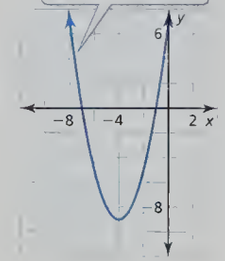
9. $(2a - 6)(3a + 15) = 0$
10. $(4q + 3)(q + 2) = 0$
11. $(5m + 4)^2 = 0$
12. $(h - 8)^2 = 0$
13. $(3 - 2g)(7 - g) = 0$
14. $(2 - 4d)(2 + 4d) = 0$
15. $z(z + 2)(z - 1) = 0$
16. $5p(2p - 3)(p + 7) = 0$
17. $(r - 4)^2(r + 8) = 0$
18. $w(w - 6)^2 = 0$
19. $(15 - 5c)(5c + 5)(-c + 6) = 0$
20. $(2 - n)\left(6 + \frac{2}{3}n\right)(n - 2) = 0$

In Exercises 21–24, find the x-coordinates of the points where the graph crosses the x-axis.

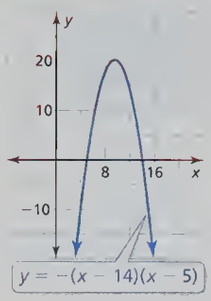
21. $y = (x - 8)(x + 8)$



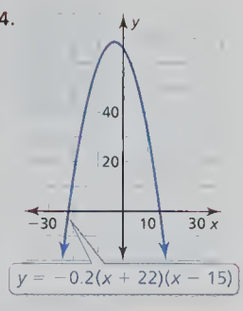
22. $y = (x + 1)(x + 7)$



23.



24.



In Exercises 25–30, factor the polynomial. *(See Example 3.)*

25. $5z^2 + 45z$
26. $6d^2 - 21d$
27. $3y^3 - 9y^2$
28. $20x^3 + 30x^2$
29. $5n^6 + 2n^5$
30. $12a^4 + 8a$

In Exercises 31–36, solve the equation. *(See Example 4.)*

31. $4p^2 - p = 0$
32. $6m^2 + 12m = 0$
33. $25c + 10c^2 = 0$
34. $18q - 2q^2 = 0$
35. $3n^2 = 9n$
36. $-28r = 4r^2$

37. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

✗
$6x(x + 5) = 0$
$x + 5 = 0$
$x = -5$
The root is $x = -5$.

34. $q = 0, q = 9$
35. $n = 0, n = 3$
36. $r = 0, r = -7$
37. also need to set $6x = 0$ and solve; $6x = 0$ or $x + 5 = 0$; $x = 0$ or $x = -5$; The roots are $x = 0$ and $x = -5$.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–39 odd, 42, 44, 48–53

Average: 1, 2–40 even, 41, 42, 44, 48–53

Advanced: 1, 2, 6, 8, 16–24 even, 28–38 even, 40–53

HOMEWORK CHECK

Basic: 3, 9, 25, 33, 39

Average: 4, 10, 26, 32, 40

Advanced: 8, 16, 28, 32, 40

ANSWERS

1. Set $3x = 0$ and $x - 6 = 0$, then solve both of the equations to get the solutions $x = 0$ and $x = 6$.
2. Find the value of k for which $(2k + 4) + (k - 3) = 0$; $k = -\frac{1}{3}$; $k = -2$, $k = 3$
3. $x = 0, x = -7$
4. $r = 0, r = 10$
5. $t = 0, t = 5$
6. $v = 0, v = -1$
7. $s = 9, s = 1$
8. $y = -2, y = 6$
9. $a = 3, a = -5$
10. $q = -\frac{3}{4}, q = -2$
11. $m = -\frac{4}{5}$
12. $h = 8$
13. $g = \frac{3}{2}, g = 7$
14. $d = \frac{1}{2}, d = -\frac{1}{2}$
15. $z = 0, z = -2, z = 1$
16. $p = 0, p = \frac{3}{2}, p = -7$
17. $r = 4, r = -8$
18. $w = 0, w = 6$
19. $c = 3, c = -1, c = 6$
20. $n = 2, n = -9$
21. $x = 8, x = -8$
22. $x = -1, x = -7$
23. $x = 14, x = 5$
24. $x = -22, x = 15$
25. $5z(z + 9)$
26. $3d(2d - 7)$
27. $3y^2(y - 3)$
28. $10x^2(2x + 3)$
29. $n^5(5n + 2)$
30. $4a(3a^3 + 2)$
31. $p = 0, p = \frac{1}{4}$
32. $m = 0, m = -2$
33. $c = 0, c = -\frac{5}{2}$

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ANSWERS

38. cannot divide both sides by y, because y could be 0 and division by 0 is undefined; $3y^2 - 21y = 0$; $3y(y - 7) = 0$; $3y = 0$ or $y - 7 = 0$; $y = 0$ or $y = 7$; The roots are $y = 0$ and $y = 7$.

39. 20 ft

40. **a.** 630 ft

b. 630 ft

41. $x = 0$, $x = 0.3$ sec; The roots represent the times when the penguin is at water level. $x = 0$ is when it leaves the water, and $x = 0.3$ second is when it returns to the water after the leap.

42. $-$; $+$; The point of intersection on the positive x-axis is farther from the origin than the point of intersection on the negative x-axis, so the positive solution to the equation will be the one with the greater absolute value.

43. 2; x-intercepts occur when $y = 0$ and the equation has 2 roots when $y = 0$.

44. no; If $a = b$, then there is a repeated root and the graph will only have one x-intercept.

45–52. See Additional Answers.

Mini-Assessment

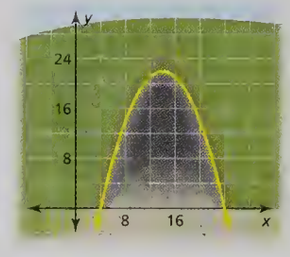
Solve the equation.

1. $5x(x - 4) = 0$
 The roots are $x = 0$ and $x = 4$.
2. $(3x + 8)(3x - 8) = 0$
 The roots are $x = -\frac{8}{3}$ and $x = \frac{8}{3}$.
3. $3x^2 + 21x = 0$
 The roots are $x = 0$ and $x = -7$.
4. Factor out the greatest common monomial factor from $6x^4 + 8x^3$.
 $2x^3(3x + 4)$
5. You can model the entrance to a train tunnel using the equation $y = -\frac{5}{16}(x + 8)(x - 8)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the entrance at ground level. 16 feet

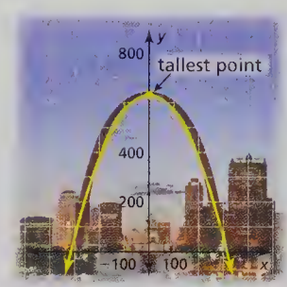
38. ERROR ANALYSIS Describe and correct the error in solving the equation.

✗
$3y^2 = 21y$
$3y = 21$
$y = 7$
The root is $y = 7$.

39. MODELING WITH MATHEMATICS The entrance of a tunnel can be modeled by $y = -\frac{11}{50}(x - 4)(x - 24)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the tunnel at ground level. *(See Example 5.)*



40. MODELING WITH MATHEMATICS The Gateway Arch in St. Louis can be modeled by $y = -\frac{2}{315}(x + 315)(x - 315)$, where x and y are measured in feet. The x-axis represents the ground.

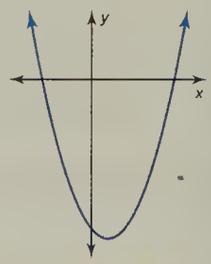


a. Find the width of the arch at ground level.

b. How tall is the arch?

41. MODELING WITH MATHEMATICS A penguin leaps out of the water while swimming. This action is called porpoising. The height y (in feet) of a porpoising penguin can be modeled by $y = -16x^2 + 4.8x$, where x is the time (in seconds) since the penguin leaped out of the water. Find the roots of the equation when $y = 0$. Explain what the roots mean in this situation.

42. HOW DO YOU SEE IT? Use the graph to fill in each blank in the equation with the symbol $+$ or $-$. Explain your reasoning.



$y = (x \quad 5)(x \quad 3)$

43. CRITICAL THINKING How many x-intercepts does the graph of $y = (2x + 5)(x - 9)^2$ have? Explain.

44. MAKING AN ARGUMENT Your friend says that the graph of the equation $y = (x - a)(x - b)$ always has two x-intercepts for any values of a and b. Is your friend correct? Explain.

45. CRITICAL THINKING Does the equation $(x^2 + 3)(x^4 + 1) = 0$ have any real roots? Explain.

46. THOUGHT PROVOKING Write a polynomial equation of degree 4 whose only roots are $x = 1$, $x = 2$, and $x = 3$.

47. REASONING Find the values of x in terms of y that are solutions of each equation.

a. $(x + y)(2x - y) = 0$

b. $(x^2 - y^2)(4x + 16y) = 0$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

List the factor pairs of the number. *(Skills Review Handbook)*

48. 10 **49.** 18

50. 30 **51.** 48

52. 36 **53.** 52

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.1–7.5 What Did You Learn?

Core Vocabulary

monomial, *p. 338*
degree of a monomial, *p. 338*
polynomial, *p. 339*
binomial, *p. 339*
trinomial, *p. 339*
degree of a polynomial, *p. 339*
standard form, *p. 339*
leading coefficient, *p. 339*
closed, *p. 340*
FOIL Method, *p. 347*
polynomial long division, *p. 358*
synthetic division, *p. 360*
factored form, *p. 364*
Zero-Product Property, *p. 364*
roots, *p. 364*
repeated roots, *p. 365*

Core Concepts

Section 7.1

Polynomials, *p. 339*
Adding Polynomials, *p. 340*
Subtracting Polynomials, *p. 340*

Section 7.2

Multiplying Binomials, *p. 346*
FOIL Method, *p. 347*
Multiplying Binomials and Trinomials, *p. 348*

Section 7.3

Square of a Binomial Pattern, *p. 352*
Sum and Difference Pattern, *p. 353*

Section 7.4

Polynomial Long Division, *p. 358*
Synthetic Division, *p. 360*

Section 7.5

Zero-Product Property, *p. 364*
Factoring Polynomials Using the GCF, *p. 365*

Mathematical Thinking

1. Explain how you wrote the polynomial in Exercise 11 on page 355. Is there another method you can use to write the same polynomial?
2. Find a shortcut for exercises like Exercise 7 on page 367 when the variable has a coefficient of 1. Does your shortcut work when the coefficient is *not* 1?

Study Skills

Preparing for a Test

- Review examples of each type of problem that could appear on the test.
- Review the homework problems your teacher assigned.
- Take a practice test.



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ANSWERS

1. *Sample answer:* Represent the sides as $x + 4$ and use the square of a binomial pattern; Yes, you could calculate the area of each shaded region and add those areas together.
2. The solutions are the constant terms with the opposite sign; no

ANSWERS

1. $-8q^3$; 3; -8; monomial
2. $3z^4 + 7.4z^2 - 1.3z$; 4; 3; trinomial
3. $x + 9$
4. $-5n + 16$
5. $-2p^2 + 7p - 15$
6. $-2ab + 2b^2$
7. $w^2 + 13w + 42$
8. $-8d^2 + 26d - 15$
9. $y^3 + 11y^2 + 15y - 27$
10. $9z^2 - 25$
11. $t^2 + 10t + 25$
12. $4q^4 - 24q^2 + 36$
13. $4 + \frac{6}{2x - 1}$
14. $3x - 7 + \frac{43}{3x + 5}$
15. $7x + 28 + \frac{96}{x - 4}$
16. $x = 0, x = 3$
17. $g = 8$
18. $p = -\frac{7}{3}, p = \frac{7}{3}, p = -8$
19. $y = 0, y = 8, y = -\frac{1}{2}$
20. a. $(8x + 240)$ in.
 b. $(4x^2 + 240x + 3456)$ in.^{2}
 c. 272 in.; 4480 in.^{2}
21. a. $1000r^2 + 2000r + 1000$
 b. \$1060.90
 c. no; The balance after 3 years is represented by the expression $1000(1 + 0.03)^3$, which is only about \$1092.73.
22. 144 in.

7.1–7.5 Quiz

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms. *(Section 7.1)*

1. $-8q^3$
2. $-1.3z + 3z^4 + 7.4z^2$

Find the sum or difference. *(Section 7.1)*

3. $(2x + 5) + (-x + 4)$
4. $(-3n + 9) - (2n - 7)$
5. $(-p^2 + 4p) - (p^2 - 3p + 15)$
6. $(a^2 - 3ab + b^2) + (-a^2 + ab + b^2)$

Find the product. *(Section 7.2 and Section 7.3)*

7. $(w + 6)(w + 7)$
8. $(3 - 4d)(2d - 5)$
9. $(y + 9)(y^2 + 2y - 3)$
10. $(3z - 5)(3z + 5)$
11. $(t + 5)^2$
12. $(2q^2 - 6)^2$

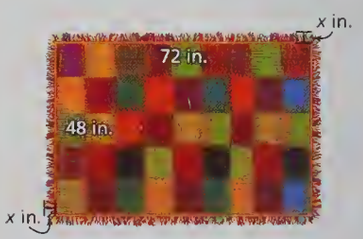
Divide. *(Section 7.4)*

13. $(8x + 2) \div (2x - 1)$
14. $(9x^2 - 6x + 8) \div (3x + 5)$
15. $(7x^2 - 16) \div (x - 4)$

Solve the equation. *(Section 7.5)*

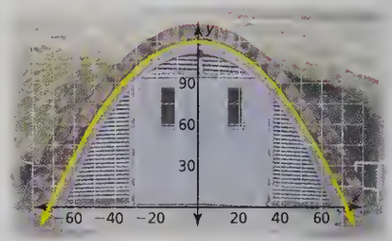
16. $5x^2 - 15x = 0$
17. $(8 - g)(8 - g) = 0$
18. $(3p + 7)(3p - 7)(p + 8) = 0$
19. $-3y(y - 8)(2y + 1) = 0$

20. You are making a blanket with a fringe border of equal width on each side. *(Section 7.1 and Section 7.2)*
 a. Write a polynomial that represents the perimeter of the blanket including the fringe.
 b. Write a polynomial that represents the area of the blanket including the fringe.
 c. Find the perimeter and the area of the blanket including the fringe when the width of the fringe is 4 inches.



21. You are saving money to buy an electric guitar. You deposit \$1000 in an account that earns interest compounded annually. The expression $1000(1 + r)^2$ represents the balance after 2 years, where r is the annual interest rate in decimal form. *(Section 7.3)*
 a. Write the polynomial in standard form that represents the balance of your account after 2 years.
 b. The interest rate is 3%. What is the balance of your account after 2 years?
 c. The guitar costs \$1100. Do you have enough money in your account *after 3 years*? Explain.

22. The front of a storage bunker can be modeled by $y = -\frac{5}{216}(x - 72)(x + 72)$, where x and y are measured in inches. The x-axis represents the ground. Find the width of the bunker at ground level. *(Section 7.5)*



7.6 Factoring $x^2 + bx + c$

Texas Essential Knowledge and Skills
A.8.A
A.10.E

Essential Question How can you use algebra tiles to factor the trinomial $x^2 + bx + c$ into the product of two binomials?

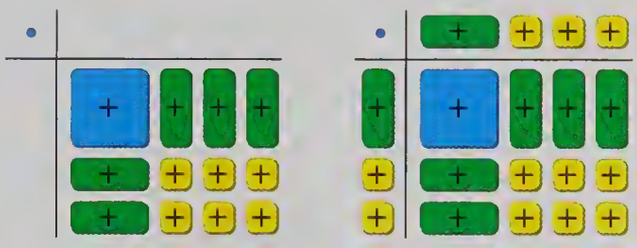
EXPLORATION 1 Finding Binomial Factors

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

Sample $x^2 + 5x + 6$

Step 1 Arrange algebra tiles that model $x^2 + 5x + 6$ into a rectangular array.

Step 2 Use additional algebra tiles to model the dimensions of the rectangle.



Step 3 Write the polynomial in factored form using the dimensions of the rectangle.

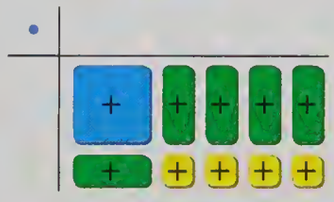
width, length

Area $= x^2 + 5x + 6 = (x + 2)(x + 3)$

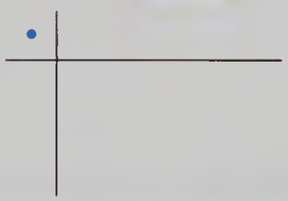
a. $x^2 - 3x + 2 =$ [redacted]



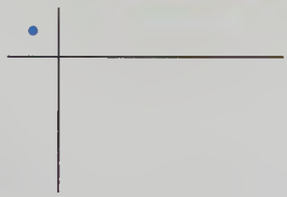
b. $x^2 + 5x + 4 =$ [redacted]



c. $x^2 - 7x + 12 =$ [redacted]



d. $x^2 + 7x + 12 =$ [redacted]



REASONING
To be proficient in math, you need to understand a situation abstractly and represent it symbolically.

Communicate Your Answer

2. How can you use algebra tiles to factor the trinomial $x^2 + bx + c$ into the product of two binomials?
3. Describe a strategy for factoring the trinomial $x^2 + bx + c$ that does not use algebra tiles.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

3. Find two integer factors of c that have a sum of b, then write the binomial factors by adding each integer factor to x.

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- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.8.A The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

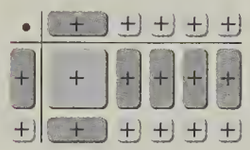
A.10.E The student is expected to factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

ANSWERS

1. **a.** $(x - 1)(x - 2)$



b. $(x + 1)(x + 4)$



c. $(x - 4)(x - 3)$



d. $(x + 4)(x + 3)$



2. Arrange algebra tiles that model the trinomial into a rectangular array, use additional algebra tiles to model the dimensions of the rectangle, then write the polynomial in factored form using the dimensions of the rectangle.

Extra Example 1

Factor $x^2 + 9x + 14$.

$(x + 2)(x + 7)$

MONITORING PROGRESS ANSWERS

1. $(x + 1)(x + 6)$
2. $(x + 1)(x + 8)$

7.6 Lesson

What You Will Learn

- Factor $x^2 + bx + c$.
- Use factoring to solve real-life problems.

Core Vocabulary

Previous

polynomial

FOIL Method

Zero-Product Property

Factoring $x^2 + bx + c$

Writing a polynomial as a product of factors is called *factoring*. To factor $x^2 + bx + c$ as $(x + p)(x + q)$, you need to find p and q such that $p + q = b$ and $pq = c$.

$$(x + p)(x + q) = x^2 + px + qx + pq$$
$$= x^2 + (p + q)x + pq$$

Core Concept

Factoring $x^2 + bx + c$ When c Is Positive

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is positive, p and q have the same sign as b.

Examples $x^2 + 6x + 5 = (x + 1)(x + 5)$

$x^2 - 6x + 5 = (x - 1)(x - 5)$

EXAMPLE 1 Factoring $x^2 + bx + c$ When b and c Are Positive

Factor $x^2 + 10x + 16$.

SOLUTION

Notice that $b = 10$ and $c = 16$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is also positive, p and q must each be positive so that $p + q$ is positive.

Find two positive integer factors of 16 whose sum is 10.

Factors of 16	Sum of factors
1, 16	17
2, 8	10
4, 4	8

The values of p and q are 2 and 8.

So, $x^2 + 10x + 16 = (x + 2)(x + 8)$.

Check

Use the FOIL Method.

$(x + 2)(x + 8)$

$= x^2 + 8x + 2x + 16$

$= x^2 + 10x + 16$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

1. $x^2 + 7x + 6$
2. $x^2 + 9x + 8$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

Pair Activity

Have students work in pairs to factor polynomials. Each student factors a different polynomial. Then students take turns explaining their solutions to their partners.

EXAMPLE 2 Factoring $x^2 + bx + c$ When b Is Negative and c Is Positive

Factor $x^2 - 8x + 12$.

SOLUTION

Notice that $b = -8$ and $c = 12$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is negative, p and q must each be negative so that $p + q$ is negative.

Find two negative integer factors of 12 whose sum is -8.

Factors of 12	$-1, -12$	$-2, -6$	$-3, -4$
Sum of factors	-13	-8	-7

The values of p and q are -2 and -6.

▶ So, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Check

Use the FOIL Method.

$(x - 2)(x - 6)$

$= x^2 - 6x - 2x + 12$

$= x^2 - 8x + 12$ ✓

Core Concept

Factoring $x^2 + bx + c$ When c Is Negative

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is negative, p and q have different signs.

Example $x^2 - 4x - 5 = (x + 1)(x - 5)$

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor $x^2 + 4x - 21$.

SOLUTION

Notice that $b = 4$ and $c = -21$. Because c is negative, the factors p and q must have different signs so that pq is negative.

Find two integer factors of -21 whose sum is 4.

Factors of -21	$-21, 1$	$-1, 21$	$-7, 3$	$-3, 7$
Sum of factors	-20	20	-4	4

The values of p and q are -3 and 7.

▶ So, $x^2 + 4x - 21 = (x - 3)(x + 7)$.

Check

Use the FOIL Method.

$(x - 3)(x + 7)$

$= x^2 + 7x - 3x - 21$

$= x^2 + 4x - 21$ ✓

Monitoring Progress

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Factor the polynomial.

3. $w^2 - 4w + 3$
4. $n^2 - 12n + 35$
5. $x^2 - 14x + 24$
6. $x^2 + 2x - 15$
7. $y^2 + 13y - 30$
8. $v^2 - v - 42$

Extra Example 2

Factor $x^2 - 12x + 27$.

$(x - 3)(x - 9)$

Extra Example 3

Factor $x^2 + 7x - 8$.

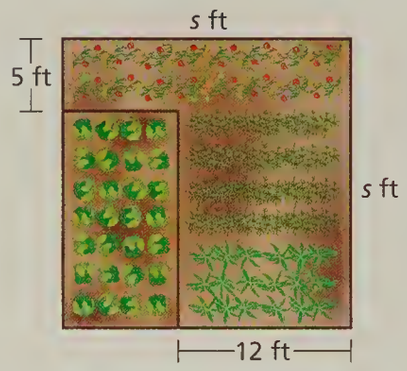
$(x - 1)(x + 8)$

MONITORING PROGRESS ANSWERS

3. $(w - 1)(w - 3)$
4. $(n - 5)(n - 7)$
5. $(x - 2)(x - 12)$
6. $(x - 3)(x + 5)$
7. $(y - 2)(y + 15)$
8. $(v + 6)(v - 7)$

Extra Example 4

A gardener plants a rectangular lettuce patch in the southwest corner of a square garden. The area of the lettuce patch is 120 square feet. What is the area of the square garden?



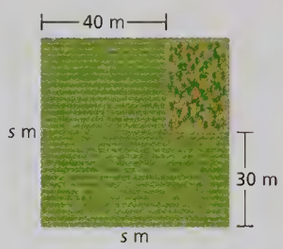
400 square feet

MONITORING PROGRESS ANSWERS

9. 2500 m^2

Solving Real-Life Problems

EXAMPLE 4 **Solving a Real-Life Problem**



A farmer plants a rectangular pumpkin patch in the northeast corner of a square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?

SOLUTION

1. **Understand the Problem** You are given the area of the pumpkin patch, the difference of the side length of the square plot and the length of the pumpkin patch, and the difference of the side length of the square plot and the width of the pumpkin patch.
2. **Make a Plan** The length of the pumpkin patch is $(s - 30)$ meters and the width is $(s - 40)$ meters. Write and solve an equation to find the side length s. Then use the solution to find the area of the square plot of land.
3. **Solve the Problem** Use the equation for the area of a rectangle to write and solve an equation to find the side length s of the square plot of land.

$$\begin{aligned} 600 &= (s - 30)(s - 40) && \text{Write an equation.} \\ 600 &= s^2 - 70s + 1200 && \text{Multiply.} \\ 0 &= s^2 - 70s + 600 && \text{Subtract 600 from each side.} \\ 0 &= (s - 10)(s - 60) && \text{Factor the polynomial.} \\ s - 10 = 0 \quad &\text{or} \quad s - 60 = 0 && \text{Zero-Product Property} \\ s = 10 \quad &\text{or} \quad s = 60 && \text{Solve for } s. \end{aligned}$$

▶ So, the area of the square plot of land is $60(60) = 3600$ square meters.

4. **Look Back** Use the diagram to check that you found the correct side length. Using $s = 60$, the length of the pumpkin patch is $60 - 30 = 30$ meters and the width is $60 - 40 = 20$ meters. So, the area of the pumpkin patch is 600 square meters. This matches the given information and confirms the side length is 60 meters, which gives an area of 3600 square meters.

STUDY TIP

The diagram shows that the side length is more than 40 meters, so a side length of 10 meters does not make sense in this situation. The side length is 60 meters.

Monitoring Progress

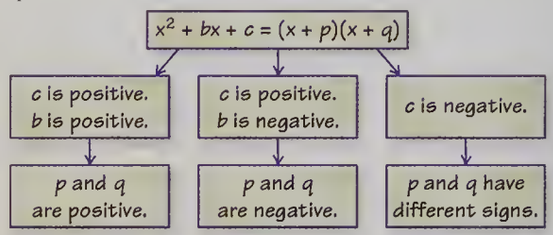
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9. **WHAT IF?** The area of the pumpkin patch is 200 square meters. What is the area of the square plot of land?

Concept Summary

Factoring $x^2 + bx + c$ as $(x + p)(x + q)$

The diagram shows the relationships between the signs of b and c and the signs of p and q.



Closure

- Explain your strategy for factoring the following polynomials.
 a. $x^2 - 2x - 15$ $(x - 5)(x + 3)$
 b. $x^2 + 2x - 15$ $(x + 5)(x - 3)$
 c. $x^2 + 8x + 15$ $(x + 5)(x + 3)$
 d. $x^2 - 8x + 15$ $(x - 5)(x - 3)$

SUPPORTING English Language Learners

Present the material on page 374 orally. Have students work in pairs and collaborate with one another to complete Monitoring Progress Question 9. Remind students to refer to Example 4 as a guide for their work.

Beginning/Advanced Advanced students clarify information for beginners as they work through Question 9.

Intermediate/Advanced High Students clarify information for one another as needed.

ELPS 2.I.4 Demonstrate listening comprehension of increasingly complex spoken English by collaborating with peers commensurate with content and grade-level needs.

7.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** You are factoring $x^2 + 11x - 26$. What do the signs of the terms tell you about the factors? Explain.

2. **OPEN-ENDED** Write a trinomial that can be factored as $(x + p)(x + q)$, where p and q are positive.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $x^2 + 8x + 7$
4. $z^2 + 10z + 21$
5. $n^2 + 9n + 20$
6. $s^2 + 11s + 30$
7. $h^2 + 11h + 18$
8. $y^2 + 13y + 40$

In Exercises 9–14, factor the polynomial. *(See Example 2.)*

9. $v^2 - 5v + 4$
10. $x^2 - 13x + 22$
11. $d^2 - 5d + 6$
12. $k^2 - 10k + 24$
13. $w^2 - 17w + 72$
14. $j^2 - 13j + 42$

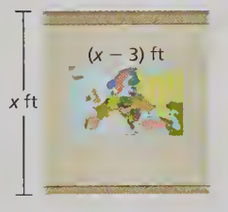
In Exercises 15–24, factor the polynomial. *(See Example 3.)*

15. $x^2 + 3x - 4$
16. $z^2 + 7z - 18$
17. $n^2 + 4n - 12$
18. $s^2 + 3s - 40$
19. $y^2 + 2y - 48$
20. $h^2 + 6h - 27$
21. $x^2 - x - 20$
22. $m^2 - 6m - 7$
23. $-6t - 16 + t^2$
24. $-7y + y^2 - 30$

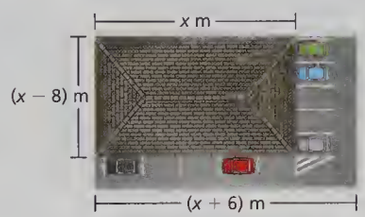
25. **MODELING WITH MATHEMATICS** A projector displays an image on a wall. The area (in square feet) of the projection is represented by $x^2 - 8x + 15$.

a. Write a binomial that represents the height of the projection.

b. Find the perimeter of the projection when the height of the wall is 8 feet.



26. **MODELING WITH MATHEMATICS** A dentist's office and parking lot are on a rectangular piece of land. The area (in square meters) of the land is represented by $x^2 + x - 30$.



a. Write a binomial that represents the width of the land.

b. Find the area of the land when the length of the dentist's office is 20 meters.

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in factoring the polynomial.**

27. ✗ $x^2 + 14x + 48 = (x + 4)(x + 12)$

28. ✗ $s^2 - 17s - 60 = (s - 5)(s - 12)$

In Exercises 29–38, solve the equation.

29. $m^2 + 3m + 2 = 0$
30. $n^2 - 9n + 18 = 0$
31. $x^2 + 5x - 14 = 0$
32. $v^2 + 11v - 26 = 0$
33. $t^2 + 15t = -36$
34. $n^2 - 5n = 24$
35. $a^2 + 5a - 20 = 30$
36. $y^2 - 2y - 8 = 7$
37. $m^2 + 10 = 15m - 34$
38. $b^2 + 5 = 8b - 10$

31. $x = 2, x = -7$
32. $v = 2, v = -13$
33. $t = -3, t = -12$
34. $n = -3, n = 8$
35. $a = 5, a = -10$
36. $y = -3, y = 5$
37. $m = 4, m = 11$
38. $b = 3, b = 5$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–29 odd, 39, 41, 46, 52–55

Average: 1, 2–40 even, 41, 46, 47, 52–55

Advanced: 1, 2, 8, 14, 22–28 even, 36–38 even, 40–55

HOMEWORK CHECK

Basic: 3, 9, 15, 39

Average: 4, 10, 16, 40

Advanced: 8, 14, 22, 40

ANSWERS

1. They have opposite signs; When factoring $x^2 + bx + c = (x + p)(x + q)$, if c is negative, p and q must have opposite signs.
2. *Sample answer:* $x^2 + 5x + 6$
3. $(x + 1)(x + 7)$
4. $(z + 3)(z + 7)$
5. $(n + 4)(n + 5)$
6. $(s + 5)(s + 6)$
7. $(h + 2)(h + 9)$
8. $(y + 5)(y + 8)$
9. $(v - 1)(v - 4)$
10. $(x - 2)(x - 11)$
11. $(d - 2)(d - 3)$
12. $(k - 4)(k - 6)$
13. $(w - 8)(w - 9)$
14. $(j - 6)(j - 7)$
15. $(x - 1)(x + 4)$
16. $(z - 2)(z + 9)$
17. $(n - 2)(n + 6)$
18. $(s - 5)(s + 8)$
19. $(y - 6)(y + 8)$
20. $(h - 3)(h + 9)$
21. $(x + 4)(x - 5)$
22. $(m + 1)(m - 7)$
23. $(t + 2)(t - 8)$
24. $(y + 3)(y - 10)$
25. a. $(x - 5)$ ft
 b. 16 ft
26. a. $(x - 5)$ m
 b. 390 m^2
27. $4 + 12$ is not 14; $= (x + 6)(x + 8)$
28. c is -60, so the signs of the factors must be opposite; $= (s + 3)(s - 20)$
29. $m = -1, m = -2$
30. $n = 3, n = 6$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

39. 100 in.^{2}

40. **a.** 6 in.

b. length: 13 in., width: 8 in.

41. yes; p and q must be factors of -12 that have a sum of b, and -12 has 6 sets of integer factors, -1 and 12, -2 and 6, -3 and 4, -4 and 3, -6 and 2, and -12 and 1.

42. **a.**



$x^2 - x - 6 = (x + 2)(x - 3)$

b.



$x^2 + 2x - 8 = (x - 2)(x + 4)$

43. length: 11 ft, width: 4 ft

44. base: 10 m, height: 7 m

45. $x^2 - 2x - 24 = 0$; Multiply $(x + 4)(x - 6)$.

46. **a.** Look for the x-intercepts and substitute them for p and q in $(x - p)(x - q)$.

b. $(x - 2)(x + 3)$

47. **a.** $-x^2 + 38x$

b. $-x^2 + 38x = 280$; 10 m

48. $(x + 2y)(x + 4y)$

49. $(r + 3s)(r + 4s)$

50. $(a - 2b)(a + 13b)$

51. $(x + 5y)(x - 7y)$

52. *Sample answer:* $2x + 2y = -12$, $3x + 3y = -18$

53. *Sample answer:* $4x + 2y = 6$, $2x + y = 3$

54–55. See Additional Answers.

Mini-Assessment

Factor the polynomial.

1. $x^2 + 13x + 12$ $(x + 1)(x + 12)$
2. $x^2 - 14x + 40$ $(x - 4)(x - 10)$
3. $x^2 + 3x - 54$ $(x - 6)(x + 9)$

39. MODELING WITH MATHEMATICS You trimmed a large square picture so that you could fit it into a frame. The area of the cut picture is 20 square inches. What is the area of the original picture? *(See Example 4.)*



40. MODELING WITH MATHEMATICS A web browser is open on your computer screen.



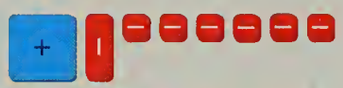
a. The area of the browser window is 24 square inches. Find the length of the browser window x.

b. The browser covers $\frac{3}{13}$ of the screen. What are the dimensions of the screen?

41. MAKING AN ARGUMENT Your friend says there are six integer values of b for which the trinomial $x^2 + bx - 12$ has two binomial factors of the form $(x + p)$ and $(x + q)$. Is your friend correct? Explain.

42. THOUGHT PROVOKING Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a.

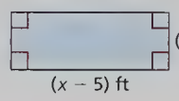


b.

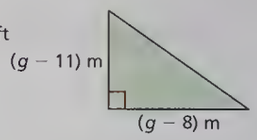


MATHEMATICAL CONNECTIONS **In Exercises 43 and 44, find the dimensions of the polygon with the given area.**

43. Area $= 44$ ft^2

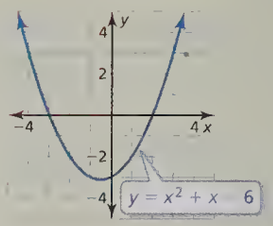


44. Area $= 35$ m^2



45. REASONING Write an equation of the form $x^2 + bx + c = 0$ that has the solutions $x = -4$ and $x = 6$. Explain how you found your answer.

46. HOW DO YOU SEE IT? The graph of $y = x^2 + x - 6$ is shown.



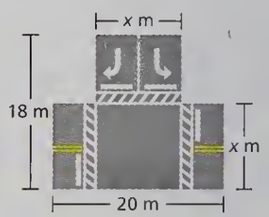
a. Explain how you can use the graph to factor the polynomial $x^2 + x - 6$.

b. Factor the polynomial.

47. PROBLEM SOLVING Road construction workers are paving the area shown.

a. Write an expression that represents the area being paved.

b. The area being paved is 280 square meters. Write and solve an equation to find the width of the road x.



USING STRUCTURE **In Exercises 48–51, factor the polynomial.**

48. $x^2 + 6xy + 8y^2$

49. $r^2 + 7rs + 12s^2$

50. $a^2 + 11ab - 26b^2$

51. $x^2 - 2xy - 35y^2$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write two equations in standard form that are equivalent to the given equation. *(Section 4.3)*

52. $x + y = -6$

53. $8x + 4y = 12$

54. $5x - 3y = -7$

55. $-2x + 6y = 10$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.7 Factoring $ax^2 + bx + c$

Texas Essential Knowledge and Skills
A.8.A
A.10.D
A.10.E

Essential Question How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

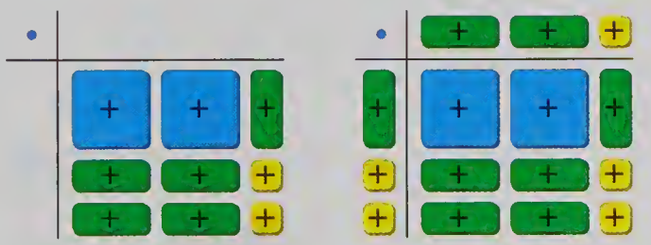
EXPLORATION 1 **Finding Binomial Factors**

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

Sample $2x^2 + 5x + 2$

Step 1 Arrange algebra tiles that model $2x^2 + 5x + 2$ into a rectangular array.

Step 2 Use additional algebra tiles to model the dimensions of the rectangle.

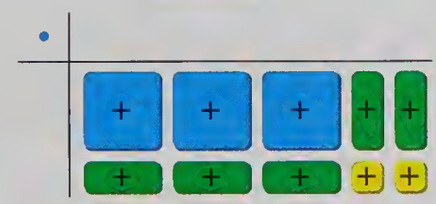


Step 3 Write the polynomial in factored form using the dimensions of the rectangle.

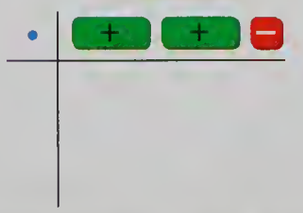
width, length

Area $= 2x^2 + 5x + 2 = (x + 2)(2x + 1)$

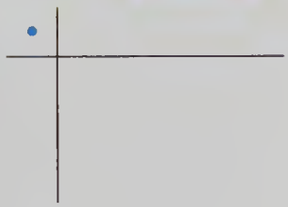
a. $3x^2 + 5x + 2 =$ [redacted]



b. $4x^2 + 4x - 3 =$ [redacted]



c. $2x^2 - 11x + 5 =$ [redacted]



SELECTING TOOLS
To be proficient in math, you need to consider the available tools, including concrete models, when solving a mathematical problem.

Communicate Your Answer

2. How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?
3. Is it possible to factor the trinomial $2x^2 + 2x + 1$? Explain your reasoning.

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

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Texas Essential Knowledge and Skills

A.8.A The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

A.10.D The student is expected to rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

A.10.E The student is expected to factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

ANSWERS

1. **a.** $(x + 1)(3x + 2)$



b. $(2x + 3)(2x - 1)$



c. $(x - 5)(2x - 1)$



2. Arrange algebra tiles that model the trinomial into a rectangular array, use additional algebra tiles to model the dimensions of the rectangle, then write the polynomial in factored form using the dimensions of the rectangle.
3. no; There is no way to model this expression as a rectangular array.

Extra Example 1

Factor $4x^2 + 32x + 60$.

$4(x + 3)(x + 5)$

Extra Example 2

Factor each polynomial.

a. $2x^2 + 7x + 6$

$(2x + 3)(x + 2)$

b. $4x^2 - 7x + 3$

$(4x - 3)(x - 1)$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

7.7 Lesson

What You Will Learn

- Factor $ax^2 + bx + c$.
- Use factoring to solve real-life problems.

Core Vocabulary

Previous

polynomial

greatest common factor (GCF)

Zero-Product Property

Factoring $ax^2 + bx + c$

In Section 7.6, you factored polynomials of the form $ax^2 + bx + c$, where $a = 1$. To factor polynomials of the form $ax^2 + bx + c$, where $a \neq 1$, first look for the GCF of the terms of the polynomial and then factor further if possible.

EXAMPLE 1 **Factoring Out the GCF**

Factor $5x^2 + 15x + 10$.

SOLUTION

Notice that the GCF of the terms $5x^2$, $15x$, and 10 is 5.

$5x^2 + 15x + 10 = 5(x^2 + 3x + 2)$ Factor out GCF.

$= 5(x + 1)(x + 2)$ Factor $x^2 + 3x + 2$.

▶ So, $5x^2 + 15x + 10 = 5(x + 1)(x + 2)$.

When there is no GCF, consider the possible factors of a and c.

EXAMPLE 2 **Factoring $ax^2 + bx + c$ When ac Is Positive**

Factor each polynomial.

a. $4x^2 + 13x + 3$ b. $3x^2 - 7x + 2$

SOLUTION

a. There is no GCF, so you need to consider the possible factors of a and c. Because b and c are both positive, the factors of c must be positive. Use a table to organize information about the factors of a and c.

Factors of 4	Factors of 3	Possible factorization	Middle term	
1, 4	1, 3	$(x + 1)(4x + 3)$	$3x + 4x = 7x$	✗
1, 4	3, 1	$(x + 3)(4x + 1)$	$x + 12x = 13x$	✓
2, 2	1, 3	$(2x + 1)(2x + 3)$	$6x + 2x = 8x$	✗

▶ So, $4x^2 + 13x + 3 = (x + 3)(4x + 1)$.

b. There is no GCF, so you need to consider the possible factors of a and c. Because b is negative and c is positive, both factors of c must be negative. Use a table to organize information about the factors of a and c.

Factors of 3	Factors of 2	Possible factorization	Middle term	
1, 3	−1, −2	$(x - 1)(3x - 2)$	$-2x - 3x = -5x$	✗
1, 3	−2, −1	$(x - 2)(3x - 1)$	$-x - 6x = -7x$	✓

▶ So, $3x^2 - 7x + 2 = (x - 2)(3x - 1)$.

STUDY TIP

You must consider the order of the factors of 3, because the middle terms formed by the possible factorizations are different.

Differentiated Instruction

Visual

Some students may benefit from seeing different colors used to highlight a, b, and c in a polynomial and to show the corresponding factors and middle term in a table. Provide a polynomial and table like the one shown below for students to use as a visual reference.

$4x^2 + 13x + 3$ (in Example 2)

Factors of 4	Factors of 3	Possible factorization	Middle term
1, 4	1, 3	$(1x + 1)(4x + 3)$	$3x + 4x = 7x$
1, 4	3, 1	$(1x + 3)(4x + 1)$	$x + 12x = 13x$
2, 2	1, 3	$(2x + 1)(2x + 3)$	$6x + 2x = 8x$

EXAMPLE 3 Factoring $ax^2 + bx + c$ When ac Is Negative

Factor $2x^2 - 5x - 7$.

SOLUTION

There is no GCF, so you need to consider the possible factors of a and c. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 2	Factors of −7	Possible factorization	Middle term	
1, 2	1, −7	$(x + 1)(2x - 7)$	$-7x + 2x = -5x$	✓
1, 2	7, −1	$(x + 7)(2x - 1)$	$-x + 14x = 13x$	✗
1, 2	−1, 7	$(x - 1)(2x + 7)$	$7x - 2x = 5x$	✗
1, 2	−7, 1	$(x - 7)(2x + 1)$	$x - 14x = -13x$	✗

▶ So, $2x^2 - 5x - 7 = (x + 1)(2x - 7)$.

STUDY TIP

When a is negative, factor -1 from each term of $ax^2 + bx + c$. Then factor the resulting trinomial as in the previous examples.

EXAMPLE 4 Factoring $ax^2 + bx + c$ When a Is Negative

Factor $-4x^2 - 8x + 5$.

SOLUTION

Step 1 Factor -1 from each term of the trinomial.

$$-4x^2 - 8x + 5 = -(4x^2 + 8x - 5)$$

Step 2 Factor the trinomial $4x^2 + 8x - 5$. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 4	Factors of −5	Possible factorization	Middle term	
1, 4	1, −5	$(x + 1)(4x - 5)$	$-5x + 4x = -x$	✗
1, 4	5, −1	$(x + 5)(4x - 1)$	$-x + 20x = 19x$	✗
1, 4	−1, 5	$(x - 1)(4x + 5)$	$5x - 4x = x$	✗
1, 4	−5, 1	$(x - 5)(4x + 1)$	$x - 20x = -19x$	✗
2, 2	1, −5	$(2x + 1)(2x - 5)$	$-10x + 2x = -8x$	✗
2, 2	−1, 5	$(2x - 1)(2x + 5)$	$10x - 2x = 8x$	✓

▶ So, $-4x^2 - 8x + 5 = -(2x - 1)(2x + 5)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

1. $8x^2 - 56x + 48$
2. $14x^2 + 31x + 15$
3. $2x^2 - 7x + 5$
4. $3x^2 - 14x + 8$
5. $4x^2 - 19x - 5$
6. $6x^2 + x - 12$
7. $-2y^2 - 5y - 3$
8. $-5m^2 + 6m - 1$
9. $-3x^2 - x + 2$

Extra Example 3

Factor $3x^2 - 7x - 6$.

$(x - 3)(3x + 2)$

Extra Example 4

Factor $-9x^2 - 3x + 2$.

$-(3x - 1)(3x + 2)$

MONITORING PROGRESS ANSWERS

1. $8(x - 1)(x - 6)$
2. $(7x + 5)(2x + 3)$
3. $(x - 1)(2x - 5)$
4. $(3x - 2)(x - 4)$
5. $(4x + 1)(x - 5)$
6. $(2x + 3)(3x - 4)$
7. $-(y + 1)(2y + 3)$
8. $-(m - 1)(5m - 1)$
9. $-(x + 1)(3x - 2)$

Extra Example 5

The length of a rectangular garden is 4 yards less than twice the width. The area of the garden is 96 square yards. What is the width of the garden? The width of the garden is 8 yards.

MONITORING PROGRESS ANSWERS

10. 8 mi

Solving Real-Life Problems

EXAMPLE 5 **Solving a Real-Life Problem**

The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. What is the width of the reserve?



SOLUTION

Use the formula for the area of a rectangle to write an equation for the area of the reserve. Let w represent the width. Then $2w + 1$ represents the length. Solve for w.

$w(2w + 1) = 55$ Area of the reserve

$2w^2 + w = 55$ Distributive Property

$2w^2 + w - 55 = 0$ Subtract 55 from each side.

Factor the left side of the equation. There is no GCF, so you need to consider the possible factors of a and c. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 2	Factors of −55	Possible factorization	Middle term	
1, 2	1, −55	$(w + 1)(2w - 55)$	$-55w + 2w = -53w$	✗
1, 2	55, −1	$(w + 55)(2w - 1)$	$-w + 110w = 109w$	✗
1, 2	−1, 55	$(w - 1)(2w + 55)$	$55w - 2w = 53w$	✗
1, 2	−55, 1	$(w - 55)(2w + 1)$	$w - 110w = -109w$	✗
1, 2	5, −11	$(w + 5)(2w - 11)$	$-11w + 10w = -w$	✗
1, 2	11, −5	$(w + 11)(2w - 5)$	$-5w + 22w = 17w$	✗
1, 2	−5, 11	$(w - 5)(2w + 11)$	$11w - 10w = w$	✓
1, 2	−11, 5	$(w - 11)(2w + 5)$	$5w - 22w = -17w$	✗

Check

Use mental math.

The width is 5 miles, so the length is $5(2) + 1 = 11$ miles and the area is $5(11) = 55$ square miles. ✓

So, you can rewrite $2w^2 + w - 55$ as $(w - 5)(2w + 11)$. Write the equation with the left side factored and continue solving for w.

$(w - 5)(2w + 11) = 0$ Rewrite equation with left side factored.

$w - 5 = 0$ *or* $2w + 11 = 0$ Zero-Product Property

$w = 5$ *or* $w = -\frac{11}{2}$ Solve for w.

A negative width does not make sense, so you should use the positive solution.

▶ So, the width of the reserve is 5 miles.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

10. WHAT IF? The area of the reserve is 136 square miles. How wide is the reserve?

Closure

- **Exit Ticket:** Factor $2x^2 - 7x + 3$. $(2x - 1)(x - 3)$

7.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** What is the greatest common factor of the terms of $3y^2 - 21y + 36$?
2. **WRITING** Compare factoring $6x^2 - x - 2$ with factoring $x^2 - x - 2$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $3x^2 + 3x - 6$
4. $8v^2 + 8v - 48$
5. $4k^2 + 28k + 48$
6. $6y^2 - 24y + 18$
7. $7b^2 - 63b + 140$
8. $9r^2 - 36r - 45$

In Exercises 9–16, factor the polynomial. *(See Examples 2 and 3.)*

9. $3h^2 + 11h + 6$
10. $8m^2 + 30m + 7$
11. $6x^2 - 5x + 1$
12. $10w^2 - 31w + 15$
13. $3n^2 + 5n - 2$
14. $4z^2 + 4z - 3$
15. $8g^2 - 10g - 12$
16. $18v^2 - 15v - 18$

In Exercises 17–22, factor the polynomial. *(See Example 4.)*

17. $-3t^2 + 11t - 6$
18. $-7v^2 - 25v - 12$
19. $-4c^2 + 19c + 5$
20. $-8h^2 - 13h + 6$
21. $-15w^2 - w + 28$
22. $-22d^2 + 29d - 9$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in factoring the polynomial.**

23. ✗ $2x^2 - 2x - 24 = 2(x^2 - 2x - 24)$
$= 2(x - 6)(x + 4)$

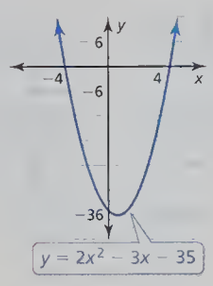
24. ✗ $6x^2 - 7x - 3 = (3x - 3)(2x + 1)$

In Exercises 25–28, solve the equation.

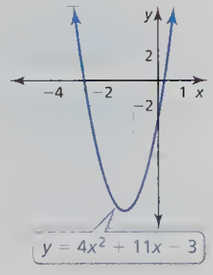
25. $5x^2 - 5x - 30 = 0$
26. $2k^2 - 5k - 18 = 0$
27. $-12n^2 - 11n = -15$
28. $14b^2 - 2 = -3b$

In Exercises 29–32, find the x-coordinates of the points where the graph crosses the x-axis.

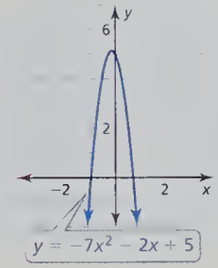
29.



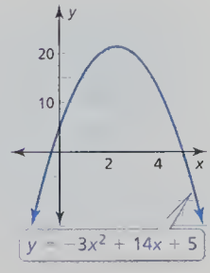
30.



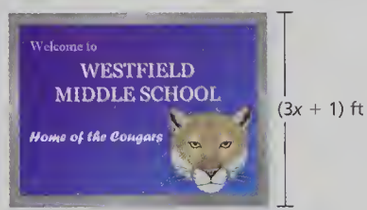
31.



32.



33. **MODELING WITH MATHEMATICS** The area (in square feet) of the school sign can be represented by $15x^2 - x - 2$.
 a. Write an expression that represents the length of the sign.
 b. Describe two ways to find the area of the sign when $x = 3$.



SUPPORTING English Language Learners

Have students share with the class their answers for the Vocabulary and Core Concept Check.

Beginning Use simple phrases to share answers.
Intermediate Use complete sentences to share information.
Advanced Use multiple complete sentences arranged appropriately.
Advanced High Explain answers using multiple complex sentences.

ELPS 3.G.1 Express opinions ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–25 odd, 33, 35, 38, 40, 49–56

Average: 1, 2–40 even, 49–56

Advanced: 1, 2, 8, 12, 14, 20–34 even, 36–56

HOMEWORK CHECK

Basic: 3, 9, 13, 17, 35

Average: 4, 10, 14, 18, 36

Advanced: 8, 12, 14, 20, 36

ANSWERS

1. 3
2. Factoring $6x^2 - x - 2$ requires considering factors of 6 and -2 in different combinations until the combination is found that produces the correct middle term. Factoring $x^2 - x - 2$ only requires finding the factors of -2 that add up to -1.
3. $3(x - 1)(x + 2)$
4. $8(v - 2)(v + 3)$
5. $4(k + 3)(k + 4)$
6. $6(y - 1)(y - 3)$
7. $7(b - 4)(b - 5)$
8. $9(r + 1)(r - 5)$
9. $(3h + 2)(h + 3)$
10. $(2m + 7)(4m + 1)$
11. $(2x - 1)(3x - 1)$
12. $(2w - 5)(5w - 3)$
13. $(n + 2)(3n - 1)$
14. $(2z - 1)(2z + 3)$
15. $2(g - 2)(4g + 3)$
16. $3(2v - 3)(3v + 2)$
17. $-(t - 3)(3t - 2)$
18. $-(v + 3)(7v + 4)$
19. $-(c - 5)(4c + 1)$
20. $-(h + 2)(8h - 3)$
21. $-(3w - 4)(5w + 7)$
22. $-(2d - 1)(11d - 9)$
23. need to factor 2 out of every term; $= 2(x^2 - x - 12) = 2(x + 3)(x - 4)$
24. These factors do not give the correct middle term; $= (2x - 3)(3x + 1)$
25. $x = -2, x = 3$
26. $k = -2, k = \frac{9}{2}$
27. $n = -\frac{5}{3}, n = \frac{3}{4}$
28. $b = -\frac{1}{2}, b = \frac{2}{7}$

29–33. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

34. 2.5 sec

35. length: 70 m, width: 31 m

36. yes; The length of the invitation is 5 inches, which is less than $5\frac{1}{8}$ inches. The width of the invitation is 3 inches, which is less than $3\frac{5}{8}$ inches.

37. *Sample answer:* $6x^2 + 3x$

38. The graph of g represents function k, and the graph of h represents function ℓ; Because c is positive, the constant terms in the factors must have the same sign. Because g has a positive value of b, the constant terms of the factors will both be positive, which results in negative roots, and parabola k has two negative x-intercepts. Because h has a negative value of b, the constant terms of the factors will both be negative, which results in positive roots, and parabola ℓ has two positive x-intercepts.

39. when no combination of factors of a and c produce the correct middle term; *Sample answer:* $2x^2 + x + 1$

40. no; To use the Zero-Product Property, one side of the equation needs to be 0. So, you must first subtract 2 from each side of the equation, then factor.

41. $\pm 9, \pm 12, \pm 21$

42. See Additional Answers.

43. 3.5 in.

44. 4 ft

45. $(k + 2j)(4k - j)$

46–56. See Additional Answers.

Mini-Assessment

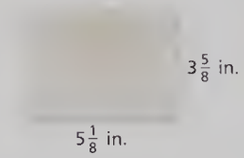
Factor the polynomial.

1. $2x^2 + 14x + 24$ $2(x + 3)(x + 4)$
2. $3x^2 + 11x + 10$ $(3x + 5)(x + 2)$
3. $2x^2 - 7x - 4$ $(2x + 1)(x - 4)$
4. $-3x^2 - 4x + 7$ $-(3x + 7)(x - 1)$
5. The length of a rectangular state park is 2 miles longer than twice the width. The area of the park is 84 square miles. What is the width of the park? The width of the park is 6 miles.

34. **MODELING WITH MATHEMATICS** The height h (in feet) above the water of a cliff diver is modeled by $h = -16t^2 + 8t + 80$, where t is the time (in seconds). How long is the diver in the air?

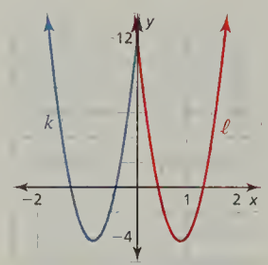
35. **MODELING WITH MATHEMATICS** The Parthenon in Athens, Greece, is an ancient structure that has a rectangular base. The length of the base of the Parthenon is 8 meters more than twice its width. The area of the base is about 2170 square meters. Find the length and width of the base. *(See Example 5.)*

36. **MODELING WITH MATHEMATICS** The length of a rectangular birthday party invitation is 1 inch less than twice its width. The area of the invitation is 15 square inches. Will the invitation fit in the envelope shown without being folded? Explain.



37. **OPEN-ENDED** Write a binomial whose terms have a GCF of $3x$.

38. **HOW DO YOU SEE IT?** Without factoring, determine which of the graphs represents the function $g(x) = 21x^2 + 37x + 12$ and which represents the function $h(x) = 21x^2 - 37x + 12$. Explain your reasoning.



39. **REASONING** When is it not possible to factor $ax^2 + bx + c$, where $a \neq 1$? Give an example.

40. **MAKING AN ARGUMENT** Your friend says that to solve the equation $5x^2 + x - 4 = 2$, you should start by factoring the left side as $(5x - 4)(x + 1)$. Is your friend correct? Explain.

41. **REASONING** For what values of t can $2x^2 + tx + 10$ be written as the product of two binomials?

42. **THOUGHT PROVOKING** Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a.

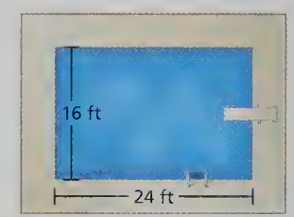


b.



43. **MATHEMATICAL CONNECTIONS** The length of a rectangle is 1 inch more than twice its width. The value of the area of the rectangle (in square inches) is 5 more than the value of the perimeter of the rectangle (in inches). Find the width.

44. **PROBLEM SOLVING** A rectangular swimming pool is bordered by a concrete patio. The width of the patio is the same on every side. The area of the surface of the pool is equal to the area of the patio. What is the width of the patio?



In Exercises 45–48, factor the polynomial.

45. $4k^2 + 7jk - 2j^2$ **46.** $6x^2 + 5xy - 4y^2$

47. $-6a^2 + 19ab - 14b^2$ **48.** $18m^3 + 39m^2n - 15mn^2$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the square root(s). *(Skills Review Handbook)*

49. $\pm\sqrt{64}$ **50.** $\sqrt{4}$ **51.** $-\sqrt{225}$ **52.** $\pm\sqrt{81}$

Solve the system of linear equations by substitution. Check your solution. *(Section 5.2)*

53. $y = 3 + 7x$, $y - x = -3$ **54.** $2x = y + 2$, $-x + 3y = 14$ **55.** $5x - 2y = 14$, $-7 = -2x + y$ **56.** $-x - 8 = -y$, $9y - 12 + 3x = 0$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.8 Factoring Special Products

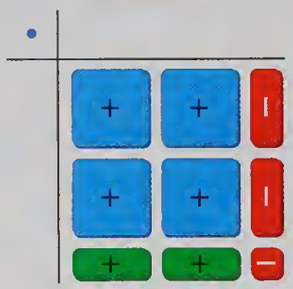
Texas Essential Knowledge and Skills
A.8.A
A.10.E
A.10.F

Essential Question How can you recognize and factor special products?

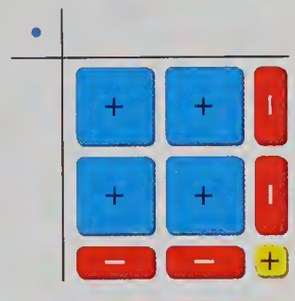
EXPLORATION 1 Factoring Special Products

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a "special product" that you studied in Section 7.3.

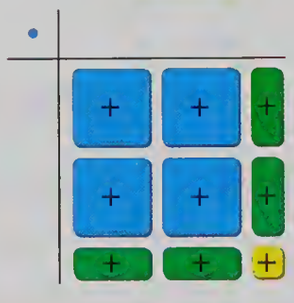
a. $4x^2 - 1 =$ [redacted]



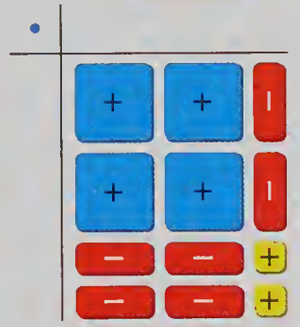
b. $4x^2 - 4x + 1 =$ [redacted]



c. $4x^2 + 4x + 1 =$ [redacted]

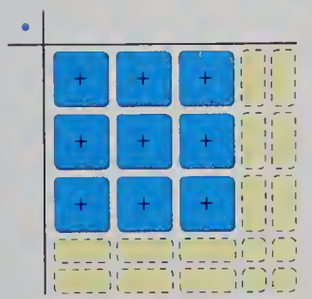


d. $4x^2 - 6x + 2 =$ [redacted]



ANALYZING MATHEMATICAL RELATIONSHIPS
To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

EXPLORATION 2 Factoring Special Products



Work with a partner. Use algebra tiles to complete the rectangular array at the left in three different ways, so that each way represents a different special product. Write each special product in standard form and in factored form.

Communicate Your Answer

3. How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.
4. Use the strategy you described in Question 3 to factor each polynomial.
 a. $25x^2 + 10x + 1$ **b.** $25x^2 - 10x + 1$ **c.** $25x^2 - 1$

SUPPORTING English Language Learners

Present the Essential Question. Have students work in pairs to complete the explorations and Communicate Your Answer.

Beginning/Advanced Beginning students read the headings and the polynomials while asking Advanced students for help as needed. Advanced students read the exploration instructions and Communicate Your Answer questions, requesting help from teachers or classroom aides as needed.

Intermediate/Advanced High Intermediate and Advanced High students alternate reading the explorations and Communicate Your Answer questions and request clarification from each other, teachers, or classroom aides as needed.

ELPS 4.F.9 Use support from peers and teachers to develop grasp of language structures needed to comprehend increasingly challenging language.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.8.A The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

A.10.E The student is expected to factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

A.10.F The student is expected to decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.

ANSWERS

1–2. See Additional Answers.

3. *Sample answer:* The algebra tiles for special patterns will always form a square array, and the x^2 tiles will also form a square array. Factor special products by using the special patterns studied in Lesson 7.3, but in reversed order; For the sum and difference pattern, the number of x and $-x$ tiles will be the same, and there will only be -1 tiles to complete the array. For the square of a binomial pattern, the x tiles will either be all positive or all negative and there will only be $+1$ tiles to complete the array.

4. a. $(5x + 1)^2$
b. $(5x - 1)^2$
c. $(5x + 1)(5x - 1)$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Extra Example 1

a. Factor $x^2 - 64$.
$(x + 8)(x - 8)$

b. Factor $25b^2 - 36$.
$(5b + 6)(5b - 6)$

Extra Example 2

Use a special product pattern to evaluate the expression $68^2 - 62^2$.
$(68 + 62)(68 - 62) = 130(6) = 780$

MONITORING PROGRESS ANSWERS

1. $(x + 6)(x - 6)$
2. $(10 + m)(10 - m)$
3. $(3n + 4)(3n - 4)$
4. $(4h + 7)(4h - 7)$
5. 140
6. 273
7. 525
8. 208

7.8 Lesson

Core Vocabulary

Previous
polynomial
trinomial

What You Will Learn

- Factor the difference of two squares.
- Factor perfect square trinomials.
- Use factoring to solve real-life problems.

Factoring the Difference of Two Squares

You can use special product patterns to factor polynomials.

Core Concept

Difference of Two Squares Pattern

Algebra	Example
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

EXAMPLE 1 Factoring the Difference of Two Squares

Factor (a) $x^2 - 25$ and (b) $4z^2 - 1$.

SOLUTION

a. $x^2 - 25 = x^2 - 5^2$ Write as $a^2 - b^2$.
$= (x + 5)(x - 5)$ Difference of two squares pattern

▶ So, $x^2 - 25 = (x + 5)(x - 5)$.

b. $4z^2 - 1 = (2z)^2 - 1^2$ Write as $a^2 - b^2$.
$= (2z + 1)(2z - 1)$ Difference of two squares pattern

▶ So, $4z^2 - 1 = (2z + 1)(2z - 1)$.

EXAMPLE 2 Evaluating a Numerical Expression

Use a special product pattern to evaluate the expression $54^2 - 48^2$.

SOLUTION

Notice that $54^2 - 48^2$ is a difference of two squares. So, you can rewrite the expression in a form that it is easier to evaluate using the difference of two squares pattern.

$54^2 - 48^2 = (54 + 48)(54 - 48)$ Difference of two squares pattern
$= 102(6)$ Simplify.
$= 612$ Multiply.

▶ So, $54^2 - 48^2 = 612$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

1. $x^2 - 36$
2. $100 - m^2$
3. $9n^2 - 16$
4. $16h^2 - 49$

Use a special product pattern to evaluate the expression.

5. $36^2 - 34^2$
6. $47^2 - 44^2$
7. $55^2 - 50^2$
8. $28^2 - 24^2$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Factoring Perfect Square Trinomials

Core Concept

Perfect Square Trinomial Pattern

Algebra	Example
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$ $= (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$ $= (x - 3)^2$

EXAMPLE 3 Factoring Perfect Square Trinomials

Factor each polynomial.

a. $n^2 + 8n + 16$ b. $4x^2 - 12x + 9$

SOLUTION

a. $n^2 + 8n + 16 = n^2 + 2(n)(4) + 4^2$ Write as $a^2 + 2ab + b^2$.

$= (n + 4)^2$ Perfect square trinomial pattern

▶ So, $n^2 + 8n + 16 = (n + 4)^2$.

b. $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ Write as $a^2 - 2ab + b^2$.

$= (2x - 3)^2$ Perfect square trinomial pattern

▶ So, $4x^2 - 12x + 9 = (2x - 3)^2$.

EXAMPLE 4 Solving a Polynomial Equation

Solve $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$.

SOLUTION

$x^2 + \frac{2}{3}x + \frac{1}{9} = 0$ Write equation.

$9x^2 + 6x + 1 = 0$ Multiply each side by 9.

$(3x)^2 + 2(3x)(1) + 1^2 = 0$ Write left side as $a^2 + 2ab + b^2$.

$(3x + 1)^2 = 0$ Perfect square trinomial pattern

$3x + 1 = 0$ Zero-Product Property

$x = -\frac{1}{3}$ Solve for x.

▶ The solution is $x = -\frac{1}{3}$.

ANALYZING MATHEMATICAL RELATIONSHIPS

Equations of the form $(x + a)^2 = 0$ always have repeated roots of $x = -a$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

9. $m^2 - 2m + 1$
10. $d^2 - 10d + 25$
11. $9z^2 + 36z + 36$

Solve the equation.

12. $a^2 + 6a + 9 = 0$
13. $w^2 - \frac{7}{3}w + \frac{49}{36} = 0$
14. $n^2 - 81 = 0$

Extra Example 3

Factor each polynomial.

a. $x^2 + 26x + 169$

$(x + 13)^2$

b. $9x^2 - 24x + 16$

$(3x - 4)^2$

Extra Example 4

Solve $x^2 + 3x + \frac{9}{4} = 0$. $x = -\frac{3}{2}$

MONITORING PROGRESS ANSWERS

9. $(m - 1)^2$
10. $(d - 5)^2$
11. $(3z + 6)^2$
12. $a = -3$
13. $w = \frac{7}{6}$
14. $n = -9, n = 9$

Differentiated Instruction

Inclusion

Some students may have difficulty writing a trinomial such as $4x^2 - 12x + 9$ in the form $a^2 - 2ab + b^2$ or $a^2 + 2ab + b^2$. Pair each of these students with one who understands the process. Have the students rewrite a trinomial together while discussing the process. Then provide a second trinomial for each student to rewrite, and have partners compare and justify their answers.

Extra Example 5

In Example 5, suppose the golf ball does not hit the pine tree. It hits the top of a 17-foot oak tree. After how many seconds does the ball hit the oak tree? The golf ball hits the oak tree after 2 seconds.

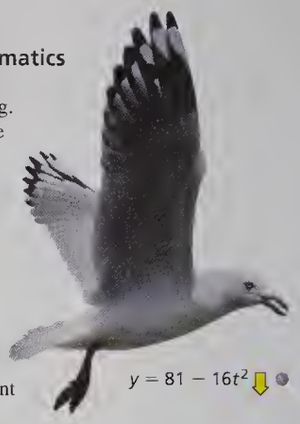
MONITORING PROGRESS ANSWERS

15. 2.25 sec

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**

A bird picks up a golf ball and drops it while flying. The function represents the height y (in feet) of the golf ball t seconds after it is dropped. The ball hits the top of a 32-foot-tall pine tree. After how many seconds does the ball hit the tree?



SOLUTION

1. **Understand the Problem** You are given the height of the golf ball as a function of the amount of time after it is dropped and the height of the tree that the golf ball hits. You are asked to determine how many seconds it takes for the ball to hit the tree.

2. **Make a Plan** Use the function for the height of the golf ball. Substitute the height of the tree for y and solve for the time t.

3. **Solve the Problem** Substitute 32 for y and solve for t.

$$y = 81 - 16t^2 \quad \text{Write equation.}$$
$$32 = 81 - 16t^2 \quad \text{Substitute 32 for } y.$$
$$0 = 49 - 16t^2 \quad \text{Subtract 32 from each side.}$$
$$0 = 7^2 - (4t)^2 \quad \text{Write as } a^2 - b^2.$$
$$0 = (7 + 4t)(7 - 4t) \quad \text{Difference of two squares pattern}$$
$$7 + 4t = 0 \quad \text{or} \quad 7 - 4t = 0 \quad \text{Zero-Product Property}$$
$$t = -\tfrac{7}{4} \quad \text{or} \quad t = \tfrac{7}{4} \quad \text{Solve for } t.$$

A negative time does not make sense in this situation.

▶ So, the golf ball hits the tree after $\frac{7}{4}$, or 1.75 seconds.

4. **Look Back** Check your solution, as shown, by substituting $t = \frac{7}{4}$ into the equation $32 = 81 - 16t^2$. Then verify that a time of $\frac{7}{4}$ seconds gives a height of 32 feet.

Check

$$32 = 81 - 16t^2$$
$$32 \stackrel{?}{=} 81 - 16\left(\tfrac{7}{4}\right)^2$$
$$32 \stackrel{?}{=} 81 - 16\left(\tfrac{49}{16}\right)$$
$$32 \stackrel{?}{=} 81 - 49$$
$$32 = 32 \checkmark$$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

15. WHAT IF? The golf ball does not hit the pine tree. After how many seconds does the ball hit the ground?

Closure

- **Exit Ticket:** Factor each polynomial.

 a. $81 - x^2$ $(9 - x)(9 + x)$

 b. $x^2 - 50x + 625$ $(x - 25)^2$

7.8 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** Can you use the perfect square trinomial pattern to factor $y^2 + 16y + 64$? Explain.

2. **WHICH ONE DOESN'T BELONG?** Which polynomial does *not* belong with the other three? Explain your reasoning.

$n^2 - 4$ | $g^2 - 6g + 9$ | $r^2 + 12r + 36$ | $k^2 + 25$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $m^2 - 49$
4. $z^2 - 81$
5. $64 - 81d^2$
6. $25 - 4x^2$
7. $225a^2 - 36b^2$
8. $16x^2 - 169y^2$

In Exercises 9–14, use a special product pattern to evaluate the expression. *(See Example 2.)*

9. $12^2 - 9^2$
10. $19^2 - 11^2$
11. $78^2 - 72^2$
12. $54^2 - 52^2$
13. $53^2 - 47^2$
14. $39^2 - 36^2$

In Exercises 15–22, factor the polynomial. *(See Example 3.)*

15. $h^2 + 12h + 36$
16. $p^2 + 30p + 225$
17. $y^2 - 22y + 121$
18. $x^2 - 4x + 4$
19. $a^2 - 28a + 196$
20. $m^2 + 24m + 144$
21. $25n^2 + 20n + 4$
22. $49a^2 - 14a + 1$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in factoring the polynomial.**

23. ✗ $n^2 - 64 = n^2 - 8^2$
$= (n - 8)^2$

24. ✗ $y^2 - 6y + 9 = y^2 - 2(y)(3) + 3^2$
$= (y - 3)(y + 3)$

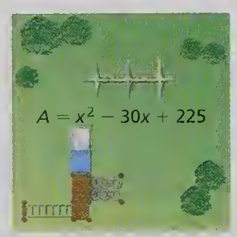
25. **MODELING WITH MATHEMATICS** The area (in square centimeters) of a square coaster can be represented by $d^2 + 8d + 16$.



a. Write an expression that represents the side length of the coaster.

b. Write an expression for the perimeter of the coaster.

26. **MODELING WITH MATHEMATICS** The polynomial represents the area (in square feet) of the square playground.



a. Write a polynomial that represents the side length of the playground.

b. Write an expression for the perimeter of the playground.

In Exercises 27–34, solve the equation. *(See Example 4.)*

27. $z^2 - 4 = 0$
28. $4x^2 = 49$
29. $k^2 - 16k + 64 = 0$
30. $s^2 + 20s + 100 = 0$
31. $n^2 + 9 = 6n$
32. $y^2 = 12y - 36$
33. $y^2 + \frac{1}{2}y = -\frac{1}{16}$
34. $-\frac{4}{3}x + \frac{4}{9} = -x^2$

In Exercises 35–40, factor the polynomial.

35. $3z^2 - 27$
36. $2m^2 - 50$
37. $4y^2 - 16y + 16$
38. $8k^2 + 80k + 200$
39. $50y^2 + 120y + 72$
40. $27m^2 - 36m + 12$

33. $y = -\frac{1}{4}$
34. $x = \frac{2}{3}$
35. $3(z + 3)(z - 3)$
36. $2(m + 5)(m - 5)$
37. $4(y - 2)^2$
38. $8(k + 5)^2$
39. $2(5y + 6)^2$
40. $3(3m - 2)^2$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–35 odd, 41, 46, 49–56

Average: 1, 2–42 even, 46, 47, 49–56

Advanced: 1, 2, 8, 14, 20–26 even, 32–40 even, 42–56

HOMEWORK CHECK

Basic: 3, 9, 15, 29, 41

Average: 4, 12, 16, 30, 42

Advanced: 8, 14, 22, 32, 42

ANSWERS

1. yes; The square roots of the first and last terms are y and 8, and the middle term is $2 \cdot y \cdot 8$, so it fits the pattern.
2. $k^2 + 25$; It is the only one that cannot be factored using a special pattern.
3. $(m + 7)(m - 7)$
4. $(z + 9)(z - 9)$
5. $(8 + 9d)(8 - 9d)$
6. $(5 + 2x)(5 - 2x)$
7. $(15a + 6b)(15a - 6b)$
8. $(4x + 13y)(4x - 13y)$
9. 63
10. 240
11. 900
12. 212
13. 600
14. 225
15. $(h + 6)^2$
16. $(p + 15)^2$
17. $(y - 11)^2$
18. $(x - 2)^2$
19. $(a - 14)^2$
20. $(m + 12)^2$
21. $(5n + 2)^2$
22. $(7a - 1)^2$
23. should follow the difference of two squares pattern; $= (n + 8)(n - 8)$
24. should follow the perfect square trinomial pattern; $= (y - 3)^2$
25. a. $(d + 4)$ cm
 b. $(4d + 16)$ cm
26. a. $(x - 15)$ ft
 b. $(4x - 60)$ ft
27. $z = -2, z = 2$
28. $x = -\frac{7}{2}, x = \frac{7}{2}$
29. $k = 8$
30. $s = -10$
31. $n = 3$
32. $y = 6$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

41. 1.25 sec

42. 0.25 sec

43. **a.** no; $w^2 + 18w + 81$

b. no; $y^2 - 10y + 25$

44. **a.**



$4x^2 - 1 = (2x + 1)(2x - 1)$

b.



$4x^2 - 4x + 1 = (2x - 1)^2$

45. Square each binomial, then combine like terms; Use the difference of two squares pattern with each binomial as one of the terms, then simplify; *Sample answer:* The difference of two squares pattern; You do not need to square any binomials.

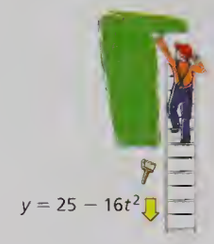
46–56. See Additional Answers.

Mini-Assessment

Factor the polynomial.

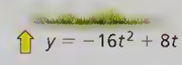
1. $9x^2 - 25$ $(3x + 5)(3x - 5)$
2. $x^2 - 40x + 400$ $(x - 20)^2$
3. Use a special product pattern to evaluate the expression $26^2 - 22^2$. $(26 + 22)(26 - 22) = 48(4) = 192$
4. Solve $16x^2 + 8x + 1 = 0$. $x = -\frac{1}{4}$
5. A bird picks up a stone and drops it while flying. The function $y = 49 - 16t^2$ represents the height y (in feet) of the stone t seconds after it is dropped. The stone hits the top of a 13-foot-tall building. After how many seconds does the stone hit the building? The stone hits the building after $\frac{3}{2}$, or 1.5, seconds.

41. MODELING WITH MATHEMATICS While standing on a ladder, you drop a paintbrush. The function represents the height y (in feet) of the paintbrush t seconds after it is dropped. After how many seconds does the paintbrush land on the ground? *(See Example 5.)*



42. MODELING WITH MATHEMATICS The function represents the height y (in feet) of a grasshopper jumping straight up from the ground t seconds after the start of the jump. After how many seconds is the grasshopper 1 foot off the ground?





43. REASONING Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial is a perfect square trinomial.

a. $w^2 + 18w + 84$ **b.** $y^2 - 10y + 23$

44. THOUGHT PROVOKING Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a.

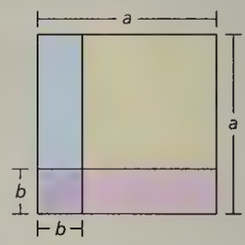


b.



45. COMPARING METHODS Describe two methods you can use to simplify $(2x - 5)^2 - (x - 4)^2$. Which one would you use? Explain.

46. HOW DO YOU SEE IT? The figure shows a large square with an area of a^2 that contains a smaller square with an area of b^2.



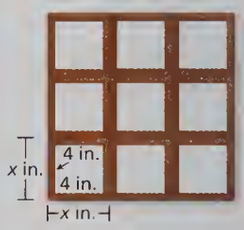
a. Describe the regions that represent $a^2 - b^2$. How can you rearrange these regions to show that $a^2 - b^2 = (a + b)(a - b)$?

b. How can you use the figure to show that $(a - b)^2 = a^2 - 2ab + b^2$?

47. PROBLEM SOLVING You hang nine identical square picture frames on a wall.

a. Write a polynomial that represents the area of the picture frames, not including the pictures.

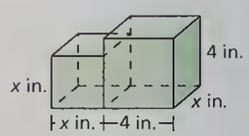
b. The area in part (a) is 81 square inches. What is the side length of one of the picture frames? Explain your reasoning.



48. MATHEMATICAL CONNECTIONS The composite solid is made up of a cube and a rectangular prism.

a. Write a polynomial that represents the volume of the composite solid.

b. The volume of the composite solid is equal to $25x$. What is the value of x? Explain your reasoning.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the prime factorization of the number. *(Skills Review Handbook)*

49. 50 **50.** 44 **51.** 85 **52.** 96

Graph the inequality in a coordinate plane. *(Section 5.6)*

53. $y \leq 4x - 1$ **54.** $y > -\frac{1}{2}x + 3$ **55.** $4y - 12 \geq 8x$ **56.** $3y + 3 < x$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.9 Factoring Polynomials Completely

Texas Essential Knowledge and Skills
A.10.D
A.10.E

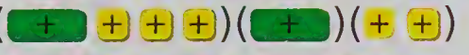
Essential Question
How can you factor a polynomial completely?

EXPLORATION 1 Writing a Product of Linear Factors

Work with a partner. Write the product represented by the algebra tiles. Then multiply to write the polynomial in standard form.

a. 

b. 

c. 

d. 

e. 

f. 

REASONING
To be proficient in math, you need to know and flexibly use different properties of operations and objects.

EXPLORATION 2 Matching Standard and Factored Forms

Work with a partner. Match the standard form of the polynomial with the equivalent factored form. Explain your strategy.

a. $x^3 + x^2$	A. $x(x + 1)(x - 1)$
b. $x^3 - x$	B. $x(x - 1)^2$
c. $x^3 + x^2 - 2x$	C. $x(x + 1)^2$
d. $x^3 - 4x^2 + 4x$	D. $x(x + 2)(x - 1)$
e. $x^3 - 2x^2 - 3x$	E. $x(x - 1)(x - 2)$
f. $x^3 - 2x^2 + x$	F. $x(x + 2)(x - 2)$
g. $x^3 - 4x$	G. $x(x - 2)^2$
h. $x^3 + 2x^2$	H. $x(x + 2)^2$
i. $x^3 - x^2$	I. $x^2(x - 1)$
j. $x^3 - 3x^2 + 2x$	J. $x^2(x + 1)$
k. $x^3 + 2x^2 - 3x$	K. $x^2(x - 2)$
l. $x^3 - 4x^2 + 3x$	L. $x^2(x + 2)$
m. $x^3 - 2x^2$	M. $x(x + 3)(x - 1)$
n. $x^3 + 4x^2 + 4x$	N. $x(x + 1)(x - 3)$
o. $x^3 + 2x^2 + x$	O. $x(x - 1)(x - 3)$

Communicate Your Answer

3. How can you factor a polynomial completely?
4. Use your answer to Question 3 to factor each polynomial completely.
 a. $x^3 + 4x^2 + 3x$ b. $x^3 - 6x^2 + 9x$ c. $x^3 + 6x^2 + 9x$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.10.D The student is expected to rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

A.10.E The student is expected to factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

ANSWERS

1. a. $(x + 1)(x + 1)(-2)$; $-2x^2 - 4x - 2$
 b. $(x + 2)(x + 1)(-x)$; $-x^3 - 3x^2 - 2x$
 c. $(x + 3)(x)(2)$; $2x^2 + 6x$
 d. $(x + 1)(x - 1)(x)$; $x^3 - x$
 e. $(-x + 1)(x + 1)(-x)$; $x^3 - x$
 f. $(-x - 1)(x + 1)(-2)$; $2x^2 + 4x + 2$
2. a. J
 b. A
 c. D
 d. G
 e. N
 f. B
 g. F
 h. L
 i. I
 j. E
 k. M
 l. O
 m. K
 n. H
 o. C

 Factor out the greatest common monomial factor first, then factor the remaining expression if possible.
3. Factor out the greatest common monomial factor first, then factor the remaining expression if possible.
4. a. $x(x + 1)(x + 3)$
 b. $x(x - 3)^2$
 c. $x(x + 3)^2$

Extra Example 1

Factor each polynomial by grouping.

a. $x^3 + 4x^2 + 2x + 8$

$(x^2 + 2)(x + 4)$

b. $x^2 + 4y + 2x + 2xy$

$(x + 2)(x + 2y)$

MONITORING PROGRESS ANSWERS

1. $(a^2 + 1)(a + 3)$
2. $(y + 2)(y + x)$

7.9 Lesson

Core Vocabulary

factoring by grouping, *p. 390*
factored completely, *p. 390*

Previous
polynomial
binomial

What You Will Learn

- Factor polynomials by grouping
- Factor polynomials completely.
- Use factoring to solve real-life problems.

Factoring Polynomials by Grouping

You have used the Distributive Property to factor out a greatest common monomial from a polynomial. Sometimes, you can factor out a common binomial. You may be able to use the Distributive Property to factor polynomials with four terms, as described below.

Core Concept

Factoring by Grouping

To factor a polynomial with four terms, group the terms into pairs. Factor the GCF out of each pair of terms. Look for and factor out the common binomial factor. This process is called **factoring by grouping**.

EXAMPLE 1 Factoring by Grouping

Factor each polynomial by grouping.

a. $x^3 + 3x^2 + 2x + 6$ b. $x^2 + y + x + xy$

SOLUTION

a. $x^3 + 3x^2 + 2x + 6 = (x^3 + 3x^2) + (2x + 6)$ Group terms with common factors.

Common binomial factor is $x + 3$. → $= x^2(x + 3) + 2(x + 3)$ Factor out GCF of each pair of terms.

$= (x + 3)(x^2 + 2)$ Factor out $(x + 3)$.

▶ So, $x^3 + 3x^2 + 2x + 6 = (x + 3)(x^2 + 2)$.

b. $x^2 + y + x + xy = x^2 + x + xy + y$ Rewrite polynomial.

$= (x^2 + x) + (xy + y)$ Group terms with common factors.

Common binomial factor is $x + 1$. → $= x(x + 1) + y(x + 1)$ Factor out GCF of each pair of terms.

$= (x + 1)(x + y)$ Factor out $(x + 1)$.

▶ So, $x^2 + y + x + xy = (x + 1)(x + y)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial by grouping.

1. $a^3 + 3a^2 + a + 3$
2. $y^2 + 2x + yx + 2y$

Factoring Polynomials Completely

You have seen that the polynomial $x^2 - 1$ can be factored as $(x + 1)(x - 1)$. This polynomial is factorable. Notice that the polynomial $x^2 + 1$ cannot be written as the product of polynomials with integer coefficients. This polynomial is unfactorable. A factorable polynomial with integer coefficients is **factored completely** when it is written as a product of unfactorable polynomials with integer coefficients.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Concept Summary

Guidelines for Factoring Polynomials Completely

To factor a polynomial completely, you should try each of these steps.

1.	Factor out the greatest common monomial factor.	$3x^2 + 6x = 3x(x + 2)$
2.	Look for a difference of two squares or a perfect square trinomial.	$x^2 + 4x + 4 = (x + 2)^2$
3.	Factor a trinomial of the form $ax^2 + bx + c$ into a product of binomial factors.	$3x^2 - 5x - 2 = (3x + 1)(x - 2)$
4.	Factor a polynomial with four terms by grouping.	$x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)$

EXAMPLE 2 Factoring Completely

Factor (a) $3x^3 + 6x^2 - 18x$ and (b) $7x^4 - 28x^2$.

SOLUTION

a. $3x^3 + 6x^2 - 18x = 3x(x^2 + 2x - 6)$ Factor out $3x$.

$x^2 + 2x - 6$ is unfactorable, so the polynomial is factored completely.

▶ So, $3x^3 + 6x^2 - 18x = 3x(x^2 + 2x - 6)$.

b. $7x^4 - 28x^2 = 7x^2(x^2 - 4)$ Factor out $7x^2$.

$= 7x^2(x^2 - 2^2)$ Write as $a^2 - b^2$.

$= 7x^2(x + 2)(x - 2)$ Difference of two squares pattern

▶ So, $7x^4 - 28x^2 = 7x^2(x + 2)(x - 2)$.

EXAMPLE 3 Solving an Equation by Factoring Completely

Solve $2x^3 + 8x^2 = 10x$.

SOLUTION

$2x^3 + 8x^2 = 10x$ Original equation

$2x^3 + 8x^2 - 10x = 0$ Subtract $10x$ from each side.

$2x(x^2 + 4x - 5) = 0$ Factor out $2x$.

$2x(x + 5)(x - 1) = 0$ Factor $x^2 + 4x - 5$.

$2x = 0$ *or* $x + 5 = 0$ *or* $x - 1 = 0$ Zero-Product Property

$x = 0$ *or* $x = -5$ *or* $x = 1$ Solve for x.

▶ The roots are $x = -5$, $x = 0$, and $x = 1$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial completely.

3. $3x^3 - 12x$
4. $2y^3 - 12y^2 + 18y$
5. $m^3 - 2m^2 - 8m$

Solve the equation.

6. $w^3 - 8w^2 + 16w = 0$
7. $x^3 - 25x = 0$
8. $c^3 - 7c^2 + 12c = 0$

Extra Example 2

a. Factor $2x^3 + 6x^2 - 2x$.
$2x(x^2 + 3x - 1)$

b. Factor $5x^4 - 45x^2$.
$5x^2(x + 3)(x - 3)$

Extra Example 3

Solve $3x^3 + 6x^2 = 24x$.
The roots are $x = -4$, $x = 0$, and $x = 2$.

MONITORING PROGRESS ANSWERS

3. $3x(x + 2)(x - 2)$
4. $2y(y - 3)^2$
5. $m(m + 2)(m - 4)$
6. $w = 0, w = 4$
7. $x = 0, x = -5, x = 5$
8. $c = 0, c = 3, c = 4$

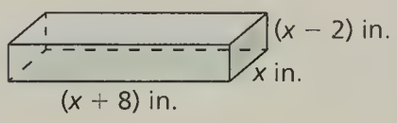
English Language Learners

Organization

Students will benefit by writing down the steps for factoring polynomials completely as shown on page 391. Have students write the steps in their notebooks. Consider also displaying a poster with the same information in the classroom.

Extra Example 4

A box in the shape of a rectangular prism has a volume of 96 cubic inches. The dimensions of the box in terms of its width are shown. Find the length, width, and height of the box.



The width is 4 inches, the length is 12 inches, and the height is 2 inches.

MONITORING PROGRESS ANSWERS

9. length: 3 ft, width: 2 ft, height: 12 ft

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**



A terrarium in the shape of a rectangular prism has a volume of 4608 cubic inches. Its length is more than 10 inches. The dimensions of the terrarium in terms of its width are shown. Find the length, width, and height of the terrarium.

SOLUTION

1. **Understand the Problem** You are given the volume of a terrarium in the shape of a rectangular prism and a description of the length. The dimensions are written in terms of its width. You are asked to find the length, width, and height of the terrarium.

2. **Make a Plan** Use the formula for the volume of a rectangular prism to write and solve an equation for the width of the terrarium. Then substitute that value in the expressions for the length and height of the terrarium.

3. **Solve the Problem**

$\text{Volume} = \text{length} \cdot \text{width} \cdot \text{height}$ — Volume of a rectangular prism

$4608 = (36 - w)(w)(w + 4)$ — Write equation.

$4608 = 32w^2 + 144w - w^3$ — Multiply.

$0 = 32w^2 + 144w - w^3 - 4608$ — Subtract 4608 from each side.

$0 = (-w^3 + 32w^2) + (144w - 4608)$ — Group terms with common factors.

$0 = -w^2(w - 32) + 144(w - 32)$ — Factor out GCF of each pair of terms.

$0 = (w - 32)(-w^2 + 144)$ — Factor out $(w - 32)$.

$0 = -1(w - 32)(w^2 - 144)$ — Factor -1 from $-w^2 + 144$.

$0 = -1(w - 32)(w - 12)(w + 12)$ — Difference of two squares pattern

$w - 32 = 0$ or $w - 12 = 0$ or $w + 12 = 0$ — Zero-Product Property

$w = 32$ or $w = 12$ or $w = -12$ — Solve for w.

Disregard $w = -12$ because a negative width does not make sense. You know that the length is more than 10 inches. Test the solutions of the equation, 12 and 32, in the expression for the length.

$\text{length} = 36 - w = 36 - 12 = 24$ ✓ or $\text{length} = 36 - w = 36 - 32 = 4$ ✗

The solution 12 gives a length of 24 inches, so 12 is the correct value of w.

Use $w = 12$ to find the height, as shown.

$\text{height} = w + 4 = 12 + 4 = 16$

▶ The width is 12 inches, the length is 24 inches, and the height is 16 inches.

Check

$V = \ell wh$

$4608 \stackrel{?}{=} 24(12)(16)$

$4608 = 4608$ ✓

4. **Look Back** Check your solution. Substitute the values for the length, width, and height when the width is 12 inches into the formula for volume. The volume of the terrarium should be 4608 cubic inches.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

9. A box in the shape of a rectangular prism has a volume of 72 cubic feet. The box has a length of x feet, a width of $(x - 1)$ feet, and a height of $(x + 9)$ feet. Find the dimensions of the box.

SUPPORTING English Language Learners

Ask students to share what they know about terrariums or aquariums. Explain that many of these are rectangular glass prisms. Have students practice reading comprehension by reading aloud Example 4 in groups of mixed language abilities. Students should consult the group or the teacher when there is a concept they do not understand.

Beginning Read the headings.
Intermediate Read the problem statement.
Advanced Read Steps 1 and 2 of the solution.
Advanced High Read Steps 3 and 4 of the solution.

ELPS 4.F.10 Use support from peers and teachers to develop background knowledge needed to comprehend increasingly challenging language.

Closure

- **Exit Ticket:**
 Solve $3x^5 - 6x^4 - 45x^3 = 0$.
 $x = -3, x = 0, x = 5$

7.9 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What does it mean for a polynomial to be factored completely?
2. **WRITING** Explain how to choose which terms to group together when factoring by grouping.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, factor the polynomial by grouping. *(See Example 1.)*

3. $x^3 + x^2 + 2x + 2$
4. $y^3 - 9y^2 + y - 9$
5. $3z^3 + 2z - 12z^2 - 8$
6. $2s^3 - 27 - 18s + 3s^2$
7. $x^2 + xy + 8x + 8y$
8. $q^2 + q + 5pq + 5p$
9. $m^2 - 3m + mn - 3n$
10. $2a^2 + 8ab - 3a - 12b$

In Exercises 11–22, factor the polynomial completely. *(See Example 2.)*

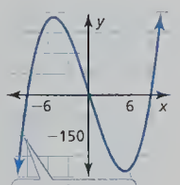
11. $2x^3 - 2x$
12. $36a^4 - 4a^2$
13. $2c^2 - 7c + 19$
14. $m^2 - 5m - 35$
15. $6g^3 - 24g^2 + 24g$
16. $-15d^3 + 21d^2 - 6d$
17. $3r^5 + 3r^4 - 90r^3$
18. $5w^4 - 40w^3 + 80w^2$
19. $-4c^4 + 8c^3 - 28c^2$
20. $8t^2 + 8t - 72$
21. $b^3 - 5b^2 - 4b + 20$
22. $h^3 + 4h^2 - 25h - 100$

In Exercises 23–28, solve the equation. *(See Example 3.)*

23. $5n^3 - 30n^2 + 40n = 0$
24. $k^4 - 100k^2 = 0$
25. $x^3 + x^2 = 4x + 4$
26. $2t^5 + 2t^4 - 144t^3 = 0$
27. $12s - 3s^3 = 0$
28. $4y^3 - 7y^2 + 28 = 16y$

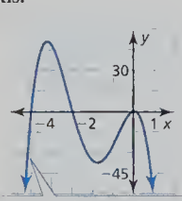
In Exercises 29–32, find the x-coordinates of the points where the graph crosses the x-axis.

29.



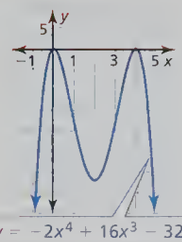
$y = x^3 - 81x$

30.



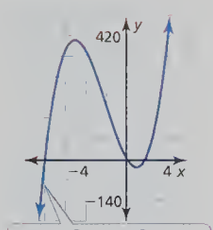
$y = -3x^4 - 24x^3 - 45x^2$

31.



$y = -2x^4 + 16x^3 - 32x^2$

32.



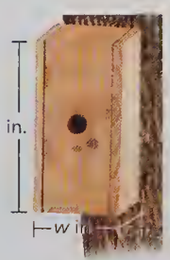
$y = 4x^3 + 25x^2 - 56x$

ERROR ANALYSIS **In Exercises 33 and 34, describe and correct the error in factoring the polynomial completely.**

33. ✗ $a^3 + 8a^2 - 6a - 48 = a^2(a + 8) + 6(a + 8)$
$= (a + 8)(a^2 + 6)$

34. ✗ $x^3 - 6x^2 - 9x + 54 = x^2(x - 6) - 9(x - 6)$
$= (x - 6)(x^2 - 9)$

35. **MODELING WITH MATHEMATICS** You are building a birdhouse in the shape of a rectangular prism that has a volume of 128 cubic inches. The dimensions of the birdhouse in terms of its width are shown. *(See Example 4.)*



a. Write a polynomial that represents the volume of the birdhouse.

b. What are the dimensions of the birdhouse?

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–35 odd, 42, 44, 50–57

Average: 1, 2–44 even, 50–57

Advanced: 1, 2, 8, 10, 20–40 even, 41–57

HOMEWORK CHECK

Basic: 3, 11, 23, 35

Average: 4, 12, 26, 36

Advanced: 8, 20, 26, 36

ANSWERS

1. It is written as a product of unfactorable polynomials with integer coefficients.
2. Look for terms with common factors.
3. $(x + 1)(x^2 + 2)$
4. $(y - 9)(y^2 + 1)$
5. $(z - 4)(3z^2 + 2)$
6. $(2s + 3)(s + 3)(s - 3)$
7. $(x + y)(x + 8)$
8. $(q + 1)(q + 5p)$
9. $(m - 3)(m + n)$
10. $(2a - 3)(a + 4b)$
11. $2x(x + 1)(x - 1)$
12. $4a^2(3a + 1)(3a - 1)$
13. unfactorable
14. unfactorable
15. $6g(g - 2)^2$
16. $-3d(d - 1)(5d - 2)$
17. $3r^3(r + 6)(r - 5)$
18. $5w^2(w - 4)^2$
19. $-4c^2(c^2 - 2c + 7)$
20. $8(t^2 + t - 9)$
21. $(b - 5)(b + 2)(b - 2)$
22. $(h + 4)(h + 5)(h - 5)$
23. $n = 0, n = 2, n = 4$
24. $k = 0, k = -10, k = 10$
25. $x = -1, x = -2, x = 2$
26. $t = 0, t = -9, t = 8$
27. $s = 0, s = 2, s = -2$
28. $y = \frac{7}{4}, y = -2, y = 2$
29. $x = 0, x = -9, x = 9$
30. $x = 0, x = -5, x = -3$
31. $x = 0, x = 4$
32. $x = 0, x = -8, x = \frac{7}{4}$
33. In the second group, factor out -6 instead of 6;
$= a^2(a + 8) - 6(a + 8)$
$= (a + 8)(a^2 - 6)$
34. It is not factored completely because $x^2 - 9$ can be factored; $= (x - 6)(x + 3)(x - 3)$
35. a. $(4w^2 + 16w)$ in.^{3}
b. length: 4 in., width: 4 in., height: 8 in.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

36. length: 16 in., width: 6 in., height: 12 in.

37. $(x + 2y)(x + 1)(x - 1)$

38. $(2b - a)(2b + 3)(2b - 3)$

39. $(4s - 1)(s + 3t)$

40. $(m^2 - 2n)(6m + n)$

41. no; The factors of the polynomial are $x^2 + 3$ and $x + 2$. Using the Zero Product Property, $x + 2 = 0$ will give 1 real solution, but $x^2 + 3 = 0$ has no real solutions.

42. The x-intercepts occur when $y = 0$, so set each factor equal to 0 and solve for x to get the x-coordinates of the x-intercepts.

43. **a.** *Sample answer:* $x^3 + x^2 + x + 2$

b. *Sample answer:* $x^3 + x^2 + x + 1$

44. no; It is possible that the terms will have a common monomial factor.

45. $3z$, $2z + 3$, $2z - 3$

46. **a.** $(-h^3 + 5h^2 + 36h)$ in.^{3}

b. length: 3 in., width: 10 in., height: 6 in.; length: 4 in., width: 9 in., height: 5 in.

c. length: 4 in., width: 9 in., height: 5 in.; These dimensions result in a surface area of 202 square inches. The other possible dimensions result in a surface area of 216 square inches.

47. radius: 5, height: 8

48. $(x - 1)(x - 1)(x + 1)(x + 2)(x - 2)$

49. See Additional Answers.

50. $(2, -2)$

51. $(2, 3)$

52. $(4, 8)$

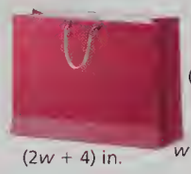
53. $(-6, -2)$

54–57. See Additional Answers.

Mini-Assessment

1. Factor $x^2 + 15y + 5x + 3xy$ by grouping. $(x + 5)(x + 3y)$
2. Factor $3x^5 - 48x^3$ completely. $3x^3(x + 4)(x - 4)$
3. Solve $6x^3 - 30x^2 = 36x$. The roots are $x = -1$, $x = 0$, and $x = 6$.

36. MODELING WITH MATHEMATICS A gift bag shaped like a rectangular prism has a volume of 1152 cubic inches. The dimensions of the gift bag in terms of its width are shown. The height is greater than the width. What are the dimensions of the gift bag?



In Exercises 37–40, factor the polynomial completely.

37. $x^3 + 2x^2y - x - 2y$

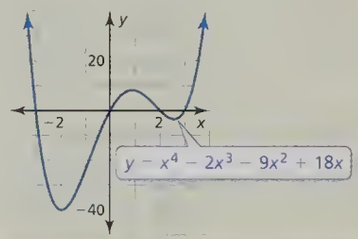
38. $8b^3 - 4b^2a - 18b + 9a$

39. $4s^2 - s + 12st - 3t$

40. $6m^3 - 12mn + m^2n - 2n^2$

41. WRITING Is it possible to find three real solutions of the equation $x^3 + 2x^2 + 3x + 6 = 0$? Explain your reasoning.

42. HOW DO YOU SEE IT? How can you use the factored form of the polynomial $x^4 - 2x^3 - 9x^2 + 18x = x(x - 3)(x + 3)(x - 2)$ to find the x-intercepts of the graph of the function?



43. OPEN-ENDED Write a polynomial of degree 3 that satsifies each of the given conditions.

a. is not factorable **b.** can be factored by grouping

44. MAKING AN ARGUMENT Your friend says that if a trinomial cannot be factored as the product of two binomials, then the trinomial is factored completely. Is your friend correct? Explain.

45. PROBLEM SOLVING The volume (in cubic feet) of a room is represented by $12z^3 - 27z$. Find expressions that could represent the dimensions of the room.

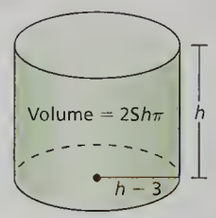
46. MATHEMATICAL CONNECTIONS The width of a box is 4 inches more than the height h. The length is the difference of 9 inches and the height.

a. Write a polynomial that represents the volume of the box in terms of its height (in inches).

b. The volume of the box is 180 cubic inches. What are the possible dimensions of the box?

c. Which dimensions result in a box with the least possible surface area? Explain your reasoning.

47. MATHEMATICAL CONNECTIONS The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. Find the dimensions of the cylinder.



48. THOUGHT PROVOKING Factor the polynomial $x^5 - x^4 - 5x^3 + 5x^2 + 4x - 4$ completely.

49. REASONING Find a value for w so that the equation has (a) two solutions and (b) three solutions. Explain your reasoning.

$$5x^3 + wx^2 + 80x = 0$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations by graphing. *(Section 5.1)*

50. $y = x - 4$, $y = -2x + 2$

51. $y = \frac{1}{2}x + 2$, $y = 3x - 3$

52. $5x - y = 12$, $\frac{1}{4}x + y = 9$

53. $x = 3y$, $y - 10 = 2x$

Graph the function. Describe the domain and range. *(Section 6.3)*

54. $f(x) = 5^x$

55. $y = 9\left(\frac{1}{3}\right)^x$

56. $y = -3(0.5)^x$

57. $f(x) = -3(4)^x$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

7.6–7.9 What Did You Learn?

Core Vocabulary

factoring by grouping, *p. 390*
factored completely, *p. 390*

Core Concepts

Section 7.6

Factoring $x^2 + bx + c$ When c Is Positive, *p. 372*
Factoring $x^2 + bx + c$ When c Is Negative, *p. 373*

Section 7.7

Factoring $ax^2 + bx + c$ When ac Is Positive, *p. 378*
Factoring $ax^2 + bx + c$ When ac Is Negative, *p. 379*

Section 7.8

Difference of Two Squares Pattern, *p. 384*
Perfect Square Trinomial Pattern, *p. 385*

Section 7.9

Factoring by Grouping, *p. 390*
Factoring Polynomials Completely, *p. 390*

Mathematical Thinking

1. How are the solutions of Exercise 29 on page 375 related to the graph of $y = m^2 + 3m + 2$?
2. The equation in part (b) of Exercise 47 on page 376 has two solutions. Are both solutions of the equation reasonable in the context of the problem? Explain your reasoning.

Performance Task

The View Matters



The way an equation or expression is written can help you interpret and solve problems. Which representation would you rather have when trying to solve for specific information? Why?

To explore the answers to these questions and more, go to **BigIdeasMath.com**.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

1. The solutions are the m-coordinates of the m-intercepts of the graph.
2. no; The solution $x = 28$ is not reasonable in the context of the problem because the diagram shows that $x < 18$.

ANSWERS

1. $2x^2 + 6$; 2; 2; binomial
2. $5p^6 - 3p^3 - 4$; 6; 5; trinomial
3. $9x^7 + 13x^5 - 6x^2$; 7; 9; trinomial
4. $8y^3 - 12y$; 3; 8; binomial
5. $4a + 6$
6. $3x^2 + 6x + 10$
7. $-2y^2 + 6y + 4$
8. $-5p - 6$
9. $x^2 + 2x - 24$
10. $3y^2 - 7y - 40$
11. $x^4 + 7x^3 + 4x^2 + 28x$
12. $-12y^3 + 7y^2 + 20y - 7$
13. $x^2 - 81$
14. $4y^2 - 16$
15. $p^2 + 8p + 16$
16. $4d^2 - 4d + 1$

7 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

7.1 Adding and Subtracting Polynomials *(pp. 337–344)*

Find $(2x^3 + 6x^2 - x) - (-3x^3 - 2x^2 - 9x)$.

$$\begin{aligned}(2x^3 + 6x^2 - x) - (-3x^3 - 2x^2 - 9x) &= (2x^3 + 6x^2 - x) + (3x^3 + 2x^2 + 9x)\\ &= (2x^3 + 3x^3) + (6x^2 + 2x^2) + (-x + 9x)\\ &= 5x^3 + 8x^2 + 8x\end{aligned}$$

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

1. $6 + 2x^2$
2. $-3p^3 + 5p^6 - 4$
3. $9x^7 - 6x^2 + 13x^5$
4. $-12y + 8y^3$

Find the sum or difference.

5. $(3a + 7) + (a - 1)$
6. $(x^2 + 6x - 5) + (2x^2 + 15)$
7. $(-y^2 + y + 2) - (y^2 - 5y - 2)$
8. $(p + 7) - (6p + 13)$

7.2 Multiplying Polynomials *(pp. 345–350)*

Find $(x + 7)(x - 9)$.

$$\begin{aligned}(x + 7)(x - 9) &= x(x - 9) + 7(x - 9) && \text{Distribute } (x - 9) \text{ to each term of } (x + 7).\\ &= x(x) + x(-9) + 7(x) + 7(-9) && \text{Distributive Property}\\ &= x^2 + (-9x) + 7x + (-63) && \text{Multiply.}\\ &= x^2 - 2x - 63 && \text{Combine like terms.}\end{aligned}$$

Find the product.

9. $(x + 6)(x - 4)$
10. $(y - 5)(3y + 8)$
11. $(x^2 + 4)(x^2 + 7x)$
12. $(-3y + 1)(4y^2 - y - 7)$

7.3 Special Products of Polynomials *(pp. 351–356)*

Find each product.

a. $(6x + 4y)^2$

$$\begin{aligned}(6x + 4y)^2 &= (6x)^2 + 2(6x)(4y) + (4y)^2 && \text{Square of a binomial pattern}\\ &= 36x^2 + 48xy + 16y^2 && \text{Simplify.}\end{aligned}$$

b. $(2x + 3y)(2x - 3y)$

$$\begin{aligned}(2x + 3y)(2x - 3y) &= (2x)^2 - (3y)^2 && \text{Sum and difference pattern}\\ &= 4x^2 - 9y^2 && \text{Simplify.}\end{aligned}$$

Find the product.

13. $(x + 9)(x - 9)$
14. $(2y + 4)(2y - 4)$
15. $(p + 4)^2$
16. $(-1 + 2d)^2$

7.4 Dividing Polynomials *(pp. 357–362)*

Find $(20x^2 + 19x - 3) \div (-4x + 1)$.

$$
\begin{array}{rl}
 & -5x - 6 \\
-4x + 1 \overline{)} & 20x^2 + 19x - 3 \\
 & \underline{20x^2 - 5x} \\
 & 24x - 3 \\
 & \underline{24x - 6} \\
 & 3
\end{array}
$$

Multiply divisor by $\frac{20x}{-4x} = -5x$.

Subtract. Bring down next term.

Multiply divisor by $\frac{24x}{-4x} = -6$.

Subtract.

▶ $(20x^2 + 19x - 3) \div (-4x + 1) = -5x - 6 + \frac{3}{-4x + 1}$

Divide.

17. $(36x + 3) \div (9x + 4)$

18. $(-16x^2 - 26x - 13) \div (2x - 3)$

19. $(11x^2 - 3x + 16) \div (x - 3)$

20. $(7x + 14x^2 - 4) \div (-7x^2 + 5)$

7.5 Solving Polynomial Equations in Factored Form *(pp. 363–368)*

Solve $(x + 6)(x - 8) = 0$.

$(x + 6)(x - 8) = 0$ Write equation.

$x + 6 = 0$ *or* $x - 8 = 0$ Zero-Product Property

$x = -6$ *or* $x = 8$ Solve for x.

Solve the equation.

21. $x^2 + 5x = 0$

22. $(z + 3)(z - 7) = 0$

23. $(b + 13)^2 = 0$

24. $2y(y - 9)(y + 4) = 0$

7.6 Factoring $x^2 + bx + c$ *(pp. 371–376)*

Factor $x^2 + 6x - 27$.

Notice that $b = 6$ and $c = -27$. Because c is negative, the factors p and q must have different signs so that pq is negative.

Find two integer factors of -27 whose sum is 6.

Factors of −27	−27, 1	−1, 27	−9, 3	−3, 9
Sum of factors	−26	26	−6	6

The values of p and q are -3 and 9.

▶ So, $x^2 + 6x - 27 = (x - 3)(x + 9)$.

Factor the polynomial.

25. $p^2 + 2p - 35$ **26.** $b^2 + 18b + 80$ **27.** $z^2 - 4z - 21$ **28.** $x^2 - 11x + 28$

ANSWERS

17. $4 + \frac{-13}{9x + 4}$

18. $-8x - 25 + \frac{-88}{2x - 3}$

19. $11x + 30 + \frac{106}{x - 3}$

20. $-2 + \frac{7x + 6}{-7x^2 + 6}$

21. $x = 0, x = -5$

22. $z = -3, z = 7$

23. $b = -13$

24. $y = 0, y = 9, y = -4$

25. $(p + 7)(p - 5)$

26. $(b + 8)(b + 10)$

27. $(z + 3)(z - 7)$

28. $(x - 7)(x - 4)$

ANSWERS

29. $(t + 6)(3t - 2)$
30. $-2(2y - 3)(y - 2)$
31. $(2x + 1)(3x + 7)$
32. $(x + 3)(x - 3)$
33. $(y + 10)(y - 10)$
34. $(z - 3)^2$
35. $(m + 8)^2$
36. $n(n + 3)(n - 3)$
37. $(x - 3)(x + 4a)$
38. $2x^2(x^2 + x - 10)$
39. $x = 0, x = 6, x = -3$
40. $x = -\frac{3}{2}, x = \frac{3}{2}$
41. $z = -3, z = 5, z = -5$
42. length: 12 ft, width: 4 ft, height: 2 ft

7.7 Factoring $ax^2 + bx + c$ *(pp. 377–382)*

Factor $5x^2 + 36x + 7$.

There is no GCF, so you need to consider the possible factors of a and c. Because b and c are both positive, the factors of c must be positive. Use a table to organize information about the factors of a and c.

Factors of 5	Factors of 7	Possible factorization	Middle term	
1, 5	1, 7	$(x + 1)(5x + 7)$	$7x + 5x = 12x$	✗
1, 5	7, 1	$(x + 7)(5x + 1)$	$x + 35x = 36x$	✓

▶ So, $5x^2 + 36x + 7 = (x + 7)(5x + 1)$.

Factor the polynomial.

29. $3t^2 + 16t - 12$ **30.** $-4y^2 + 14y - 12$ **31.** $6x^2 + 17x + 7$

7.8 Factoring Special Products *(pp. 383–388)*

Factor $25x^2 - 30x + 9$.

$25x^2 - 30x + 9 = (5x)^2 - 2(5x)(3) + 3^2$ Write as $a^2 - 2ab + b^2$.

$= (5x - 3)^2$ Perfect square trinomial pattern

Factor the polynomial.

32. $x^2 - 9$ **33.** $y^2 - 100$ **34.** $z^2 - 6z + 9$ **35.** $m^2 + 16m + 64$

7.9 Factoring Polynomials Completely *(pp. 389–394)*

Factor $x^3 + 4x^2 - 3x - 12$.

$x^3 + 4x^2 - 3x - 12 = (x^3 + 4x^2) + (-3x - 12)$ Group terms with common factors.

$= x^2(x + 4) + (-3)(x + 4)$ Factor out GCF of each pair of terms.

$= (x + 4)(x^2 - 3)$ Factor out $(x + 4)$.

Factor the polynomial completely.

36. $n^3 - 9n$ **37.** $x^2 - 3x + 4ax - 12a$ **38.** $2x^4 + 2x^3 - 20x^2$

Solve the equation.

39. $3x^3 - 9x^2 - 54x = 0$ **40.** $16x^2 - 36 = 0$ **41.** $z^3 + 3z^2 - 25z - 75 = 0$

42. A box in the shape of a rectangular prism has a volume of 96 cubic feet. The box has a length of $(x + 8)$ feet, a width of x feet, and a height of $(x - 2)$ feet. Find the dimensions of the box.

7 Chapter Test

Find the sum or difference. Then identify the degree of the sum or difference and classify it by the number of terms.

1. $(-2p + 4) - (p^2 - 6p + 8)$
2. $(9c^6 - 5b^4) - (4c^6 - 5b^4)$
3. $(4s^4 + 2st + t) + (2s^4 - 2st - 4t)$

Find the product.

4. $(h - 5)(h - 8)$
5. $(2w - 3)(3w + 5)$
6. $(z + 11)(z - 11)$
7. Explain how you can determine whether a polynomial is a perfect square trinomial.
8. Is 18 a polynomial? Explain your reasoning.

Divide.

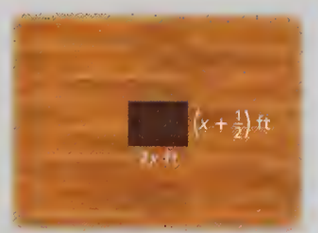
9. $(-3x^2 + 12x - 8) \div (x + 1)$
10. $(45x - 6) \div (15x - 5)$
11. $(5x^2 + 4 + 17x) \div (-x^2 + 7)$

Factor the polynomial completely.

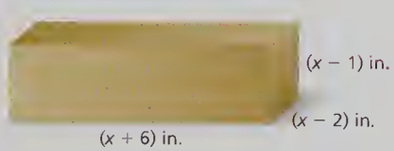
12. $s^2 - 15s + 50$
13. $h^3 + 2h^2 - 9h - 18$
14. $-5k^2 - 22k + 15$

Solve the equation.

15. $(n - 1)(n + 6)(n + 5) = 0$
16. $d^2 + 14d + 49 = 0$
17. $6x^4 + 8x^2 = 26x^3$
18. The expression $\pi(r - 3)^2$ represents the area covered by the hour hand on a clock in one rotation, where r is the radius of the entire clock. Write a polynomial in standard form that represents the area covered by the hour hand in one rotation.
19. A magician's stage has a trapdoor.
 a. The total area (in square feet) of the stage can be represented by $x^2 + 27x + 176$. Write an expression for the width of the stage.
 b. Write an expression for the perimeter of the stage.
 c. The area of the trapdoor is 10 square feet. Find the value of x.
 d. The magician wishes to have the area of the stage be at least 20 times the area of the trapdoor. Does this stage satisfy his requirement? Explain.



20. You are jumping on a trampoline. For one jump, your height y (in feet) above the trampoline after t seconds can be represented by $y = -16t^2 + 24t$. How many seconds are you in the air?
21. A cardboard box in the shape of a rectangular prism has the dimensions shown.
 a. Write a polynomial that represents the volume of the box.
 b. The volume of the box is 60 cubic inches. What are the length, width, and height of the box?



If students need help...	If students got it...
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

ANSWERS

1. $-p^2 + 4p - 4$; 2; trinomial
2. $5c^6$; 6; monomial
3. $6s^4 - 3t$; 4; binomial
4. $h^2 - 13h + 40$
5. $6w^2 + w - 15$
6. $z^2 - 121$
7. Check the first and last terms to verify they are perfect squares. If they are, find 2 times the product of their square roots. If this matches the middle term, or the opposite of the middle term, it is a perfect square trinomial.
8. yes; A monomial is a type of polynomial, and a number is a type of monomial.
9. $-3x + 15 + \frac{-23}{x + 1}$
10. $3 + \frac{9}{15x - 5}$
11. $-5 + \frac{17x + 39}{-x^2 + 7}$
12. $(s - 5)(s - 10)$
13. $(h + 2)(h + 3)(h - 3)$
14. $-(5k - 3)(k + 5)$
15. $n = 1, n = -6, n = -5$
16. $d = -7$
17. $x = 0, x = \frac{1}{3}, x = 4$
18. $\pi r^2 - 6\pi r + 9\pi$
19. a. $(x + 11)$ ft
 b. $(4x + 54)$ ft
 c. $x = 2$
 d. yes; The area of the stage is 234 square feet, and twenty times the area of the trap door is 200 square feet, so the area of the stage is greater than twenty times the area of the trap door.
20. 1.5 sec
21. a. $(x^3 + 3x^2 - 16x + 12)$ in.^{3}
 b. length: 10 in., width: 2 in., height: 3 in.

ANSWERS

1. C
2. F
3. B
4. J
5. C

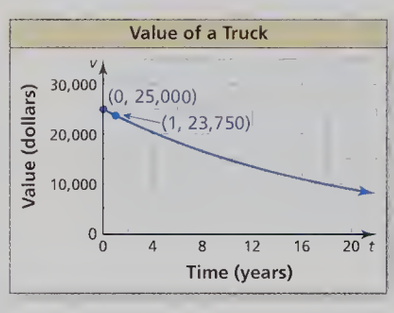
7 Standards Assessment

1. Which expression is equivalent to $-2x + 15x^2 - 8$? *(TEKS A.10.E)*

 Ⓐ $(3x - 2)(5x - 4)$ Ⓑ $(5x + 4)(3x + 2)$

 Ⓒ $(5x - 4)(3x + 2)$ Ⓓ $(3x - 2)(5x + 4)$

2. The graph shows the value v (in dollars) of a truck over time t (in years). Based on the graph, which statement is true? *(TEKS A.9.B, TEKS A.9.C)*



 Ⓕ The value of the truck decreases by 5% each year.

 Ⓖ The value of the truck decreases by 95% each year.

 Ⓗ The value of the truck decreases by $1250 each year.

 Ⓙ The value of the truck decreases by $2250 each year.

3. Which polynomial represents the product of $2x - 4$ and $x^2 + 6x - 2$? *(TEKS A.10.B)*

 Ⓐ $2x^3 + 8x^2 - 4x + 8$ Ⓑ $2x^3 + 8x^2 - 28x + 8$

 Ⓒ $2x^3 + 8$ Ⓓ $2x^3 - 24x - 2$

4. Which of the following correctly describes a line that is perpendicular to the x-axis? *(TEKS A.2.G)*

 Ⓕ $y = -4$; slope $= 0$ Ⓖ $x = -4$; slope $= 0$

 Ⓗ $y = -4$; The slope is undefined. Ⓙ $x = -4$; The slope is undefined.

5. Which of the following are solutions of the equation $x^3 + 6x^2 - 4x = 24$? *(TEKS A.10.F)*

 I. -6
 II. -2
 III. 0
 IV. 2

 Ⓐ I and IV only Ⓑ I and II only

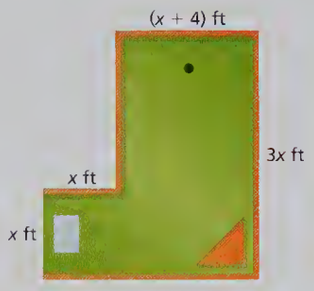
 Ⓒ I, II, and IV only Ⓓ I, II, III, and IV

6. Which of the following is a direct variation equation? *(TEKS A.2.D)*

Ⓕ $4y - 2x = 6$ Ⓖ $5y = 5x + 10$

Ⓗ $3y + 12x = 0$ Ⓙ $y + 4 = 0$

7. **GRIDDED ANSWER** You are playing miniature golf on the hole shown. The area of the golf hole is 216 square feet. What is the perimeter (in feet) of the golf hole? *(TEKS A.8.A, TEKS A.10.E)*

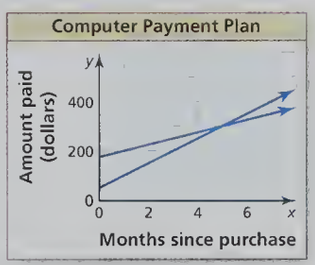


8. What is the 4th term of the geometric sequence $f(1) = 6, f(n) = -\frac{1}{2}f(n-1)$? *(TEKS A.12.C)*

Ⓐ $-\frac{3}{2}$ Ⓑ $-\frac{3}{4}$

Ⓒ $\frac{3}{8}$ Ⓓ $\frac{3}{4}$

9. At an electronics store, two customers each buy a computer on the same day. Each customer arranges a payment plan. The graph shows the amount y (in dollars) paid for the computers after x months. What does the intersection of the lines represent? *(TEKS A.3.G)*



Ⓕ After 5 months, each customer will have paid the same amount of $300.

Ⓖ After 5 months, each customer will have paid the same amount of $350.

Ⓗ After 6 months, each customer will have paid the same amount of $350.

Ⓙ none of the above

10. What is the remainder of $(4x^2 + 7x - 1) \div (4 + x)$? *(TEKS A.10.C)*

Ⓐ $-9x - 1$ Ⓑ $23x - 1$

Ⓒ 35 Ⓓ -37

ANSWERS

6. H
7. 68
8. B
9. F
10. C

Chapter 8 Pacing Guide

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Quiz	1 Day
Section 4	2 Days
Section 5	3 Days
Section 6	3 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 8	18 Days
Year-to-Date	139 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
8.1	A.6.A, A.7.A, A.7.C
8.2	A.7.A, A.7.C
8.3	A.6.A, A.7.A
8.4	A.6.B, A.7.A, A.7.C
8.5	A.6.A, A.6.B, A.6.C, A.7.A, A.7.B
8.6	A.2.C, A.6.C, A.9.C

8 Graphing Quadratic Functions

8.1 Graphing $f(x) = ax^2$
8.2 Graphing $f(x) = ax^2 + c$
8.3 Graphing $f(x) = ax^2 + bx + c$
8.4 Graphing $f(x) = a(x - h)^2 + k$
8.5 Using Intercept Form
8.6 Comparing Linear, Exponential, and Quadratic Functions



Town Population *(p. 450)*


Satellite Dish *(p. 443)*


Roller Coaster *(p. 434)*


Firework Explosion *(p. 423)*


Garden Waterfalls *(p. 416)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

For an overview of this chapter, formative assessment tips, and teaching strategies, see Laurie's Notes at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

Graphing Linear Equations (A.3.C)

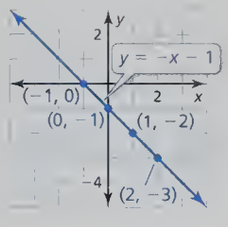
Example 1 Graph $y = -x - 1$.

Step 1 Make a table of values.

x	$y = -x - 1$	y	(x, y)
-1	$y = -(-1) - 1$	0	$(-1, 0)$
0	$y = -(0) - 1$	-1	$(0, -1)$
1	$y = -(1) - 1$	-2	$(1, -2)$
2	$y = -(2) - 1$	-3	$(2, -3)$

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points.



Graph the linear equation.

1. $y = 2x - 3$
2. $y = -3x + 4$
3. $y = -\frac{1}{2}x - 2$
4. $y = x + 5$

Evaluating Expressions (A.11.B)

Example 2 Evaluate $2x^2 + 3x - 5$ when $x = -1$.

$2x^2 + 3x - 5 = 2(-1)^2 + 3(-1) - 5$ Substitute -1 for x.

$= 2(1) + 3(-1) - 5$ Evaluate the power.

$= 2 - 3 - 5$ Multiply.

$= -6$ Subtract.

Evaluate the expression when $x = -2$.

5. $5x^2 - 9$
6. $3x^2 + x - 2$
7. $-x^2 + 4x + 1$
8. $x^2 + 8x + 5$
9. $-2x^2 - 4x + 3$
10. $-4x^2 + 2x - 6$

11. **ABSTRACT REASONING** Complete the table. Find a pattern in the differences of consecutive y-values. Use the pattern to write an expression for y when $x = 6$.

x	1	2	3	4	5
$y = ax^2$					

Dynamic Solutions available at *BigIdeasMath.com*

For suggestions on Maintaining Mathematical Proficiency in your classroom, see Laurie's Notes at *BigIdeasMath.com*.

Vocabulary Review

Have students make Process Diagrams for the following topics.

- Graphing Linear Equations
- Evaluating Algebraic Expressions

Dynamic Teaching Tools

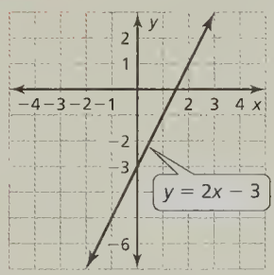
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

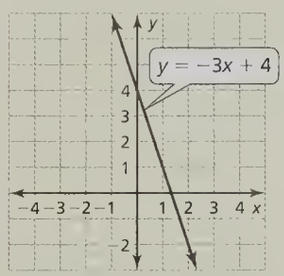
Texas Essential Knowledge and Skills

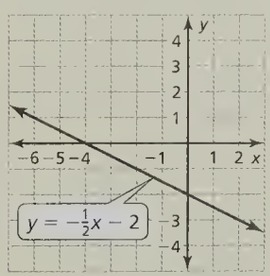
A.3.C The student is expected to graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

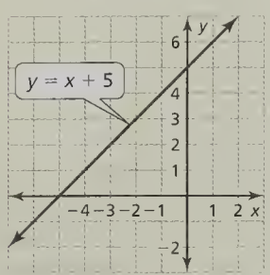
A.11.B The student is expected to simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

ANSWERS

1.


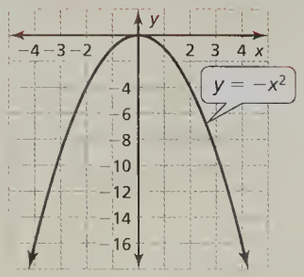
2.


3.


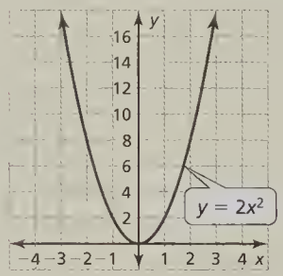
4.


5–11. See Additional Answers.

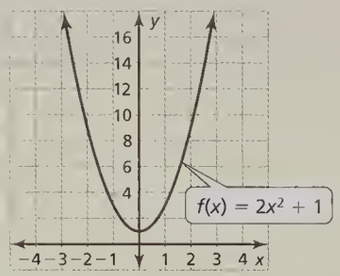
MONITORING PROGRESS ANSWERS

1. 

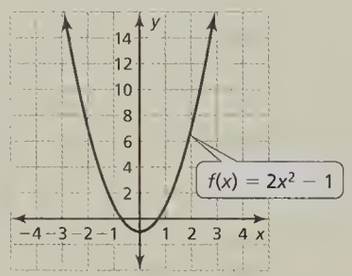
The graph opens down and the highest point is at the origin.

2. 

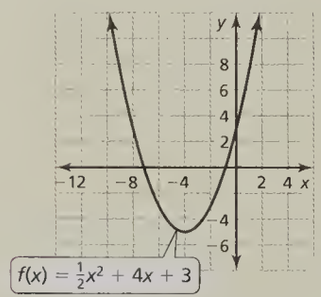
The graph opens up and the lowest point is at the origin.

3. 

The graph opens up and the lowest point is at (0, 1).

4. 

The graph opens up and the lowest point is at (0, −1).

5. 

The graph opens up and the lowest point is at (−4, −5).

6–9. See Additional Answers.

Mathematical Thinking

Mathematically proficient students use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. **(A.1.B)**

Problem-Solving Strategies

Core Concept

Trying Special Cases

When solving a problem in mathematics, it can be helpful to try special cases of the original problem. For instance, in this chapter, you will learn to graph a quadratic function of the form $f(x) = ax^2 + bx + c$. The problem-solving strategy used is to first graph quadratic functions of the form $f(x) = ax^2$. From there, you progress to other forms of quadratic functions.

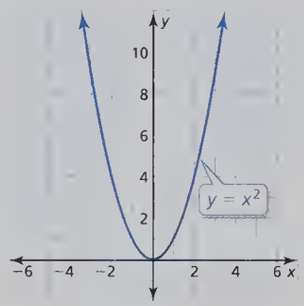
$f(x) = ax^2$	Section 8.1
$f(x) = ax^2 + c$	Section 8.2
$f(x) = ax^2 + bx + c$	Section 8.3
$f(x) = a(x - h)^2 + k$	Section 8.4

EXAMPLE 1 **Graphing the Parent Quadratic Function**

Graph the parent quadratic function $y = x^2$. Then describe its graph.

SOLUTION

The function is of the form $y = ax^2$, where $a = 1$. By plotting several points, you can see that the graph is U-shaped, as shown.



▶ The graph opens up, and the lowest point is at the origin.

Monitoring Progress

Graph the quadratic function. Then describe its graph.

1. $y = -x^2$
2. $y = 2x^2$
3. $f(x) = 2x^2 + 1$
4. $f(x) = 2x^2 - 1$
5. $f(x) = \frac{1}{2}x^2 + 4x + 3$
6. $f(x) = \frac{1}{2}x^2 - 4x + 3$
7. $y = -2(x + 1)^2 + 1$
8. $y = -2(x - 1)^2 + 1$
9. How are the graphs in Monitoring Progress Questions 1–8 similar? How are they different?

For insights into Mathematical Thinking, see Laurie's Notes at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

8.1 Graphing $f(x) = ax^2$

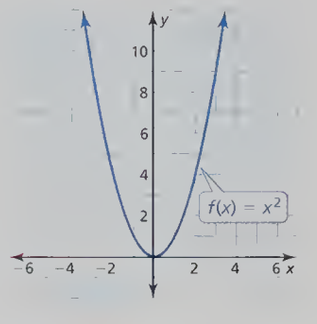
Texas Essential Knowledge and Skills
A.6.A
A.7.A
A.7.C

Essential Question What are some of the characteristics of the graph of a quadratic function of the form $f(x) = ax^2$?

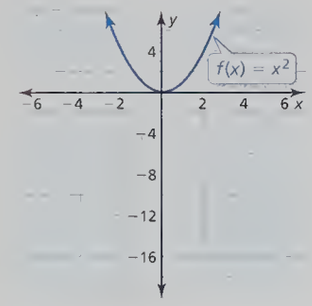
EXPLORATION 1 Graphing Quadratic Functions

Work with a partner. Graph each quadratic function. Compare each graph to the graph of $f(x) = x^2$.

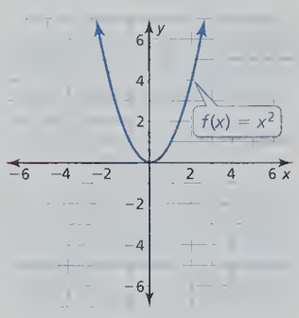
a. $g(x) = 3x^2$



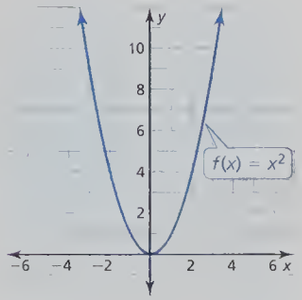
b. $g(x) = -5x^2$



c. $g(x) = -0.2x^2$



d. $g(x) = \frac{1}{10}x^2$

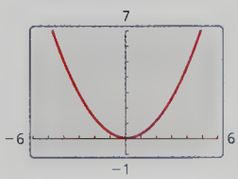


REASONING

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

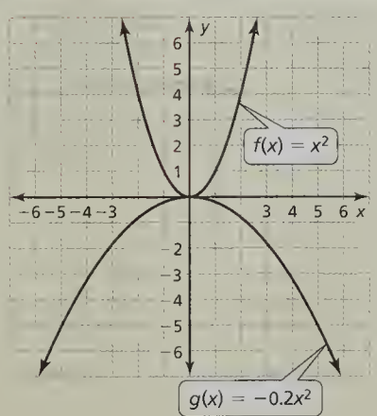
Communicate Your Answer

2. What are some of the characteristics of the graph of a quadratic function of the form $f(x) = ax^2$?
3. How does the value of a affect the graph of $f(x) = ax^2$? Consider $0 < a < 1$, $a > 1$, $-1 < a < 0$, and $a < -1$. Use a graphing calculator to verify your answers.
4. The figure shows the graph of a quadratic function of the form $y = ax^2$. Which of the intervals in Question 3 describes the value of a? Explain your reasoning.



For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

1. c.



The graph of g is a vertical shrink by a factor of 0.2 and a reflection in the x-axis of the graph of f.

1. d. See Additional Answers.

2. *Sample answer:* They are U-shaped and symmetric. They either open up with the lowest point at the origin, or they open down with the highest point at the origin.

3–4. See Additional Answers.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

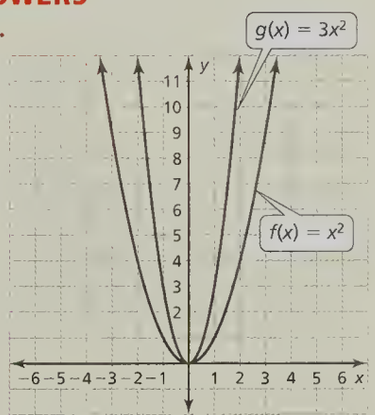
A.6.A The student is expected to determine the domain and range of quadratic functions and represent the domain and range using inequalities.

A.7.A The student is expected to graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

A.7.C The student is expected to determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

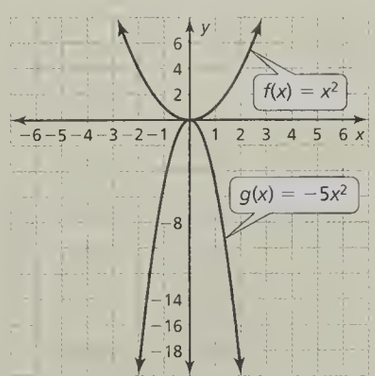
ANSWERS

1. a.



The graph of g is a vertical stretch by a factor of 3 of the graph of f.

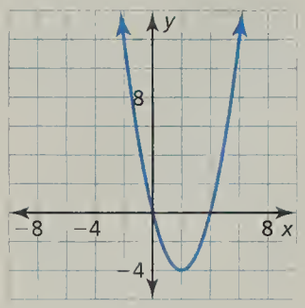
b.



The graph of g is a vertical stretch by a factor of 5 and a reflection in the x-axis of the graph of f.

Extra Example 1

Identify characteristics of the quadratic function and its graph.



The vertex is (2, −4), the axis of symmetry is $x = 2$, the domain is all real numbers, the range is $y \geq -4$; when $x < 2$, y increases as x decreases, and when $x > 2$, y increases as x increases.

MONITORING PROGRESS ANSWERS

1. The vertex is (2, −3). The axis of symmetry is $x = 2$. The domain is all real numbers. The range is $y \geq -3$. When $x < 2$, y increases as x decreases. When $x > 2$, y increases as x increases.
2. The vertex is (−3, 7). The axis of symmetry is $x = -3$. The domain is all real numbers. The range is $y \leq 7$. When $x < -3$, y increases as x increases. When $x > -3$, y increases as x decreases.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

8.1 Lesson

What You Will Learn

- Identify characteristics of quadratic functions.
- Graph and use quadratic functions of the form $f(x) = ax^2$.

Core Vocabulary

quadratic function, *p. 406*
parabola, *p. 406*
vertex, *p. 406*
axis of symmetry, *p. 406*

Previous
domain
range
vertical shrink
vertical stretch
reflection

REMEMBER

The notation $f(x)$ is another name for y.

Identifying Characteristics of Quadratic Functions

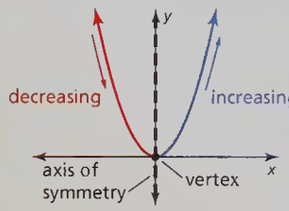
A **quadratic function** is a nonlinear function that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$. The U-shaped graph of a quadratic function is called a **parabola**. In this lesson, you will graph quadratic functions, where b and c equal 0.

Core Concept

Characteristics of Quadratic Functions

The *parent quadratic function* is $f(x) = x^2$. The graphs of all other quadratic functions are *transformations* of the graph of the parent quadratic function.

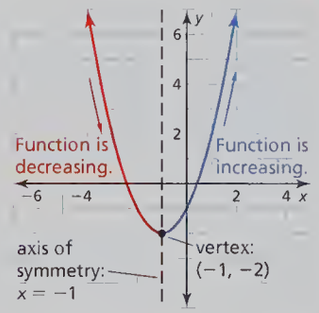
The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex**. The vertex of the graph of $f(x) = x^2$ is (0, 0).



The vertical line that divides the parabola into two symmetric parts is the **axis of symmetry**. The axis of symmetry passes through the vertex. For the graph of $f(x) = x^2$, the axis of symmetry is the y-axis, or $x = 0$.

EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.



Using the graph, you can identify characteristics such as the vertex, axis of symmetry, and the behavior of the graph, as shown.

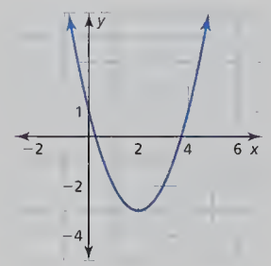
You can also determine the following:

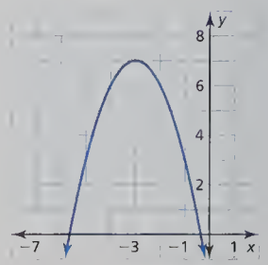
- The domain is all real numbers.
- The range is all real numbers greater than or equal to −2.
- When $x < -1$, y increases as x decreases.
- When $x > -1$, y increases as x increases.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Identify characteristics of the quadratic function and its graph.

1.


2.


SUPPORTING English Language Learners

Present the information about Identifying Characteristics of Quadratic Functions and the Core Concept that precede Example 1. Have students follow along as you read. Explain that the sentences containing the highlighted Core Vocabulary terms provide definitions of those terms. Review each term. Have students close their books. State a definition and ask them to state the Core Vocabulary term that is defined.

Beginning Choose the correct term from a list in multiple-choice form.
Intermediate/Advanced/Advanced High State the correct Core Vocabulary term in short answer form.

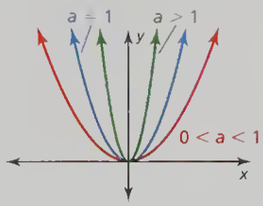
ELPS 2.E.3 Use linguistic support to enhance and confirm understanding of increasingly complex and elaborated spoken language.

Graphing and Using $f(x) = ax^2$

Core Concept

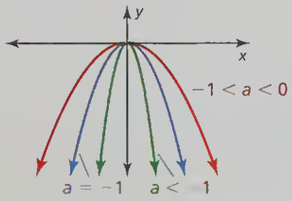
Graphing $f(x) = ax^2$ When $a > 0$

- When $0 < a < 1$, the graph of $f(x) = ax^2$ is a vertical shrink of the graph of $f(x) = x^2$.
- When $a > 1$, the graph of $f(x) = ax^2$ is a vertical stretch of the graph of $f(x) = x^2$.



Graphing $f(x) = ax^2$ When $a < 0$

- When $-1 < a < 0$, the graph of $f(x) = ax^2$ is a vertical shrink with a reflection in the x-axis of the graph of $f(x) = x^2$.
- When $a < -1$, the graph of $f(x) = ax^2$ is a vertical stretch with a reflection in the x-axis of the graph of $f(x) = x^2$.



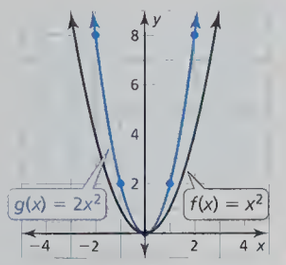
REMEMBER

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$.

The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.

EXAMPLE 2 Graphing $y = ax^2$ When $a > 0$

Graph $g(x) = 2x^2$. Compare the graph to the graph of $f(x) = x^2$.



SOLUTION

Step 1 Make a table of values.

x	−2	−1	0	1	2
$g(x)$	8	2	0	2	8

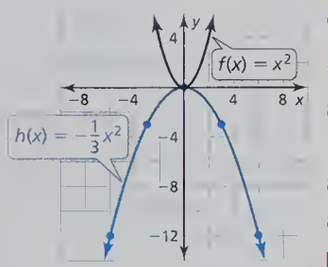
Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

▶ Both graphs open up and have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of g is narrower than the graph of f because the graph of g is a vertical stretch by a factor of 2 of the graph of f.

EXAMPLE 3 Graphing $y = ax^2$ When $a < 0$

Graph $h(x) = -\frac{1}{3}x^2$. Compare the graph to the graph of $f(x) = x^2$.



SOLUTION

Step 1 Make a table of values.

x	−6	−3	0	3	6
$h(x)$	−12	−3	0	−3	−12

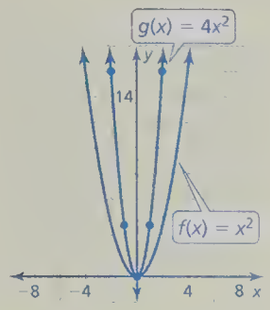
Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

▶ The graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$, but the graph of h opens down and is wider than the graph of f. So, the graph of h is a vertical shrink by a factor of $\frac{1}{3}$ and a reflection in the x-axis of the graph of f.

Extra Example 2

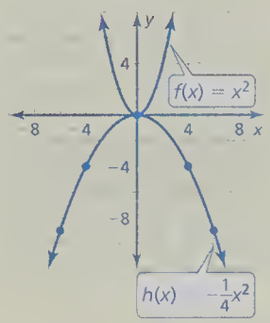
Graph $g(x) = 4x^2$. Compare the graph to the graph of $f(x) = x^2$.



Both graphs open up and have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of g is a vertical stretch by a factor of 4 of the graph of f.

Extra Example 3

Graph $h(x) = -\frac{1}{4}x^2$. Compare the graph to the graph of $f(x) = x^2$.



The graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of h opens down and is a vertical shrink by a factor of $\frac{1}{4}$ and a reflection in the x-axis of the graph of f.

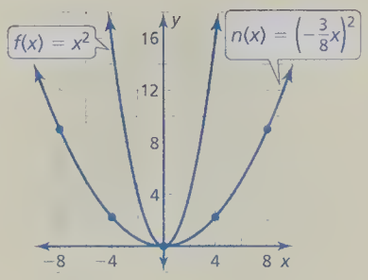
Differentiated Instruction

Kinesthetic/Visual

Some students may benefit by having access to graphing calculators or graphing software programs available on many computers. Students can enter the quadratic function $f(x) = x^2$ and the given quadratic equation, view the graphs, and then compare the graphs.

Extra Example 4

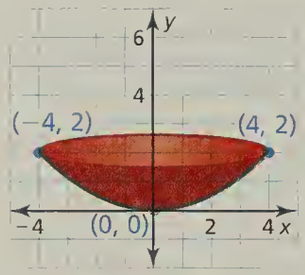
Graph $n(x) = \left(-\frac{3}{8}x\right)^2$. Compare the graph to the graph of $f(x) = x^2$.



Both graphs open up and have the same vertex, (0, 0), and axis of symmetry, $x = 0$. The graph of n is a horizontal stretch by a factor of $\frac{8}{3}$ of the graph of f.

Extra Example 5

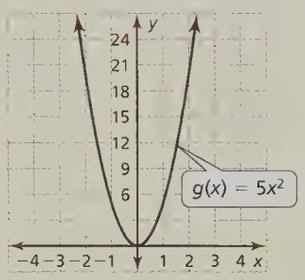
The diagram shows the cross section of a soup bowl, where x and y are measured in inches. Find the width and depth of the bowl.



8 inches wide, 2 inches deep

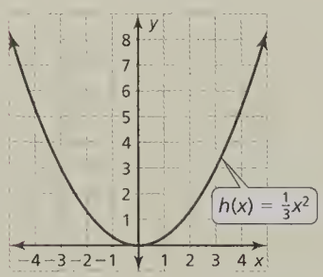
MONITORING PROGRESS ANSWERS

3.



The graph of g is a vertical stretch by a factor of 5 of the graph of f.

4.



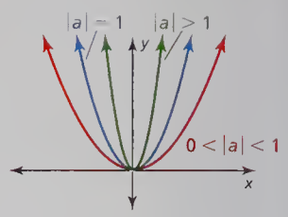
The graph of h is a vertical shrink by a factor of $\frac{1}{3}$ of the graph of f.

5–9. See Additional Answers.

Core Concept

Graphing $f(x) = (ax)^2$

- When $0 < |a| < 1$, the graph of $f(x) = (ax)^2$ is a horizontal stretch of the graph of $f(x) = x^2$.
- When $|a| > 1$, the graph of $f(x) = (ax)^2$ is a horizontal shrink of the graph of $f(x) = x^2$.



EXAMPLE 4 Graphing $y = (ax)^2$

Graph $n(x) = \left(-\frac{1}{4}x\right)^2$. Compare the graph to the graph of $f(x) = x^2$.

SOLUTION

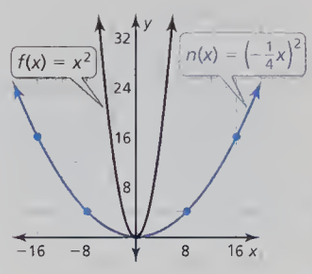
Rewrite n as $n(x) = \left(-\frac{1}{4}x\right)^2 = \frac{1}{16}x^2$.

Step 1 Make a table of values.

x	−16	−8	0	8	16
$n(x)$	16	4	0	4	16

Step 2 Plot the ordered pairs.

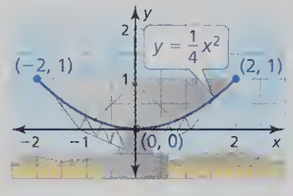
Step 3 Draw a smooth curve through the points.



▶ Both graphs open up and have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of n is wider than the graph of f because the graph of n is a horizontal stretch by a factor of 4 of the graph of f.

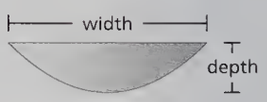
EXAMPLE 5 Solving a Real-Life Problem

The diagram at the left shows the cross section of a satellite dish, where x and y are measured in meters. Find the width and depth of the dish.



SOLUTION

Use the domain of the function to find the width of the dish. Use the range to find the depth.



The leftmost point on the graph is (−2, 1), and the rightmost point is (2, 1). So, the domain is $-2 \le x \le 2$, which represents 4 meters.

The lowest point on the graph is (0, 0), and the highest points on the graph are (−2, 1) and (2, 1). So, the range is $0 \le y \le 1$, which represents 1 meter.

▶ So, the satellite dish is 4 meters wide and 1 meter deep.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

3. $g(x) = 5x^2$
4. $h(x) = \frac{1}{3}x^2$
5. $p(x) = -3x^2$
6. $q(x) = -0.1x^2$
7. $n(x) = (3x)^2$
8. $g(x) = \left(-\frac{1}{2}x\right)^2$
9. The cross section of a spotlight can be modeled by the graph of $y = 0.5x^2$, where x and y are measured in inches and $-2 \le x \le 2$. Find the width and depth of the spotlight.

Closure

- **Exit Ticket:** Describe the differences between the graphs of $y = -3x^2$ and $y = \frac{1}{3}x^2$. The graph of $y = -3x^2$ opens down and the graph of $y = \frac{1}{3}x^2$ opens up. The graph of $y = -3x^2$ is narrower than the graph of $y = \frac{1}{3}x^2$. The range of $y = -3x^2$ is $y \le 0$. The range of $y = \frac{1}{3}x^2$ is $y \ge 0$.

8.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

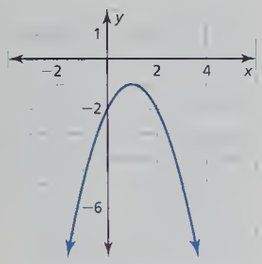
Vocabulary and Core Concept Check

1. **VOCABULARY** What is the U-shaped graph of a quadratic function called?
2. **WRITING** When does the graph of a quadratic function open up? open down?

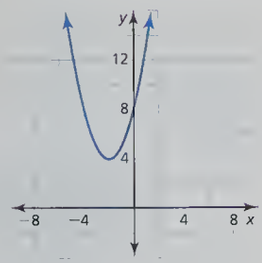
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, identify characteristics of the quadratic function and its graph. *(See Example 1.)*

3.



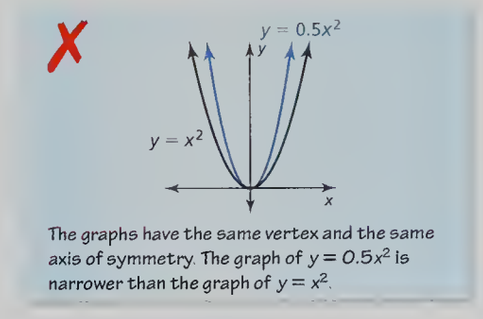
4.



In Exercises 5–16, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Examples 2, 3, and 4.)*

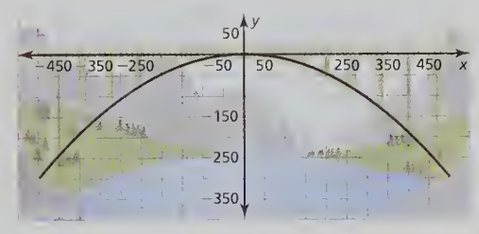
5. $g(x) = 6x^2$
6. $b(x) = 2.5x^2$
7. $h(x) = \frac{1}{4}x^2$
8. $j(x) = 0.75x^2$
9. $m(x) = -2x^2$
10. $q(x) = -\frac{9}{2}x^2$
11. $k(x) = -0.2x^2$
12. $p(x) = -\frac{2}{3}x^2$
13. $n(x) = (2x)^2$
14. $d(x) = (-4x)^2$
15. $c(x) = \left(-\frac{1}{3}x\right)^2$
16. $r(x) = (0.1x)^2$

17. **ERROR ANALYSIS** Describe and correct the error in graphing and comparing $y = x^2$ and $y = 0.5x^2$.

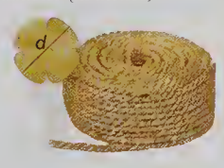


The graphs have the same vertex and the same axis of symmetry. The graph of $y = 0.5x^2$ is narrower than the graph of $y = x^2$.

18. **MODELING WITH MATHEMATICS** The arch support of a bridge can be modeled by $y = -0.0012x^2$, where x and y are measured in feet. Find the height and width of the arch. *(See Example 5.)*



19. **PROBLEM SOLVING** The breaking strength z (in pounds) of a manila rope can be modeled by $z = 8900d^2$, where d is the diameter (in inches) of the rope.



a. Describe the domain and range of the function.

b. Graph the function using the domain in part (a).

c. A manila rope has four times the breaking strength of another manila rope. Does the stronger rope have four times the diameter? Explain.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–17 odd, 18, 20–22, 31–35

Average: 1, 2–16 even, 17–23, 31–35

Advanced: 1, 2, 4, 8, 12–16 even, 17–23, 24–30 even, 31–35

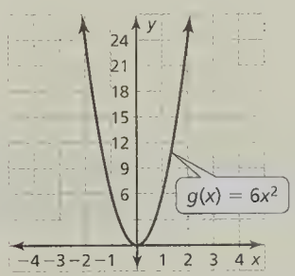
HOMEWORK CHECK

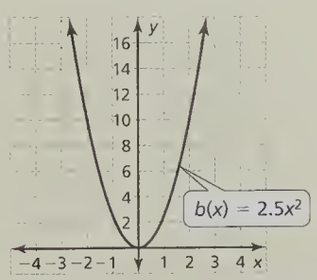
Basic: 3, 5, 9, 18

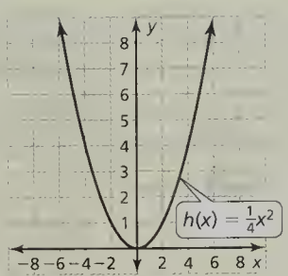
Average: 4, 6, 12, 18

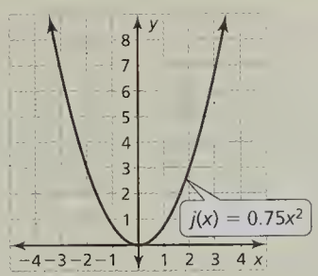
Advanced: 4, 8, 12, 18

ANSWERS

1. parabola
2. when $a > 0$; when $a < 0$
3. The vertex is $(1, -1)$. The axis of symmetry is $x = 1$. The domain is all real numbers. The range is $y \leq -1$. When $x < 1$, y increases as x increases. When $x > 1$, y increases as x decreases.
4. The vertex is $(-2, 4)$. The axis of symmetry is $x = -2$. The domain is all real numbers. The range is $y \geq 4$. When $x < -2$, y increases as x decreases. When $x > -2$, y increases as x increases.
5. 

The graph of g is a vertical stretch by a factor of 6 of the graph of f.
6. 

The graph of b is a vertical stretch by a factor of 2.5 of the graph of f.
7. 

The graph of h is a vertical shrink by a factor of $\frac{1}{4}$ of the graph of f.
8. 

The graph of j is a vertical shrink by a factor of 0.75 of the graph of f.

9–19. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

20. a. $a > 1$

b. $-1 < a < 0$

21. f is increasing when $x > 0$. g is increasing when $x < 0$.

22. f is decreasing when $x < 0$. g is decreasing when $x > 0$.

23. f; $a = \frac{3}{4}$

24. yes; *Sample answer:* At an x-intercept, $y = 0$, so $ax^2 = 0$. By the definition of a quadratic function, $a \neq 0$, so by the Zero-Product Property, $x^2 = 0$, which means $x = 0$.

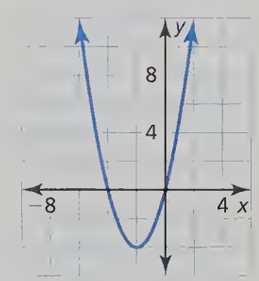
25. *Sample answer:* The vertex of a parabola that opens up is the minimum point, so its y-coordinate is the minimum value of y. The graph passes through $(6, -3)$, so 2 is not the minimum value of y.

26. sometimes; *Sample answer:* The graph of f will be narrower than the graph of g when $a > 1$, but it will be wider when $0 < a < 1$.

27–35. See Additional Answers.

Mini-Assessment

1. Identify characteristics of the quadratic function and its graph.



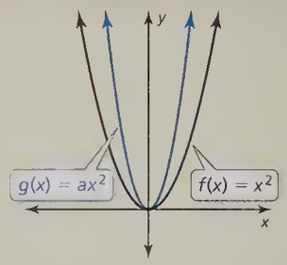
vertex: $(2, -4)$, axis of symmetry: $x = -2$, domain: real numbers, range: $y \geq -4$; when $x < -2$, y increases as x decreases, when $x > -2$, y increases as x increases.

2. Graph $g(x) = 3.5x^2$. Compare the graph to the graph of $f(x) = x^2$.

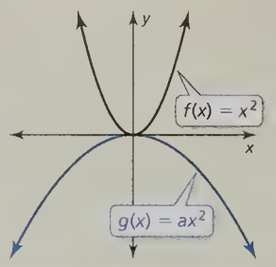
See Additional Answers.

20. HOW DO YOU SEE IT? Describe the possible values of a.

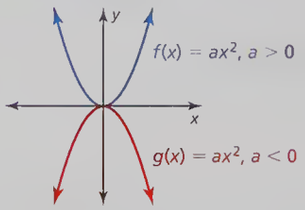
a.



b.



ANALYZING GRAPHS In Exercises 21–23, use the graph.



21. When is each function increasing?

22. When is each function decreasing?

23. Which function could include the point $(-2, 3)$? Find the value of a when the graph passes through $(-2, 3)$.

24. REASONING Is the x-intercept of the graph of $y = ax^2$ always 0? Justify your answer.

25. REASONING A parabola opens up and passes through $(-4, 2)$ and $(6, -3)$. How do you know that $(-4, 2)$ is not the vertex?

ABSTRACT REASONING In Exercises 26–29, determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

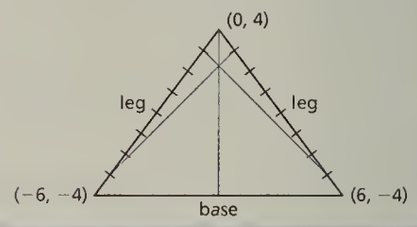
26. The graph of $f(x) = ax^2$ is narrower than the graph of $g(x) = x^2$ when $a > 0$.

27. The graph of $f(x) = ax^2$ is narrower than the graph of $g(x) = x^2$ when $|a| > 1$.

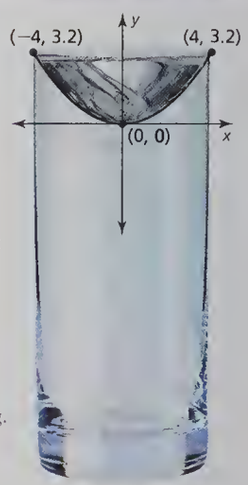
28. The graph of $f(x) = ax^2$ is wider than the graph of $g(x) = x^2$ when $0 < |a| < 1$.

29. The graph of $f(x) = ax^2$ is wider than the graph of $g(x) = dx^2$ when $|a| > |d|$.

30. THOUGHT PROVOKING Draw the isosceles triangle shown. Divide each leg into eight congruent segments. Connect the highest point of one leg with the lowest point of the other leg. Then connect the second highest point of one leg to the second lowest point of the other leg. Continue this process. Write a quadratic equation whose graph models the shape that appears.



31. MAKING AN ARGUMENT The diagram shows the parabolic cross section of a swirling glass of water, where x and y are measured in centimeters.



a. About how wide is the mouth of the glass?

b. Your friend claims that the rotational speed of the water would have to increase for the cross section to be modeled by $y = 0.1x^2$. Is your friend correct? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when $n = 3$ and $x = -2$. *(Skills Review Handbook)*

32. $n^2 + 5$ **33.** $3x^2 - 9$ **34.** $-4n^2 + 11$ **35.** $n + 2x^2$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

8.2 Graphing $f(x) = ax^2 + c$

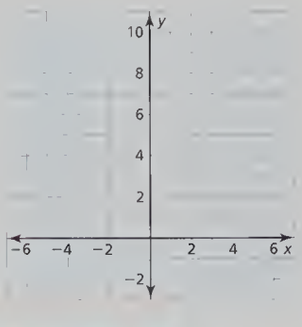
Texas Essential Knowledge and Skills
A.7.A
A.7.C

Essential Question How does the value of c affect the graph of $f(x) = ax^2 + c$?

EXPLORATION 1 Graphing $y = ax^2 + c$

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. What do you notice?

a. $f(x) = x^2$ and $g(x) = x^2 + 2$



b. $f(x) = 2x^2$ and $g(x) = 2x^2 - 2$



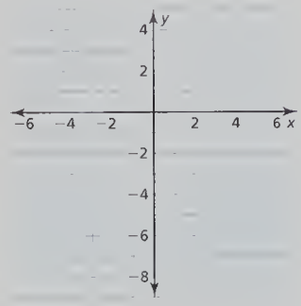
EXPLORATION 2 Finding x-Intercepts of Graphs

Work with a partner. Graph each function. Find the x-intercepts of the graph. Explain how you found the x-intercepts.

a. $y = x^2 - 7$



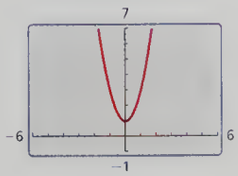
b. $y = -x^2 + 1$



SELECTING TOOLS
To be proficient in math, you need to consider the available tools, such as a graphing calculator, when solving a mathematical problem.

Communicate Your Answer

3. How does the value of c affect the graph of $f(x) = ax^2 + c$?
4. Use a graphing calculator to verify your answers to Question 3.
5. The figure shows the graph of a quadratic function of the form $y = ax^2 + c$. Describe possible values of a and c. Explain your reasoning.



For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the Essential Question. Have students work in pairs to complete the explorations and Communicate Your Answer. When they have finished, ask them to share their answers.

Beginning Use simple phrases to share answers and demonstrate visually when possible.
Intermediate Use simple sentences to share answers.
Advanced/Advanced High Explain answers using multiple complex sentences.

ELPS 3.G.2 Express ideas ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

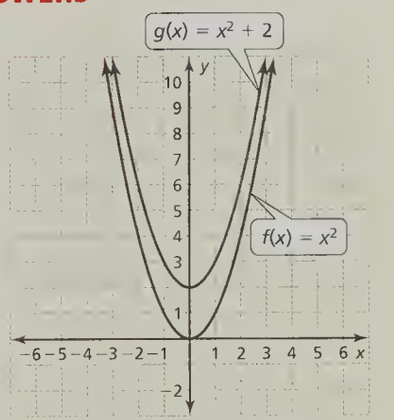
Texas Essential Knowledge and Skills

A.7.A The student is expected to graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

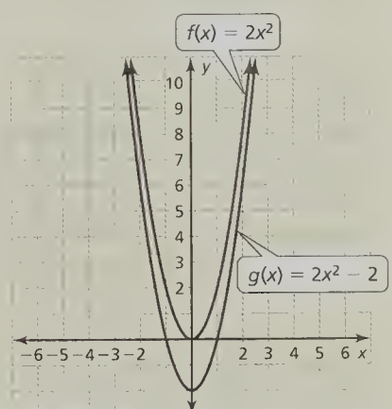
A.7.C The student is expected to determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

ANSWERS

1. a.



b.

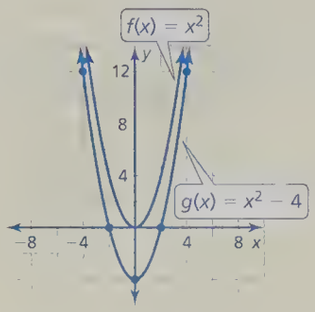


Sample answer: In $g(x) = ax^2 + c$, c causes a vertical shift in the graph of $f(x) = ax^2$.

2–5. See Additional Answers.

Extra Example 1

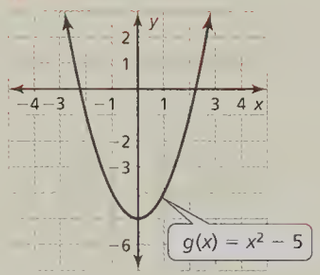
Graph $g(x) = x^2 - 4$. Compare the graph to the graph of $f(x) = x^2$.



Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of g, $(0, -4)$, is below the vertex of the graph of f, $(0, 0)$, because the graph of g is a vertical translation 4 units down of the graph of f.

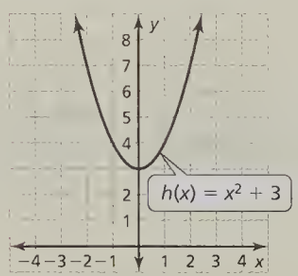
MONITORING PROGRESS ANSWERS

1.



The graph of g is a vertical translation 5 units down of the graph of f.

2.



The graph of h is a vertical translation 3 units up of the graph of f.

8.2 Lesson

What You Will Learn

- Graph quadratic functions of the form $f(x) = ax^2 + c$.
- Solve real-life problems involving functions of the form $f(x) = ax^2 + c$.

Core Vocabulary

Previous
translation
vertex of a parabola
axis of symmetry
vertical stretch
vertical shrink
zero of a function

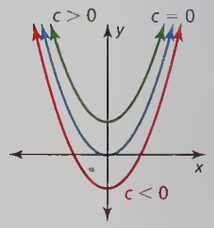
Graphing $f(x) = ax^2 + c$

Core Concept

Graphing $f(x) = ax^2 + c$

- When $c > 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation c units up of the graph of $f(x) = ax^2$.
- When $c < 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation $|c|$ units down of the graph of $f(x) = ax^2$.

The vertex of the graph of $f(x) = ax^2 + c$ is $(0, c)$, and the axis of symmetry is $x = 0$.



EXAMPLE 1 Graphing $y = x^2 + c$

Graph $g(x) = x^2 - 2$. Compare the graph to the graph of $f(x) = x^2$.

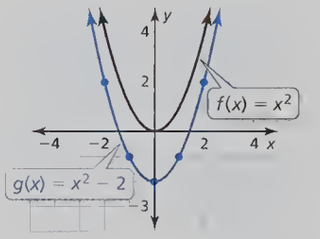
SOLUTION

Step 1 Make a table of values.

x	−2	−1	0	1	2
$g(x)$	2	−1	−2	−1	2

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



REMEMBER

The graph of $y = f(x) + k$ is a vertical translation, and the graph of $y = f(x - h)$ is a horizontal translation of the graph of f.

- Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of g, $(0, -2)$, is below the vertex of the graph of f, $(0, 0)$, because the graph of g is a vertical translation 2 units down of the graph of f.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

1. $g(x) = x^2 - 5$
2. $h(x) = x^2 + 3$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

English Language Learners

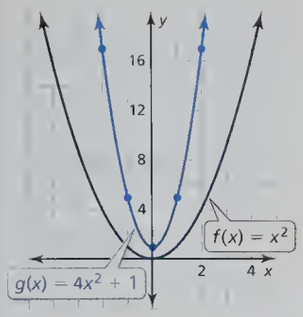
Pair Activity

Pair each English learner with an English speaker. Have the pairs graph quadratic functions and compare the graphs to the graph of the function $f(x) = x^2$. Have one student make a table of values and the other student use the ordered pairs to draw the graph. Have partners work together to compare the graph of each function to the graph of $f(x) = x^2$.

EXAMPLE 2 Graphing $y = ax^2 + c$

Graph $g(x) = 4x^2 + 1$. Compare the graph to the graph of $f(x) = x^2$.

SOLUTION



Step 1 Make a table of values.

x	-2	-1	0	1	2
$g(x)$	17	5	1	5	17

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

▶ Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of g is narrower, and its vertex, $(0, 1)$, is above the vertex of the graph of f, $(0, 0)$. So, the graph of g is a vertical stretch by a factor of 4 and a vertical translation 1 unit up of the graph of f.

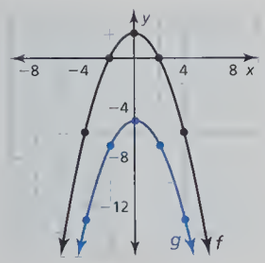
EXAMPLE 3 Translating the Graph of $y = ax^2 + c$

Let $f(x) = -0.5x^2 + 2$ and $g(x) = f(x) - 7$.

a. Describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane.

b. Write an equation that represents g in terms of x.

SOLUTION



a. The function g is of the form $y = f(x) + k$, where $k = -7$. So, the graph of g is a vertical translation 7 units down of the graph of f.

x	-4	-2	0	2	4
$f(x)$	-6	0	2	0	-6
$g(x)$	-13	-7	-5	-7	-13

$-0.5x^2 + 2$

$f(x) - 7$

b. $g(x) = f(x) - 7$ Write the function g.

$= -0.5x^2 + 2 - 7$ Substitute for $f(x)$.

$= -0.5x^2 - 5$ Subtract.

▶ So, the equation $g(x) = -0.5x^2 - 5$ represents g in terms of x.

Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = x^2$.

3. $g(x) = 2x^2 - 5$

4. $h(x) = -\frac{1}{4}x^2 + 4$

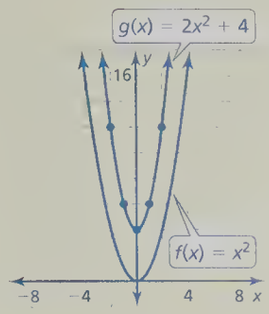
5. Let $f(x) = 3x^2 - 1$ and $g(x) = f(x) + 3$.

a. Describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane.

b. Write an equation that represents g in terms of x.

Extra Example 2

Graph $g(x) = 2x^2 + 4$. Compare the graph to the graph of $f(x) = x^2$.

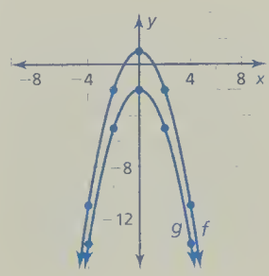


Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of g is narrower, and its vertex, $(0, 4)$, is above the vertex of the graph of f, $(0, 0)$. So, the graph of g is a vertical stretch by a factor of 2 and a vertical translation 4 units up of the graph of f.

Extra Example 3

Let $f(x) = -0.75x^2 + 1$ and $g(x) = f(x) - 3$.

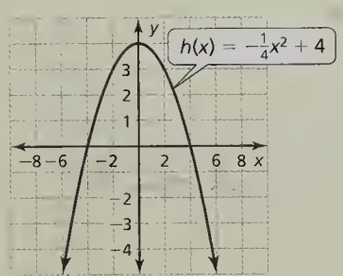
a. Describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane. The graph of g is a vertical translation 3 units down of the graph of f.



b. Write an equation that represents g in terms of x. $g(x) = -0.75x^2 - 2$

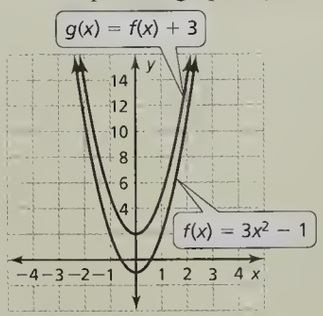
MONITORING PROGRESS ANSWERS

4.



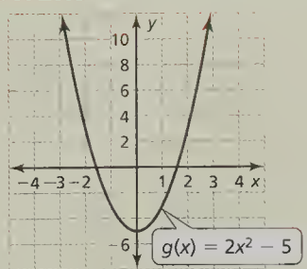
The graph of h is a vertical shrink by a factor of $\frac{1}{4}$, a reflection in the x-axis, and a vertical translation 4 units up of the graph of f.

5. a. The graph of g is a vertical translation 3 units up of the graph of f.



b. $g(x) = 3x^2 + 2$

3.



The graph of g is a vertical stretch by a factor of 2 and a vertical translation 5 units down of the graph of f.

Extra Example 4

The function $h(t) = -4.9t^2 + s_0$ represents the approximate height (in meters) of a falling object t seconds after it is dropped from an initial height s_0 (in meters). A coin is dropped from an initial height of 44.1 meters.

a. After how many seconds does the coin hit the ground?
The coin hits the ground 3 seconds after it is dropped.

b. Suppose the initial height is m meters lower. How will this affect part (a)?
The coin will hit the ground in less than 3 seconds.

MONITORING PROGRESS ANSWERS

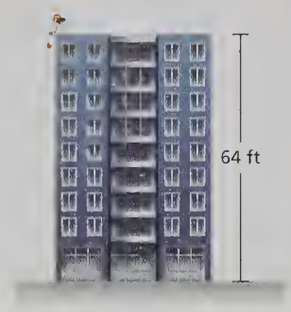
6. *Sample answer:* Negative values of t would represent times before the egg was dropped, which have no meaning in the context of this problem.

7. 2.5 sec

Solving Real-Life Problems

Recall that a zero of a function f is an x-value for which $f(x) = 0$. A zero of a function is an x-intercept of the graph of the function.

EXAMPLE 4 **Solving a Real-Life Problem**



The function $f(t) = -16t^2 + s_0$ represents the approximate height (in feet) of a falling object t seconds after it is dropped from an initial height s_0 (in feet). An egg is dropped from a height of 64 feet.

a. After how many seconds does the egg hit the ground?

b. Suppose the initial height is adjusted by k feet. How will this affect part (a)?

SOLUTION

1. **Understand the Problem** You know the function that models the height of a falling object and the initial height of an egg. You are asked to find how many seconds it takes the egg to hit the ground when dropped from the initial height. Then you need to describe how a change in the initial height affects how long it takes the egg to hit the ground.

2. **Make a Plan** Use the initial height to write a function that models the height of the egg. Use a table to graph the function. Find the zero(s) of the function to answer the question. Then explain how vertical translations of the graph affect the zero(s) of the function.

COMMON ERROR

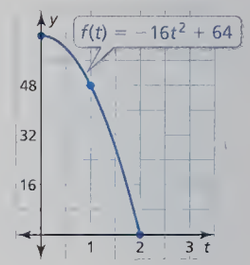
The graph in Step 1 shows the height of the object over time, not the path of the object.

3. **Solve the Problem**

a. The initial height is 64 feet. So, the function $f(t) = -16t^2 + 64$ represents the height of the egg t seconds after it is dropped. The egg hits the ground when $f(t) = 0$.

Step 1 Make a table of values and sketch the graph.

t	0	1	2
$f(t)$	64	48	0



Step 2 Find the positive zero of the function. When $t = 2, f(t) = 0$. So, the zero is 2.

▶ The egg hits the ground 2 seconds after it is dropped.

b. When the initial height is adjusted by k feet, the graph of f is translated up k units when $k > 0$ or down $|k|$ units when $k < 0$. So, the x-intercept of the graph of f will move right when $k > 0$ or left when $k < 0$.

▶ When $k > 0$, the egg will take more than 2 seconds to hit the ground. When $k < 0$, the egg will take less than 2 seconds to hit the ground.

4. **Look Back** To check that the egg hits the ground 2 seconds after it is dropped, you can solve $0 = -16t^2 + 64$ by factoring.

Monitoring Progress

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6. Explain why only nonnegative values of t are used in Example 4.

7. **WHAT IF?** The egg is dropped from a height of 100 feet. After how many seconds does the egg hit the ground?

Closure

- **Writing Prompt:** The graph of $y = -4x^2 + 12$ is ... *Sample answer:* a vertical stretch by a factor of 4, a reflection in the x-axis of the graph of $y = x^2$, and a vertical translation 12 units up.

8.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** State the vertex and axis of symmetry of the graph of $y = ax^2 + c$.
2. **WRITING** How does the graph of $y = ax^2 + c$ compare to the graph of $y = ax^2$?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 1.)*

3. $g(x) = x^2 + 6$
4. $h(x) = x^2 + 8$
5. $p(x) = x^2 - 3$
6. $q(x) = x^2 - 1$

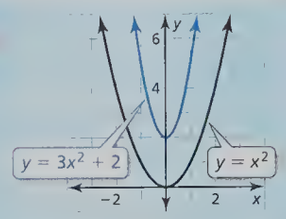
In Exercises 7–12, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 2.)*

7. $g(x) = -x^2 + 3$
8. $h(x) = -x^2 - 7$
9. $s(x) = 2x^2 - 4$
10. $t(x) = -3x^2 + 1$
11. $p(x) = -\frac{1}{3}x^2 - 2$
12. $q(x) = \frac{1}{2}x^2 + 6$

In Exercises 13–16, describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane. Write an equation that represents g in terms of x. *(See Example 3.)*

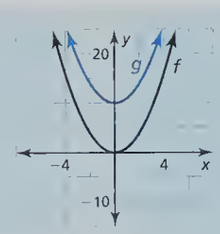
13. $f(x) = 3x^2 + 4$, $g(x) = f(x) + 2$
14. $f(x) = \frac{1}{2}x^2 + 1$, $g(x) = f(x) - 4$
15. $f(x) = -\frac{1}{4}x^2 - 6$, $g(x) = f(x) - 3$
16. $f(x) = 4x^2 - 5$, $g(x) = f(x) + 7$

17. **ERROR ANALYSIS** Describe and correct the error in comparing the graphs.



The graph of $y = 3x^2 + 2$ is a vertical shrink by a factor of 3 and a translation 2 units up of the graph of $y = x^2$.

18. **ERROR ANALYSIS** Describe and correct the error in graphing and comparing $f(x) = x^2$ and $g(x) = x^2 - 10$.



Both graphs open up and have the same axis of symmetry. However, the vertex of the graph of g, $(0, 10)$, is 10 units above the vertex of the graph of f, $(0, 0)$.

In Exercises 19–26, find the zeros of the function.

19. $y = x^2 - 1$
20. $y = x^2 - 36$
21. $f(x) = -x^2 + 25$
22. $f(x) = -x^2 + 49$
23. $f(x) = 4x^2 - 16$
24. $f(x) = 3x^2 - 27$
25. $f(x) = -12x^2 + 3$
26. $f(x) = -8x^2 + 98$

27. **MODELING WITH MATHEMATICS** A water balloon is dropped from a height of 144 feet. *(See Example 4.)*
 a. After how many seconds does the water balloon hit the ground?
 b. Suppose the initial height is adjusted by k feet. How does this affect part (a)?

28. **MODELING WITH MATHEMATICS** The function $y = -16x^2 + 36$ represents the height y (in feet) of an apple x seconds after falling from a tree. Find and interpret the x- and y-intercepts.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–19 odd, 27, 29, 34, 36, 42–45

Average: 1, 2–22 even, 27–30, 34–38 even, 42–45

Advanced: 1, 2, 6, 10–26 even, 27, 28–32 even, 33–45

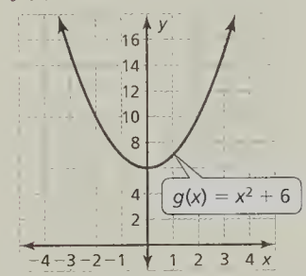
HOMEWORK CHECK

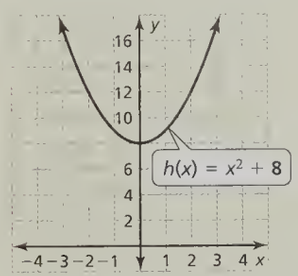
Basic: 3, 7, 13, 27

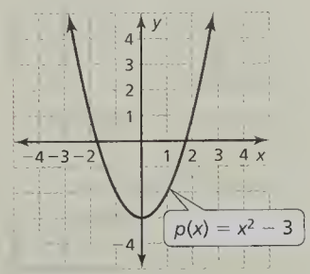
Average: 4, 10, 14, 27

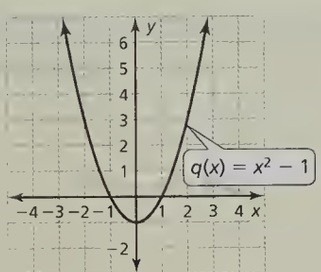
Advanced: 6, 10, 16, 27

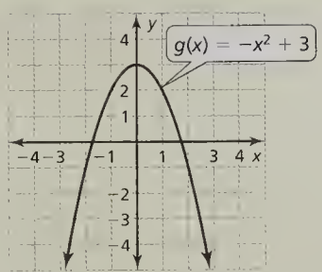
ANSWERS

1. $(0, c)$, $x = 0$
2. When $c > 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation c units up of the graph of $f(x) = ax^2$. When $c < 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation $|c|$ units down of the graph of $f(x) = ax^2$.
3. 

The graph of g is a vertical translation 6 units up of the graph of f.
4. 

The graph of h is a vertical translation 8 units up of the graph of f.
5. 

The graph of p is a vertical translation 3 units down of the graph of f.
6. 

The graph of q is a vertical translation 1 unit down of the graph of f.
7. 

The graph of g is a reflection in the x-axis, and a vertical translation 3 units up of the graph of f.

8–28. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

29–45. See Additional Answers.

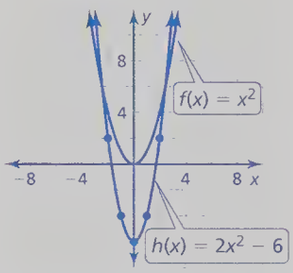
Mini-Assessment

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 4$

 See Additional Answers.

2. $h(x) = 2x^2 - 6$



 Both graphs open up and have the same axis of symmetry. The graph of h is a vertical stretch by a factor of 2 and a vertical translation 6 units down of the graph of f.

3. The function $f(t) = -16t^2 + s_0$ represents the approximate height (in feet) of a falling object t seconds after it is dropped from an initial height s_0 (in feet). A marble is dropped from a height of 36 feet.

 a. After how many seconds does the marble hit the ground?
 1.5 seconds

 b. Suppose the initial height is adjusted by k feet. How will this affect part (a)? When $k > 0$, the marble will take more than 1.5 seconds to hit the ground. When $k < 0$, the marble will take less than 1.5 seconds to hit the ground.

In Exercises 29–32, sketch a parabola with the given characteristics.

29. The parabola opens up, and the vertex is (0, 3).

30. The vertex is (0, 4), and one of the x-intercepts is 2.

31. The related function is increasing when $x < 0$, and the zeros are -1 and 1.

32. The highest point on the parabola is (0, -5).

33. **DRAWING CONCLUSIONS** You and your friend both drop a ball at the same time. The function $h(x) = -16x^2 + 256$ represents the height (in feet) of your ball after x seconds. The function $g(x) = -16x^2 + 300$ represents the height (in feet) of your friend's ball after x seconds.

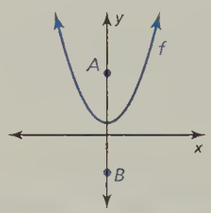
 a. Write the function $T(x) = h(x) - g(x)$. What does $T(x)$ represent?

 b. When your ball hits the ground, what is the height of your friend's ball? Use a graph to justify your answer.

34. **MAKING AN ARGUMENT** Your friend claims that in the equation $y = ax^2 + c$, the vertex changes when the value of a changes. Is your friend correct? Explain your reasoning.

35. **MATHEMATICAL CONNECTIONS** The area A (in square feet) of a square patio is represented by $A = x^2$, where x is the length of one side of the patio. You add 48 square feet to the patio, resulting in a total area of 192 square feet. What are the dimensions of the original patio? Use a graph to justify your answer.

36. **HOW DO YOU SEE IT?** The graph of $f(x) = ax^2 + c$ is shown. Points A and B are the same distance from the vertex of the graph of f. Which point is closer to the vertex of the graph of f as c increases?



37. **REASONING** Describe two algebraic methods you can use to find the zeros of the function $f(t) = -16t^2 + 400$. Check your answer by graphing.

38. **PROBLEM SOLVING** The paths of water from three different garden waterfalls are given below. Each function gives the height h (in feet) and the horizontal distance d (in feet) of the water.



 Waterfall 1 $h = -3.1d^2 + 4.8$

 Waterfall 2 $h = -3.5d^2 + 1.9$

 Waterfall 3 $h = -1.1d^2 + 1.6$

 a. Which waterfall drops water from the highest point?

 b. Which waterfall follows the narrowest path?

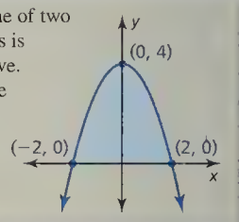
 c. Which waterfall sends water the farthest?

39. **WRITING EQUATIONS** Two acorns fall to the ground from an oak tree. One falls 45 feet, while the other falls 32 feet.

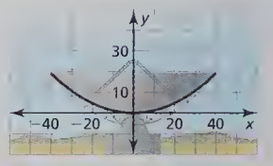
 a. For each acorn, write an equation that represents the height h (in feet) as a function of the time t (in seconds).

 b. Describe how the graphs of the two equations are related.

40. **THOUGHT PROVOKING** One of two classic problems in calculus is to find the area under a curve. Approximate the area of the region bounded by the parabola and the x-axis. Show your work.



41. **CRITICAL THINKING** A cross section of the parabolic surface of the antenna shown can be modeled by $y = 0.012x^2$, where x and y are measured in feet. The antenna is moved up so that the outer edges of the dish are 25 feet above the ground. Where is the vertex of the cross section located? Explain.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when $a = 4$ and $b = -3$. *(Skills Review Handbook)*

42. $\frac{a}{4b}$

43. $-\frac{b}{2a}$

44. $\frac{a - b}{3a + b}$

45. $-\frac{b + 2a}{ab}$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

8.3 Graphing $f(x) = ax^2 + bx + c$

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.6.A
A.7.A

Essential Question How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

EXPLORATION 1 Comparing x-Intercepts with the Vertex

Work with a partner.

a. Sketch the graphs of $y = 2x^2 - 8x$ and $y = 2x^2 - 8x + 6$.

b. What do you notice about the x-coordinate of the vertex of each graph?

c. Use the graph of $y = 2x^2 - 8x$ to find its x-intercepts. Verify your answer by solving $0 = 2x^2 - 8x$.

d. Compare the value of the x-coordinate of the vertex with the values of the x-intercepts.

EXPLORATION 2 Finding x-Intercepts

Work with a partner.

a. Solve $0 = ax^2 + bx$ for x by factoring.

b. What are the x-intercepts of the graph of $y = ax^2 + bx$?

c. Copy and complete the table to verify your answer.

x	$y = ax^2 + bx$
0	
$-\frac{b}{a}$	

MAKING MATHEMATICAL ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements.

EXPLORATION 3 Deductive Reasoning

Work with a partner. Complete the following logical argument.

The x-intercepts of the graph of $y = ax^2 + bx$ are 0 and $-\frac{b}{a}$.

The vertex of the graph of $y = ax^2 + bx$ occurs when $x =$ [redacted].

The vertices of the graphs of $y = ax^2 + bx$ and $y = ax^2 + bx + c$ have the same x-coordinate.

The vertex of the graph of $y = ax^2 + bx + c$ occurs when $x =$ [redacted].

Communicate Your Answer

4. How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?
5. Without graphing, find the vertex of the graph of $f(x) = x^2 - 4x + 3$. Check your result by graphing.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

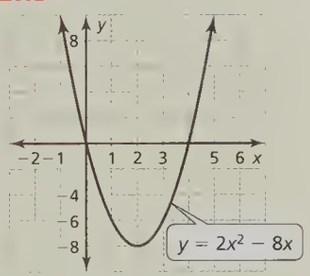
Texas Essential Knowledge and Skills

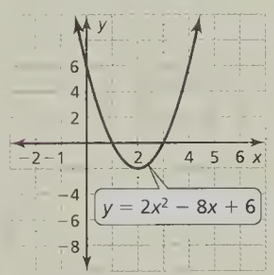
A.6.A The student is expected to determine the domain and range of quadratic functions and represent the domain and range using inequalities.

A.7.A The student is expected to graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

ANSWERS

1. a.





b. They are the same.

c. $x = 0, x = 4$

d. *Sample answer:* The value of the x-coordinate of the vertex is the average of the values of the x-intercepts.

2. a. $x = 0, x = -\frac{b}{a}$

b. $x = 0, x = -\frac{b}{a}$

c. 0, 0

3. $-\frac{b}{2a}, -\frac{b}{2a}$

4. *Sample answer:* Find the x-coordinate using $x = -\frac{b}{2a}$, then use the function to find the y-coordinate.

5. $(2, -1)$

Extra Example 1

a. Find the axis of symmetry of the graph of $f(x) = 2x^2 - 4x + 5$. $x = 1$

b. Find the vertex of the graph of $f(x) = 2x^2 - 4x + 5$. $(1, 3)$

MONITORING PROGRESS ANSWERS

1. a. $x = \frac{1}{3}$
 b. $\left(\frac{1}{3}, -\frac{1}{3}\right)$
2. a. $x = -3$
 b. $(-3, -4)$
3. a. $x = 7$
 b. $\left(7, \frac{41}{2}\right)$

8.3 Lesson

Core Vocabulary

maximum value, *p. 419*
minimum value, *p. 419*

Previous
independent variable
dependent variable

What You Will Learn

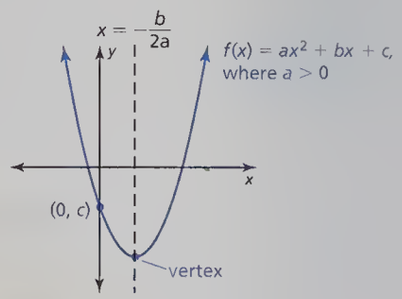
- Graph quadratic functions of the form $f(x) = ax^2 + bx + c$.
- Find maximum and minimum values of quadratic functions.

Graphing $f(x) = ax^2 + bx + c$

Core Concept

Graphing $f(x) = ax^2 + bx + c$

- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.
- The y-intercept is c.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is $x = -\frac{b}{2a}$.



EXAMPLE 1 Finding the Axis of Symmetry and the Vertex

Find (a) the axis of symmetry and (b) the vertex of the graph of $f(x) = 2x^2 + 8x - 1$.

SOLUTION

a. Find the axis of symmetry when $a = 2$ and $b = 8$.

$x = -\frac{b}{2a}$ — Write the equation for the axis of symmetry.

$x = -\frac{8}{2(2)}$ — Substitute 2 for a and 8 for b.

$x = -2$ — Simplify.

▶ The axis of symmetry is $x = -2$.

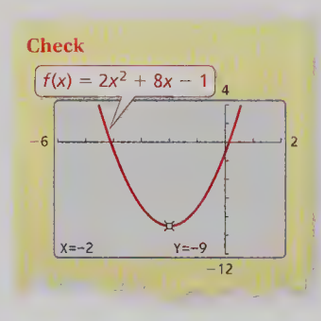
b. The axis of symmetry is $x = -2$, so the x-coordinate of the vertex is -2. Use the function to find the y-coordinate of the vertex.

$f(x) = 2x^2 + 8x - 1$ — Write the function.

$f(-2) = 2(-2)^2 + 8(-2) - 1$ — Substitute -2 for x.

$= -9$ — Simplify.

▶ The vertex is $(-2, -9)$.



Monitoring Progress

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Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. $f(x) = 3x^2 - 2x$
2. $g(x) = x^2 + 6x + 5$
3. $h(x) = -\frac{1}{2}x^2 + 7x - 4$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

EXAMPLE 2 **Graphing $f(x) = ax^2 + bx + c$**

Graph $f(x) = 3x^2 - 6x + 5$. Describe the domain and range.

SOLUTION

Step 1 Find and graph the axis of symmetry.

$$x = -\frac{b}{2a} = -\frac{(-6)}{2(3)} = 1$$ Substitute and simplify.

Step 2 Find and plot the vertex.

The axis of symmetry is $x = 1$, so the x-coordinate of the vertex is 1. Use the function to find the y-coordinate of the vertex.

$f(x) = 3x^2 - 6x + 5$ Write the function.

$f(1) = 3(1)^2 - 6(1) + 5$ Substitute 1 for x.

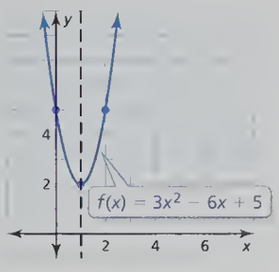
$= 2$ Simplify.

So, the vertex is (1, 2).

Step 3 Use the y-intercept to find two more points on the graph.

Because $c = 5$, the y-intercept is 5. So, (0, 5) lies on the graph. Because the axis of symmetry is $x = 1$, the point (2, 5) also lies on the graph.

Step 4 Draw a smooth curve through the points.



▶ The domain is all real numbers. The range is $y \geq 2$.

COMMON ERROR

Be sure to include the negative sign before the fraction when finding the axis of symmetry.

REMEMBER

The domain is the set of all possible input values of the independent variable x. The range is the set of all possible output values of the dependent variable y.

Monitoring Progress Help in English and Spanish at *BigIdeasMath.com*

Graph the function. Describe the domain and range.

4. $h(x) = 2x^2 + 4x + 1$ 5. $k(x) = x^2 - 8x + 7$ 6. $p(x) = -5x^2 - 10x - 2$

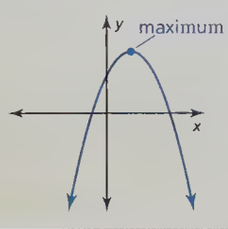
Finding Maximum and Minimum Values

Core Concept

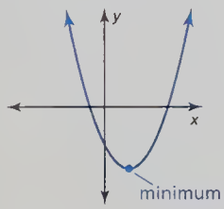
Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is the **maximum value** of the function when $a < 0$ or the **minimum value** of the function when $a > 0$.

$f(x) = ax^2 + bx + c, a < 0$

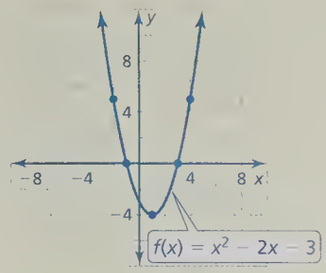


$f(x) = ax^2 + bx + c, a > 0$



Extra Example 2

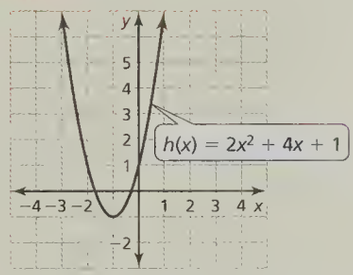
Graph $f(x) = x^2 - 2x - 3$. Describe the domain and range.



The domain is all real numbers. The range is $y \geq -4$.

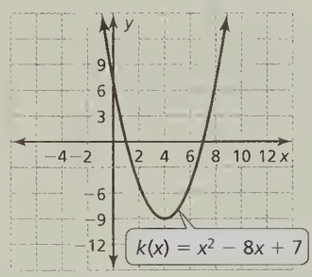
MONITORING PROGRESS ANSWERS

4.



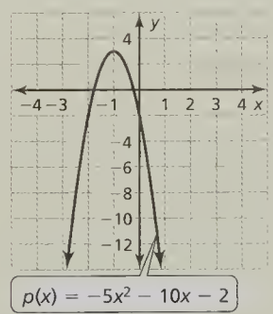
domain: all real numbers, range: $y \geq -1$

5.



domain: all real numbers, range: $y \geq -9$

6.



domain: all real numbers, range: $y \leq 3$

Differentiated Instruction

Organization

Have students copy this table in their notes to help them determine whether a function of the form $y = ax^2 + bx + c$ has a maximum value or a minimum value. Suggest that they also include a sketch of each type of function.

Value of a	Graph Opens	Maximum or Minimum?
$a < 0$	down	maximum
$a > 0$	up	minimum

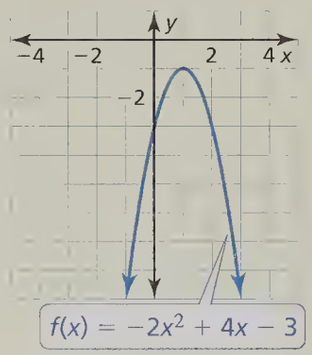
Extra Example 3

Tell whether the function $f(x) = 5x^2 + 15x - 10$ has a minimum value or a maximum value. Then find the value. The minimum value is -21.25.

Extra Example 4

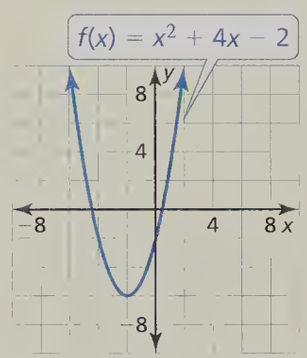
Estimate the y-intercept of the graph and the maximum or minimum value of the function represented by the graph.

a.



From the graph, you can estimate that the y-intercept is -3 and the function has a maximum value of -1.

b.



From the graph, you can estimate that the y-intercept is -2 and the function has a minimum value of -6.

MONITORING PROGRESS ANSWERS

7. minimum value; 4
8. maximum value; 10
9. -4; maximum value: -1

EXAMPLE 3 Finding a Maximum or Minimum Value

Tell whether the function $f(x) = -4x^2 - 24x - 19$ has a minimum value or a maximum value. Then find the value.

SOLUTION

For $f(x) = -4x^2 - 24x - 19$, $a = -4$ and $-4 < 0$. So, the parabola opens down and the function has a maximum value. To find the maximum value, find the y-coordinate of the vertex.

First, find the x-coordinate of the vertex. Use $a = -4$ and $b = -24$.

$$x = -\frac{b}{2a} = -\frac{(-24)}{2(-4)} = -3$$ Substitute and simplify.

Then evaluate the function when $x = -3$ to find the y-coordinate of the vertex.

$f(-3) = -4(-3)^2 - 24(-3) - 19$ Substitute -3 for x.

$= 17$ Simplify.

▶ The maximum value is 17.

Check

y-intercept:

$f(0) = 2(0)^2 - 8(0) + 8$

$= 8$ ✓

vertex at: $x = -\frac{(-8)}{2(2)} = 2$

minimum value:

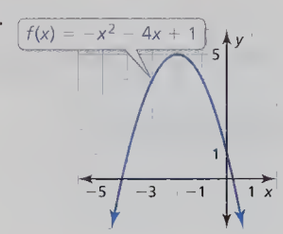
$f(2) = 2(2)^2 - 8(2) + 8$

$= 0$ ✓

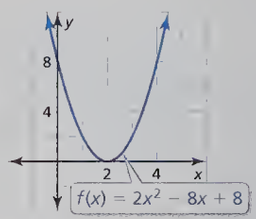
EXAMPLE 4 Finding a Maximum or Minimum Value

Estimate the y-intercept of the graph and the maximum or minimum value of the function represented by the graph.

a.



b.



SOLUTION

a. From the graph, you can estimate that the y-intercept is 1 and the function has a maximum value of 5.

b. From the graph, you can estimate that the y-intercept is 8 and the function has a minimum value of 0. You can check your estimates as shown.

Monitoring Progress

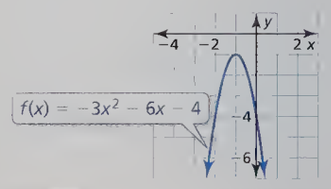
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Tell whether the function has a minimum value or a maximum value. Then find the value.

7. $g(x) = 8x^2 - 8x + 6$

8. $h(x) = -\frac{1}{4}x^2 + 3x + 1$

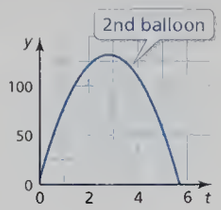
9. Estimate the y-intercept of the graph and the maximum or minimum value of the function represented by the graph.



EXAMPLE 5 Modeling with Mathematics



A group of friends is launching water balloons. The function $f(t) = -16t^2 + 80t + 5$ represents the height (in feet) of the first water balloon t seconds after it is launched. The height of the second water balloon t seconds after it is launched is shown in the graph. Which water balloon went higher?

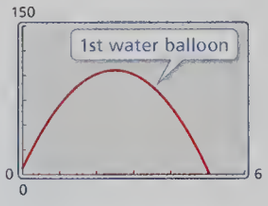


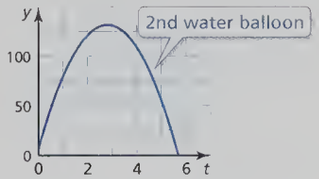
SOLUTION

1. **Understand the Problem** You are given a function that represents the height of the first water balloon. The height of the second water balloon is represented graphically. You need to find and compare the maximum heights of the water balloons.

2. **Make a Plan** To compare the maximum heights, represent both functions graphically. Use a graphing calculator to graph $f(t) = -16t^2 + 80t + 5$ in an appropriate viewing window. Then visually compare the heights of the water balloons.

3. **Solve the Problem** Enter the function $f(t) = -16t^2 + 80t + 5$ into your calculator and graph it. Compare the graphs to determine which function has a greater maximum value.

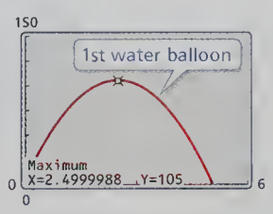


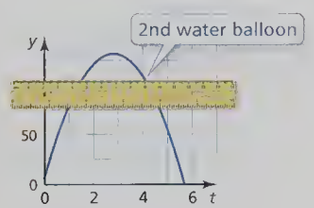


You can see that the second water balloon reaches a height of about 125 feet, while the first water balloon reaches a height of only about 100 feet.

▶ So, the second water balloon went higher.

4. **Look Back** Use the *maximum* feature to determine that the maximum value of $f(t) = -16t^2 + 80t + 5$ is 105. Use a straightedge to represent a height of 105 feet on the graph that represents the second water balloon to clearly see that the second water balloon went higher.





APPLYING MATHEMATICS

Because time cannot be negative, use only nonnegative values of t.

Monitoring Progress

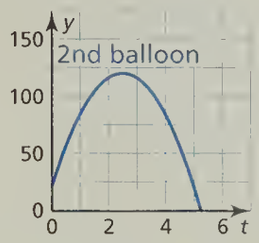
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10. Which balloon is in the air longer? Explain your reasoning.

11. Which balloon reaches its maximum height faster? Explain your reasoning.

Extra Example 5

A group of friends is launching water balloons off the roof of a building. The function $f(t) = -16t^2 + 96t + 20$ represents the height (in feet) of the first balloon t seconds after it is launched. The height of the second balloon t seconds after it is launched is shown in the graph. Which balloon went higher?



The first water balloon went higher.

MONITORING PROGRESS ANSWERS

10. second water balloon; *Sample answer:* Comparing the graphs, the first water balloon has a height of 0 after about 5 seconds, and the second water balloon has a height of 0 after about 5.7 seconds.

11. first water balloon; *Sample answer:* Comparing the graphs, the first water balloon is at its maximum height after about 2.5 seconds, and the second water balloon is at its maximum height after about 2.8 seconds.

SUPPORTING English Language Learners

Explain that *launching* water balloons in Example 5 means *throwing* them using a sling, as shown in the photo. Ask if they have ever thrown a water balloon and why someone might throw one. Explain that they should study the graphs that represent the height over time of each balloon. Read aloud the problem statement for the example and have students read the solution.

Beginning Read the headings and graph labels.
Intermediate Read Step 1.
Advanced Read Step 2.
Advanced High Read Steps 3 and 4.

ELPS 4.F.5 Use visual and contextual support to develop background knowledge needed to comprehend increasingly challenging language.

Closure

- Write an equation of a quadratic function that opens up, has a negative y-intercept, and is wider than the graph of $y = x^2$.
 Sample answer: $y = 0.25x^2 - 2$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–27 odd, 30, 31, 33, 37, 42, 50–52

Average: 1, 2–36 even, 37, 38, 42, 45, 50–52

Advanced: 1, 2, 6, 12, 16–20 even, 24–38 even, 39–52

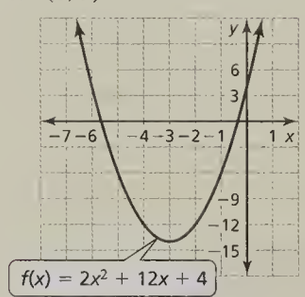
HOMEWORK CHECK

Basic: 7, 13, 23, 31, 37

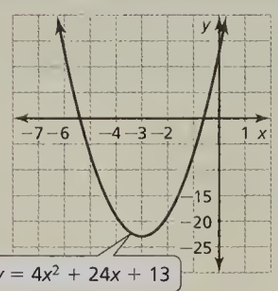
Average: 8, 16, 24, 32, 37

Advanced: 12, 16, 24, 32, 38

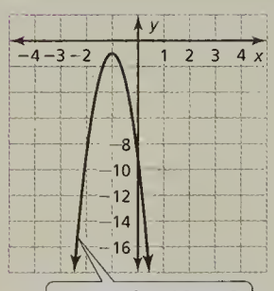
ANSWERS

1. *Sample answer:* If the leading coefficient is positive, the graph has a minimum value. If the leading coefficient is negative, the graph has a maximum value.
2. What is the axis of symmetry of the graph of the function?; $x = 2$; 32
3. $(2, -1)$; $x = 2$; 1
4. $(-3, 2)$; $x = -3$; -1
5. $(-2, 0)$; $x = -2$; -3
6. $(-1, 1)$; $x = -1$; 5
7. a. $x = 1$
 b. $(1, -2)$
8. a. $x = -\frac{1}{3}$
 b. $\left(-\frac{1}{3}, -\frac{1}{3}\right)$
9. a. $x = -1$
 b. $(-1, 8)$
10. a. $x = 2$
 b. $(2, 4)$
11. a. $x = 5$
 b. $(5, 4)$
12. a. $x = 6$
 b. $(6, 9)$
13. 

domain: all real numbers, range: $y \geq -14$

14. 

domain: all real numbers, range: $y \geq -23$

15. 

domain: all real numbers, range: $y \leq -1$

16–23. See Additional Answers.

8.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Explain how you can tell whether a quadratic function has a maximum value or a minimum value without graphing the function.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the quadratic function $f(x) = -2x^2 + 8x + 24$. Which is different? Find "both" answers.

 What is the maximum value of the function?

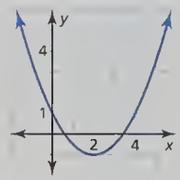
 What is the greatest number in the range of the function?

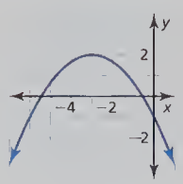
 What is the y-coordinate of the vertex of the graph of the function?

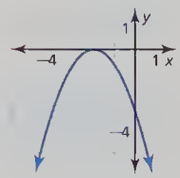
 What is the axis of symmetry of the graph of the function?

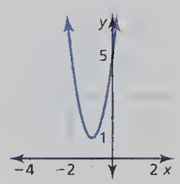
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the vertex, the axis of symmetry, and the y-intercept of the graph.

3. 

4. 

5. 

6. 

In Exercises 7–12, find (a) the axis of symmetry and (b) the vertex of the graph of the function. *(See Example 1.)*

7. $f(x) = 2x^2 - 4x$
8. $y = 3x^2 + 2x$
9. $y = -9x^2 - 18x - 1$
10. $f(x) = -6x^2 + 24x - 20$
11. $f(x) = \frac{2}{5}x^2 - 4x + 14$
12. $y = -\frac{3}{4}x^2 + 9x - 18$

In Exercises 13–18, graph the function. Describe the domain and range. *(See Example 2.)*

13. $f(x) = 2x^2 + 12x + 4$
14. $y = 4x^2 + 24x + 13$
15. $y = -8x^2 - 16x - 9$
16. $f(x) = -5x^2 + 20x - 7$
17. $y = \frac{2}{3}x^2 - 6x + 5$
18. $f(x) = -\frac{1}{2}x^2 - 3x - 4$

19. **ERROR ANALYSIS** Describe and correct the error in finding the axis of symmetry of the graph of $y = 3x^2 - 12x + 11$.

✗ $x = -\frac{b}{2a} = \frac{-12}{2(3)} = -2$

The axis of symmetry is $x = -2$.

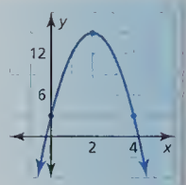
20. **ERROR ANALYSIS** Describe and correct the error in graphing the function $f(x) = x^2 + 4x + 3$.

✗ The axis of symmetry is $x = \frac{b}{2a} = \frac{4}{2(1)} = 2$.

$f(2) = 2^2 + 4(2) + 3 = 15$

So, the vertex is $(2, 15)$.

The y-intercept is 3. So, the points $(0, 3)$ and $(4, 3)$ lie on the graph.



In Exercises 21–26, tell whether the function has a minimum value or a maximum value. Then find the value. *(See Example 3.)*

21. $y = 3x^2 - 18x + 15$
22. $f(x) = -5x^2 + 10x + 7$
23. $f(x) = -4x^2 + 4x - 2$

24. $y = 2x^2 - 10x + 13$

25. $y = -\frac{1}{2}x^2 - 11x + 6$

26. $f(x) = \frac{1}{5}x^2 - 5x + 27$

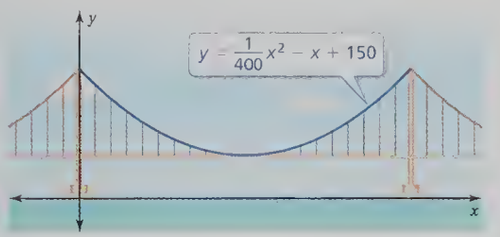
27. **MODELING WITH MATHEMATICS** The function shown represents the height h (in feet) of a firework t seconds after it is launched. The firework explodes at its highest point.



a. When does the firework explode?

b. At what height does the firework explode?

28. **MODELING WITH MATHEMATICS** The cable between two towers of a suspension bridge can be modeled by the function shown, where x and y are measured in feet. The cable is at road level midway between the towers.



a. How far from each tower shown is the lowest point of the cable?

b. How high is the road above the water?

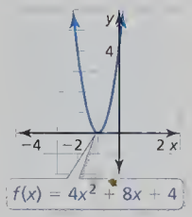
c. Describe the domain and range of the function shown.

29. **ATTENDING TO PRECISION** The vertex of a parabola is $(3, -1)$. One point on the parabola is $(6, 8)$. Find another point on the parabola. Justify your answer.

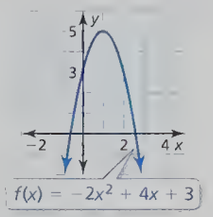
30. **MAKING AN ARGUMENT** Your friend claims that it is possible to draw a parabola through any two points with different x-coordinates. Is your friend correct? Explain.

In Exercises 31–34, estimate the y-intercept of the graph and the minimum or maximum value of the function represented by the graph. *(See Example 4.)*

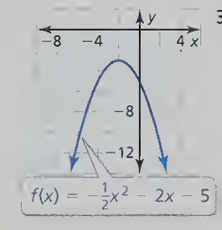
31.



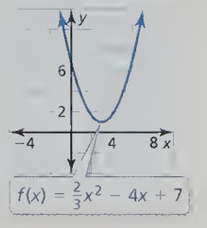
32.



33.



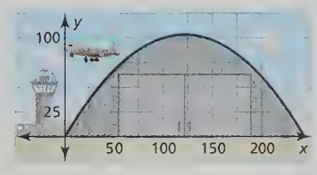
34.



USING TOOLS In Exercises 35 and 36, use the *minimum* or *maximum* feature of a graphing calculator to approximate the vertex of the graph of the function.

35. $y = 0.5x^2 + \sqrt{2}x - 3$ 36. $y = -\pi x^2 + 3x$

37. **MODELING WITH MATHEMATICS** The opening of one aircraft hangar is a parabolic arch that can be modeled by the equation $y = -0.006x^2 + 1.5x$, where x and y are measured in feet. The opening of a second aircraft hangar is shown in the graph. *(See Example 5.)*



a. Which aircraft hangar is taller?

b. Which aircraft hangar is wider?

38. **MODELING WITH MATHEMATICS** An office supply store sells about 80 graphing calculators per month for \$120 each. For each \$6 decrease in price, the store expects to sell eight more calculators. The revenue from calculator sales is given by the function $R(n) =$ (unit price)(units sold), or $R(n) = (120 - 6n)(80 + 8n)$, where n is the number of \$6 price decreases.

a. How much should the store charge to maximize monthly revenue?

b. Using a different revenue model, the store expects to sell five more calculators for each \$4 decrease in price. Which revenue model results in a greater maximum monthly revenue? Explain.

Dynamic Teaching Tools
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

ANSWERS

24. minimum value; $\frac{1}{2}$
25. maximum value; $66\frac{1}{2}$
26. minimum value; $-4\frac{1}{4}$
27. **a.** 4 sec
 b. 256 ft
28. **a.** 200 ft
 b. 50 ft
 c. domain: $0 \le x \le 400$, range: $50 \le y \le 150$
29. (0, 8); *Sample answer:* Because the axis of symmetry is $x = 3$, the point (0, 8) would also lie on the graph.
30. yes; *Sample answer:* If the two points have the same y-coordinate, then a parabola can be drawn by selecting a vertex with its x-coordinate halfway between the x-coordinates of the two points, and with a different y-coordinate. If the two points have different y-coordinates, a parabola can be drawn by making one of the points the vertex and drawing the parabola through the other point.
31. 4; minimum value: 0
32. 3; maximum value: 5
33. −5; maximum value: −3
34. 7; minimum value: 1
35. (−1.41, −4)
36. (0.48, 0.72)
37. **a.** second aircraft hangar
 b. first aircraft hangar
38. **a.** \$90
 b. the first revenue model; *Sample answer:* The first revenue model has a maximum value of \$10,800 and the second revenue model has a maximum value of \$10,580, so the first has a greater maximum value.

ANSWERS

39. a. $x = 4.5$

b. 20.25 in.^{2}

40. a. $x = 2$

b. 32 ft^2

41. The graph of g is a reflection in the y-axis of the graph of h.

42. a. 1.5 m

b. about 1.6 m

c. about 90 m

43. down; *Sample answer:* Because (3, 2) and (9, 2) have the same y-coordinate, any point with an x-coordinate between 3 and 9 lies on the part of the parabola between these two points that passes through the vertex. Because 7 is greater than 2, the vertex must be above these 2 points, so the parabola opens down.

44. the maximum value or minimum value of the function;

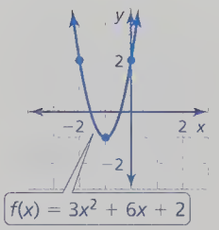
Sample answer: Because $-\frac{b}{2a}$ is the x-coordinate of the vertex, $f\left(-\frac{b}{2a}\right)$ is the y-coordinate of the vertex, which is also the maximum value or minimum value of the function.

45. $y = -2x^2 + 8x$

46–52. See Additional Answers.

Mini-Assessment

1. Graph $f(x) = 3x^2 + 6x + 2$. Find the axis of symmetry and the vertex. Describe the domain and range.



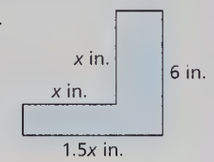
axis of symmetry: $x = -1$; vertex: $(-1, -1)$; domain: all real numbers, range: $y \geq -1$

2. Tell whether the function $f(x) = -6x^2 + 24x - 8$ has a minimum value or a maximum value. Then find the value.

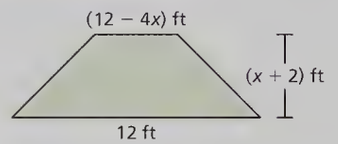
maximum value: 16

MATHEMATICAL CONNECTIONS **In Exercises 39 and 40, (a) find the value of x that maximizes the area of the figure and (b) find the maximum area.**

39.

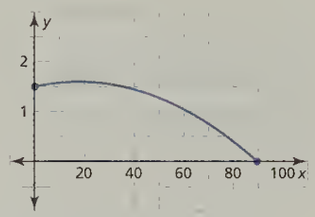


40.



41. WRITING Compare the graph of $g(x) = x^2 + 4x + 1$ with the graph of $h(x) = x^2 - 4x + 1$.

42. HOW DO YOU SEE IT? During an archery competition, an archer shoots an arrow. The arrow follows the parabolic path shown, where x and y are measured in meters.



a. What is the initial height of the arrow?

b. Estimate the maximum height of the arrow.

c. How far does the arrow travel?

43. USING TOOLS The graph of a quadratic function passes through (3, 2), (4, 7), and (9, 2). Does the graph open up or down? Explain your reasoning.

44. REASONING For a quadratic function f, what does $f\left(-\frac{b}{2a}\right)$ represent? Explain your reasoning.

45. PROBLEM SOLVING Write a function of the form $y = ax^2 + bx$ whose graph contains the points (1, 6) and (3, 6).

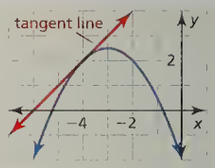
46. CRITICAL THINKING Parabolas A and B contain the points shown. Identify characteristics of each parabola, if possible. Explain your reasoning.

Parabola A	
x	y
2	3
6	4

Parabola B	
x	y
1	4
3	−4
5	4

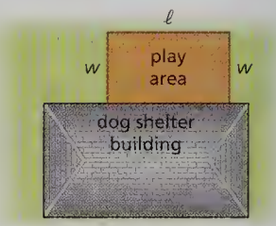
47. MODELING WITH MATHEMATICS At a basketball game, an air cannon launches T-shirts into the crowd. The function $y = -\frac{1}{8}x^2 + 4x$ represents the path of a T-shirt. The function $3y = 2x - 14$ represents the height of the bleachers. In both functions, y represents vertical height (in feet) and x represents horizontal distance (in feet). At what height does the T-shirt land in the bleachers?

48. THOUGHT PROVOKING One of two classic problems in calculus is finding the slope of a *tangent line* to a curve. An example of a tangent line, which just touches the parabola at one point, is shown.



Approximate the slope of the tangent line to the graph of $y = x^2$ at the point (1, 1). Explain your reasoning.

49. PROBLEM SOLVING The owners of a dog shelter want to enclose a rectangular play area on the side of their building. They have k feet of fencing. What is the maximum area of the outside enclosure in terms of k? (*Hint:* Find the y-coordinate of the vertex of the graph of the area function.)



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph f and h. Describe the transformations from the graph of f to the graph of h. *(Section 3.7)*

50. $f(x) = x$; $h(x) = 4x + 3$ **51.** $f(x) = x$; $h(x) = -x - 8$ **52.** $f(x) = x$; $h(x) = -\frac{1}{2}x + 5$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

8.1–8.3 What Did You Learn?

Core Vocabulary

quadratic function, *p. 406*
parabola, *p. 406*
vertex, *p. 406*
axis of symmetry, *p. 406*
maximum value, *p. 419*
minimum value, *p. 419*

Core Concepts

Section 8.1

Characteristics of Quadratic Functions, *p. 406*
Graphing $f(x) = ax^2$ When $a > 0$, *p. 407*
Graphing $f(x) = ax^2$ When $a < 0$, *p. 407*
Graphing $f(x) = (ax)^2$, *p. 408*

Section 8.2

Graphing $f(x) = ax^2 + c$, *p. 412*

Section 8.3

Graphing $f(x) = ax^2 + bx + c$, *p. 418*
Maximum and Minimum Values, *p. 419*

Mathematical Thinking

1. Explain your plan for solving Exercise 18 on page 409.
2. How does graphing the function in Exercise 27 on page 415 help you answer the questions?
3. What definition and characteristics of the graph of a quadratic function did you use to answer Exercise 44 on page 424?

Study Skills

Learning Visually

- Draw a picture of a word problem before writing a verbal model. You do not have to be an artist.
- When making a review card for a word problem, include a picture. This will help you recall the information while taking a test.
- Make sure your notes are visually neat for easy recall.



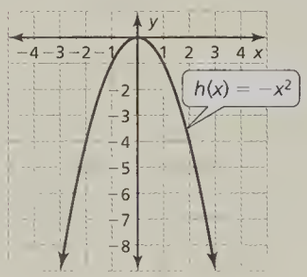
Dynamic Teaching Tools

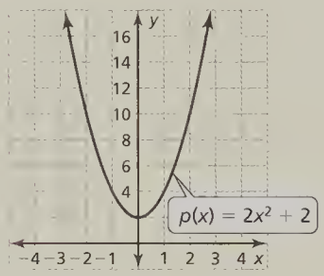
Dynamic Assessment & Progress Monitoring Tool
Interactive Whiteboard Lesson Library
Dynamic Classroom with Dynamic Investigations

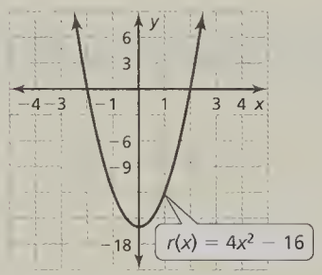
ANSWERS

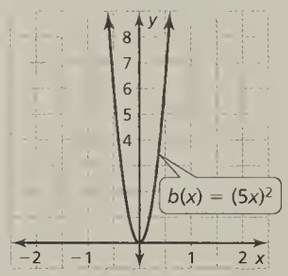
1. *Sample answer:* Because the highest point has a y-coordinate of 0, the height is the opposite of the y-coordinate of the lowest points. The width is the absolute value of the difference of the x-coordinates of the endpoints.
2. *Sample answer:* The t-intercept of the graph is the total time before the water balloon hits the ground.
3. *Sample answer:* the definition of maximum value/minimum value and the vertex

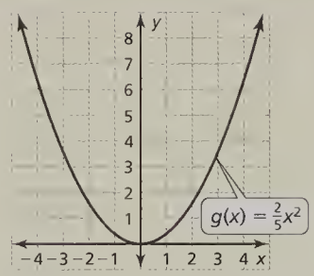
ANSWERS

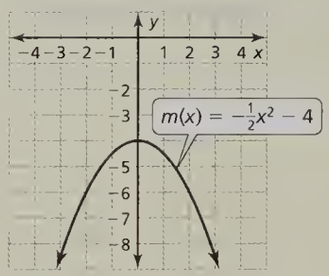
1. The vertex is (1, 4). The axis of symmetry is $x = 1$. The domain is all real numbers. The range is $y \leq 4$. When $x < 1$, y increases as x increases. When $x > 1$, y increases as x decreases.
2. The vertex is $(-2, 5)$. The axis of symmetry is $x = -2$. The domain is all real numbers. The range is $y \geq 5$. When $x < -2$, y increases as x decreases. When $x > -2$, y increases as x increases.
3. 

The graph of h is a reflection in the x-axis of the graph of f.
4. 

The graph of p is a vertical stretch by a factor of 2 and a vertical translation 2 units up of the graph of f.
5. 

The graph of r is a vertical stretch by a factor of 4 and a vertical translation 16 units down of the graph of f.
6. 

The graph of b is a horizontal shrink by a factor of $\frac{1}{5}$ of the graph of f.
7. 

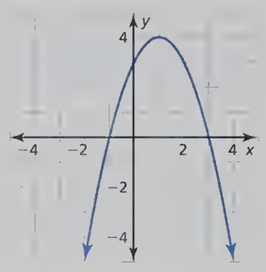
The graph of g is a vertical shrink by a factor of $\frac{2}{5}$ of the graph of f.
8. 

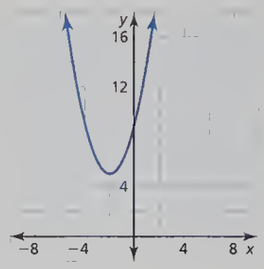
The graph of m is a vertical shrink by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a vertical translation 4 units down of the graph of f.

9–23. See Additional Answers.

8.1–8.3 Quiz

Identify characteristics of the quadratic function and its graph. *(Section 8.1)*

1. 

2. 

Graph the function. Compare the graph to the graph of $f(x) = x^2$. *(Section 8.1 and Section 8.2)*

3. $h(x) = -x^2$
4. $p(x) = 2x^2 + 2$
5. $r(x) = 4x^2 - 16$
6. $b(x) = (5x)^2$
7. $g(x) = \frac{2}{5}x^2$
8. $m(x) = -\frac{1}{2}x^2 - 4$

Describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane. Write an equation that represents g in terms of x. *(Section 8.2)*

9. $f(x) = 2x^2 + 1$; $g(x) = f(x) + 2$
10. $f(x) = -3x^2 + 12$; $g(x) = f(x) - 9$
11. $f(x) = \frac{1}{2}x^2 - 2$; $g(x) = f(x) - 6$
12. $f(x) = 5x^2 - 3$; $g(x) = f(x) + 1$

Graph the function. Describe the domain and range. *(Section 8.3)*

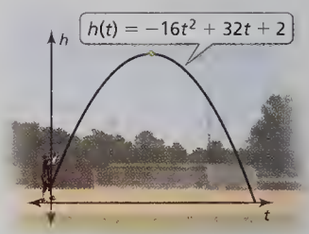
13. $f(x) = -4x^2 - 4x + 7$
14. $f(x) = 2x^2 + 12x + 5$
15. $y = x^2 + 4x - 5$
16. $y = -3x^2 + 6x + 9$

Tell whether the function has a minimum value or a maximum value. Then find the value. *(Section 8.3)*

17. $f(x) = 5x^2 + 10x - 3$
18. $f(x) = -\frac{1}{2}x^2 + 2x + 16$
19. $y = -x^2 + 4x + 12$
20. $y = 2x^2 + 8x + 3$

21. The distance y (in feet) that a coconut falls after t seconds is given by the function $y = 16t^2$. Use a graph to determine how many seconds it takes for the coconut to fall 64 feet. *(Section 8.1)*

22. The function $y = -16t^2 + 25$ represents the height y (in feet) of a pinecone t seconds after falling from a tree. *(Section 8.2)*
 a. After how many seconds does the pinecone hit the ground?
 b. A second pinecone falls from a height of 36 feet. Which pinecone hits the ground in the least amount of time? Explain.



23. The function shown models the height (in feet) of a softball t seconds after it is pitched in an underhand motion. Describe the domain and range. Find the maximum height of the softball. *(Section 8.3)*

8.4 Graphing $f(x) = a(x - h)^2 + k$

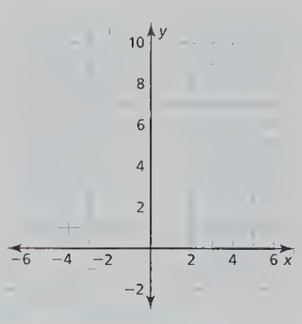
Texas Essential Knowledge and Skills
A.6.B
A.7.A
A.7.C

Essential Question How can you describe the graph of $f(x) = a(x - h)^2$?

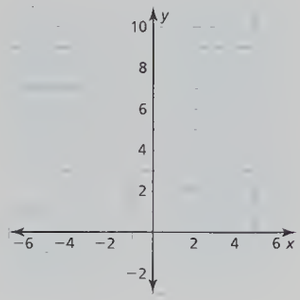
EXPLORATION 1 Graphing $y = a(x - h)^2$ When $h > 0$

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. How does the value of h affect the graph of $y = a(x - h)^2$?

a. $f(x) = x^2$ and $g(x) = (x - 2)^2$



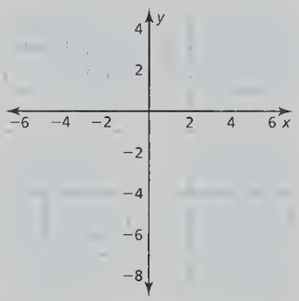
b. $f(x) = 2x^2$ and $g(x) = 2(x - 2)^2$



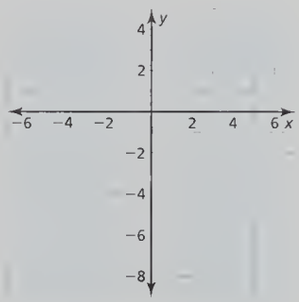
EXPLORATION 2 Graphing $y = a(x - h)^2$ When $h < 0$

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. How does the value of h affect the graph of $y = a(x - h)^2$?

a. $f(x) = -x^2$ and $g(x) = -(x + 2)^2$



b. $f(x) = -2x^2$ and $g(x) = -2(x + 2)^2$



SELECTING TOOLS
To be proficient in math, you need to consider the available tools, such as a graphing calculator, when solving a mathematical problem.

Communicate Your Answer

3. How can you describe the graph of $f(x) = a(x - h)^2$?
4. Without graphing, describe the graph of each function. Use a graphing calculator to check your answer.
 a. $y = (x - 3)^2$
 b. $y = (x + 3)^2$
 c. $y = -(x - 3)^2$

For a section overview and insights into this Exploration page, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the Essential Question. Have students work in pairs of mixed language abilities to read and complete the explorations and Communicate Your Answer. Have them summarize their answers and share them with the class.

Beginning Use simple phrases to summarize answers.
Intermediate Use complete sentences to summarize answers.
Advanced/Advanced High Use multiple complex sentences to summarize answers.

ELPS 4.G.2 Demonstrate comprehension of increasingly complex English by retelling or summarizing material commensurate with content area and grade level needs.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

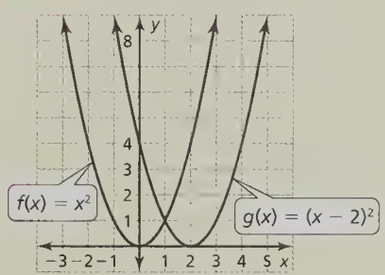
A.6.B The student is expected to write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$).

A.7.A The student is expected to graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

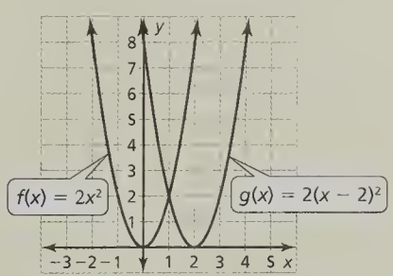
A.7.C The student is expected to determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

ANSWERS

1. a.



b.



Sample answer: The value of h causes a horizontal translation of the graph of $y = a(x - h)^2$ from the graph of $y = ax^2$.

2–4. See Additional Answers.

Extra Example 1

Determine whether each function is *even*, *odd*, or *neither*.

a. $f(x) = 3x$

The function is odd.

b. $g(x) = 2x^2 - 6$

The function is even.

c. $h(x) = 3x^2 - 2x + 4$

The function is neither even nor odd.

MONITORING PROGRESS ANSWERS

1. odd
2. neither
3. even

8.4 Lesson

What You Will Learn

- Identify even and odd functions.
- Graph quadratic functions of the form $f(x) = a(x - h)^2$.
- Graph quadratic functions of the form $f(x) = a(x - h)^2 + k$.
- Model real-life problems using $f(x) = a(x - h)^2 + k$.

Core Vocabulary

even function, *p. 428*
odd function, *p. 428*
vertex form (of a quadratic function), *p. 430*

Previous
reflection

Identifying Even and Odd Functions

Core Concept

Even and Odd Functions

A function $y = f(x)$ is **even** when $f(-x) = f(x)$ for each x in the domain of f. The graph of an even function is symmetric about the y-axis.

A function $y = f(x)$ is **odd** when $f(-x) = -f(x)$ for each x in the domain of f. The graph of an odd function is symmetric *about the origin* when it looks the same after reflections in the x-axis and then in the y-axis.

STUDY TIP

The graph of an odd function looks the same after a 180° rotation about the origin.

EXAMPLE 1 Identifying Even and Odd Functions

Determine whether each function is *even*, *odd*, or *neither*.

a. $f(x) = 2x$ **b.** $g(x) = x^2 - 2$ **c.** $h(x) = 2x^2 + x - 2$

SOLUTION

a. $f(x) = 2x$ Write the original function.

$f(-x) = 2(-x)$ Substitute $-x$ for x.

$= -2x$ Simplify.

$= -f(x)$ Substitute $f(x)$ for $2x$.

▶ Because $f(-x) = -f(x)$, the function is odd.

b. $g(x) = x^2 - 2$ Write the original function.

$g(-x) = (-x)^2 - 2$ Substitute $-x$ for x.

$= x^2 - 2$ Simplify.

$= g(x)$ Substitute $g(x)$ for $x^2 - 2$.

▶ Because $g(-x) = g(x)$, the function is even.

c. $h(x) = 2x^2 + x - 2$ Write the original function.

$h(-x) = 2(-x)^2 + (-x) - 2$ Substitute $-x$ for x.

$= 2x^2 - x - 2$ Simplify.

▶ Because $h(x) = 2x^2 + x - 2$ and $-h(x) = -2x^2 - x + 2$, you can conclude that $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$. So, the function is neither even nor odd.

STUDY TIP

Most functions are neither even nor odd.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Determine whether the function is *even*, *odd*, or *neither*.

1. $f(x) = 5x$
2. $g(x) = 2^x$
3. $h(x) = 2x^2 + 3$

For classroom suggestions on teaching this lesson, see Laurie's Notes at *BigIdeasMath.com*.

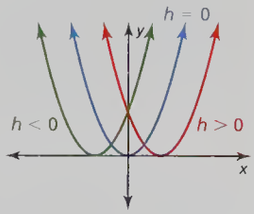
Graphing $f(x) = a(x - h)^2$

Core Concept

Graphing $f(x) = a(x - h)^2$

- When $h > 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units right of the graph of $f(x) = ax^2$.
- When $h < 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation $|h|$ units left of the graph of $f(x) = ax^2$.

The vertex of the graph of $f(x) = a(x - h)^2$ is $(h, 0)$, and the axis of symmetry is $x = h$.



EXAMPLE 2 Graphing $y = a(x - h)^2$

Graph $g(x) = \frac{1}{2}(x - 4)^2$. Compare the graph to the graph of $f(x) = x^2$.

ANOTHER WAY

In Step 3, you could instead choose two x-values greater than the x-coordinate of the vertex.

SOLUTION

Step 1 Graph the axis of symmetry. Because $h = 4$, graph $x = 4$.

Step 2 Plot the vertex. Because $h = 4$, plot $(4, 0)$.

Step 3 Find and plot two more points on the graph. Choose two x-values less than the x-coordinate of the vertex. Then find $g(x)$ for each x-value.

When $x = 0$:

$g(0) = \frac{1}{2}(0 - 4)^2$

$= 8$

When $x = 2$:

$g(2) = \frac{1}{2}(2 - 4)^2$

$= 2$

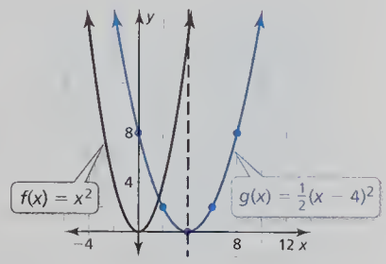
So, plot $(0, 8)$ and $(2, 2)$.

Step 4 Reflect the points plotted in Step 3 in the axis of symmetry. So, plot $(8, 8)$ and $(6, 2)$.

Step 5 Draw a smooth curve through the points.

STUDY TIP

From the graph, you can see that $f(x) = x^2$ is an even function. However, $g(x) = \frac{1}{2}(x - 4)^2$ is neither even nor odd.



▶ Both graphs open up. The graph of g is wider than the graph of f. The axis of symmetry $x = 4$ and the vertex $(4, 0)$ of the graph of g are 4 units right of the axis of symmetry $x = 0$ and the vertex $(0, 0)$ of the graph of f. So, the graph of g is a translation 4 units right and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of f.

Monitoring Progress

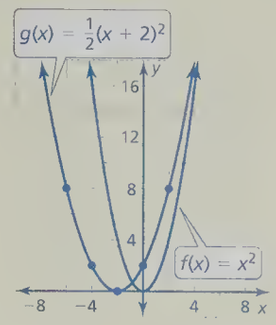
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Graph the function. Compare the graph to the graph of $f(x) = x^2$.

4. $g(x) = 2(x + 5)^2$
5. $h(x) = -(x - 2)^2$

Extra Example 2

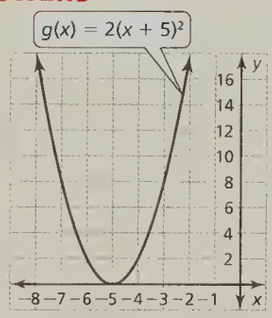
Graph $g(x) = \frac{1}{2}(x + 2)^2$. Compare the graph to the graph of $f(x) = x^2$.



Both graphs open up. The graph of g is wider than the graph of f. The axis of symmetry, $x = -2$, and the vertex, $(-2, 0)$, of the graph of g are 2 units left of the axis of symmetry, $x = 0$, and the vertex, $(0, 0)$, of the graph of f. So, the graph of g is a translation 2 units left and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of f.

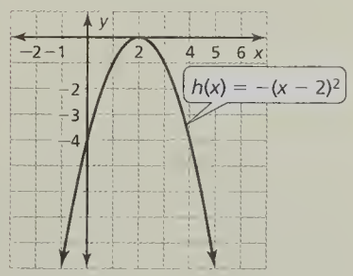
MONITORING PROGRESS ANSWERS

4.



The graph of g is a horizontal translation 5 units left and a vertical stretch by a factor of 2 of the graph of f.

5.



The graph of h is a horizontal translation 2 units right and a reflection in the x-axis of the graph of f.

English Language Learners

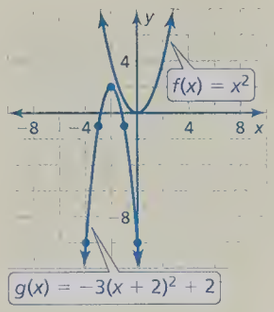
Graphic Organizer

Have students make a table like this one for Example 2 to organize the attributes of the function they are comparing to $f(x) = x^2$. Encourage them to use the information to write their comparison.

Function	$f(x) = x^2$	$f(x) = \frac{1}{2}(x - 4)^2$
Axis of symmetry	$x = 0$	$x = 4$
Vertex	$(0, 0)$	$(4, 0)$
Type of shrink or stretch (factor)		vertical shrink (0.5)
Translation		4 units right

Extra Example 3

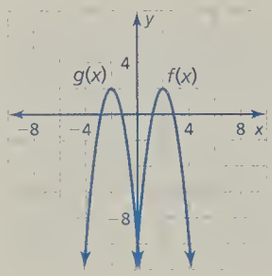
Graph $g(x) = -3(x + 2)^2 + 2$. Compare the graph to the graph of $f(x) = x^2$.



The graph of g opens down and is narrower than the graph of f. The vertex of the graph of g, $(-2, 2)$, is 2 units left and 2 units up of the vertex of the graph of f, $(0, 0)$. So, the graph of g is a vertical stretch by a factor of 3, a reflection in the x-axis, and a translation 2 units left and 2 units up of the graph of f.

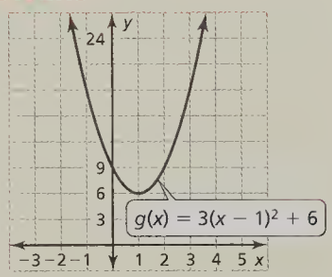
Extra Example 4

Consider function g in Extra Example 3. Graph $f(x) = g(x - 4)$.



MONITORING PROGRESS ANSWERS

6.



The graph of g is a vertical stretch by a factor of 3, and a translation 1 unit right and 6 units up of the graph of f.

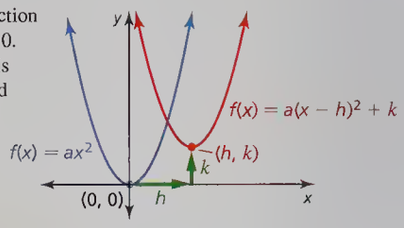
Graphing $f(x) = a(x - h)^2 + k$

Core Concept

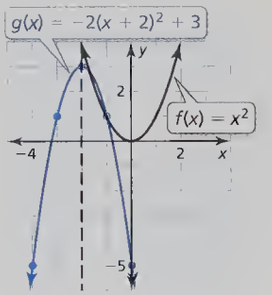
Graphing $f(x) = a(x - h)^2 + k$

The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The graph of $f(x) = a(x - h)^2 + k$ is a translation h units horizontally and k units vertically of the graph of $f(x) = ax^2$.

The vertex of the graph of $f(x) = a(x - h)^2 + k$ is (h, k), and the axis of symmetry is $x = h$.



EXAMPLE 3 Graphing $y = a(x - h)^2 + k$



Graph $g(x) = -2(x + 2)^2 + 3$. Compare the graph to the graph of $f(x) = x^2$.

SOLUTION

Step 1 Graph the axis of symmetry. Because $h = -2$, graph $x = -2$.

Step 2 Plot the vertex. Because $h = -2$ and $k = 3$, plot $(-2, 3)$.

Step 3 Find and plot two more points on the graph. Choose two x-values less than the x-coordinate of the vertex. Then find $g(x)$ for each x-value. So, plot $(-4, -5)$ and $(-3, 1)$.

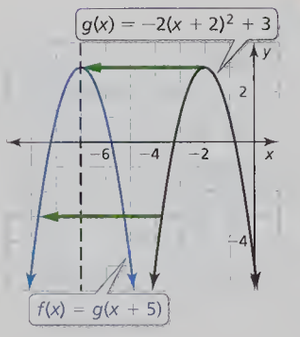
x	-4	-3
$g(x)$	-5	1

Step 4 Reflect the points plotted in Step 3 in the axis of symmetry. So, plot $(-1, 1)$ and $(0, -5)$.

Step 5 Draw a smooth curve through the points.

▶ The graph of g opens down and is narrower than the graph of f. The vertex of the graph of g, $(-2, 3)$, is 2 units left and 3 units up of the vertex of the graph of f, $(0, 0)$. So, the graph of g is a vertical stretch by a factor of 2, a reflection in the x-axis, and a translation 2 units left and 3 units up of the graph of f.

EXAMPLE 4 Transforming the Graph of $y = a(x - h)^2 + k$



Consider function g in Example 3. Graph $f(x) = g(x + 5)$.

SOLUTION

The function f is of the form $y = g(x - h)$, where $h = -5$. So, the graph of f is a horizontal translation 5 units left of the graph of g. To graph f, subtract 5 from the x-coordinates of the points on the graph of g.

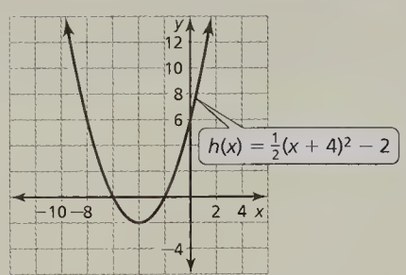
Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = x^2$.

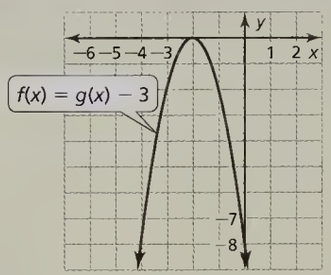
6. $g(x) = 3(x - 1)^2 + 6$
7. $h(x) = \frac{1}{2}(x + 4)^2 - 2$
8. Consider function g in Example 3. Graph $f(x) = g(x) - 3$.

7.



The graph of h is a vertical shrink by a factor of $\frac{1}{2}$, and a translation 4 units left and 2 units down of the graph of f.

8.



Modeling Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**



Water fountains are usually designed to give a specific visual effect. For example, the water fountain shown consists of streams of water that are shaped like parabolas. Notice how the streams are designed to land on the underwater spotlights. Write and graph a quadratic function that models the path of a stream of water with a maximum height of 5 feet, represented by a vertex of (3, 5), landing on a spotlight 6 feet from the water jet, represented by (6, 0).

SOLUTION

1. **Understand the Problem** You know the vertex and another point on the graph that represents the parabolic path. You are asked to write and graph a quadratic function that models the path.

2. **Make a Plan** Use the given points and the vertex form to write a quadratic function. Then graph the function.

3. **Solve the Problem**
 Use the vertex form, vertex (3, 5), and point (6, 0) to find the value of a.

$$f(x) = a(x - h)^2 + k \qquad \text{Write the vertex form of a quadratic function.}$$
$$f(x) = a(x - 3)^2 + 5 \qquad \text{Substitute 3 for } h \text{ and 5 for } k.$$
$$0 = a(6 - 3)^2 + 5 \qquad \text{Substitute 6 for } x \text{ and 0 for } f(x).$$
$$0 = 9a + 5 \qquad \text{Simplify.}$$
$$-\frac{5}{9} = a \qquad \text{Solve for } a.$$

So, $f(x) = -\frac{5}{9}(x - 3)^2 + 5$ models the path of a stream of water. Now graph the function.

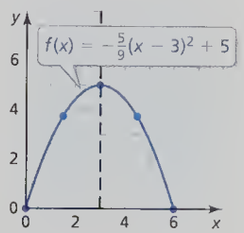
Step 1 Graph the axis of symmetry. Because $h = 3$, graph $x = 3$.

Step 2 Plot the vertex, (3, 5).

Step 3 Find and plot two more points on the graph. Because the x-axis represents the water surface, the graph should only contain points with nonnegative values of $f(x)$. You know that (6, 0) is on the graph. To find another point, choose an x-value between $x = 3$ and $x = 6$. Then find the corresponding value of $f(x)$.

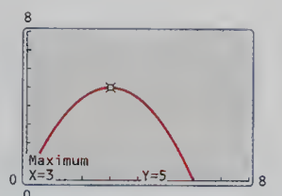
$$f(4.5) = -\tfrac{5}{9}(4.5 - 3)^2 + 5 = 3.75$$

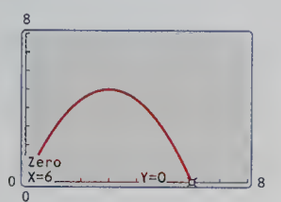
So, plot (6, 0) and (4.5, 3.75).



Step 4 Reflect the points plotted in Step 3 in the axis of symmetry. So, plot (0, 0) and (1.5, 3.75).

Step 5 Draw a smooth curve through the points.





4. **Look Back** Use a graphing calculator to graph $f(x) = -\frac{5}{9}(x - 3)^2 + 5$. Use the *maximum* feature to verify that the maximum value is 5. Then use the *zero* feature to verify that $x = 6$ is a zero of the function.

Monitoring Progress

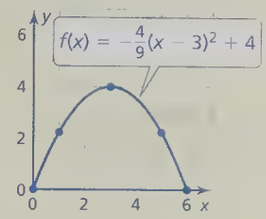
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9. **WHAT IF?** The vertex is (3, 6). Write and graph a quadratic function that models the path.

Extra Example 5

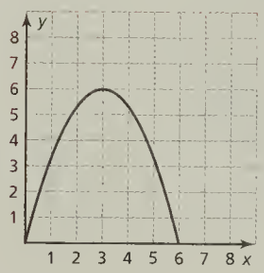
Use the information in Example 5. The vertex is (3, 4), and the water lands 6 feet from the water jet. Write and graph a quadratic function that models the path of the stream of water.

$f(x) = -\frac{4}{9}(x - 3)^2 + 4$



MONITORING PROGRESS ANSWER

9. $f(x) = -\frac{2}{3}(x - 3)^2 + 6$



Closure

- **Exit Ticket:** Given $f(x) = \frac{1}{2}(x + 8)^2 + 4$, tell what you know about the function. The graph opens up and is wider than the graph of $y = x^2$. The axis of symmetry is $x = -8$. The vertex is $(-8, 4)$. The domain is all real numbers. The range is $y \geq 4$. So, the graph of f is a vertical shrink by a factor of $\frac{1}{2}$ and a translation 8 units left and 4 units up of the graph of $y = x^2$.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–55 odd, 63, 70, 79–81

Average: 1–3, 4–70 even, 79–81

Advanced: 1–4, 12–16 even, 22, 26–38 even, 42–48 even, 52–68 even, 69–81

HOMEWORK CHECK

Basic: 5, 23, 39, 49, 63

Average: 8, 24, 40, 50, 64

Advanced: 12, 26, 42, 52, 64

ANSWERS

1. *Sample answer:* The graph of an even function is symmetric about the y-axis. The graph of an odd function is symmetric about the origin.
2. *Sample answer:* $y = 5(x - 1)^2 + 2$
3. The graph of g is a horizontal translation h units right if h is positive or $|h|$ units left if h is negative, and a vertical translation k units up if k is positive or $|k|$ units down if k is negative of the graph of f.
4. $f(x) = 2(x + 0)^2$; It is the only function that is not a horizontal translation of the parent function $g(x) = x^2$.
5. neither
6. even
7. neither
8. neither
9. even
10. odd
11. neither
12. even
13. even
14. neither
15. neither
16. even
17. odd
18. neither
19. $(-1, 0)$; $x = -1$
20. $(6, 0)$; $x = 6$
21. $(4, 0)$; $x = 4$
22. $(-9, 0)$; $x = -9$

8.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Compare the graph of an even function with the graph of an odd function.
2. **OPEN-ENDED** Write a quadratic function whose graph has a vertex of $(1, 2)$.
3. **WRITING** Describe the transformation from the graph of $f(x) = ax^2$ to the graph of $g(x) = a(x - h)^2 + k$.
4. **WHICH ONE DOESN'T BELONG?** Which function does *not* belong with the other three? Explain your reasoning.

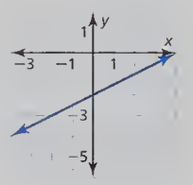
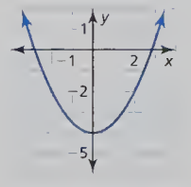
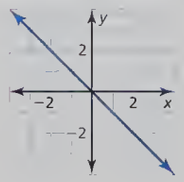
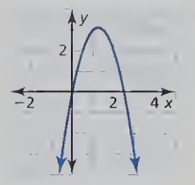
$f(x) = 8(x + 4)^2$ $\quad$ $f(x) = (x - 2)^2 + 4$ $\quad$ $f(x) = 2(x + 0)^2$ $\quad$ $f(x) = 3(x + 1)^2 + 1$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, determine whether the function is *even*, *odd*, or *neither*. *(See Example 1.)*

5. $f(x) = 4x + 3$
6. $g(x) = 3x^2$
7. $h(x) = 5^x + 2$
8. $m(x) = 2x^2 - 7x$
9. $p(x) = -x^2 + 8$
10. $f(x) = -\frac{1}{2}x$
11. $n(x) = 2x^2 - 7x + 3$
12. $r(x) = -6x^2 + 5$

In Exercises 13–18, determine whether the function represented by the graph is *even*, *odd*, or *neither*.

13. 
14. 
15. 
16. 
17. 
18. 

In Exercises 19–22, find the vertex and the axis of symmetry of the graph of the function.

19. $f(x) = 3(x + 1)^2$
20. $f(x) = \frac{1}{4}(x - 6)^2$
21. $y = -\frac{1}{8}(x - 4)^2$
22. $y = -5(x + 9)^2$

In Exercises 23–28, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 2.)*

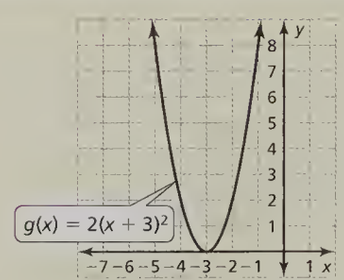
23. $g(x) = 2(x + 3)^2$
24. $p(x) = 3(x - 1)^2$
25. $r(x) = \frac{1}{4}(x + 10)^2$
26. $n(x) = \frac{1}{3}(x - 6)^2$
27. $d(x) = \frac{1}{5}(x - 5)^2$
28. $q(x) = 6(x + 2)^2$

29. **ERROR ANALYSIS** Describe and correct the error in determining whether the function $f(x) = x^2 + 3$ is even, odd, or neither.

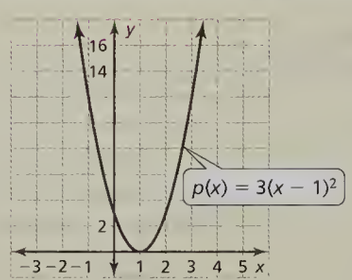
✗
$$f(x) = x^2 + 3$$
$$f(-x) = (-x)^2 + 3$$
$$= x^2 + 3$$
$$= f(x)$$
So, $f(x)$ is an odd function.

30. **ERROR ANALYSIS** Describe and correct the error in finding the vertex of the graph of the function.

✗ $y = -(x + 8)^2$
Because $h = -8$, the vertex is $(0, -8)$.

23. 

The graph of g is a horizontal translation 3 units left and a vertical stretch by a factor of 2 of the graph of f.

24. 

The graph of p is a horizontal translation 1 unit right and a vertical stretch by a factor of 3 of the graph of f.

25–30. See Additional Answers.

In Exercises 31–34, find the vertex and the axis of symmetry of the graph of the function.

31. $y = -6(x + 4)^2 - 3$ **32.** $f(x) = 3(x - 3)^2 + 6$

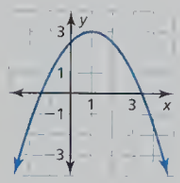
33. $f(x) = -4(x + 3)^2 + 1$ **34.** $y = -(x - 6)^2 - 5$

In Exercises 35–38, match the function with its graph.

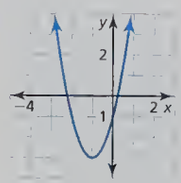
35. $y = -(x + 1)^2 - 3$ **36.** $y = -\frac{1}{2}(x - 1)^2 + 3$

37. $y = \frac{1}{3}(x - 1)^2 + 3$ **38.** $y = 2(x + 1)^2 - 3$

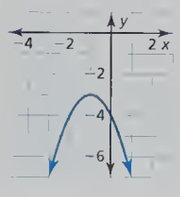
A.



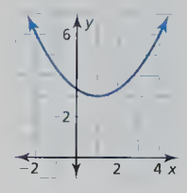
B.



C.



D.



In Exercises 39–44, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 3.)*

39. $h(x) = (x - 2)^2 + 4$ **40.** $g(x) = (x + 1)^2 - 7$

41. $r(x) = 4(x - 1)^2 - 5$ **42.** $n(x) = -(x + 4)^2 + 2$

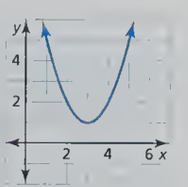
43. $g(x) = -\frac{1}{3}(x + 3)^2 - 2$ **44.** $r(x) = \frac{1}{2}(x - 2)^2 - 4$

In Exercises 45–48, let $f(x) = (x - 2)^2 + 1$. Match the function with its graph.

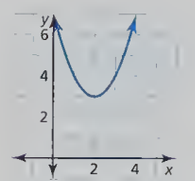
45. $g(x) = f(x - 1)$ **46.** $r(x) = f(x + 2)$

47. $h(x) = f(x) + 2$ **48.** $p(x) = f(x) - 3$

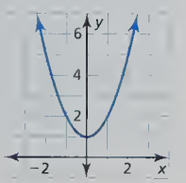
A.



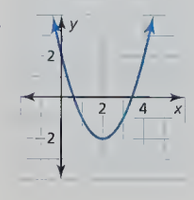
B.



C.



D.



In Exercises 49–54, graph g. *(See Example 4.)*

49. $f(x) = 2(x - 1)^2 + 1$; $g(x) = f(x + 3)$

50. $f(x) = -(x + 1)^2 + 2$; $g(x) = \frac{1}{2}f(x)$

51. $f(x) = -3(x + 5)^2 - 6$; $g(x) = 2f(x)$

52. $f(x) = 5(x - 3)^2 - 1$; $g(x) = f(x) - 6$

53. $f(x) = (x + 3)^2 + 5$; $g(x) = f(x - 4)$

54. $f(x) = -2(x - 4)^2 - 8$; $g(x) = -f(x)$

55. MODELING WITH MATHEMATICS The height (in meters) of a bird diving to catch a fish is represented by $h(t) = 5(t - 2.5)^2$, where t is the number of seconds after beginning the dive.

a. Graph h.

b. Another bird's dive is represented by $r(t) = 2h(t)$. Graph r.

c. Compare the graphs. Which bird starts its dive from a greater height? Explain.



56. MODELING WITH MATHEMATICS A kicker punts a football. The height (in yards) of the football is represented by $f(x) = -\frac{1}{9}(x - 30)^2 + 25$, where x is the horizontal distance (in yards) from the kicker's goal line.

a. Graph f. Describe the domain and range.

b. On the next possession, the kicker punts the football. The height of the football is represented by $g(x) = f(x + 5)$. Graph g. Describe the domain and range.

c. Compare the graphs. On which possession does the kicker punt closer to his goal line? Explain.

In Exercises 57–62, write a quadratic function in vertex form whose graph has the given vertex and passes through the given point.

57. vertex: (1, 2); passes through (3, 10)

58. vertex: (−3, 5); passes through (0, −14)

59. vertex: (−2, −4); passes through (−1, −6)

60. vertex: (1, 8); passes through (3, 12)

61. vertex: (5, −2); passes through (7, 0)

62. vertex: (−5, −1); passes through (−2, 2)

Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool

Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

ANSWERS

31. $(-4, -3)$; $x = -4$

32. $(3, 6)$; $x = 3$

33. $(-3, 1)$; $x = -3$

34. $(6, -5)$; $x = 6$

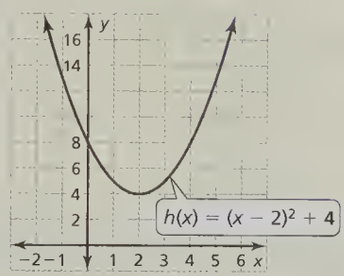
35. C

36. A

37. D

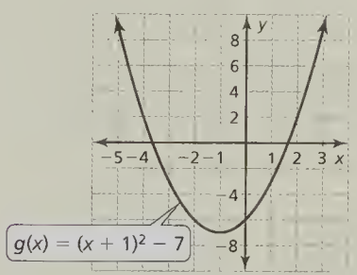
38. B

39.



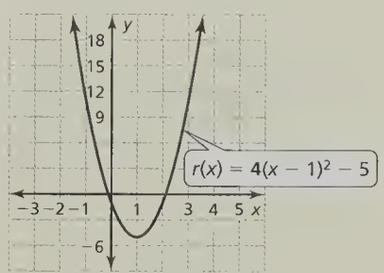
The graph of h is a translation 2 units right and 4 units up of the graph of f.

40.



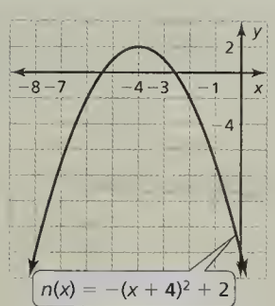
The graph of g is a translation 1 unit left and 7 units down of the graph of f.

41.



The graph of r is a vertical stretch by a factor of 4, and a translation 1 unit right and 5 units down of the graph of f.

42.

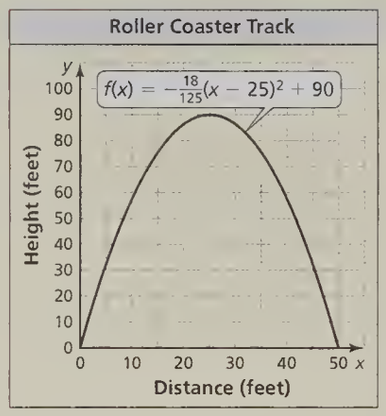


The graph of n is a reflection in the x-axis, and a translation 4 units left and 2 units up of the graph of f.

43–62. See Additional Answers.

ANSWERS

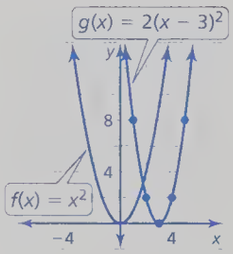
63. $f(x) = -\frac{18}{125}(x - 25)^2 + 90$



64–81. See Additional Answers.

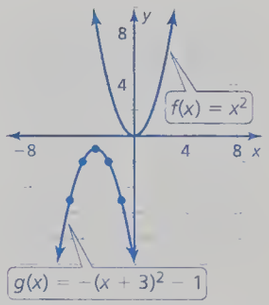
Mini-Assessment

1. Determine whether the function $f(x) = x^2 + 5$ is *even*, *odd*, or *neither*. even
2. Graph $g(x) = 2(x - 3)^2$. Compare the graph to the graph of $f(x) = x^2$.



The graph of g is a translation 3 units right and a vertical stretch by a factor of 2 of the graph of f.

3. Graph $g(x) = -(x + 3)^2 - 1$. Compare the graph to the graph of $f(x) = x^2$.



The graph of g is a reflection in the x-axis, and a translation 3 units left and 1 unit down of the graph of f.

63. MODELING WITH MATHEMATICS A portion of a roller coaster track is in the shape of a parabola. Write and graph a quadratic function that models this portion of the roller coaster with a maximum height of 90 feet, represented by a vertex of (25, 90), passing through the point (50, 0). *(See Example 5.)*



64. MODELING WITH MATHEMATICS A flare is launched from a boat and travels in a parabolic path until reaching the water. Write and graph a quadratic function that models the path of the flare with a maximum height of 300 meters, represented by a vertex of (59, 300), landing in the water at the point (119, 0).

In Exercises 65–68, rewrite the quadratic function in vertex form.

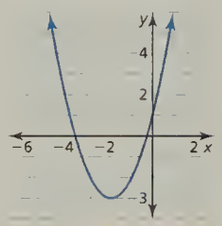
65. $y = 2x^2 - 8x + 4$ **66.** $y = 3x^2 + 6x - 1$

67. $f(x) = -5x^2 + 10x + 3$

68. $f(x) = -x^2 - 4x + 2$

69. REASONING Can a function be symmetric about the x-axis? Explain.

70. HOW DO YOU SEE IT? The graph of a quadratic function is shown. Determine which symbols to use to complete the vertex form of the quadratic function. Explain your reasoning.



$y = a(x \quad 2)^2 \quad 3$

In Exercises 71–74, describe the transformation from the graph of f to the graph of h. Write an equation that represents h in terms of x.

71. $f(x) = -(x + 1)^2 - 2$
$h(x) = f(x) + 4$

72. $f(x) = 2(x - 1)^2 + 1$
$h(x) = f(x - 5)$

73. $f(x) = 4(x - 2)^2 + 3$
$h(x) = 2f(x)$

74. $f(x) = -(x + 5)^2 - 6$
$h(x) = \frac{1}{3}f(x)$

75. REASONING The graph of $y = x^2$ is translated 2 units right and 5 units down. Write an equation for the function in vertex form and in standard form. Describe advantages of writing the function in each form.

76. THOUGHT PROVOKING Which of the following are true? Justify your answers.

a. Any constant multiple of an even function is even.

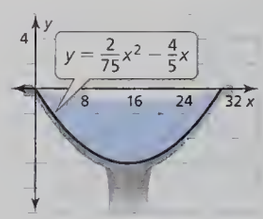
b. Any constant multiple of an odd function is odd.

c. The sum or difference of two even functions is even.

d. The sum or difference of two odd functions is odd.

e. The sum or difference of an even function and an odd function is odd.

77. COMPARING FUNCTIONS A cross section of a birdbath can be modeled by $y = \frac{1}{81}(x - 18)^2 - 4$, where x and y are measured in inches. The graph shows the cross section of another birdbath.



a. Which birdbath is deeper? Explain.

b. Which birdbath is wider? Explain.

78. REASONING Compare the graphs of $y = 2x^2 + 8x + 8$ and $y = x^2$ without graphing the functions. How can factoring help you compare the parabolas? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. *(Section 7.5)*

79. $x(x - 1) = 0$ **80.** $(x + 3)(x - 8) = 0$ **81.** $(3x - 9)(4x + 12) = 0$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

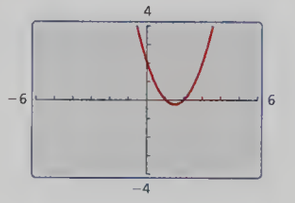
8.5 Using Intercept Form

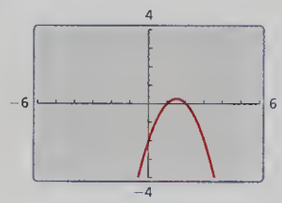
TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.6.A
A.6.B
A.6.C
A.7.A
A.7.B

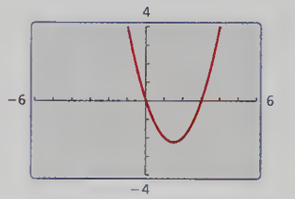
Essential Question What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?

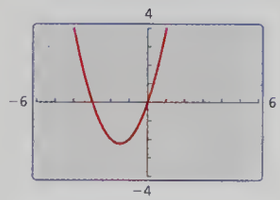
EXPLORATION 1 **Using Zeros to Write Functions**

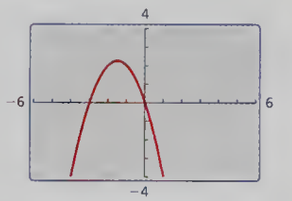
Work with a partner. Each graph represents a function of the form $f(x) = (x - p)(x - q)$ or $f(x) = -(x - p)(x - q)$. Write the function represented by each graph. Explain your reasoning.

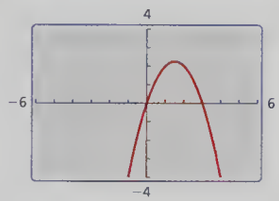
a. 

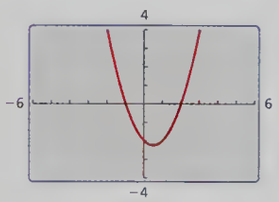
b. 

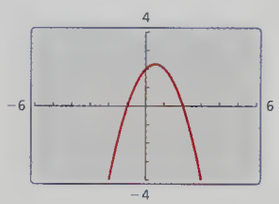
c. 

d. 

e. 

f. 

g. 

h. 

MAKING MATHEMATICAL ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

Communicate Your Answer

2. What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?
3. Consider the graph of $f(x) = a(x - p)(x - q)$.
 a. Does changing the sign of a change the x-intercepts? Does changing the sign of a change the y-intercept? Explain your reasoning.
 b. Does changing the value of p change the x-intercepts? Does changing the value of p change the y-intercept? Explain your reasoning.

SUPPORTING English Language Learners

Present the Essential Question. Have students work in pairs to complete the exploration and Communicate Your Answer. Have them share their answers with the class.

Beginning/Advanced Beginning students read headings. Advanced students read exploration instructions and Communicate Your Answer questions. When sharing answers, Beginning students use simple phrases and Advanced students use multiple sentences.

Intermediate/Advanced High Intermediate students read Exploration 1. Advanced High students read Communicate Your Answer questions. When sharing answers, Intermediate students use complete simple sentences and Advanced High students use multiple sentences.

ELPS 4.G.3 Demonstrate comprehension of increasingly complex English by responding to questions commensurate with content area and grade level needs.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.6.A The student is expected to determine the domain and range of quadratic functions and represent the domain and range using inequalities.

A.6.B The student is expected to write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$).

A.6.C The student is expected to write quadratic functions when given real solutions and graphs of their related equations.

A.7.A The student is expected to graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

A.7.B The student is expected to describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions.

ANSWERS

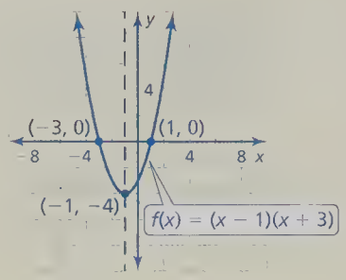
1. a. $y = (x - 1)(x - 2)$; *Sample answer:* The graph opens up and the x-intercepts are 1 and 2.
 b. $y = -(x - 1)(x - 2)$; *Sample answer:* The graph opens down and the x-intercepts are 1 and 2.
 c. $y = x(x - 3)$; *Sample answer:* The graph opens up and the x-intercepts are 0 and 3.
 d. $y = x(x + 3)$; *Sample answer:* The graph opens up and the x-intercepts are 0 and -3.
 e. $y = -x(x + 3)$; *Sample answer:* The graph opens down and the x-intercepts are 0 and -3.

1f–3. See Additional Answers.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Extra Example 1

Graph $f(x) = (x - 1)(x + 3)$. Describe the domain and range.



The domain is all real numbers. The range is $y \geq -4$.

8.5 Lesson

Core Vocabulary

intercept form, *p. 436*

What You Will Learn

- Graph quadratic functions of the form $f(x) = a(x - p)(x - q)$.
- Use intercept form to find zeros of functions.
- Graphing and writing quadratic functions.
- Graphing and writing cubic functions.

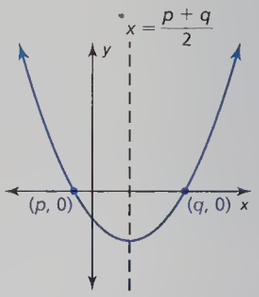
Graphing $f(x) = a(x - p)(x - q)$

You have already graphed quadratic functions written in several different forms, such as $f(x) = ax^2 + bx + c$ (standard form) and $g(x) = a(x - h)^2 + k$ (vertex form). Quadratic functions can also be written in **intercept form**, $f(x) = a(x - p)(x - q)$, where $a \neq 0$. In this form, the polynomial that defines a function is in factored form and the x-intercepts of the graph can be easily determined.

Core Concept

Graphing $f(x) = a(x - p)(x - q)$

- The x-intercepts are p and q.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p + q}{2}$.
- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.



EXAMPLE 1 Graphing $f(x) = a(x - p)(x - q)$

Graph $f(x) = -(x + 1)(x - 5)$. Describe the domain and range.

SOLUTION

Step 1 Identify the x-intercepts. Because the x-intercepts are $p = -1$ and $q = 5$, plot $(-1, 0)$ and $(5, 0)$.

Step 2 Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-1 + 5}{2} = 2$$

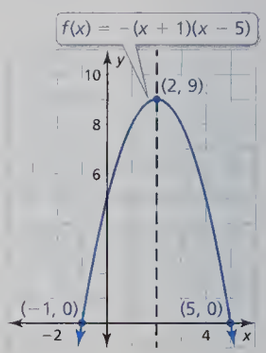
Step 3 Find and plot the vertex.

The x-coordinate of the vertex is 2. To find the y-coordinate of the vertex, substitute 2 for x and simplify.

$$f(2) = -(2 + 1)(2 - 5) = 9$$

So, the vertex is $(2, 9)$.

Step 4 Draw a parabola through the vertex and the points where the x-intercepts occur.



▶ The domain is all real numbers. The range is $y \leq 9$.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

EXAMPLE 2 Graphing a Quadratic Function

Graph $f(x) = 2x^2 - 8$. Describe the domain and range.

SOLUTION

Step 1 Rewrite the quadratic function in intercept form.

$f(x) = 2x^2 - 8$ Write the function.

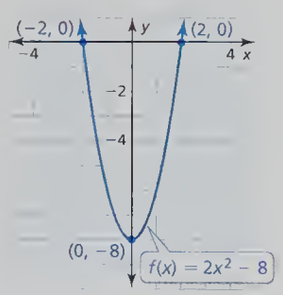
$= 2(x^2 - 4)$ Factor out common factor.

$= 2(x + 2)(x - 2)$ Difference of two squares pattern

Step 2 Identify the x-intercepts. Because the x-intercepts are $p = -2$ and $q = 2$, plot $(-2, 0)$ and $(2, 0)$.

Step 3 Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-2 + 2}{2} = 0$$



Step 4 Find and plot the vertex.

The x-coordinate of the vertex is 0.
The y-coordinate of the vertex is

$f(0) = 2(0)^2 - 8 = -8$.

So, the vertex is $(0, -8)$.

Step 5 Draw a parabola through the vertex and the points where the x-intercepts occur.

▶ The domain is all real numbers. The range is $y \geq -8$.

Monitoring Progress

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Graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function.

1. $f(x) = (x + 2)(x - 3)$ **2.** $g(x) = -2(x - 4)(x + 1)$ **3.** $h(x) = 4x^2 - 36$

REMEMBER

Functions have zeros, and *graphs* have x-intercepts.

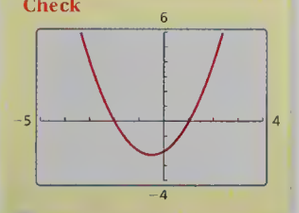
Using Intercept Form to Find Zeros of Functions

In Section 3.4, you learned that a zero of a function is an x-value for which $f(x) = 0$. You can use the intercept form of a function to find the zeros of the function.

EXAMPLE 3 Finding Zeros of a Function

Find the zeros of $f(x) = (x - 1)(x + 2)$.

Check



SOLUTION

To find the zeros, determine the x-values for which $f(x)$ is 0.

$f(x) = (x - 1)(x + 2)$ Write the function.

$0 = (x - 1)(x + 2)$ Substitute 0 for $f(x)$.

$x - 1 = 0$ or $x + 2 = 0$ Zero-Product Property

$x = 1$ or $x = -2$ Solve for x.

▶ So, the zeros of the function are -2 and 1.

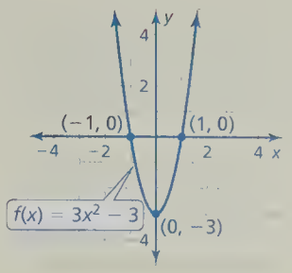
Differentiated Instruction

Kinesthetic

Ask two students to assist you at the board to solve a problem like Example 1 or Example 2. Assign finding the x-intercepts, the axis of symmetry, and the vertex of a function to one student. Assign graphing the x-intercepts, axis of symmetry, the vertex, and the parabola to the second student. Have students discuss how the intercepts, axis, and vertex relate to the function and its graph.

Extra Example 2

Graph $f(x) = 3x^2 - 3$. Describe the domain and range.



The domain is all real numbers. The range is $y \geq -3$.

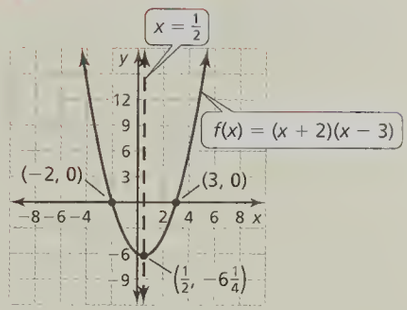
Extra Example 3

Find the zeros of $f(x) = (x - 5)(x + 7)$.

The zeros of the function are -7 and 5.

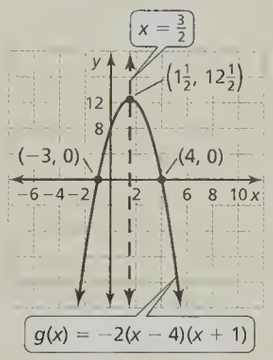
MONITORING PROGRESS ANSWERS

1.



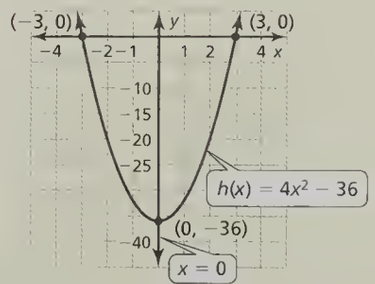
domain: all real numbers, range: $y \geq -6\frac{1}{4}$

2.



domain: all real numbers, range: $y \leq 12\frac{1}{2}$

3.



domain: all real numbers, range: $y \geq -36$

Extra Example 4

Find the zeros of each function.

a. $f(x) = 2x^2 - 8x - 24$

The zeros of the function are -2 and 6.

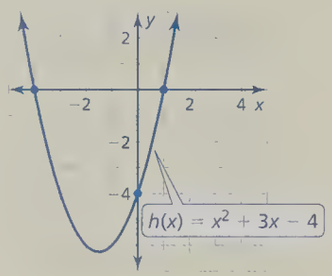
b. $h(x) = (x - 2)(x^2 - 25)$

The zeros of the function are -5, 2, and 5.

Extra Example 5

Use zeros to graph $h(x) = x^2 + 3x - 4$.

The zeros are -4 and 1.



MONITORING PROGRESS ANSWERS

4. 1, 6
5. 2
6. -1, 0, 1

Core Concept

Factors and Zeros

For any factor $x - n$ of a polynomial, n is a zero of the function defined by the polynomial.

EXAMPLE 4 Finding Zeros of Functions

Find the zeros of each function.

a. $f(x) = -2x^2 - 10x - 12$ b. $h(x) = (x - 1)(x^2 - 16)$

SOLUTION

Write each function in intercept form to identify the zeros.

a. $f(x) = -2x^2 - 10x - 12$ Write the function.

$= -2(x^2 + 5x + 6)$ Factor out common factor.

$= -2(x + 3)(x + 2)$ Factor the trinomial.

▶ So, the zeros of the function are -3 and -2.

b. $h(x) = (x - 1)(x^2 - 16)$ Write the function.

$= (x - 1)(x + 4)(x - 4)$ Difference of two squares pattern

▶ So, the zeros of the function are -4, 1, and 4.

ANALYZING MATHEMATICAL RELATIONSHIPS

The function in Example 4(b) is called a *cubic function*. You can extend the concept of intercept form to cubic functions. You will graph a cubic function in Example 8.

Monitoring Progress

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Find the zero(s) of the function.

4. $f(x) = (x - 6)(x - 1)$ 5. $g(x) = 3x^2 - 12x + 12$ 6. $h(x) = x(x^2 - 1)$

Graphing and Writing Quadratic Functions

EXAMPLE 5 Graphing a Quadratic Function Using Zeros

Use zeros to graph $h(x) = x^2 - 2x - 3$.

SOLUTION

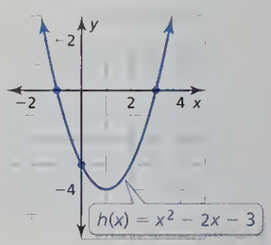
The function is in standard form. You know that the parabola opens up ($a > 0$) and the y-intercept is -3. So, begin by plotting $(0, -3)$.

Notice that the polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

$h(x) = x^2 - 2x - 3$ Write the function.

$= (x + 1)(x - 3)$ Factor the trinomial.

The zeros of the function are -1 and 3. So, plot $(-1, 0)$ and $(3, 0)$. Draw a parabola through the points.



USING PRECISE MATHEMATICAL LANGUAGE

To sketch a more precise graph, make a table of values and plot other points on the graph.

STUDY TIP

In part (a), many possible functions satisfy the given condition. The value a can be *any* nonzero number. To allow easier calculations, let a = 1. By letting $a = 2$, the resulting function would be $f(x) = 2x^2 - 6x - 56$.

EXAMPLE 6 Writing Quadratic Functions

Write a quadratic function in standard form whose graph satisfies the given condition(s).

a. x-intercepts: -4 and 7 **b.** vertex: $(-3, 4)$

SOLUTION

a. Because you know the x-intercepts, use intercept form to write a function.

$f(x) = a(x - p)(x - q)$ Intercept form

$= 1(x + 4)(x - 7)$ Substitute for a, p, and q.

$= x^2 - 7x + 4x - 28$ FOIL Method

$= x^2 - 3x - 28$ Combine like terms.

b. Because you know the vertex, use vertex form to write a function.

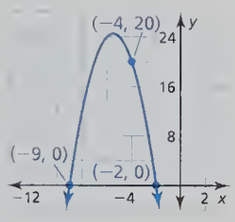
$f(x) = a(x - h)^2 + k$ Vertex form

$= 1(x + 3)^2 + 4$ Substitute for a, h, and k.

$= x^2 + 6x + 9 + 4$ Find the product $(x + 3)^2$.

$= x^2 + 6x + 13$ Combine like terms.

EXAMPLE 7 Writing a Quadratic Function



The graph represents a quadratic function. Write the function.

SOLUTION

From the graph, you can see that the x-intercepts are -9 and -2. Use intercept form to write a function.

$f(x) = a(x - p)(x - q)$ Intercept form

$= a(x + 9)(x + 2)$ Substitute for p and q.

Use the other given point, $(-4, 20)$, to find the value of a.

$20 = a(-4 + 9)(-4 + 2)$ Substitute -4 for x and 20 for $f(x)$.

$20 = a(5)(-2)$ Simplify.

$-2 = a$ Solve for a.

Use the value of a to write the function.

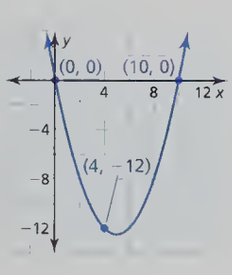
$f(x) = -2(x + 9)(x + 2)$ Substitute -2 for a.

$= -2x^2 - 22x - 36$ Simplify.

▶ The function represented by the graph is $f(x) = -2x^2 - 22x - 36$.

Monitoring Progress

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Use zeros to graph the function.

7. $f(x) = (x - 1)(x - 4)$ **8.** $g(x) = x^2 + x - 12$

Write a quadratic function in standard form whose graph satisfies the given condition(s).

9. x-intercepts: -1 and 1 **10.** vertex: $(8, 8)$

11. The graph at the left represents a quadratic function. Write the function.

Extra Example 6

Write a quadratic function in standard form whose graph satisfies the given condition(s).

a. x-intercepts: -1 and 5

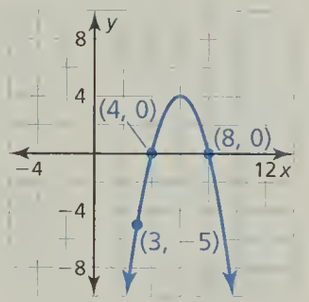
Sample answer: $f(x) = x^2 - 4x - 5$

b. vertex: $(-3, -2)$

Sample answer: $f(x) = x^2 + 6x + 7$

Extra Example 7

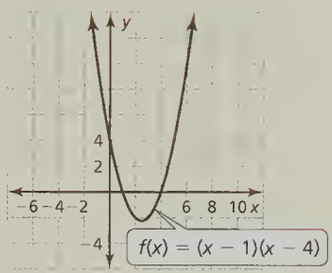
The graph represents a quadratic function. Write the function.



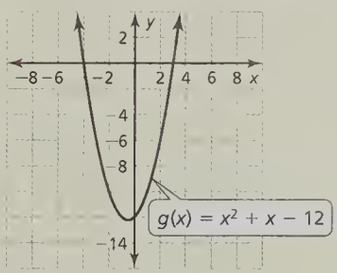
$f(x) = -x^2 + 12x - 32$

MONITORING PROGRESS ANSWERS

7.



8.



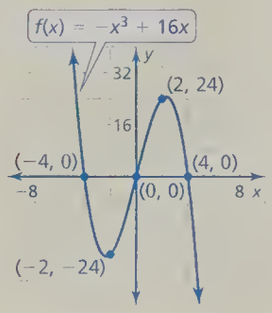
9. *Sample answer:* $f(x) = x^2 - 1$

10. *Sample answer:* $f(x) = x^2 - 16x + 72$

11. $f(x) = \frac{1}{2}x^2 - 5x$

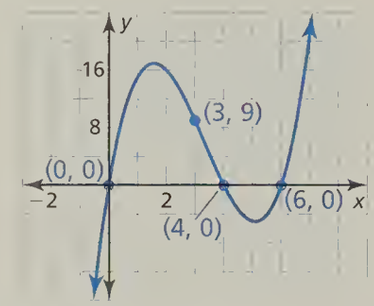
Extra Example 8

Use zeros to graph $f(x) = -x^3 + 16x$.



Extra Example 9

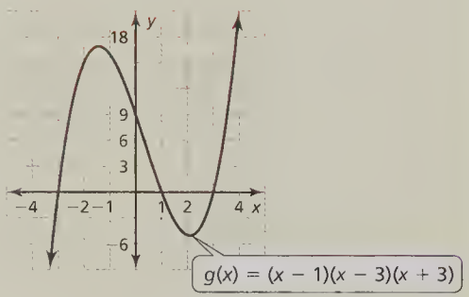
The graph represents a cubic function. Write the function.



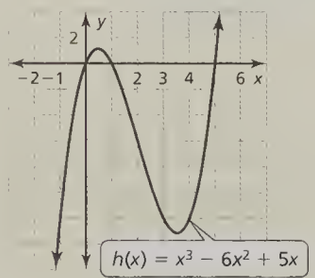
$f(x) = x^3 - 10x^2 + 24x$

MONITORING PROGRESS ANSWERS

12.



13.



14. $f(x) = x^3 + 3x^2 - x - 3$

Graphing and Writing Cubic Functions

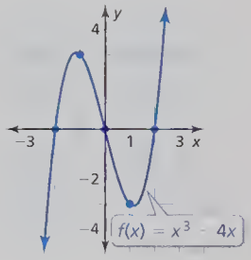
In Example 4, you extended the concept of intercept form to cubic functions.

$f(x) = a(x - p)(x - q)(x - r), a \neq 0$ Intercept form of a cubic function

The x-intercepts of the graph of f are p, q, and r.

EXAMPLE 8 Graphing a Cubic Function Using Zeros

Use zeros to graph $f(x) = x^3 - 4x$.



SOLUTION

Notice that the polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

$f(x) = x^3 - 4x$ Write the function.

$= x(x^2 - 4)$ Factor out x.

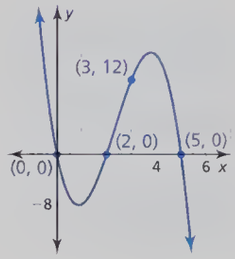
$= x(x + 2)(x - 2)$ Difference of two squares pattern

The zeros of the function are -2, 0, and 2. So, plot $(-2, 0)$, $(0, 0)$, and $(2, 0)$.

To help determine the shape of the graph, find points between the zeros. Plot $(-1, 3)$ and $(1, -3)$. Draw a smooth curve through the points.

x	-1	1
$f(x)$	3	-3

EXAMPLE 9 Writing a Cubic Function



The graph represents a cubic function. Write the function.

SOLUTION

From the graph, you can see that the x-intercepts are 0, 2, and 5. Use intercept form to write a function.

$f(x) = a(x - p)(x - q)(x - r)$ Intercept form

$= a(x - 0)(x - 2)(x - 5)$ Substitute for p, q, and r.

$= a(x)(x - 2)(x - 5)$ Simplify.

Use the other given point, (3, 12), to find the value of a.

$12 = a(3)(3 - 2)(3 - 5)$ Substitute 3 for x and 12 for $f(x)$.

$-2 = a$ Solve for a.

Use the value of a to write the function.

$f(x) = -2(x)(x - 2)(x - 5)$ Substitute -2 for a.

$= -2x^3 + 14x^2 - 20x$ Simplify.

▶ The function represented by the graph is $f(x) = -2x^3 + 14x^2 - 20x$.

Monitoring Progress

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Use zeros to graph the function.

12. $g(x) = (x - 1)(x - 3)(x + 3)$

13. $h(x) = x^3 - 6x^2 + 5x$

14. The zeros of a cubic function are -3, -1, and 1. The graph of the function passes through the point $(0, -3)$. Write the function.

Closure

- **Exit Ticket:** Use zeros to sketch the graphs of $f(x) = -(x + 2)(x - 3)$ and $g(x) = x^2 + 7x + 10$. Check students' work. The graph of $f(x)$ passes through points $(-2, 0)$, $(0, 6)$, and $(3, 0)$. The graph of $g(x)$ passes through the points $(-5, 0)$, $(-2, 0)$, and $(0, 10)$.

8.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

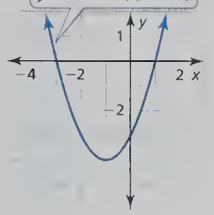
1. **COMPLETE THE SENTENCE** The values p and q are _____ of the graph of the function $f(x) = a(x - p)(x - q)$.

2. **WRITING** Explain how to find the maximum value or minimum value of a quadratic function when the function is given in intercept form.

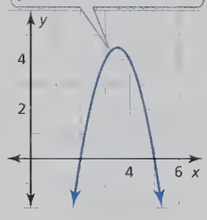
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the x-intercepts and axis of symmetry of the graph of the function.

3. $y = (x - 1)(x + 3)$



4. $y = -2(x - 2)(x - 5)$



5. $f(x) = -5(x + 7)(x - 5)$

6. $g(x) = \frac{2}{3}x(x + 8)$

In Exercises 7–12, graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function. *(See Example 1.)*

7. $f(x) = (x + 4)(x + 1)$

8. $y = (x - 2)(x + 2)$

9. $y = -(x + 6)(x - 4)$

10. $h(x) = -4(x - 7)(x - 3)$

11. $g(x) = 5(x + 1)(x + 2)$

12. $y = -2(x - 3)(x + 4)$

In Exercises 13–20, graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function. *(See Example 2.)*

13. $y = x^2 - 9$

14. $f(x) = x^2 - 8x$

15. $h(x) = -5x^2 + 5x$

16. $y = 3x^2 - 48$

17. $q(x) = x^2 + 9x + 14$

18. $p(x) = x^2 + 6x - 27$

19. $y = 4x^2 - 36x + 32$

20. $y = -2x^2 - 4x + 30$

In Exercises 21–30, find the zero(s) of the function. *(See Examples 3 and 4.)*

21. $y = -2(x - 2)(x - 10)$

22. $f(x) = \frac{1}{3}(x + 5)(x - 1)$

23. $g(x) = x^2 + 5x - 24$

24. $y = x^2 - 17x + 52$

25. $y = 3x^2 - 15x - 42$

26. $g(x) = -4x^2 - 8x - 4$

27. $f(x) = (x + 5)(x^2 - 4)$

28. $h(x) = (x^2 - 36)(x - 11)$

29. $y = x^3 - 49x$

30. $y = x^3 - x^2 - 9x + 9$

In Exercises 31–36, match the function with its graph.

31. $y = (x + 5)(x + 3)$

32. $y = (x + 5)(x - 3)$

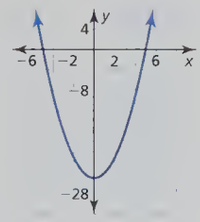
33. $y = (x - 5)(x + 3)$

34. $y = (x - 5)(x - 3)$

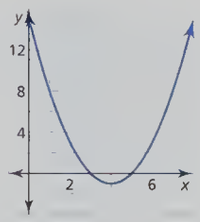
35. $y = (x + 5)(x - 5)$

36. $y = (x + 3)(x - 3)$

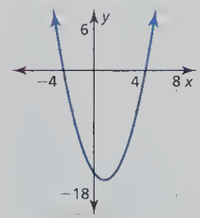
A.



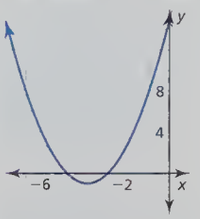
B.



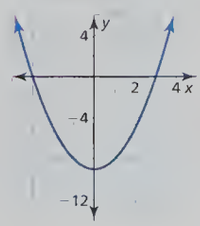
C.



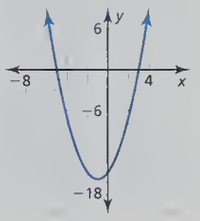
D.



E.



F.



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–71 odd, 85, 92, 96, 102–108

Average: 1, 2–90 even, 92, 96, 102–108

Advanced: 1, 2, 10–44 even, 52–60 even, 66–90 even, 91–108

HOMEWORK CHECK

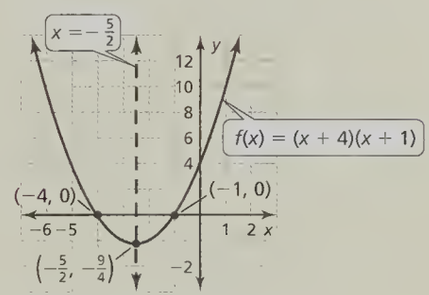
Basic: 7, 21, 45, 57, 61

Average: 8, 24, 46, 58, 62

Advanced: 10, 24, 52, 58, 66

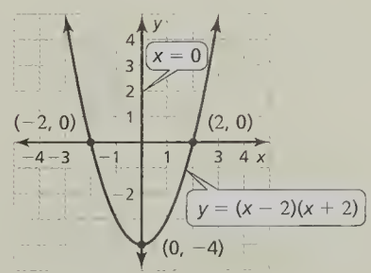
ANSWERS

1. x-intercepts
2. *Sample answer:* Find the x-coordinate of the vertex using $x = \frac{p + q}{2}$, then evaluate the function to determine the maximum value or minimum value.
3. $-3, 1; x = -1$
4. $2, 5; x = \frac{7}{2}$
5. $-7, 5; x = -1$
6. $-8, 0; x = -4$
7.



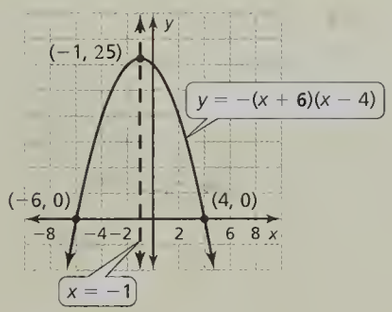
domain: all real numbers, range: $y \geq -\frac{9}{4}$

8.



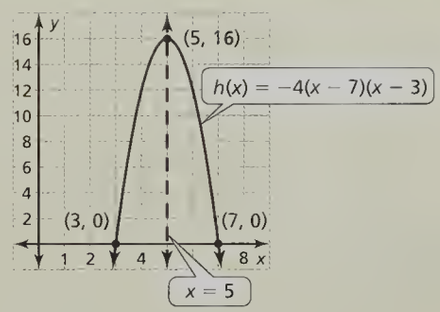
domain: all real numbers, range: $y \geq -4$

9.



domain: all real numbers, range: $y \leq 25$

10.



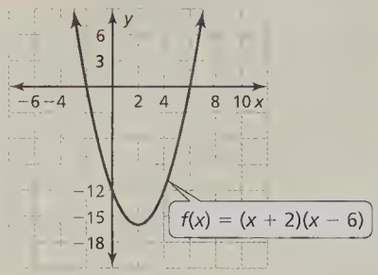
domain: all real numbers, range: $y \leq 16$

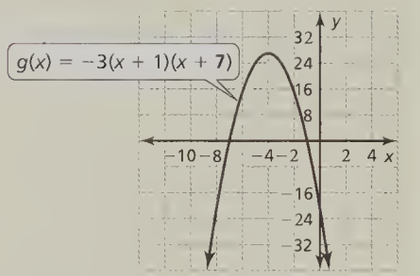
11–36. See Additional Answers.

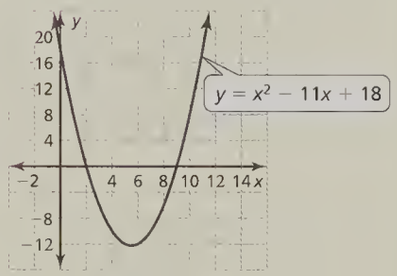
Dynamic Teaching Tools

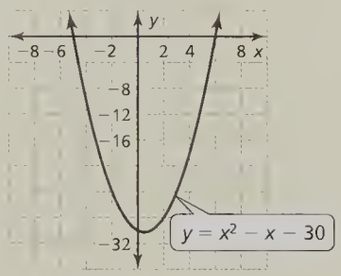
- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

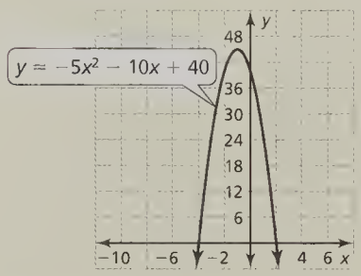
ANSWERS

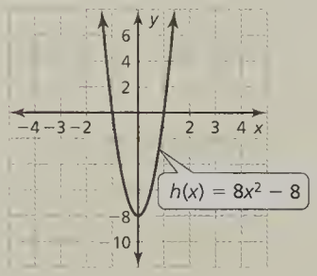
37. 

38. 

39. 

40. 

41. 

42. 

In Exercises 37–42, use zeros to graph the function. *(See Example 5.)*

37. $f(x) = (x + 2)(x - 6)$ **38.** $g(x) = -3(x + 1)(x + 7)$

39. $y = x^2 - 11x + 18$ **40.** $y = x^2 - x - 30$

41. $y = -5x^2 - 10x + 40$ **42.** $h(x) = 8x^2 - 8$

ERROR ANALYSIS **In Exercises 43 and 44, describe and correct the error in finding the zeros of the function.**

43.

✗ $y = 5(x + 3)(x - 2)$

The zeros of the function are 3 and −2.

44.

✗ $y = (x + 4)(x^2 - 9)$

The zeros of the function are −4 and 9.

In Exercises 45–56, write a quadratic function in standard form whose graph satisfies the given condition(s). *(See Example 6.)*

45. vertex: (7, −3) **46.** vertex: (4, 8)

47. x-intercepts: 1 and 9 **48.** x-intercepts: −2 and −5

49. passes through (−4, 0), (3, 0), and (2, −18)

50. passes through (−5, 0), (−1, 0), and (−4, 3)

51. passes through (7, 0)

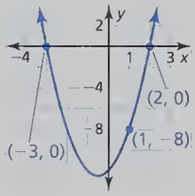
52. passes through (0, 0) and (6, 0)

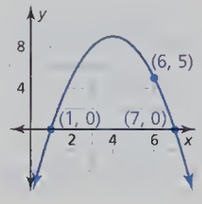
53. axis of symmetry: $x = -5$

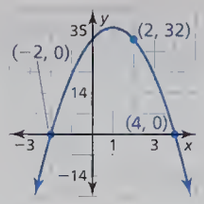
54. y increases as x increases when $x < 4$; y decreases as x increases when $x > 4$.

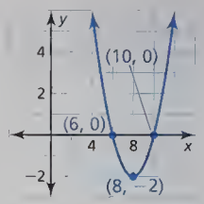
55. range: $y \geq -3$ **56.** range: $y \leq 10$

In Exercises 57–60, write the quadratic function represented by the graph. *(See Example 7.)*

57. 

58. 

59. 

60. 

In Exercises 61–68, use zeros to graph the function. *(See Example 8.)*

61. $y = 5x(x + 2)(x - 6)$ **62.** $f(x) = -x(x + 9)(x + 3)$

63. $h(x) = (x - 2)(x + 2)(x + 7)$

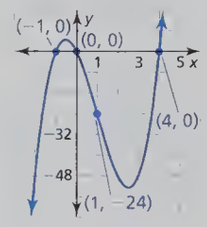
64. $y = (x + 1)(x - 5)(x - 4)$

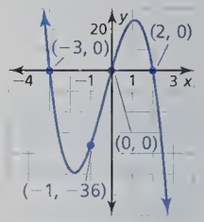
65. $f(x) = 3x^3 - 48x$ **66.** $y = -2x^3 + 20x^2 - 50x$

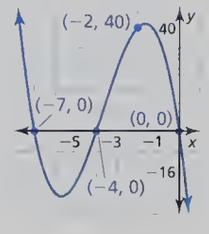
67. $y = -x^3 - 16x^2 - 28x$

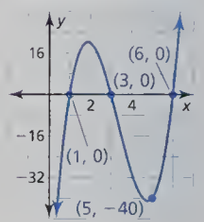
68. $g(x) = 6x^3 + 30x^2 - 36x$

In Exercises 69–72, write the cubic function represented by the graph. *(See Example 9.)*

69. 

70. 

71. 

72. 

In Exercises 73–76, write a cubic function whose graph satisfies the given condition(s).

73. x-intercepts: −2, 3, and 8

74. x-intercepts: −7, −5, and 0

75. passes through (1, 0) and (7, 0)

76. passes through (0, 6)

43. The factors need to be set equal to 0 and solved to find the zeros; $x + 3 = 0$ or $x - 2 = 0$; $x = -3$ or $x = 2$; The zeros of the function are −3 and 2.

44. $x^2 - 9$ needs to be factored; $y = (x + 4)(x + 3)(x - 3)$; The zeros of the function are −4, −3, and 3.

45. *Sample answer:* $f(x) = x^2 - 14x + 46$

46. *Sample answer:* $f(x) = x^2 - 8x + 24$

47. *Sample answer:* $f(x) = x^2 - 10x + 9$

48. *Sample answer:* $f(x) = x^2 + 7x + 10$

49. $f(x) = 3x^2 + 3x - 36$

50. $f(x) = -x^2 - 6x - 5$

51. *Sample answer:* $f(x) = x^2 - 7x$

52. *Sample answer:* $f(x) = x^2 - 6x$

53. *Sample answer:* $f(x) = x^2 + 10x + 25$

54. *Sample answer:* $f(x) = -x^2 + 8x - 16$

55. *Sample answer:* $f(x) = x^2 - 3$

56. *Sample answer:* $f(x) = -x^2 + 10$

57. $f(x) = 2x^2 + 2x - 12$

58. $f(x) = -x^2 + 8x - 7$

59. $f(x) = -4x^2 + 8x + 32$

60. $f(x) = \frac{1}{2}x^2 - 8x + 30$

61–76. See Additional Answers.

In Exercises 77–80, all the zeros of a function are given. Use the zeros and the other point given to write a quadratic or cubic function represented by the table.

77.

x	y
0	0
2	30
7	0

78.

x	y
−3	0
1	−72
4	0

79.

x	y
−4	0
−3	0
0	−180
3	0

80.

x	y
−8	0
−6	−36
−3	0
0	0

In Exercises 81–84, sketch a parabola that satisfies the given conditions.

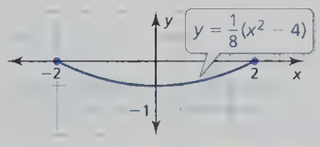
81. x-intercepts: -4 and 2; range: $y \geq -3$

82. axis of symmetry: $x = 6$; passes through (4, 15)

83. range: $y \leq 5$; passes through (0, 2)

84. x-intercept: 6; y-intercept: 1; range: $y \geq -4$

85. MODELING WITH MATHEMATICS Satellite dishes are shaped like parabolas to optimally receive signals. The cross section of a satellite dish can be modeled by the function shown, where x and y are measured in feet. The x-axis represents the top of the opening of the dish.



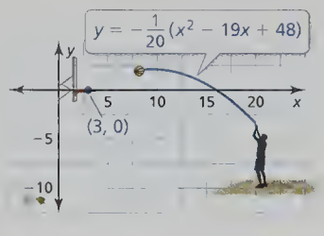
a. How wide is the satellite dish?

b. How deep is the satellite dish?

c. Write a quadratic function in standard form that models the cross section of a satellite dish that is 6 feet wide and 1.5 feet deep.



86. MODELING WITH MATHEMATICS A professional basketball player's shot is modeled by the function shown, where x and y are measured in feet.



a. Does the player make the shot? Explain.

b. The basketball player releases another shot from the point (13, 0) and makes the shot. The shot also passes through the point (10, 1.4). Write a quadratic function in standard form that models the path of the shot.

USING STRUCTURE In Exercises 87–90, match the function with its graph.

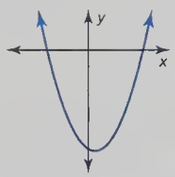
87. $y = -x^2 + 5x$

88. $y = x^2 - x - 12$

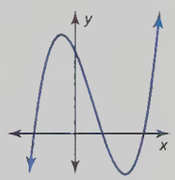
89. $y = x^3 - 2x^2 - 8x$

90. $y = x^3 - 4x^2 - 11x + 30$

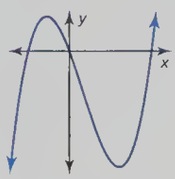
A.



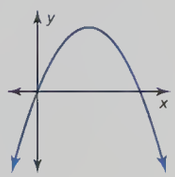
B.



C.



D.



91. CRITICAL THINKING Write a quadratic function represented by the table, if possible. If not, explain why.

x	−5	−3	−1	1
y	0	12	4	0

ANSWERS

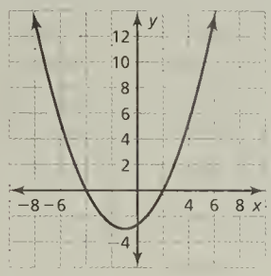
77. $f(x) = -3x^2 + 21x$

78. $f(x) = 6x^2 - 6x - 72$

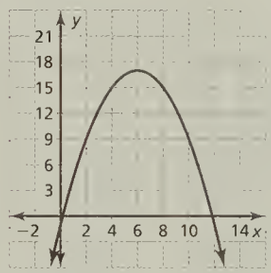
79. $f(x) = 5x^3 + 20x^2 - 45x - 180$

80. $f(x) = -x^3 - 11x^2 - 24x$

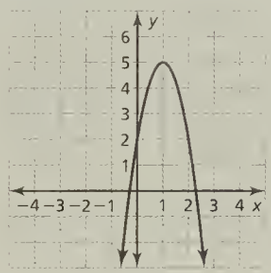
81.



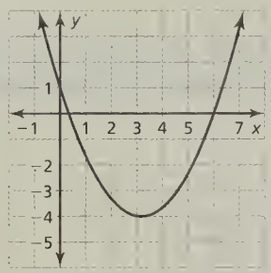
82. *Sample answer:*



83. *Sample answer:*



84. *Sample answer:*



85. a. 4 ft

b. $\frac{1}{2}$ ft

c. $y = \frac{1}{6}(x^2 - 9)$

86. a. yes; *Sample answer:* Using the function, when the horizontal distance of the ball is 3 feet, the vertical distance is 0, which is the location of the basket.

b. $y = -\frac{1}{15}x^2 + \frac{16}{15}x - \frac{39}{15}$

87. D

88. A

89. C

90. B

91. not possible; *Sample answer:* Because -5 and 1 are the x-intercepts, the axis of symmetry is $x = -2$. The points $(-3, 12)$ and $(-1, 4)$ are the same horizontal distance from the axis of symmetry, so for both of them to lie on the parabola they would have to have the same y-coordinate.

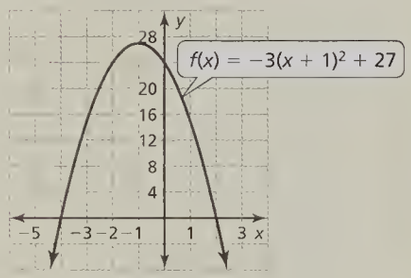
ANSWERS

92. **a.** *Sample answer:* $p = 75, q = 425$

b. *Sample answer:* 350 feet wide, 61 feet high; Use the height as the value of $f(x)$ from part (a) along with the values of p and q estimated in part (a). Use the values of p and q to calculate the value of the x-coordinate of the vertex to substitute for x in the function in part (a), and solve the function for a.

93. **a.** $f(x) = -3(x + 4)(x - 2)$

b.

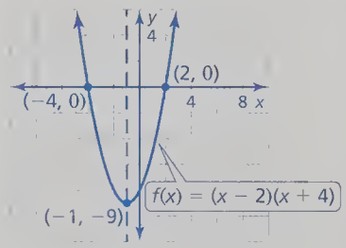


Sample answer: Plot the vertex $(-1, 27)$, which can be determined from the vertex form. Then plot the x-intercepts $(-4, 0)$ and $(2, 0)$, which can be determined from the intercept form. Draw a smooth curve through these points.

94–108. See Additional Answers.

Mini-Assessment

1. Graph $f(x) = (x - 2)(x + 4)$. Describe the domain and range.



domain: all real numbers; range: $y \geq -9$

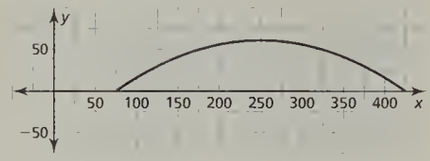
2. Find the zeros of the function. $f(x) = 2x^2 + 2x - 60$

-6 and 5

3. Write a quadratic function in standard form whose graph has x-intercepts -4 and -3.

Sample answer: $f(x) = x^2 + 7x + 12$

92. HOW DO YOU SEE IT? The graph shows the parabolic arch that supports the roof of a convention center, where x and y are measured in feet.



a. The arch can be represented by a function of the form $f(x) = a(x - p)(x - q)$. Estimate the values of p and q.

b. Estimate the width and height of the arch. Explain how you can use your height estimate to calculate a.

ANALYZING EQUATIONS **In Exercises 93 and 94, (a) rewrite the quadratic function in intercept form and (b) graph the function using any method. Explain the method you used.**

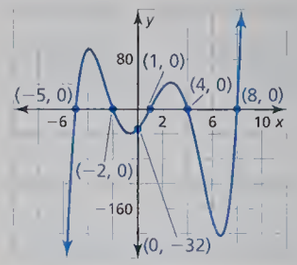
93. $f(x) = -3(x + 1)^2 + 27$

94. $g(x) = 2(x - 1)^2 - 2$

95. WRITING Can a quadratic function with exactly one real zero be written in intercept form? Explain.

96. MAKING AN ARGUMENT Your friend claims that any quadratic function can be written in standard form and in vertex form. Is your friend correct? Explain.

97. PROBLEM SOLVING Write the function represented by the graph in intercept form.



98. THOUGHT PROVOKING Sketch the graph of each function. Explain your procedure.

a. $f(x) = (x^2 - 1)(x^2 - 4)$

b. $g(x) = x(x^2 - 1)(x^2 - 4)$

99. REASONING Let k be a constant. Find the zeros of the function $f(x) = kx^2 - k^2x - 2k^3$ in terms of k.

PROBLEM SOLVING **In Exercises 100 and 101, write a system of two quadratic equations whose graphs intersect at the given points. Explain your reasoning.**

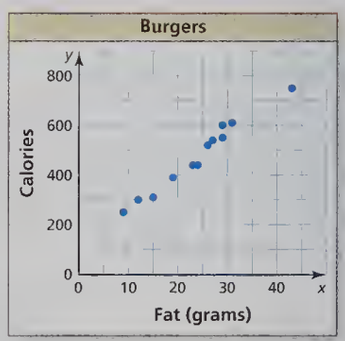
100. $(-4, 0)$ and $(2, 0)$

101. $(3, 6)$ and $(7, 6)$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

The scatter plot shows the amounts x (in grams) of fat and the numbers y of calories in 12 burgers at a fast-food restaurant. *(Section 4.5)*



102. How many calories are in the burger that contains 12 grams of fat?

103. How many grams of fat are in the burger that contains 600 calories?

104. What tends to happen to the number of calories as the number of grams of fat increases?

Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning. *(Section 6.5)*

105. 3, 11, 21, 33, 47, . . .

106. $-2, -6, -18, -54, \ldots$

107. 26, 18, 10, 2, $-6, \ldots$

108. 4, 5, 9, 14, 23, . . .

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

8.6 Comparing Linear, Exponential, and Quadratic Functions

Texas Essential Knowledge and Skills
A.2.C
A.6.C
A.9.C

Essential Question How can you compare the growth rates of linear, exponential, and quadratic functions?

EXPLORATION 1 Comparing Speeds

Work with a partner. Three cars start traveling at the same time. The distance traveled in t minutes is y miles. Complete each table and sketch all three graphs in the same coordinate plane. Compare the speeds of the three cars. Which car has a constant speed? Which car is accelerating the most? Explain your reasoning.

t	$y = t$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

t	$y = 2^t - 1$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

t	$y = t^2$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

APPLYING MATHEMATICS
To be proficient in math, you need to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

EXPLORATION 2 Comparing Speeds

Work with a partner. Analyze the speeds of the three cars over the given time periods. The distance traveled in t minutes is y miles. Which car eventually overtakes the others?

t	$y = t$
1	
2	
3	
4	
5	
6	
7	
8	
9	

t	$y = 2^t - 1$
1	
2	
3	
4	
5	
6	
7	
8	
9	

t	$y = t^2$
1	
2	
3	
4	
5	
6	
7	
8	
9	

Communicate Your Answer

3. How can you compare the growth rates of linear, exponential, and quadratic functions?
4. Which function has a growth rate that is eventually much greater than the growth rates of the other two functions? Explain your reasoning.

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

3. *Sample answer:* Graph functions of each type in the same coordinate plane and compare the shapes of the graphs.
4. exponential; *Sample answer:* An exponential graph begins with a small change in each interval, but the change increases more rapidly as the value of the exponent increases.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

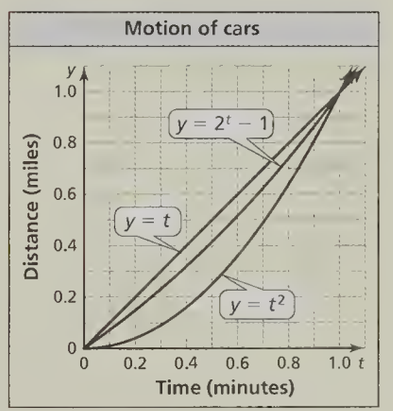
A.2.C The student is expected to write linear equations in two variables given a table of values, a graph, and a verbal description.

A.6.C The student is expected to write quadratic functions when given real solutions and graphs of their related equations.

A.9.C The student is expected to write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

ANSWERS

1. 0, 0.2, 0.4, 0.6, 0.8, 1.0; 0, 0.15, 0.32, 0.52, 0.74, 1.00; 0, 0.04, 0.16, 0.36, 0.64, 1.00



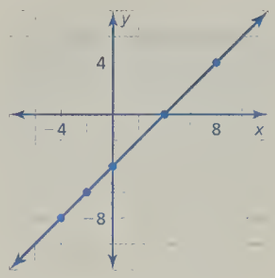
Sample answer: All three cars have the same average speed over the interval; first car; third car; The car modeled by a linear function is traveling at the same speed because it is traveling the same distance in each time interval. The car modeled by the quadratic function is accelerating the most because it has the greatest change in speed over each time interval.

2. 1, 2, 3, 4, 5, 6, 7, 8, 9; 1, 3, 7, 15, 31, 63, 127, 255, 511; 1, 4, 9, 16, 25, 36, 49, 64, 81; The first car has a constant speed. The second and third car are accelerating; The car modeled by the exponential function eventually overtakes the others.

Extra Example 1

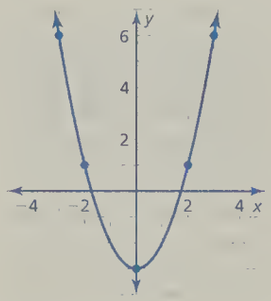
Plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

a. (8, 4), (4, 0), (0, −4), (−2, −6), (−4, −8)



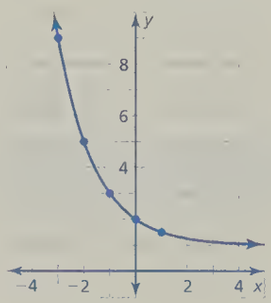
linear function

b. (−3, 6), (−2, 1), (0, −3), (2, 1), (3, 6)



quadratic function

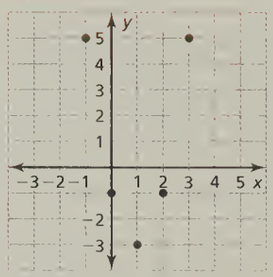
c. (−3, 9), (−2, 5), (−1, 3), (0, 2), (1, 1.5)



exponential function

MONITORING PROGRESS ANSWERS

1.



quadratic

8.6 Lesson

What You Will Learn

- Choose functions to model data.
- Write functions to model data.
- Compare functions using average rates of change.
- Solve real-life problems involving different function types.

Core Vocabulary

average rate of change, *p. 448*

Previous

zero of a function

slope

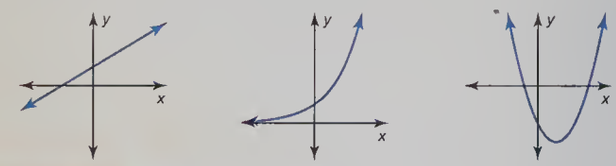
Choosing Functions to Model Data

So far, you have studied linear functions, exponential functions, and quadratic functions. You can use these functions to model data.

Core Concept

Linear, Exponential, and Quadratic Functions

Linear Function	Exponential Function	Quadratic Function
$y = mx + b$	$y = ab^x$	$y = ax^2 + bx + c$



EXAMPLE 1 **Using Graphs to Identify Functions**

Plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

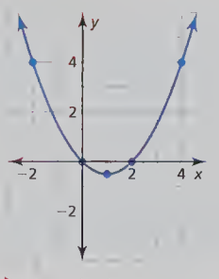
a. (4, 4), (2, 0), (0, 0), $\left(1, -\frac{1}{2}\right)$, (−2, 4)

b. (0, 1), (2, 4), (4, 7), (−2, −2), (−4, −5)

c. (0, 2), (2, 8), (1, 4), (−1, 1), $\left(-2, \frac{1}{2}\right)$

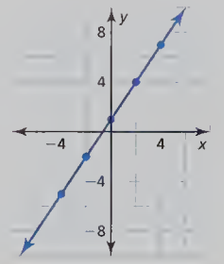
SOLUTION

a.



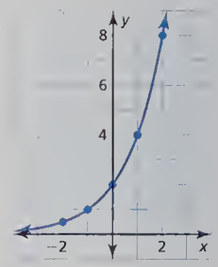
▶ quadratic

b.



▶ linear

c.



▶ exponential

Monitoring Progress

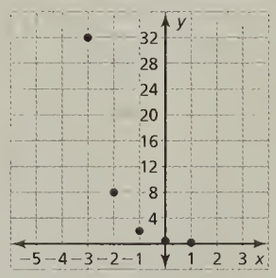
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Plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

1. (−1, 5), (2, −1), (0, −1), (3, 5), (1, −3)
2. (−1, 2), (−2, 8), (−3, 32), $\left(0, \frac{1}{2}\right)$, $\left(1, \frac{1}{8}\right)$
3. (−3, 5), (0, −1), (2, −5), (−4, 7), (1, −3)

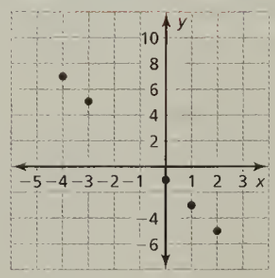
For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

2.



exponential

3.



linear

Core Concept

Differences and Ratios of Functions

You can use patterns between consecutive data pairs to determine which type of function models the data. The differences of consecutive y-values are called *first differences*. The differences of consecutive first differences are called *second differences*.

- **Linear Function** The first differences are constant.
- **Exponential Function** Consecutive y-values have a common *ratio*.
- **Quadratic Function** The second differences are constant.

In all cases, the differences of consecutive x-values need to be constant.

STUDY TIP

The first differences for exponential and quadratic functions are *not* constant.

EXAMPLE 2 Using Differences or Ratios to Identify Functions

STUDY TIP

First determine that the differences of consecutive x-values are constant. Then check whether the first differences are constant or consecutive y-values have a common ratio. If neither of these is true, check whether the second differences are constant.

Tell whether each table of values represents a *linear*, an *exponential*, or a *quadratic* function.

a.

x	−3	−2	−1	0	1
y	11	8	5	2	−1

b.

x	−2	−1	0	1	2
y	1	2	4	8	16

c.

x	−2	−1	0	1	2
y	−1	−2	−1	2	7

SOLUTION

a.

x	−3	−2	−1	0	1
y	11	8	5	2	−1

(+1, +1, +1, +1 above; −3, −3, −3, −3 below)

▶ The first differences are constant. So, the table represents a linear function.

b.

x	−2	−1	0	1	2
y	1	2	4	8	16

(+1, +1, +1, +1 above; ×2, ×2, ×2, ×2 below)

▶ Consecutive y-values have a common ratio. So, the table represents an exponential function.

c.

x	−2	−1	0	1	2
y	−1	−2	−1	2	7

(+1, +1, +1, +1 above)

first differences → −1, +1, +3, +3

second differences → +2, +2, +2

▶ The second differences are constant. So, the table represents a quadratic function.

Monitoring Progress

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4. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

x	−1	0	1	2	3
y	1	3	9	27	81

Extra Example 2

Tell whether each table of values represents a *linear*, an *exponential*, or a *quadratic* function.

a.

x	−2	−1	0	1	2
y	11	5	3	5	11

quadratic function

b.

x	−2	−1	0	1	2
y	6	8	10	12	14

linear function

c.

x	0	1	2	3	4
y	0.5	3	18	108	648

exponential function

MONITORING PROGRESS ANSWER

4. exponential

Differentiated Instruction

Inclusion

Some students may not understand the concept of "second difference." Give these students several tables of values with constant consecutive x-values. Have them find the y-value differences. Then have them find the differences of the y-value differences, or the "second differences."

Extra Example 3

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

x	−2	−1	0	1	2
y	−3	−1	1	3	5

linear function; $y = 2x + 1$

MONITORING PROGRESS ANSWER

5. exponential; $y = 8\left(\frac{1}{2}\right)^x$

Writing Functions to Model Data

EXAMPLE 3 **Writing a Function to Model Data**

x	2	4	6	8	10
y	12	0	−4	0	12

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

SOLUTION

Step 1 Determine which type of function the table of values represents.

The second differences are constant. So, the table represents a quadratic function.

		+2	+2	+2	+2
x	2	4	6	8	10
y	12	0	−4	0	12
first differences		−12	−4	+4	+12
second differences			+8	+8	+8

Step 2 Write an equation of the quadratic function. Using the table, notice that the x-intercepts are 4 and 8. So, use intercept form to write a function.

$y = a(x - 4)(x - 8)$ Substitute for p and q in intercept form.

Use another point from the table, such as (2, 12), to find a.

$12 = a(2 - 4)(2 - 8)$ Substitute 2 for x and 12 for y.

$1 = a$ Solve for a.

Use the value of a to write the function.

$y = (x - 4)(x - 8)$ Substitute 1 for a.

$= x^2 - 12x + 32$ Use the FOIL Method and combine like terms.

So, the quadratic function is $y = x^2 - 12x + 32$.

STUDY TIP

To check your function in Example 3, substitute the other points from the table to verify that they satisfy the function.

Monitoring Progress

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x	−1	0	1	2	3
y	16	8	4	2	1

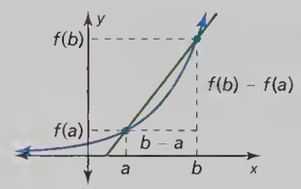
5. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

Comparing Functions Using Average Rates of Change

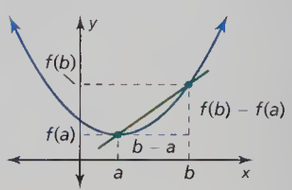
For nonlinear functions, the rate of change is not constant. You can compare two nonlinear functions over the same interval using their *average rates of change*. The **average rate of change** of a function $y = f(x)$ between $x = a$ and $x = b$ is the slope of the line through $(a, f(a))$ and $(b, f(b))$.

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$

Exponential Function



Quadratic Function



Core Concept

Comparing Functions Using Average Rates of Change

- Over the same interval, the average rate of change of a function increasing quadratically eventually exceeds the average rate of change of a function increasing linearly. So, the value of the quadratic function eventually exceeds the value of the linear function.
- Over the same interval, the average rate of change of a function increasing exponentially eventually exceeds the average rate of change of a function increasing linearly or quadratically. So, the value of the exponential function eventually exceeds the value of the linear or quadratic function.

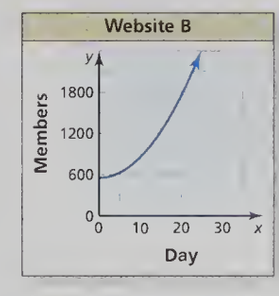
STUDY TIP

You can explore these concepts using a graphing calculator.

EXAMPLE 4 Using and Interpreting Average Rates of Change

Two social media websites open their memberships to the public. (a) Compare the websites by calculating and interpreting the average rates of change from Day 10 to Day 20. (b) Predict which website will have more members after 50 days. Explain.

Website A	
Day, x	Members, y
0	650
5	1025
10	1400
15	1775
20	2150
25	2525



SOLUTION

a. Calculate the average rates of change by using the points whose x-coordinates are 10 and 20.

Website A: Use (10, 1400) and (20, 2150).

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a} = \frac{2150 - 1400}{20 - 10} = \frac{750}{10} = 75$$

Website B: Use the graph to estimate the points when $x = 10$ and $x = 20$. Use (10, 850) and (20, 1800).

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a} \approx \frac{1800 - 850}{20 - 10} = \frac{950}{10} = 95$$

▶ From Day 10 to Day 20, Website A membership increases at an average rate of 75 people per day, and Website B membership increases at an average rate of about 95 people per day. So, Website B membership is growing faster.

b. After 25 days, the memberships of both websites are about equal. Using the table, the average rates of change of Website A are constant, so membership will increase linearly. Using the graph, the average rates of change of Website B are increasing, so membership appears to increase quadratically or exponentially. So, Website B will have more members after 50 days.

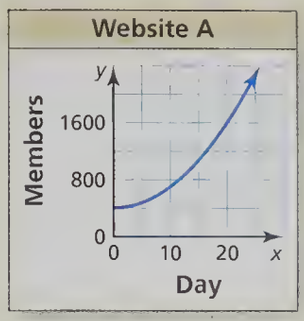
Monitoring Progress

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6. Compare the websites in Example 4 by calculating and interpreting the average rates of change from Day 0 to Day 10.

Extra Example 4

Two news websites open their memberships to the public.



Website B	
Day, x	Members, y
0	500
5	850
10	1200
15	1550
20	1900
25	2250

a. Compare the websites by calculating and interpreting the average rates of change from Day 10 to Day 20. From Day 10 to Day 20, Website A membership increases at an average rate of about 90 people per day, and Website B membership increases at an average rate of 70 people per day. So, Website A membership is growing faster.

b. Predict which website will have more members after 50 days. Explain. After 25 days, the memberships of both websites are about equal. Using the table, the average rates of change of Website B are constant, so membership will increase linearly. Using the graph, the average rates of change of Website A are increasing, so membership appears to increase quadratically or exponentially. So, Website A will have more members after 50 days.

MONITORING PROGRESS ANSWER

6. From Day 0 to Day 10, Website A membership increases at an average rate of 75 people per day and Website B membership increases at an average rate of about 30 people per day. So, Website A membership is growing faster.

Extra Example 5

A mathematics teacher is considering two different websites that provide online practice for students. Website A charges $10 to join, plus $2 for each user. Website B has no fee to join. Website B charges $2 for the first user, plus 10% more for each additional user.

a. For how many users is the cost for Website A greater than the cost for Website B?
The cost for Website A is greater than the cost for Website B when there are fewer than 40 users.

b. Suppose Website A has no fee to join. For how many users are the website costs about equal?
The costs for the websites are about the same for 38 users.

c. Suppose Website A has no fee to join and lowers the charge per user to $1.50. For how many users is the cost for Website A less than the cost for Website B? Explain.
The cost for Website A is less than the cost for Website B when there are more than 34 users. Because Website A's cost increases linearly and Website B's cost increases exponentially, Website A's cost will eventually be less than Website B's cost.

MONITORING PROGRESS ANSWER

7. 1917

Solving Real-Life Problems



EXAMPLE 5 **Comparing Different Function Types**

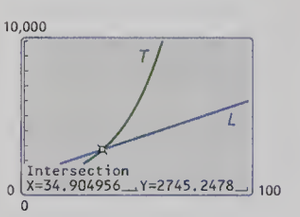
In 1900, Littleton had a population of 1000 people. Littleton's population increased by 50 people each year. In 1900, Tinyville had a population of 500 people. Tinyville's population increased by 5% each year.

a. In what year were the populations about equal?

b. Suppose Littleton's initial population doubled to 2000 and maintained a constant rate of increase of 50 people each year. Did Tinyville's population still catch up to Littleton's population? If so, in which year?

c. Suppose Littleton's rate of increase doubled to 100 people each year, in addition to doubling the initial population. Did Tinyville's population still catch up to Littleton's population? Explain.

SOLUTION



a. Let x represent the number of years since 1900. Write a function to model the population of each town.

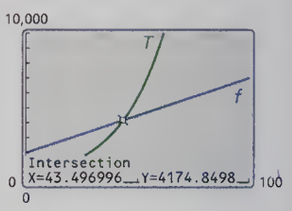
Littleton: $L(x) = 50x + 1000$ Linear function

Tinyville: $T(x) = 500(1.05)^x$ Exponential function

Use a graphing calculator to graph each function in the same viewing window. Use the *intersect* feature to find the value of x for which $L(x) \approx T(x)$. The graphs intersect when $x \approx 34.9$.

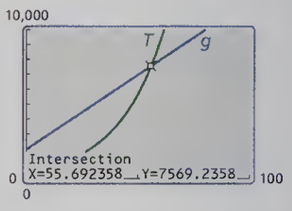
▶ So, the populations were about equal in 1934.

b. Littleton's new population function is $f(x) = 50x + 2000$. Use a graphing calculator to graph f and T in the same viewing window. Use the *intersect* feature to find the value of x for which $f(x) \approx T(x)$. The graphs intersect when $x \approx 43.5$.



▶ So, Tinyville's population caught Littleton's population in 1943.

c. Littleton's new population function is $g(x) = 100x + 2000$. Use a graphing calculator to graph g and T in the same viewing window. Use the *intersect* feature to find the value of x for which $g(x) \approx T(x)$. The graphs intersect when $x \approx 55.7$.



▶ So, Tinyville's population caught Littleton's population in 1955. Because Littleton's population increased linearly and Tinyville's population increased exponentially, Tinyville's population eventually exceeded Littleton's regardless of Littleton's constant rate or initial value.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. **WHAT IF?** Tinyville's population increased by 8% each year. In what year were the populations about equal?

Closure

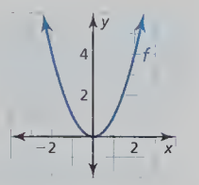
- Write a linear, an exponential, and a quadratic function that each has a y-intercept of 2.
Sample answer: $y = \frac{1}{2}x + 2$, $y = 2(3)^x$, $y = x^2 - 4x + 2$

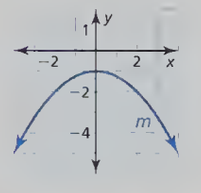
8.6 Exercises

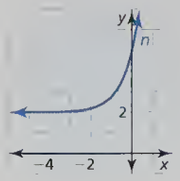
Dynamic Solutions available at *BigIdeasMath.com*

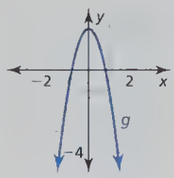
Vocabulary and Core Concept Check

1. **WRITING** Name three types of functions that you can use to model data. Describe the equation and graph of each function.
2. **WRITING** How can you decide whether to use a linear, an exponential, or a quadratic function to model a data set?
3. **VOCABULARY** Describe how to find the average rate of change of a function $y = f(x)$ between $x = a$ and $x = b$.
4. **WHICH ONE DOESN'T BELONG?** Which graph does *not* belong with the other three? Explain your reasoning.



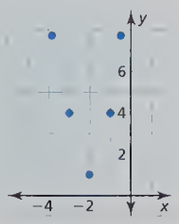


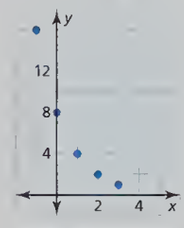


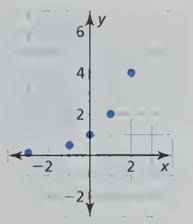


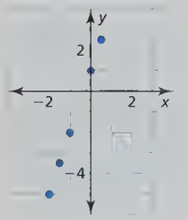
Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

5. 

6. 

7. 

8. 

In Exercises 9–14, plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function. *(See Example 1.)*

9. $(-2, -1), (-1, 0), (1, 2), (2, 3), (0, 1)$
10. $\left(0, \frac{1}{4}\right), (1, 1), (2, 4), (3, 16), \left(-1, \frac{1}{16}\right)$
11. $(0, -3), (1, 0), (2, 9), (-2, 9), (-1, 0)$
12. $(-1, -3), (-3, 5), (0, -1), (1, 5), (2, 15)$
13. $(-4, -4), (-2, -3.4), (0, -3), (2, -2.6), (4, -2)$
14. $(0, 8), (-4, 0.25), (-3, 0.4), (-2, 1), (-1, 3)$

In Exercises 15–18, tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. *(See Example 2.)*

15.

x	−2	−1	0	1	2
y	0	0.5	1	1.5	2

16.

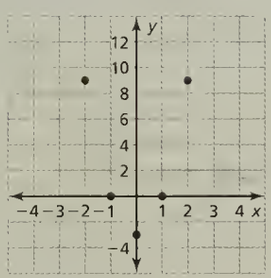
x	−1	0	1	2	3
y	0.2	1	5	25	125

17.

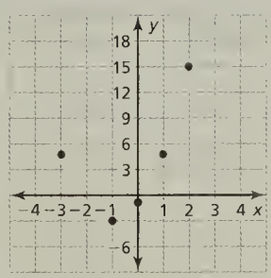
x	2	3	4	5	6
y	2	6	18	54	162

18.

x	−3	−2	−1	0	1
y	2	4.5	8	12.5	18

11. 

quadratic

12. 

quadratic

13–18. See Additional Answers.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–27 odd, 32–36 even, 41, 43–50

Average: 1–3, 4–36 even, 41, 43–50

Advanced: 1–4, 12–36 even, 37–50

HOMEWORK CHECK

Basic: 9, 15, 21, 32, 34

Average: 12, 16, 22, 32, 34

Advanced: 14, 18, 24, 32, 38

ANSWERS

1. linear, exponential, quadratic; $y = mx + b$, $y = ab^x$, $y = ax^2 + bx + c$; a straight line, a continuously increasing or decreasing curve, a parabola
2. *Sample answer:* Plot the points from the data set and determine what shape they appear to take.
3. Find the slope of the line through $(a, f(a))$ and $(b, f(b))$.
4. n; n is an exponential function, but the rest are quadratic functions.
5. quadratic
6. exponential
7. exponential
8. linear
9. 

linear
10. 

exponential

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

19. linear; The first differences are constant.

20. quadratic; The second differences are constant.

21. quadratic; $y = 2x^2 - 2x - 4$

22. exponential; $y = \left(\frac{1}{2}\right)^x$

23. linear; $y = -3x - 2$

24. exponential; $y = 5(2)^x$

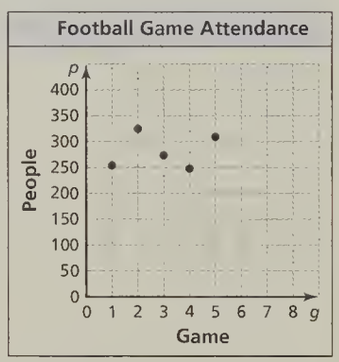
25. quadratic; $y = -3x^2$

26. linear; $y = -2x - 2$

27. Consecutive y-values have a constant ratio. They do not change by a constant amount; Consecutive y-values change by a constant ratio. So, the table represents an exponential function.

28. The factor for the first x-intercept should be $(x + 2)$, not $(x - 2)$;
$f(x) = a(x + 2)(x - 1)$;
$4 = a(-3 + 2)(-3 - 1)$; $1 = a$;
$f(x) = 1(x + 2)(x - 1)$;
$= x^2 + x - 2$;
So, the function is $f(x) = x^2 + x - 2$.

29. a.

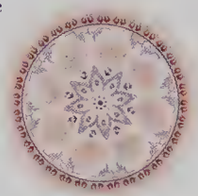


b. no; The points do not appear to follow the shape of any of these types of functions.

19. MODELING WITH MATHEMATICS A student takes a subway to a public library. The table shows the distances d (in miles) the student travels in t minutes. Let the time t represent the independent variable. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function. Explain.

Time, t	0.5	1	3	5
Distance, d	0.335	0.67	2.01	3.35

20. MODELING WITH MATHEMATICS A store sells custom circular rugs. The table shows the costs c (in dollars) of rugs that have diameters of d feet. Let the diameter d represent the independent variable. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function. Explain.



Diameter, d	3	4	5	6
Cost, c	63.90	113.60	177.50	255.60

In Exercises 21–26, tell whether the data represent a *linear*, an *exponential*, or a *quadratic* function. Then write the function. *(See Example 3.)*

21. $(-2, 8), (-1, 0), (0, -4), (1, -4), (2, 0), (3, 8)$

22. $(-3, 8), (-2, 4), (-1, 2), (0, 1), (1, 0.5)$

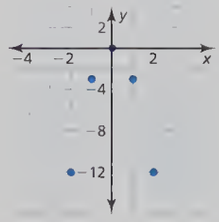
23.

x	-2	-1	0	1	2
y	4	1	-2	-5	-8

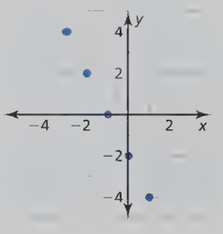
24.

x	-1	0	1	2	3
y	2.5	5	10	20	40

25.



26.



27. ERROR ANALYSIS Describe and correct the error in determining whether the table represents a linear, an exponential, or a quadratic function.

x	1	2	3	4	5
y	3	9	27	81	243

(+1, +1, +1, +1 over the x-row; ×3, ×3, ×3, ×3 over the y-row)

Consecutive y-values change by a constant amount. So, the table represents a linear function.

28. ERROR ANALYSIS Describe and correct the error in writing the function represented by the table.

x	-3	-2	-1	0	1
y	4	0	-2	-2	0

(+1, +1, +1, +1 over the x-row)

first differences → -4 -2 $+0$ $+2$

second differences → $+2$ $+2$ $+2$

The table represents a quadratic function.

$f(x) = a(x - 2)(x - 1)$

$4 = a(-3 - 2)(-3 - 1)$

$\frac{1}{5} = a$

$f(x) = \frac{1}{5}(x - 2)(x - 1)$

$= \frac{1}{5}x^2 - \frac{3}{5}x + \frac{2}{5}$

So, the function is $f(x) = \frac{1}{5}x^2 - \frac{3}{5}x + \frac{2}{5}$.

29. REASONING The table shows the numbers of people attending the first five football games at a high school.

Game, g	1	2	3	4	5
People, p	252	325	270	249	310

a. Plot the points. Let the game g represent the independent variable.

b. Can a linear, an exponential, or a quadratic function represent this situation? Explain.

30. MODELING WITH MATHEMATICS The table shows the breathing rates y (in liters of air per minute) of a cyclist traveling at different speeds x (in miles per hour).

Speed, x	20	21	22	23	24
Breathing rate, y	51.4	57.1	63.3	70.3	78.0



a. Plot the points. Let the speed x represent the independent variable. Then determine the type of function that best represents this situation.

b. Write a function that models the data.

c. Find the breathing rate of a cyclist traveling 18 miles per hour. Round your answer to the nearest tenth.

31. ANALYZING RATES OF CHANGE The function $f(t) = -16t^2 + 48t + 3$ represents the height (in feet) of a volleyball t seconds after it is hit into the air.

a. Copy and complete the table.

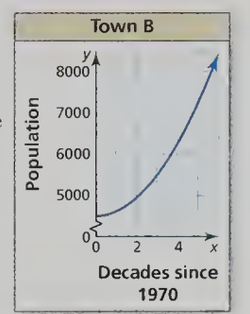
t	0	0.5	1	1.5	2	2.5	3
$f(t)$							

b. Plot the ordered pairs and draw a smooth curve through the points.

c. Describe where the function is increasing and decreasing.

d. Find the average rate of change for each 0.5-second interval in the table. What do you notice about the average rates of change when the function is increasing? decreasing?

32. ANALYZING RELATIONSHIPS The population of Town A in 1970 was 3000. The population of Town A increased by 20% every decade. Let x represent the number of decades since 1970. The graph shows the population of Town B. *(See Example 4.)*

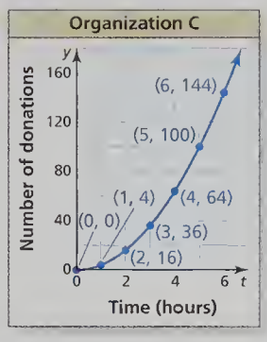


a. Compare the populations of the towns by calculating and interpreting the average rates of change from 1990 to 2010.

b. Predict which town will have a greater population after 2020. Explain.

33. ANALYZING RELATIONSHIPS Three organizations are collecting donations for a cause. Organization A begins with one donation, and the number of donations quadruples each hour. The table shows the numbers of donations collected by Organization B. The graph shows the numbers of donations collected by Organization C.

Time (hours), t	Number of donations, y
0	0
1	4
2	8
3	12
4	16
5	20
6	24

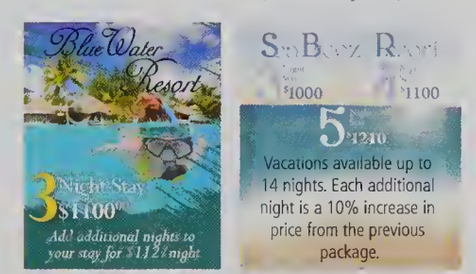


a. What type of function represents the numbers of donations collected by Organization A? B? C?

b. Find the average rates of change of each function for each 1-hour interval from $t = 0$ to $t = 6$.

c. For which function does the average rate of change increase most quickly? What does this tell you about the numbers of donations collected by the three organizations?

34. COMPARING FUNCTIONS The room expenses for two different resorts are shown. *(See Example 5.)*



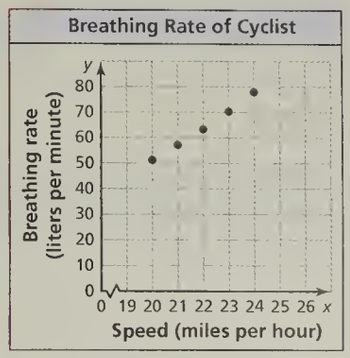
a. For what length of vacation does each resort cost about the same?

b. Suppose Blue Water Resort charges $1450 for the first three nights and $105 for each additional night. Would Sea Breeze Resort ever be more expensive than Blue Water Resort? Explain.

c. Suppose Sea Breeze Resort charges $1200 for the first three nights. The charge increases 10% for each additional night. Would Blue Water Resort ever be more expensive than Sea Breeze Resort? Explain.

ANSWERS

30. a.



exponential function

b. $y = 51.4(1.11)^{x-20}$

c. about 41.7 L/min

31. a. 3, 23, 35, 39, 35, 23, 3

b. See Additional Answers.

c. The function is increasing between 0 and 1.5 seconds, and the function is decreasing between 1.5 seconds and about 3 seconds.

d. 40, 24, 8, −8, −24, −40; The average rate of change decreases when the function is increasing, and the average rate of change increases in the negative direction when the function is decreasing.

32. a. From 1990 to 2010, the population of Town A increased at an average rate of about 950 people per decade and the population of Town B increased at an average rate of 660 people per decade. So, the population of Town A is growing faster.

b. Town A; The exponential growth of Town A will eventually cause it to be greater than the quadratic growth of Town B.

33–34. See Additional Answers.

SUPPORTING English Language Learners

Point out the photo in Exercise 30. Ask students if they have ever ridden a bicycle. Ask them what happens to their breathing when they do something that requires physical effort. Explain that the table represents the cyclist's speed and breathing rate. Have students read Exercise 30.

Beginning Read the headings and table.
Intermediate Read the problem setup.
Advanced Read part (a).
Advanced High Read parts (b) and (c).

ELPS 4.F.5 Use visual and contextual support to develop background knowledge needed to comprehend increasingly challenging language.

ANSWERS

35–50. See Additional Answers.

Mini-Assessment

1. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

x	0	1	2	3	4
y	8	3	0	−1	0

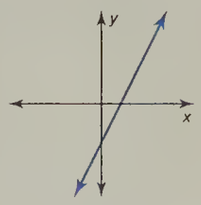
quadratic function; $y = x^2 - 6x + 8$

2. Use the table and graph in Example 4. Compare the social media websites by calculating and interpreting the average rates of change from Day 0 to Day 20. From Day 0 to Day 20, Website A membership increases at an average rate of 75 people per day, and Website B membership increases at an average rate of about 60 people per day. So, Website A membership is growing faster.
3. In 1900, Smalltown had a population of 2000 people. Smalltown's population increased by 100 people each year. In 1900, Growton had a population of 1000 people. Growton's population increased by 6% each year.
 a. In what year were the populations about equal? 1926
 b. Suppose Smalltown's initial population doubled to 4000 and maintained a constant rate of increase of 100 people each year. Did Growton's population still catch up to Smalltown's population? If so, in which year? Yes; 1934
 c. Suppose Smalltown's rate of increase doubled to 200 people each year, in addition to doubling the initial population. Did Growton's population still catch up to Smalltown's population? Explain. Yes, in 1943; Smalltown's population increased linearly and Growton's population increased exponentially.

35. REASONING Explain why the average rate of change of a linear function is constant and the average rate of change of a quadratic or exponential function is not constant.

36. HOW DO YOU SEE IT? Match each graph with its function. Explain your reasoning.

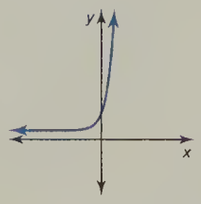
a.



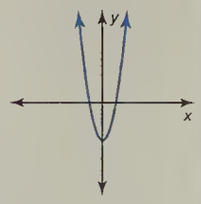
b.



c.



d.



A. $y = 2x^2 - 4$

B. $y = 2(4)^x + 1$

C. $y = 2\left(\frac{3}{4}\right)^x + 1$

D. $y = 2x - 4$

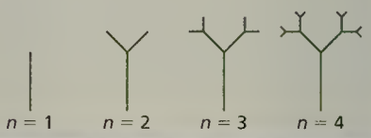
37. CRITICAL THINKING In the ordered pairs below, the y-values are given in terms of n. Tell whether the ordered pairs represent a *linear*, an *exponential*, or a *quadratic* function. Explain.

$(1, 3n - 1), (2, 10n + 2), (3, 26n),$
$(4, 51n - 7), (5, 85n - 19)$

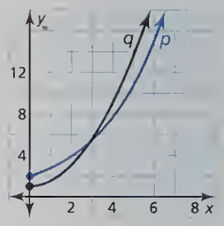
38. USING STRUCTURE Write a function that has constant second differences of 3.

39. CRITICAL THINKING Is the graph of a set of points enough to determine whether the points represent a linear, an exponential, or a quadratic function? Justify your answer.

40. THOUGHT PROVOKING Find four different patterns in the figure. Determine whether each pattern represents a *linear*, an *exponential*, or a *quadratic* function. Write a model for each pattern.



41. MAKING AN ARGUMENT Function p is an exponential function and function q is a quadratic function. Your friend says that after about $x = 3$, function q will always have a greater y-value than function p. Is your friend correct? Explain.



42. USING TOOLS The table shows the amount a (in billions of dollars) United States residents spent on pets or pet-related products and services each year for a 5-year period. Let the year x represent the independent variable. Using technology, find a function that models the data. How did you choose the model? Predict how much residents will spend on pets or pet-related products and services in Year 7.

Year, x	1	2	3	4	5
Amount, a	53.1	56.9	61.8	65.7	67.1

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression. *(Section 6.2)*

43. $\sqrt{121}$

44. $\sqrt[3]{125}$

45. $\sqrt[3]{512}$

46. $\sqrt[5]{243}$

Find the product. *(Section 7.3)*

47. $(x + 8)(x - 8)$

48. $(4y + 2)(4y - 2)$

49. $(3a - 5b)(3a + 5b)$

50. $(-2r + 6s)(-2r - 6s)$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

8.4–8.6 What Did You Learn?

Core Vocabulary

even function, *p. 428*
odd function, *p. 428*
vertex form (of a quadratic function), *p. 430*
intercept form, *p. 436*
average rate of change, *p. 448*

Core Concepts

Section 8.4

Even and Odd Functions, *p. 428*
Graphing $f(x) = a(x - h)^2$, *p. 429*
Graphing $f(x) = a(x - h)^2 + k$, *p. 430*
Writing Quadratic Functions of the Form $f(x) = a(x - h)^2 + k$, *p. 431*

Section 8.5

Graphing $f(x) = a(x - p)(x - q)$, *p. 436*
Factors and Zeros, *p. 438*
Graphing and Writing Quadratic Functions, *p. 438*
Graphing and Writing Cubic Functions, *p. 440*

Section 8.6

Linear, Exponential, and Quadratic Functions, *p. 446*
Differences and Ratios of Functions, *p. 447*
Writing Functions to Model Data, *p. 448*
Comparing Functions Using Average Rates of Change, *p. 449*

Mathematical Thinking

1. How can you use technology to confirm your answer in Exercise 64 on page 434?
2. How did you use the structure of the equation in Exercise 85 on page 443 to solve the problem?
3. Describe why your answer makes sense considering the context of the data in Exercise 20 on page 452.

Performance Task

Asteroid Aim

Apps take a long time to design and program. One app in development is a game in which players shoot lasers at asteroids. They score points based on the number of hits per shot. The designer wants your feedback. Do you think students will like the game and want to play it? What changes would improve it?

To explore the answers to these questions and more, go to *BigIdeasMath.com*.



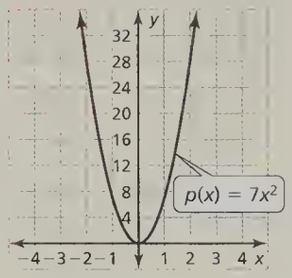
Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

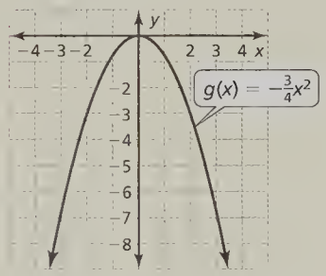
ANSWERS

1. *Sample answer:* Graph the function on a graphing calculator and compare it to your graph.
2. *Sample answer:* The equation can be factored into intercept form, and the width can be determined from the intercepts.
3. *Sample answer:* The area of a circular rug, πr^2, is a quadratic function, so it makes sense that the cost, based on the amount of material needed to make the rug, would also be a quadratic function.

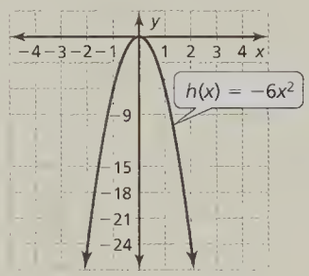
ANSWERS

1. 

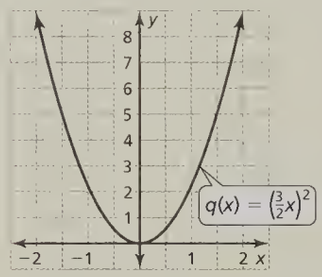
The graph of p is a vertical stretch by a factor of 7 of the graph of f.

2. 

The graph of g is a vertical shrink by a factor of $\frac{3}{4}$ and a reflection in the x-axis of the graph of f.

3. 

The graph of h is a vertical stretch by a factor of 6 and a reflection in the x-axis of the graph of f.

4. 

The graph of q is a horizontal shrink by a factor of $\frac{2}{3}$ of the graph of f.

5. The vertex is $(1, -3)$. The axis of symmetry is $x = 1$. The domain is all real numbers. The range is $y \geq -3$. When $x < 1$, y increases as x decreases. When $x > 1$, y increases as x increases.

8 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

8.1 Graphing $f(x) = ax^2$ (pp. 405–410)

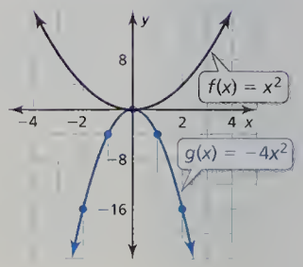
Graph $g(x) = -4x^2$. Compare the graph to the graph of $f(x) = x^2$.

Step 1 Make a table of values.

x	−2	−1	0	1	2
$g(x)$	−16	−4	0	−4	−16

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

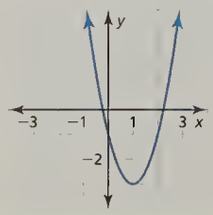


▶ The graphs have the same vertex, $(0, 0)$, and the same axis of symmetry, $x = 0$, but the graph of g opens down and is narrower than the graph of f. So, the graph of g is a vertical stretch by a factor of 4 and a reflection in the x-axis of the graph of f.

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

1. $p(x) = 7x^2$
2. $g(x) = -\frac{3}{4}x^2$
3. $h(x) = -6x^2$
4. $q(x) = \left(\frac{3}{2}x\right)^2$

5. Identify characteristics of the quadratic function and its graph.



8.2 Graphing $f(x) = ax^2 + c$ (pp. 411–416)

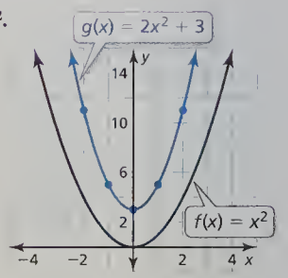
Graph $g(x) = 2x^2 + 3$. Compare the graph to the graph of $f(x) = x^2$.

Step 1 Make a table of values.

x	−2	−1	0	1	2
$g(x)$	11	5	3	5	11

Step 2 Plot the ordered pairs.

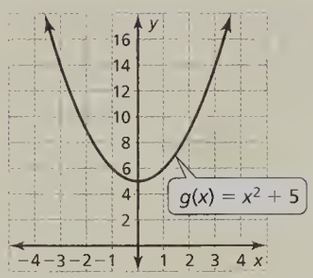
Step 3 Draw a smooth curve through the points.



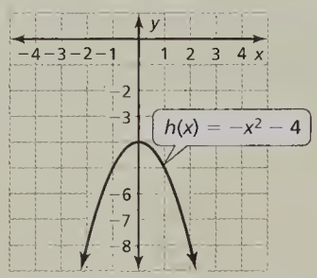
▶ Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of g is narrower, and its vertex, $(0, 3)$, is above the vertex of the graph of f, $(0, 0)$. So, the graph of g is a vertical stretch by a factor of 2 and a vertical translation 3 units up of the graph of f.

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

6. $g(x) = x^2 + 5$
7. $h(x) = -x^2 - 4$
8. $m(x) = -2x^2 + 6$
9. $n(x) = \frac{1}{3}x^2 - 5$

6. 

The graph of g is a vertical translation 5 units up of the graph of f.

7. 

The graph of h is a reflection in the x-axis, and a vertical translation 4 units down of the graph of f.

8–9. See Additional Answers.

8.3 Graphing $f(x) = ax^2 + bx + c$ *(pp. 417–424)*

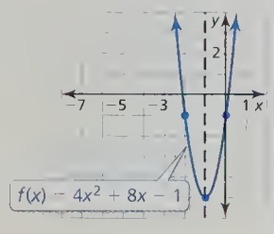
Graph $f(x) = 4x^2 + 8x - 1$. Describe the domain and range.

Step 1 Find and graph the axis of symmetry: $x = -\frac{b}{2a} = -\frac{8}{2(4)} = -1$.

Step 2 Find and plot the vertex. The axis of symmetry is $x = -1$. So, the x-coordinate of the vertex is -1. The y-coordinate of the vertex is $f(-1) = 4(-1)^2 + 8(-1) - 1 = -5$. So, the vertex is $(-1, -5)$.

Step 3 Use the y-intercept to find two more points on the graph. Because $c = -1$, the y-intercept is -1. So, $(0, -1)$ lies on the graph. Because the axis of symmetry is $x = -1$, the point $(-2, -1)$ also lies on the graph.

Step 4 Draw a smooth curve through the points.



▶ The domain is all real numbers. The range is $y \geq -5$.

Graph the function. Describe the domain and range.

10. $y = x^2 - 2x + 7$ **11.** $f(x) = -3x^2 + 3x - 4$ **12.** $y = \frac{1}{2}x^2 - 6x + 10$

13. The function $f(t) = -16t^2 + 88t + 12$ represents the height (in feet) of a pumpkin t seconds after it is launched from a catapult. When does the pumpkin reach its maximum height? What is the maximum height of the pumpkin?

8.4 Graphing $f(x) = a(x - h)^2 + k$ *(pp. 427–434)*

Determine whether $f(x) = 2x^2 + 4$ is *even*, *odd*, or *neither*.

$f(x) = 2x^2 + 4$	Write the original function.
$f(-x) = 2(-x)^2 + 4$	Substitute $-x$ for x.
$= 2x^2 + 4$	Simplify.
$= f(x)$	Substitute $f(x)$ for $2x^2 + 4$.

▶ Because $f(-x) = f(x)$, the function is even.

Determine whether the function is *even*, *odd*, or *neither*.

14. $w(x) = 5^x$ **15.** $r(x) = -8x$ **16.** $h(x) = 3x^2 - 2x$

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

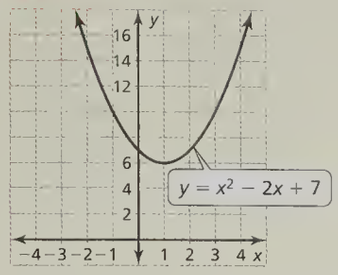
17. $h(x) = 2(x - 4)^2$ **18.** $g(x) = \frac{1}{2}(x - 1)^2 + 1$ **19.** $q(x) = -(x + 4)^2 + 7$

20. Consider the function $g(x) = -3(x + 2)^2 - 4$. Graph $h(x) = g(x - 1)$.

21. Write a quadratic function whose graph has a vertex of $(3, 2)$ and passes through the point $(4, 7)$.

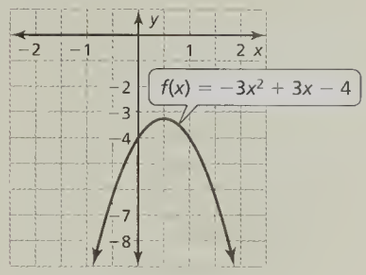
ANSWERS

10.



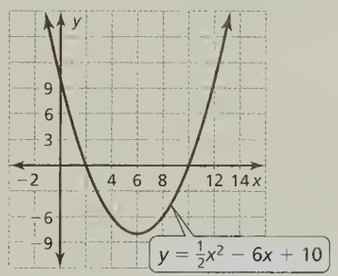
domain: all real numbers, range: $y \geq 6$

11.



domain: all real numbers, range: $y \leq -\frac{13}{4}$

12.



domain: all real numbers, range: $y \geq -8$

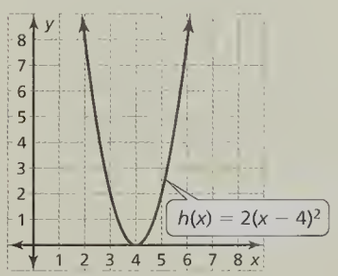
13. 2.75 sec; 133 ft

14. neither

15. odd

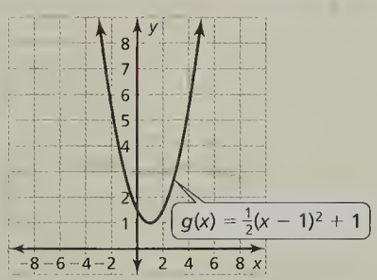
16. neither

17.



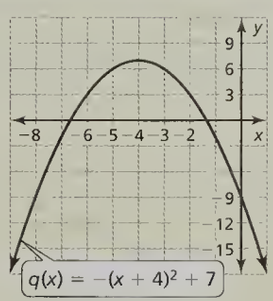
The graph of h is a vertical stretch by a factor of 2 and a horizontal translation 4 units right of the graph of f.

18.



The graph of g is a vertical shrink by a factor of $\frac{1}{2}$, and a translation 1 unit right and 1 unit up of the graph of f.

19.

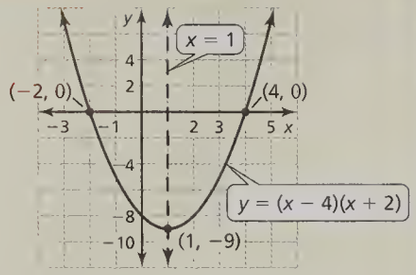


The graph of q is a reflection in the x-axis, and a translation 4 units left and 7 units up of the graph of f.

20–21. See Additional Answers.

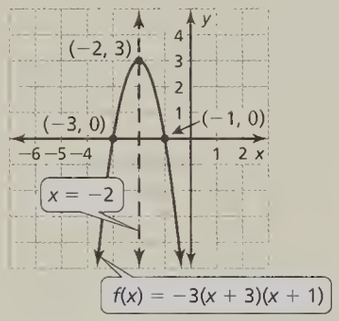
ANSWERS

22.



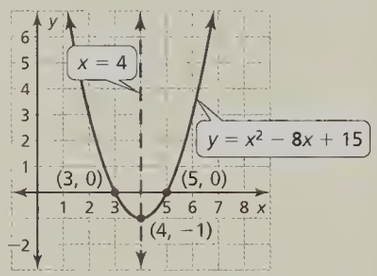
domain: all real numbers, range: $y \geq -9$

23.



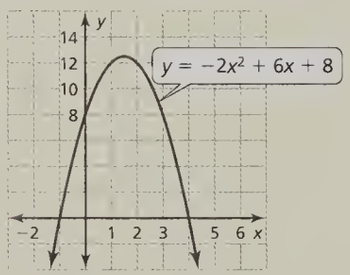
domain: all real numbers, range: $y \leq 3$

24.

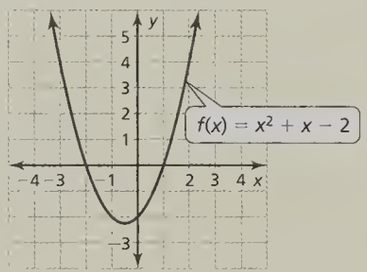


domain: all real numbers, range: $y \geq -1$

25.



26.



8.5 Using Intercept Form *(pp. 435–444)*

Use zeros to graph $h(x) = x^2 - 7x + 6$.

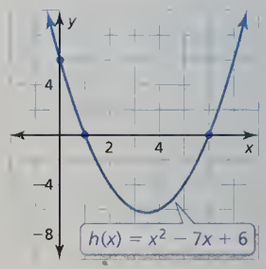
The function is in standard form. The parabola opens up ($a > 0$), and the y-intercept is 6. So, plot (0, 6).

The polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

$h(x) = x^2 - 7x + 6$ Write the function.

$= (x - 6)(x - 1)$ Factor the trinomial.

The zeros of the function are 1 and 6. So, plot (1, 0) and (6, 0). Draw a parabola through the points.



Graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function.

22. $y = (x - 4)(x + 2)$ **23.** $f(x) = -3(x + 3)(x + 1)$ **24.** $y = x^2 - 8x + 15$

Use zeros to graph the function.

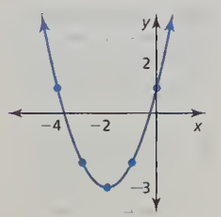
25. $y = -2x^2 + 6x + 8$ **26.** $f(x) = x^2 + x - 2$ **27.** $f(x) = 2x^3 - 18x$

28. Write a quadratic function in standard form whose graph passes through (4, 0) and (6, 0).

8.6 Comparing Linear, Exponential, and Quadratic Functions *(pp. 445–454)*

Tell whether the data represent a *linear*, an *exponential*, or a *quadratic* function.

a. (−4, 1), (−3, −2), (−2, −3), (−1, −2), (0, 1)



▶ The points appear to represent a quadratic function.

b.

x	−1	0	1	2	3
y	15	8	1	−6	−13

		+1	+1	+1	+1
x	−1	0	1	2	3
y	15	8	1	−6	−13
		−7	−7	−7	−7

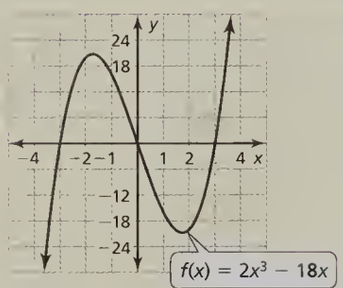
▶ The first differences are constant. So, the table represents a linear function.

29. Tell whether the table of values represents a *linear*, an *exponential*, or *a quadratic* function. Then write the function.

x	−1	0	1	2	3
y	512	128	32	8	2

30. The balance y (in dollars) of your savings account after t years is represented by $y = 200(1.1)^t$. The beginning balance of your friend's account is \$250, and the balance increases by \$20 each year. (a) Compare the account balances by calculating and interpreting the average rates of change from $t = 2$ to $t = 7$. (b) Predict which account will have a greater balance after 10 years. Explain.

27.



28. *Sample answer:* $x^2 - 10x + 24$

29. exponential; $y = 128\left(\frac{1}{4}\right)^x$

30. a. From year 2 to year 7, your account increased at an average rate of about \$29.55 per year and your friend's account increased at an average rate of \$20 per year. So, your account is growing faster.

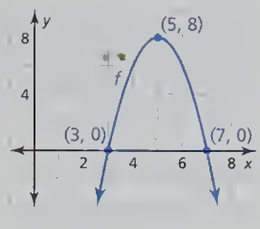
b. your account; The exponential growth of your account will continue to have a higher balance than your friend's account which has linear growth.

8 Chapter Test

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

1. $h(x) = 2x^2 - 3$
2. $g(x) = -\frac{1}{2}x^2$
3. $p(x) = \frac{1}{2}(x + 1)^2 - 1$

4. Consider the graph of the function f.
 a. Find the domain, range, and zeros of the function.
 b. Write the function f in standard form.
 c. Compare the graph of f to the graph of $g(x) = x^2$.
 d. Graph $h(x) = f(x - 6)$.



Use zeros to graph the function. Describe the domain and range of the function.

5. $f(x) = 2x^2 - 8x + 8$
6. $y = -(x + 5)(x - 1)$
7. $h(x) = 16x^2 - 4$

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Explain your reasoning. Then write the function.

8.

x	-1	0	1	2	3
y	4	8	16	32	64

9.

x	-2	-1	0	1	2
y	-8	-2	0	-2	-8

Write a quadratic function in standard form whose graph satisfies the given conditions. Explain the process you used.

10. passes through $(-8, 0)$, $(-2, 0)$, and $(-6, 4)$
11. passes through $(0, 0)$, $(10, 0)$, and $(9, -27)$
12. is even and has a range of $y \geq 3$
13. passes through $(4, 0)$ and $(1, 9)$

14. The table shows the distances d (in miles) that Earth moves in its orbit around the Sun after t seconds. Let the time t be the independent variable. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function. Explain. Then write a function that models the data.

Time, t	1	2	3	4	5
Distance, d	19	38	57	76	95

15. You are playing tennis with a friend. The path of the tennis ball after you return a serve can be modeled by the function $y = -0.005x^2 + 0.17x + 3$, where x is the horizontal distance (in feet) from where you hit the ball and y is the height (in feet) of the ball.
 a. What is the maximum height of the tennis ball?
 b. You are standing 30 feet from the net, which is 3 feet high. Will the ball clear the net? Explain your reasoning.

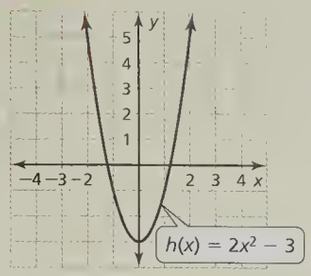
16. Find values of a, b, and c so that the function $f(x) = ax^2 + bx + c$ is (a) even, (b) odd, and (c) neither even nor odd.

17. Consider the function $f(x) = x^2 + 4$. Find the average rate of change from $x = 0$ to $x = 1$, from $x = 1$ to $x = 2$, and from $x = 2$ to $x = 3$. What do you notice about the average rates of change when the function is increasing?

If students need help...	If students got it...
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

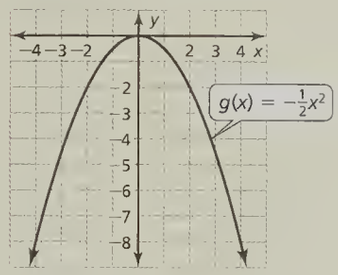
ANSWERS

1.



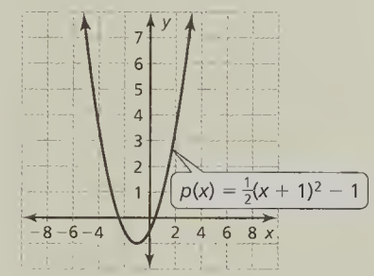
The graph of h is a vertical stretch by a factor of 2 and a vertical translation 3 units down of the graph of f.

2.



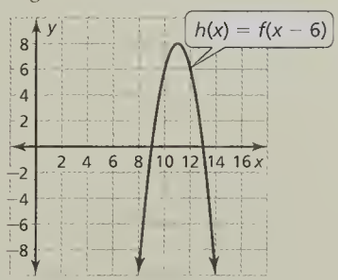
The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ and a reflection in the x-axis of the graph of f.

3.



The graph of p is a vertical shrink by a factor of $\frac{1}{2}$, and a translation 1 unit left and 1 unit down of the graph of f.

4. a. domain: all real numbers, range: $y \leq 8$; 3, 7
 b. $f(x) = -2x^2 + 20x - 42$
 c. The graph of f is a vertical stretch by a factor of 2, a reflection in the x-axis, and a translation 5 units right and 8 units up of the graph of g.
 d.



5–17. See Additional Answers.

ANSWERS

1. C
2. J
3. C
4. −4
5. F

8 Standards Assessment

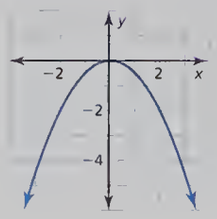
1. Which function is represented by the graph? *(TEKS A.6.C)*

 Ⓐ $y = \frac{1}{2}x^2$

 Ⓑ $y = 2x^2$

 Ⓒ $y = -\frac{1}{2}x^2$

 Ⓓ $y = -2x^2$



2. Which of the following expressions are equivalent to $(b^{-5})^{-4}$? *(TEKS A.11.B)*

 I. b^{-20}

 II. $\frac{b^6}{b^{-14}}$

 III. $b^{12}b^8$

 IV. $(b^{-4})^{-5}$

 Ⓕ I and IV only

 Ⓖ II and III only

 Ⓗ III and IV only

 Ⓙ II, III, and IV only

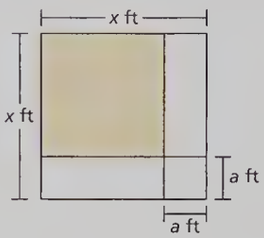
3. Which polynomial represents the area (in square feet) of the shaded region of the figure? *(TEKS A.10.B)*

 Ⓐ $a^2 - x^2$

 Ⓑ $x^2 - a^2$

 Ⓒ $x^2 - 2ax + a^2$

 Ⓓ $x^2 + 2ax + a^2$



4. **GRIDDED ANSWER** A quadratic function whose graph has a vertex of $(-2, 8)$ and passes through the point $(-4, -4)$ can be written in the standard form $f(x) = -3x^2 - 12x + c$. What is the value of c? *(TEKS A.6.B)*

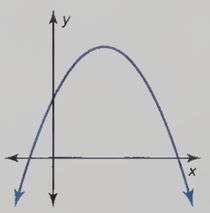
5. Which function could be represented by the graph? *(TEKS A.7.B)*

 Ⓕ $y = -(x + 1)(x - 5)$

 Ⓖ $y = 3(x + 1)(x - 5)$

 Ⓗ $y = \frac{1}{2}(x - 1)(x - 5)$

 Ⓙ $y = -2(x + 1)(x + 5)$



ANSWERS

6. B
7. F
8. A
9. F

6. Which expression represents the sum of $3x^2 + 9x - 8$ and $-5(x^2 + 8 - 3x)$? *(TEKS A.10.A)*

Ⓐ $-2x^2 + 6x$

Ⓑ $-2x^2 + 24x - 48$

Ⓒ $-2x^2 - 6x + 32$

Ⓓ $8x^2 - 6x + 32$

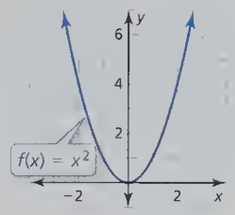
7. The graph of $f(x) = x^2$ is shown. Which statement describes the graph of $g(x) = -2f(x)$? *(TEKS A.7.C)*

Ⓕ The graph of g opens down and is narrower than the graph of f.

Ⓖ The graph of g opens down and is wider than the graph of f.

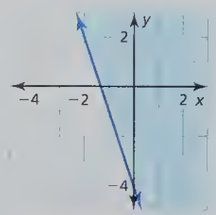
Ⓗ The graph of g opens up and is narrower than the graph of f.

Ⓙ The graph of g opens up and is wider than the graph of f.

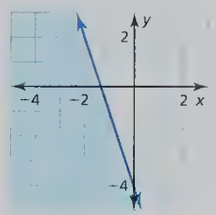


8. Which graph represents the solution of $-3y - 9x \leq 12$? *(TEKS A.3.D)*

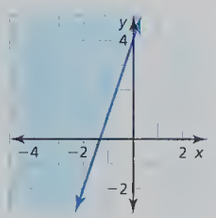
Ⓐ



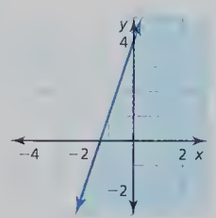
Ⓑ



Ⓒ



Ⓓ



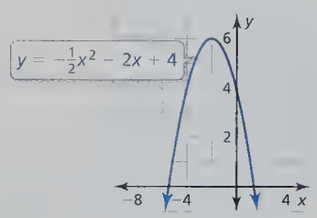
9. What is the domain and range of the quadratic function represented by the graph? *(TEKS A.6.A)*

Ⓕ domain: all real numbers; range: $y \leq 6$

Ⓖ domain: all real numbers; range: $y < 6$

Ⓗ domain: $x > -2$; range: all real numbers

Ⓙ none of the above



9 Solving Quadratic Equations

9.1 Properties of Radicals
9.2 Solving Quadratic Equations by Graphing
9.3 Solving Quadratic Equations Using Square Roots
9.4 Solving Quadratic Equations by Completing the Square
9.5 Solving Quadratic Equations Using the Quadratic Formula



Dolphin *(p. 509)*



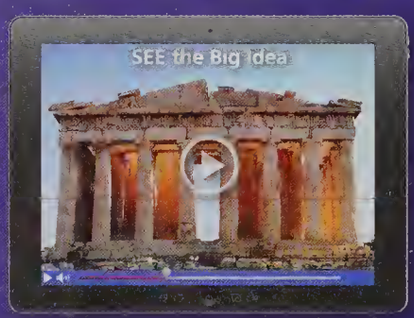
Half-pipe *(p. 501)*



Pond *(p. 489)*



Kicker *(p. 479)*



Parthenon *(p. 469)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

Chapter 9 Pacing Guide

Chapter Opener/ Mathematical Thinking	1 Day
Section 1	3 Days
Section 2	3 Days
Section 3	2 Days
Quiz	1 Day
Section 4	3 Days
Section 5	3 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 9	18 Days
Year-to-Date	157 Days

Texas Essential Knowledge and Skills Summary

Section	TEKS
9.1	A.11.A
9.2	A.7.A, A.8.B
9.3	A.8.A
9.4	A.8.A
9.5	A.8.A

For an overview of this chapter, formative assessment tips, and teaching strategies, see Laurie's Notes at *BigIdeasMath.com*.

Maintaining Mathematical Proficiency

Factoring Perfect Square Trinomials (A.10.E)

Example 1 Factor $x^2 + 14x + 49$.

$x^2 + 14x + 49 = x^2 + 2(x)(7) + 7^2$ Write as $a^2 + 2ab + b^2$.

$= (x + 7)^2$ Perfect square trinomial pattern

Factor the trinomial.

1. $x^2 + 10x + 25$
2. $x^2 - 20x + 100$
3. $x^2 + 12x + 36$
4. $x^2 - 18x + 81$
5. $x^2 + 16x + 64$
6. $x^2 - 30x + 225$

Solving Systems of Linear Equations by Graphing (A.5.C)

Example 2 Solve the system of linear equations by graphing.

$y = 2x + 1$ Equation 1

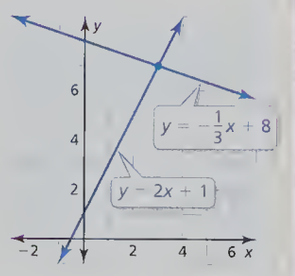
$y = -\frac{1}{3}x + 8$ Equation 2

Step 1 Graph each equation.

Step 2 Estimate the point of intersection. The graphs appear to intersect at (3, 7).

Step 3 Check your point from Step 2.

Equation 1	Equation 2
$y = 2x + 1$	$y = -\frac{1}{3}x + 8$
$7 \stackrel{?}{=} 2(3) + 1$	$7 \stackrel{?}{=} -\frac{1}{3}(3) + 8$
$7 = 7$ ✓	$7 = 7$ ✓



▶ The solution is (3, 7).

Solve the system of linear equations by graphing.

7. $y = -5x + 3$, $y = 2x - 4$
8. $y = \frac{3}{2}x - 2$, $y = -\frac{1}{4}x + 5$
9. $y = \frac{1}{2}x + 4$, $y = -3x - 3$

10. **ABSTRACT REASONING** What value of c makes $x^2 + bx + c$ a perfect square trinomial?

Dynamic Solutions available at *BigIdeasMath.com*

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For suggestions on Maintaining Mathematical Proficiency in your classroom, see **Laurie's Notes** at *BigIdeasMath.com*.

Vocabulary Review

Have students make Four Squares for the following topics.

- Factoring Perfect Square Trinomials
- Solving Systems of Linear Equations by Graphing

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations
- Real-Life STEM Videos

Texas Essential Knowledge and Skills

A.10.E The student is expected to factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

A.5.C The student is expected to solve systems of two linear equations with two variables for mathematical and real-world problems.

ANSWERS

1. $(x + 5)^2$
2. $(x - 10)^2$
3. $(x + 6)^2$
4. $(x - 9)^2$
5. $(x + 8)^2$
6. $(x - 15)^2$
7. $(1, -2)$
8. $(4, 4)$
9. $(-2, 3)$
10. $\left(\frac{b}{2}\right)^2$

MONITORING PROGRESS ANSWERS

1. $x \approx -1.618$
2. $x \approx 1.303, x \approx -2.303$

Mathematical Thinking

Mathematically proficient students use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. **(A.1.B)**

Problem-Solving Strategies

Core Concept

Guess, Check, and Revise

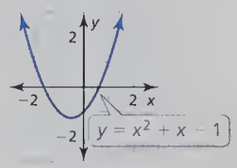
When solving a problem in mathematics, it is often helpful to estimate a solution and then observe how close that solution is to being correct. For instance, you can use the guess, check, and revise strategy to find a decimal approximation of the square root of 2.

	Guess	Check	How to revise
1.	1.4	$1.4^2 = 1.96$	Increase guess.
2.	1.41	$1.41^2 = 1.9881$	Increase guess.
3.	1.415	$1.415^2 = 2.002225$	Decrease guess.

By continuing this process, you can determine that the square root of 2 is approximately 1.4142.

EXAMPLE 1 **Approximating a Solution of an Equation**

The graph of $y = x^2 + x - 1$ is shown. Approximate the positive solution of the equation $x^2 + x - 1 = 0$ to the nearest thousandth.



SOLUTION

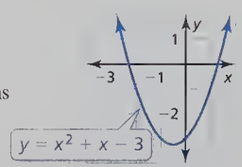
Using the graph, you can make an initial estimate of the positive solution to be $x = 0.65$.

	Guess	Check	How to revise
1.	0.65	$0.65^2 + 0.65 - 1 = 0.0725$	Decrease guess.
2.	0.62	$0.62^2 + 0.62 - 1 = 0.0044$	Decrease guess.
3.	0.618	$0.618^2 + 0.618 - 1 = -0.000076$	Increase guess.
4.	0.6181	$0.6181^2 + 0.6181 - 1 \approx 0.00015$	The solution is between 0.618 and 0.6181.

▶ So, to the nearest thousandth, the positive solution of the equation is $x = 0.618$.

Monitoring Progress

1. Use the graph in Example 1 to approximate the negative solution of the equation $x^2 + x - 1 = 0$ to the nearest thousandth.
2. The graph of $y = x^2 + x - 3$ is shown. Approximate both solutions of the equation $x^2 + x - 3 = 0$ to the nearest thousandth.



For insights into Mathematical Thinking, see **Laurie's Notes** at *BigIdeasMath.com*.

If students need help...	If students got it...
Student Journal • Maintaining Mathematical Proficiency	Game Closet at *BigIdeasMath.com*
Lesson Tutorials	Start the *next* Section
Skills Review Handbook	

9.1 Properties of Radicals

Texas Essential Knowledge and Skills
A.11.A

Essential Question
How can you multiply and divide square roots?

EXPLORATION 1 Operations with Square Roots

Work with a partner. For each operation with square roots, compare the results obtained using the two indicated orders of operations. What can you conclude?

a. Square Roots and Addition

Is $\sqrt{36} + \sqrt{64}$ equal to $\sqrt{36 + 64}$?

In general, is $\sqrt{a} + \sqrt{b}$ equal to $\sqrt{a + b}$? Explain your reasoning.

b. Square Roots and Multiplication

Is $\sqrt{4} \cdot \sqrt{9}$ equal to $\sqrt{4 \cdot 9}$?

In general, is $\sqrt{a} \cdot \sqrt{b}$ equal to $\sqrt{a \cdot b}$? Explain your reasoning.

c. Square Roots and Subtraction

Is $\sqrt{64} - \sqrt{36}$ equal to $\sqrt{64 - 36}$?

In general, is $\sqrt{a} - \sqrt{b}$ equal to $\sqrt{a - b}$? Explain your reasoning.

d. Square Roots and Division

Is $\dfrac{\sqrt{100}}{\sqrt{4}}$ equal to $\sqrt{\dfrac{100}{4}}$?

In general, is $\dfrac{\sqrt{a}}{\sqrt{b}}$ equal to $\sqrt{\dfrac{a}{b}}$? Explain your reasoning.

REASONING
To be proficient in math, you need to recognize and use counterexamples.

EXPLORATION 2 Writing Counterexamples

Work with a partner. A **counterexample** is an example that proves that a general statement is *not* true. For each general statement in Exploration 1 that is not true, write a counterexample different from the example given.

Communicate Your Answer

3. How can you multiply and divide square roots?
4. Give an example of multiplying square roots and an example of dividing square roots that are different from the examples in Exploration 1.
5. Write an algebraic rule for each operation.
 a. the product of square roots
 b. the quotient of square roots

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Dynamic Teaching Tools
- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.11.A The student is expected to simplify numerical radical expressions involving square roots.

ANSWERS

1. **a.** no; no; Because $\sqrt{36} + \sqrt{64} \neq \sqrt{36 + 64}$, the general statement cannot be true.
 b. yes; yes; By the Power of a Product Property, $a^{1/2} \cdot b^{1/2} = (a \cdot b)^{1/2}$.
 c. no; no; Because $\sqrt{64} - \sqrt{36} \neq \sqrt{64 - 36}$, the general statement cannot be true.
 d. yes; yes; By the Power of a Quotient Property, $\dfrac{a^{1/2}}{b^{1/2}} = \left(\dfrac{a}{b}\right)^{1/2}$.
2. *Sample answer:* $\sqrt{9} + \sqrt{16} \neq \sqrt{25}$, $\sqrt{16} - \sqrt{9} \neq \sqrt{7}$
3. Multiply or divide the numbers inside the square root symbols and take the square root of the product or quotient.
4. *Sample answer:* $\sqrt{9} \cdot \sqrt{16} = \sqrt{9 \cdot 16} = \sqrt{144} = 12$, $\dfrac{\sqrt{16}}{\sqrt{4}} = \sqrt{\dfrac{16}{4}} = \sqrt{4} = 2$
5. **a.** $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$
 b. $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

Extra Example 1

Simplify the expression.

a. $\sqrt{147}$ $7\sqrt{3}$

b. $\sqrt{25x^5}$ $5x^2\sqrt{x}$

MONITORING PROGRESS ANSWERS

1. $2\sqrt{6}$
2. $-4\sqrt{5}$
3. $7x\sqrt{x}$
4. $5n^2\sqrt{3n}$

9.1 Lesson

What You Will Learn

- Use properties of radicals to simplify expressions.
- Simplify expressions by rationalizing the denominator.
- Perform operations with radicals.

Core Vocabulary

counterexample, *p. 465*
radical expression, *p. 466*
simplest form, *p. 466*
rationalizing the denominator, *p. 468*
conjugates, *p. 468*
like radicals, *p. 470*

Previous
radicand
perfect cube

Using Properties of Radicals

A **radical expression** is an expression that contains a radical. A radical with index n is in **simplest form** when these three conditions are met.

- No radicands have perfect nth powers as factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

You can use the property below to simplify radical expressions involving square roots.

Core Concept

Product Property of Square Roots

Words The square root of a product equals the product of the square roots of the factors.

Numbers $\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

STUDY TIP

There can be more than one way to factor a radicand. An efficient method is to find the greatest perfect square factor.

EXAMPLE 1 **Using the Product Property of Square Roots**

a. $\sqrt{108} = \sqrt{36 \cdot 3}$ Factor using the greatest perfect square factor.

$= \sqrt{36} \cdot \sqrt{3}$ Product Property of Square Roots

$= 6\sqrt{3}$ Simplify.

STUDY TIP

In this course, whenever a variable appears in the radicand, assume that it has only *nonnegative* values.

b. $\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x}$ Factor using the greatest perfect square factor.

$= \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x}$ Product Property of Square Roots

$= 3x\sqrt{x}$ Simplify.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Simplify the expression.

1. $\sqrt{24}$
2. $-\sqrt{80}$
3. $\sqrt{49x^3}$
4. $\sqrt{75n^5}$

Core Concept

Quotient Property of Square Roots

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Numbers $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$ **Algebra** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

As you present the material on page 466, ask students to write down what they understand without reading along. This should include the objectives from What You Will Learn and the main ideas from the topic Using Properties of Radicals. Have them share what they understand.

Beginning Use simple phrases to share what they understand.
Intermediate Use simple sentences to share what they understand.
Advanced/Advanced High Use complex sentences to share what they understand.

ELPS 2.I.5 Demonstrate listening comprehension of increasingly complex spoken English by taking notes commensurate with content and grade-level needs.

EXAMPLE 2 Using the Quotient Property of Square Roots

a. $\sqrt{\frac{15}{64}} = \frac{\sqrt{15}}{\sqrt{64}}$ Quotient Property of Square Roots

$= \frac{\sqrt{15}}{8}$ Simplify.

b. $\sqrt{\frac{81}{x^2}} = \frac{\sqrt{81}}{\sqrt{x^2}}$ Quotient Property of Square Roots

$= \frac{9}{x}$ Simplify.

You can extend the Product and Quotient Properties of Square Roots to other radicals, such as cube roots. When using these *properties of cube roots*, the radicands may contain negative numbers.

EXAMPLE 3 Using Properties of Cube Roots

STUDY TIP
To write a cube root in simplest form, find factors of the radicand that are perfect cubes.

a. $\sqrt[3]{-128} = \sqrt[3]{-64 \cdot 2}$ Factor using the greatest perfect cube factor.

$= \sqrt[3]{-64} \cdot \sqrt[3]{2}$ Product Property of Cube Roots

$= -4\sqrt[3]{2}$ Simplify.

b. $\sqrt[3]{125x^7} = \sqrt[3]{125 \cdot x^6 \cdot x}$ Factor using the greatest perfect cube factors.

$= \sqrt[3]{125} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{x}$ Product Property of Cube Roots

$= 5x^2\sqrt[3]{x}$ Simplify.

c. $\sqrt[3]{\frac{y}{216}} = \frac{\sqrt[3]{y}}{\sqrt[3]{216}}$ Quotient Property of Cube Roots

$= \frac{\sqrt[3]{y}}{6}$ Simplify.

d. $\sqrt[3]{\frac{8x^4}{27y^3}} = \frac{\sqrt[3]{8x^4}}{\sqrt[3]{27y^3}}$ Quotient Property of Cube Roots

$= \frac{\sqrt[3]{8 \cdot x^3 \cdot x}}{\sqrt[3]{27 \cdot y^3}}$ Factor using the greatest perfect cube factors.

$= \frac{\sqrt[3]{8} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x}}{\sqrt[3]{27} \cdot \sqrt[3]{y^3}}$ Product Property of Cube Roots

$= \frac{2x\sqrt[3]{x}}{3y}$ Simplify.

Monitoring Progress

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Simplify the expression.

5. $\sqrt{\frac{23}{9}}$
6. $-\sqrt{\frac{17}{100}}$
7. $\sqrt{\frac{36}{z^2}}$
8. $\sqrt{\frac{4x^2}{64}}$
9. $\sqrt[3]{54}$
10. $\sqrt[3]{16x^4}$
11. $\sqrt[3]{\frac{a}{-27}}$
12. $\sqrt[3]{\frac{25c^7d^3}{64}}$

Extra Example 2

Simplify the expression.

a. $\sqrt{\frac{11}{36}}$ $\frac{\sqrt{11}}{6}$

b. $\sqrt{\frac{144}{n^2}}$ $\frac{12}{n}$

Extra Example 3

Simplify the expression.

a. $\sqrt[3]{135}$ $3\sqrt[3]{5}$

b. $\sqrt[3]{8x^4}$ $2x\sqrt[3]{x}$

c. $\sqrt[3]{\frac{n^2}{-1000}}$ $-\frac{\sqrt[3]{n^2}}{10}$

d. $\sqrt[3]{\frac{343a^8}{b^3}}$ $\frac{7a^2\sqrt[3]{a^2}}{b}$

MONITORING PROGRESS ANSWERS

5. $\frac{\sqrt{23}}{3}$
6. $-\frac{\sqrt{17}}{10}$
7. $\frac{6}{z}$
8. $\frac{x}{4}$
9. $3\sqrt[3]{2}$
10. $2x\sqrt[3]{2x}$
11. $-\frac{\sqrt[3]{a}}{3}$
12. $\frac{c^2d}{4}\sqrt[3]{25c}$

English Language Learners

Notebook Development

Have students create a radicals page in their notebooks. They should include the vocabulary and properties used in this section. For each property, have students include an example. Be sure they include, when appropriate, a cube root example, as well as a square root example.

Extra Example 4

Simplify the expression.

a. $\frac{\sqrt{7}}{\sqrt{2x}} \quad \frac{\sqrt{14x}}{2x}$

b. $\frac{5}{\sqrt[3]{4}} \quad \frac{5\sqrt[3]{2}}{2}$

Extra Example 5

Simplify $\frac{4}{\sqrt{3}+1}$. $2\sqrt{3} - 2$

MONITORING PROGRESS ANSWERS

13. $\frac{\sqrt{5}}{5}$

14. $\frac{\sqrt{30}}{3}$

15. $\frac{7\sqrt{2x}}{2x}$

16. $\frac{y\sqrt{6}}{3}$

17. $\frac{5\sqrt[3]{2}}{4}$

18. $-4 + 4\sqrt{3}$

19. $\sqrt{65} + 2\sqrt{13}$

20. $-\frac{12\sqrt{2} - 12\sqrt{7}}{5}$

Rationalizing the Denominator

When a radical is in the denominator of a fraction, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

EXAMPLE 4 Rationalizing the Denominator

STUDY TIP

Rationalizing the denominator works because you multiply the numerator and denominator by the same nonzero number a, which is the same as multiplying by $\frac{a}{a}$, or 1.

a. $\frac{\sqrt{5}}{\sqrt{3n}} = \frac{\sqrt{5}}{\sqrt{3n}} \cdot \frac{\sqrt{3n}}{\sqrt{3n}}$ Multiply by $\frac{\sqrt{3n}}{\sqrt{3n}}$.

$= \frac{\sqrt{15n}}{\sqrt{9n^2}}$ Product Property of Square Roots

$= \frac{\sqrt{15n}}{\sqrt{9} \cdot \sqrt{n^2}}$ Product Property of Square Roots

$= \frac{\sqrt{15n}}{3n}$ Simplify.

b. $\frac{2}{\sqrt[3]{9}} = \frac{2}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$ Multiply by $\frac{\sqrt[3]{3}}{\sqrt[3]{3}}$.

$= \frac{2\sqrt[3]{3}}{\sqrt[3]{27}}$ Product Property of Cube Roots

$= \frac{2\sqrt[3]{3}}{3}$ Simplify.

The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates**. You can use conjugates to simplify radical expressions that contain a sum or difference involving square roots in the denominator.

ANALYZING MATHEMATICAL RELATIONSHIPS

Notice that the product of two conjugates $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ does not contain a radical and is a *rational* number.

$(a\sqrt{b} + c\sqrt{d})(a\sqrt{b} - c\sqrt{d})$
$= (a\sqrt{b})^2 - (c\sqrt{d})^2$
$= a^2b - c^2d$

EXAMPLE 5 Rationalizing the Denominator Using Conjugates

Simplify $\frac{7}{2 - \sqrt{3}}$.

SOLUTION

$\frac{7}{2 - \sqrt{3}} = \frac{7}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$ The conjugate of $2 - \sqrt{3}$ is $2 + \sqrt{3}$.

$= \frac{7(2 + \sqrt{3})}{2^2 - (\sqrt{3})^2}$ Sum and difference pattern

$= \frac{14 + 7\sqrt{3}}{1}$ Simplify.

$= 14 + 7\sqrt{3}$ Simplify.

Monitoring Progress

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Simplify the expression.

13. $\frac{1}{\sqrt{5}}$

14. $\frac{\sqrt{10}}{\sqrt{3}}$

15. $\frac{7}{\sqrt{2x}}$

16. $\sqrt{\frac{2y^2}{3}}$

17. $\frac{5}{\sqrt[3]{32}}$

18. $\frac{8}{1 + \sqrt{3}}$

19. $\frac{\sqrt{13}}{\sqrt{5} - 2}$

20. $\frac{12}{\sqrt{2} + \sqrt{7}}$

EXAMPLE 6 Solving a Real-Life Problem



The distance d (in miles) that you can see to the horizon with your eye level h feet above the water is given by $d = \sqrt{\frac{3h}{2}}$. How far can you see when your eye level is 5 feet above the water?

SOLUTION

$$d = \sqrt{\frac{3(5)}{2}}$$ Substitute 5 for h.

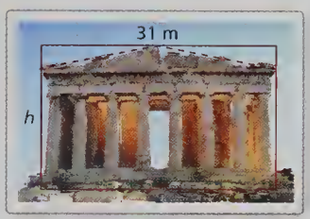
$$= \frac{\sqrt{15}}{\sqrt{2}}$$ Quotient Property of Square Roots

$$= \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$ Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$$= \frac{\sqrt{30}}{2}$$ Simplify.

▶ You can see $\frac{\sqrt{30}}{2}$, or about 2.74 miles.

EXAMPLE 7 Modeling with Mathematics



The ratio of the length to the width of a *golden rectangle* is $(1 + \sqrt{5}) : 2$. The dimensions of the face of the Parthenon in Greece form a golden rectangle. What is the height h of the Parthenon?

SOLUTION

1. **Understand the Problem** Think of the length and height of the Parthenon as the length and width of a golden rectangle. The length of the rectangular face is 31 meters. You know the ratio of the length to the height. Find the height h.
2. **Make a Plan** Use the ratio $(1 + \sqrt{5}) : 2$ to write a proportion and solve for h.
3. **Solve the Problem**

$$\frac{1 + \sqrt{5}}{2} = \frac{31}{h}$$ Write a proportion.

$$h(1 + \sqrt{5}) = 62$$ Cross Products Property

$$h = \frac{62}{1 + \sqrt{5}}$$ Divide each side by $1 + \sqrt{5}$.

$$h = \frac{62}{1 + \sqrt{5}} \cdot \frac{1 - \sqrt{5}}{1 - \sqrt{5}}$$ Multiply the numerator and denominator by the conjugate.

$$h = \frac{62 - 62\sqrt{5}}{-4}$$ Simplify.

$$h \approx 19.16$$ Use a calculator.

▶ The height is about 19 meters.

4. **Look Back** $\frac{1 + \sqrt{5}}{2} \approx 1.62$ and $\frac{31}{19.16} \approx 1.62$. So, your answer is reasonable.

Monitoring Progress

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21. **WHAT IF?** In Example 6, how far can you see when your eye level is 35 feet above the water?
22. The dimensions of a dance floor form a golden rectangle. The shorter side of the dance floor is 50 feet. What is the length of the longer side of the dance floor?

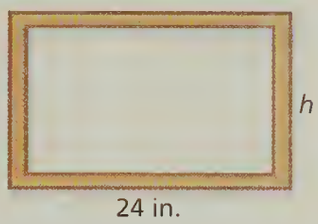
Extra Example 6

The distance d (in miles) that you can see to the horizon with your eye level h feet above the water is given by $d = \sqrt{\frac{3h}{2}}$. How far can you see when your eye level is 15 feet above the water?

You can see $\frac{3\sqrt{10}}{2}$, or about 4.74 miles.

Extra Example 7

The ratio of the length to the width of a *golden rectangle* is $(1 + \sqrt{5}) : 2$. The dimensions of the picture frame form a golden rectangle. What is the height h of the frame?



The height is about 15 inches.

MONITORING PROGRESS ANSWERS

21. about 7.25 mi
22. about 81 ft

Extra Example 8

Simplify the expression.

a. $3\sqrt{15} + 2\sqrt{13} - 7\sqrt{15} - 4\sqrt{15} + 2\sqrt{13}$

b. $5\sqrt{7} + \sqrt{63}$ $8\sqrt{7}$

c. $4\sqrt[3]{2x} + 5\sqrt[3]{2x}$ $9\sqrt[3]{2x}$

Extra Example 9

Simplify $\sqrt{7}(\sqrt{3} + \sqrt{48})$. $5\sqrt{21}$

MONITORING PROGRESS ANSWERS

23. $13\sqrt{2} - \sqrt{6}$
24. $-14\sqrt{7}$
25. $-7\sqrt[3]{5x}$
26. $36\sqrt{6}$
27. $36 - 16\sqrt{5}$
28. 2

Performing Operations with Radicals

Radicals with the same index and radicand are called **like radicals**. You can add and subtract like radicals the same way you combine like terms by using the Distributive Property.

STUDY TIP

Do not assume that radicals with different radicands cannot be added or subtracted. Always check to see whether you can simplify the radicals. In some cases, the radicals will become like radicals.

EXAMPLE 8 Adding and Subtracting Radicals

a. $5\sqrt{7} + \sqrt{11} - 8\sqrt{7} = 5\sqrt{7} - 8\sqrt{7} + \sqrt{11}$ Commutative Property of Addition

$= (5 - 8)\sqrt{7} + \sqrt{11}$ Distributive Property

$= -3\sqrt{7} + \sqrt{11}$ Subtract.

b. $10\sqrt{5} + \sqrt{20} = 10\sqrt{5} + \sqrt{4 \cdot 5}$ Factor using the greatest perfect square factor.

$= 10\sqrt{5} + \sqrt{4} \cdot \sqrt{5}$ Product Property of Square Roots

$= 10\sqrt{5} + 2\sqrt{5}$ Simplify.

$= (10 + 2)\sqrt{5}$ Distributive Property

$= 12\sqrt{5}$ Add.

c. $6\sqrt[3]{x} + 2\sqrt[3]{x} = (6 + 2)\sqrt[3]{x}$ Distributive Property

$= 8\sqrt[3]{x}$ Add.

EXAMPLE 9 Multiplying Radicals

Simplify $\sqrt{5}(\sqrt{3} - \sqrt{75})$.

SOLUTION

Method 1 $\sqrt{5}(\sqrt{3} - \sqrt{75}) = \sqrt{5} \cdot \sqrt{3} - \sqrt{5} \cdot \sqrt{75}$ Distributive Property

$= \sqrt{15} - \sqrt{375}$ Product Property of Square Roots

$= \sqrt{15} - 5\sqrt{15}$ Simplify.

$= (1 - 5)\sqrt{15}$ Distributive Property

$= -4\sqrt{15}$ Subtract.

Method 2 $\sqrt{5}(\sqrt{3} - \sqrt{75}) = \sqrt{5}(\sqrt{3} - 5\sqrt{3})$ Simplify $\sqrt{75}$.

$= \sqrt{5}[(1 - 5)\sqrt{3}]$ Distributive Property

$= \sqrt{5}(-4\sqrt{3})$ Subtract.

$= -4\sqrt{15}$ Product Property of Square Roots

Monitoring Progress

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Simplify the expression.

23. $3\sqrt{2} - \sqrt{6} + 10\sqrt{2}$
24. $4\sqrt{7} - 6\sqrt{63}$
25. $4\sqrt[3]{5x} - 11\sqrt[3]{5x}$
26. $\sqrt{3}(8\sqrt{2} + 7\sqrt{32})$
27. $(2\sqrt{5} - 4)^2$
28. $\sqrt[3]{-4}(\sqrt[3]{2} - \sqrt[3]{16})$

Closure

- **Writing Prompt:** This was a very long lesson. Right now I am feeling …

9.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The process of eliminating a radical from the denominator of a radical expression is called _____.
2. **VOCABULARY** What is the conjugate of the binomial $\sqrt{6} + 4$?
3. **WRITING** Are the expressions $\frac{1}{3}\sqrt{2x}$ and $\sqrt{\frac{2x}{9}}$ equivalent? Explain your reasoning.
4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$-\frac{1}{3}\sqrt{6}$ $\quad 6\sqrt{3}$ $\quad \frac{1}{6}\sqrt{3}$ $\quad -3\sqrt{3}$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, determine whether the expression is in simplest form. If the expression is not in simplest form, explain why.

5. $\sqrt{19}$
6. $\sqrt{\frac{1}{7}}$
7. $\sqrt{48}$
8. $\sqrt{34}$
9. $\frac{5}{\sqrt{2}}$
10. $\frac{3\sqrt{10}}{4}$
11. $\frac{1}{2 + \sqrt[3]{2}}$
12. $6 - \sqrt[3]{54}$

In Exercises 13–20, simplify the expression. *(See Example 1.)*

13. $\sqrt{20}$
14. $\sqrt{32}$
15. $\sqrt{128}$
16. $-\sqrt{72}$
17. $\sqrt{125b}$
18. $\sqrt{4x^2}$
19. $-\sqrt{81m^3}$
20. $\sqrt{48n^5}$

In Exercises 21–28, simplify the expression. *(See Example 2.)*

21. $\sqrt{\frac{4}{49}}$
22. $-\sqrt{\frac{7}{81}}$
23. $-\sqrt{\frac{23}{64}}$
24. $\sqrt{\frac{65}{121}}$
25. $\sqrt{\frac{a^3}{49}}$
26. $\sqrt{\frac{144}{k^2}}$
27. $\sqrt{\frac{100}{4x^2}}$
28. $\sqrt{\frac{25v^2}{36}}$

In Exercises 29–36, simplify the expression. *(See Example 3.)*

29. $\sqrt[3]{16}$
30. $\sqrt[3]{-108}$
31. $\sqrt[3]{-64x^5}$
32. $-\sqrt[3]{343n^2}$
33. $\sqrt[3]{\frac{6c}{-125}}$
34. $\sqrt[3]{\frac{8h^4}{27}}$
35. $-\sqrt[3]{\frac{81y^2}{1000x^3}}$
36. $\sqrt[3]{\frac{21}{-64a^3b^6}}$

ERROR ANALYSIS **In Exercises 37 and 38, describe and correct the error in simplifying the expression.**

37. ✗
$$\sqrt{72} = \sqrt{4 \cdot 18} = \sqrt{4} \cdot \sqrt{18} = 2\sqrt{18}$$

38. ✗
$$\sqrt[3]{\frac{128y^3}{125}} = \frac{\sqrt[3]{128y^3}}{125} = \frac{\sqrt[3]{64 \cdot 2 \cdot y^3}}{125} = \frac{\sqrt[3]{64} \cdot \sqrt[3]{2} \cdot \sqrt[3]{y^3}}{125} = \frac{4y\sqrt[3]{2}}{125}$$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–61 odd, 73–89 odd, 102, 104, 108–115

Average: 1–3, 4–90 even, 94, 102, 104, 108–115

Advanced: 1–4, 20, 28, 34–40 even, 52–64 even, 68–74 even, 80–90 even, 91–115

HOMEWORK CHECK

Basic: 13, 29, 55, 75, 83

Average: 14, 32, 56, 76, 84

Advanced: 20, 34, 56, 80, 84

ANSWERS

1. rationalizing the denominator
2. $\sqrt{6} - 4$
3. yes; $\sqrt{\frac{2x}{9}} = \sqrt{\frac{1}{9} \cdot 2x}$, $= \sqrt{\frac{1}{9}} \cdot \sqrt{2x} = \frac{1}{3}\sqrt{2x}$
4. $-\frac{1}{3}\sqrt{6}$; The other three expressions have like radicals of $\sqrt{3}$.
5. yes
6. no; The radicand is a fraction.
7. no; The radicand has a perfect square factor of 16.
8. yes
9. no; A radical appears in the denominator of a fraction.
10. yes
11. no; A radical appears in the denominator of a fraction.
12. no; The radicand has a perfect cube factor of 27.
13. $2\sqrt{5}$
14. $4\sqrt{2}$
15. $8\sqrt{2}$
16. $-6\sqrt{2}$
17. $5\sqrt{5b}$
18. $2x$
19. $-9m\sqrt{m}$
20. $4n^2\sqrt{3n}$
21. $\frac{2}{7}$
22. $-\frac{\sqrt{7}}{9}$
23. $-\frac{\sqrt{23}}{8}$
24. $\frac{\sqrt{65}}{11}$
25. $\frac{a\sqrt{a}}{7}$
26. $\frac{12}{k}$
27. $\frac{5}{x}$
28. $\frac{5v}{6}$
29. $2\sqrt[3]{2}$
30. $-3\sqrt[3]{4}$
31. $-4x\sqrt[3]{x^2}$
32. $-7\sqrt[3]{n^2}$
33. $-\frac{\sqrt[3]{6c}}{5}$
34. $\frac{2h\sqrt[3]{h}}{3}$
35. $-\frac{3\sqrt[3]{3y^2}}{10x}$
36. $-\frac{\sqrt[3]{21}}{4ab^2}$
37. The radicand 18 has a perfect square factor of 9; $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$
38. The denominator should be $\sqrt[3]{125}$; $\sqrt[3]{\frac{128y^3}{125}} = \frac{\sqrt[3]{128y^3}}{\sqrt[3]{125}} = \frac{4y\sqrt[3]{2}}{5}$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

39. $\frac{\sqrt{6}}{\sqrt{6}}$
40. $\frac{\sqrt{13z}}{\sqrt{13z}}$
41. $\frac{\sqrt[3]{x}}{\sqrt[3]{x}}$
42. $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$
43. $\frac{\sqrt{5}+8}{\sqrt{5}+8}$
44. $\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}-\sqrt{7}}$
45. $\sqrt{2}$
46. $\frac{4\sqrt{3}}{3}$
47. $\frac{\sqrt{15}}{12}$
48. $\frac{\sqrt{13}}{13}$
49. $\frac{3\sqrt{a}}{a}$
50. $\frac{\sqrt{2x}}{2x}$
51. $\frac{d\sqrt{15}}{5}$
52. $\frac{2\sqrt{6n}}{3n^2}$
53. $\frac{4\sqrt[3]{5}}{5}$
54. $\frac{\sqrt[3]{2y}}{6y}$
55. $\frac{\sqrt{7}-1}{6}$
56. $\frac{5+\sqrt{3}}{11}$
57. $\frac{7\sqrt{10}+2\sqrt{5}}{47}$
58. $\frac{6\sqrt{5}-5}{31}$
59. $\sqrt{5}+\sqrt{2}$
60. $\frac{\sqrt{21}-3}{4}$
61. **a.** about 1.85 sec
 b. about 0.68 sec
62. **a.** $P = d\sqrt{d}$
 b. about 11.86 Earth years

In Exercises 39–44, write a factor that you can use to rationalize the denominator of the expression.

39. $\frac{4}{\sqrt{6}}$
40. $\frac{1}{\sqrt{13z}}$
41. $\frac{2}{\sqrt[3]{x^2}}$
42. $\frac{3m}{\sqrt[3]{4}}$
43. $\frac{\sqrt{2}}{\sqrt{5}-8}$
44. $\frac{5}{\sqrt{3}+\sqrt{7}}$

In Exercises 45–54, simplify the expression. *(See Example 4.)*

45. $\frac{2}{\sqrt{2}}$
46. $\frac{4}{\sqrt{3}}$
47. $\frac{\sqrt{5}}{\sqrt{48}}$
48. $\sqrt{\frac{4}{52}}$
49. $\frac{3}{\sqrt{a}}$
50. $\frac{1}{\sqrt{2x}}$
51. $\sqrt{\frac{3d^2}{5}}$
52. $\frac{\sqrt{8}}{\sqrt{3n^3}}$
53. $\frac{4}{\sqrt[3]{25}}$
54. $\sqrt[3]{\frac{1}{108y^2}}$

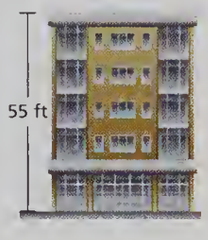
In Exercises 55–60, simplify the expression. *(See Example 5.)*

55. $\frac{1}{\sqrt{7}+1}$
56. $\frac{2}{5-\sqrt{3}}$
57. $\frac{\sqrt{10}}{7-\sqrt{2}}$
58. $\frac{\sqrt{5}}{6+\sqrt{5}}$
59. $\frac{3}{\sqrt{5}-\sqrt{2}}$
60. $\frac{\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

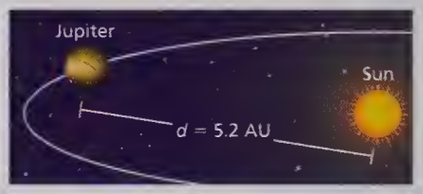
61. **MODELING WITH MATHEMATICS** The time t (in seconds) it takes an object to hit the ground is given by $t = \sqrt{\frac{h}{16}}$, where h is the height (in feet) from which the object was dropped. *(See Example 6.)*

 a. How long does it take an earring to hit the ground when it falls from the roof of the building?

 b. How much sooner does the earring hit the ground when it is dropped from two stories (22 feet) below the roof?



62. **MODELING WITH MATHEMATICS** The orbital period of a planet is the time it takes the planet to travel around the Sun. You can find the orbital period P (in Earth years) using the formula $P = \sqrt{d^3}$, where d is the average distance (in astronomical units, abbreviated AU) of the planet from the Sun.



 a. Simplify the formula.

 b. What is Jupiter's orbital period?

63. **MODELING WITH MATHEMATICS** The electric current I (in amperes) an appliance uses is given by the formula $I = \sqrt{\frac{P}{R}}$, where P is the power (in watts) and R is the resistance (in ohms). Find the current an appliance uses when the power is 147 watts and the resistance is 5 ohms.



64. **MODELING WITH MATHEMATICS** You can find the average annual interest rate r (in decimal form) of a savings account using the formula $r = \sqrt{\frac{V_2}{V_0}} - 1$, where V_0 is the initial investment and V_2 is the balance of the account after 2 years. Use the formula to compare the savings accounts. In which account would you invest money? Explain.

Account	Initial investment	Balance after 2 years
1	\$275	\$293
2	\$361	\$382
3	\$199	\$214
4	\$254	\$272
5	\$386	\$406

63. about 5.42 amperes
64. Account 3; It has the greatest interest rate.

In Exercises 65–68, evaluate the function for the given value of x. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

65. $h(x) = \sqrt{5x}; x = 10$ **66.** $g(x) = \sqrt{3x}, x = 60$

67. $r(x) = \sqrt{\dfrac{3x}{3x^2 + 6}}; x = 4$

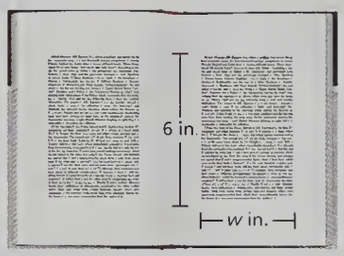
68. $p(x) = \sqrt{\dfrac{x - 1}{5x}}; x = 8$

In Exercises 69–72, evaluate the expression when $a = -2$, $b = 8$, and $c = \frac{1}{2}$. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

69. $\sqrt{a^2 + bc}$ **70.** $-\sqrt{4c - 6ab}$

71. $-\sqrt{2a^2 + b^2}$ **72.** $\sqrt{b^2 - 4ac}$

73. MODELING WITH MATHEMATICS The text in the book shown forms a golden rectangle. What is the width w of the text? *(See Example 7.)*



74. MODELING WITH MATHEMATICS The flag of Togo is approximately the shape of a golden rectangle. What is the width w of the flag?



In Exercises 75–82, simplify the expression. *(See Example 8.)*

75. $\sqrt{3} - 2\sqrt{2} + 6\sqrt{2}$ **76.** $\sqrt{5} - 5\sqrt{13} - 8\sqrt{5}$

77. $2\sqrt{6} - 5\sqrt{54}$ **78.** $9\sqrt{32} + \sqrt{2}$

79. $\sqrt{12} + 6\sqrt{3} + 2\sqrt{6}$ **80.** $3\sqrt{7} - 5\sqrt{14} + 2\sqrt{28}$

81. $\sqrt[3]{-81} + 4\sqrt[3]{3}$ **82.** $6\sqrt[3]{128t} - 2\sqrt[3]{2t}$

In Exercises 83–90, simplify the expression. *(See Example 9.)*

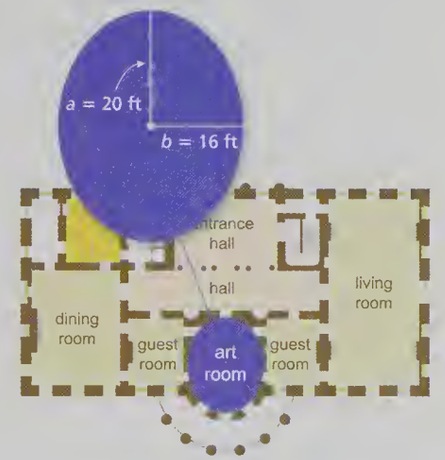
83. $\sqrt{2}(\sqrt{45} + \sqrt{5})$ **84.** $\sqrt{3}(\sqrt{72} - 3\sqrt{2})$

85. $\sqrt{5}(2\sqrt{6x} - \sqrt{96x})$ **86.** $\sqrt{7y}(\sqrt{27y} + 5\sqrt{12y})$

87. $(4\sqrt{2} - \sqrt{98})^2$ **88.** $(\sqrt{3} + \sqrt{48})(\sqrt{20} - \sqrt{5})$

89. $\sqrt[3]{3}(\sqrt[3]{4} + \sqrt[3]{32})$ **90.** $\sqrt[3]{2}(\sqrt[3]{135} - 4\sqrt[3]{5})$

91. MODELING WITH MATHEMATICS The circumference C of the art room in a mansion is approximated by the formula $C \approx 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$. Approximate the circumference of the room.



92. CRITICAL THINKING Determine whether each expression represents a *rational* or an *irrational* number. Justify your answer.

a. $4 + \sqrt{6}$ **b.** $\dfrac{\sqrt{48}}{\sqrt{3}}$

c. $\dfrac{8}{\sqrt{12}}$ **d.** $\sqrt{3} + \sqrt{7}$

e. $\dfrac{a}{\sqrt{10} - \sqrt{2}}$, where a is a positive integer

f. $\dfrac{2 + \sqrt{5}}{2b + \sqrt{5b^2}}$, where b is a positive integer

In Exercises 93–98, simplify the expression.

93. $\sqrt[5]{\dfrac{13}{5x^5}}$ **94.** $\sqrt[4]{\dfrac{10}{81}}$

95. $\sqrt[4]{256y}$ **96.** $\sqrt[5]{160x^6}$

97. $6\sqrt[4]{9} - \sqrt[5]{9} + 3\sqrt[4]{9}$ **98.** $\sqrt[5]{2}(\sqrt[4]{7} + \sqrt[5]{16})$

ANSWERS

65. $5\sqrt{2}$, about 7.07

66. $6\sqrt{5}$, about 13.42

67. $\dfrac{\sqrt{2}}{3}$, about 0.47

68. $\dfrac{\sqrt{70}}{20}$, about 0.42

69. $2\sqrt{2}$, about 2.83

70. $-7\sqrt{2}$, about -9.90

71. $-6\sqrt{2}$, about -8.49

72. $2\sqrt{17}$, about 8.25

73. about 3.71 in.

74. about 25.96 in.

75. $\sqrt{3} + 4\sqrt{2}$

76. $-7\sqrt{5} - 5\sqrt{13}$

77. $-13\sqrt{6}$

78. $37\sqrt{2}$

79. $8\sqrt{3} + 2\sqrt{6}$

80. $7\sqrt{7} - 5\sqrt{14}$

81. $\sqrt[3]{3}$

82. $22\sqrt[3]{2t}$

83. $4\sqrt{10}$

84. $3\sqrt{6}$

85. $-2\sqrt{30x}$

86. $13y\sqrt{21}$

87. 18

88. $5\sqrt{15}$

89. $3\sqrt[3]{12}$

90. $-\sqrt[3]{10}$

91. about 114 ft

92. **a.** irrational; Six is not a perfect square.

b. rational; Four is a rational number.

c. irrational; Twelve is not a perfect square.

d. irrational; Three and seven are not perfect squares.

e. irrational; Two and ten are not perfect squares.

f. rational; $\frac{1}{b}$ is rational when b is a positive integer.

93. $\dfrac{\sqrt[5]{8125}}{5x}$

94. $\dfrac{\sqrt[4]{10}}{3}$

95. $4\sqrt[4]{y}$

96. $2x\sqrt[5]{5x}$

97. $9\sqrt[4]{9} - \sqrt[5]{9}$

98. $\sqrt[5]{2}\sqrt[4]{7} + 2$

ANSWERS

99. See Additional Answers.

100. a. always; The sum of two fractions can be written as a fraction.

b. always; The sum cannot be written as the ratio of two integers.

c. sometimes; $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$, but $\sqrt{3} + (-\sqrt{3}) = 0$.

d. always; The product of two fractions can be written as a fraction.

e. always; The product cannot be written as the ratio of two integers.

f. sometimes; $\sqrt{3} \cdot \pi = \pi\sqrt{3}$, but $\sqrt{3} \cdot \sqrt{3} = 3$.

101. odd; even; When m is even, 2^m is a perfect square.

102. *Sample answer:* $\sqrt[3]{2}$

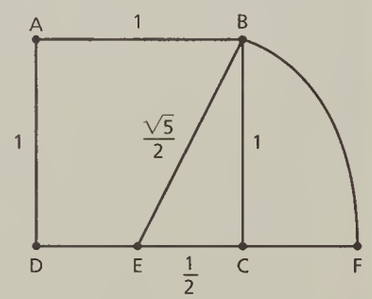
103. $a^2 < ab < b^2$ when $a < b$.

104. no; Applying the sum and difference pattern to the denominator results in $16 - \sqrt[3]{5^2}$, which still contains a radical.

105. 377

106. a. $\left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{1+\sqrt{5}}{2} - 1 = 0$, $\left(\frac{1-\sqrt{5}}{2}\right)^2 - \frac{1-\sqrt{5}}{2} - 1 = 0$

b. *Sample answer:* $DF = \frac{1+\sqrt{5}}{2}$



107–115. See Additional Answers.

REASONING In Exercises 99 and 100, use the table shown.

	2	$\frac{1}{4}$	0	$\sqrt{3}$	$-\sqrt{3}$	π
2						
$\frac{1}{4}$						
0						
$\sqrt{3}$						
$-\sqrt{3}$						
π						

99. Copy and complete the table by (a) finding each sum $\left(2 + 2, 2 + \frac{1}{4}, \text{etc.}\right)$ and (b) finding each product $\left(2 \cdot 2, 2 \cdot \frac{1}{4}, \text{etc.}\right)$.

100. Use your answers in Exercise 99 to determine whether each statement is *always*, *sometimes*, or *never* true. Justify your answer.

a. The sum of a rational number and a rational number is rational.

b. The sum of a rational number and an irrational number is irrational.

c. The sum of an irrational number and an irrational number is irrational.

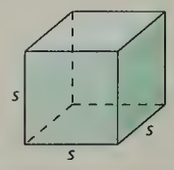
d. The product of a rational number and a rational number is rational.

e. The product of a nonzero rational number and an irrational number is irrational.

f. The product of an irrational number and an irrational number is irrational.

101. REASONING Let m be a positive integer. For what values of m will the simplified form of the expression $\sqrt{2^m}$ contain a radical? For what values will it *not* contain a radical? Explain.

102. HOW DO YOU SEE IT? The edge length s of a cube is an irrational number, the surface area is an irrational number, and the volume is a rational number. Give a possible value of s.



103. REASONING Let a and b be positive numbers. Explain why $\sqrt{ab}$ lies between a and b on a number line. (*Hint:* Let $a < b$ and multiply each side of $a < b$ by a. Then let $a < b$ and multiply each side by b.)

104. MAKING AN ARGUMENT Your friend says that you can rationalize the denominator of the expression $\frac{2}{4 + \sqrt[3]{5}}$ by multiplying the numerator and denominator by $4 - \sqrt[3]{5}$. Is your friend correct? Explain.

105. PROBLEM SOLVING The ratio of consecutive terms $\frac{a_n}{a_{n-1}}$ in the Fibonacci sequence gets closer and closer to the golden ratio $\frac{1+\sqrt{5}}{2}$ as n increases. Find the term that precedes 610 in the sequence.

106. THOUGHT PROVOKING Use the golden ratio $\frac{1+\sqrt{5}}{2}$ and the golden ratio conjugate $\frac{1-\sqrt{5}}{2}$ for each of the following.

a. Show that the golden ratio and golden ratio conjugate are both solutions of $x^2 - x - 1 = 0$.

b. Construct a geometric diagram that has the golden ratio as the length of a part of the diagram.

107. CRITICAL THINKING Use the special product pattern $(a + b)(a^2 - ab + b^2) = a^3 + b^3$ to simplify the expression $\frac{2}{\sqrt[3]{x} + 1}$. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph the linear equation. Identify the x-intercept. *(Section 3.5)*

108. $y = x - 4$ **109.** $y = -2x + 6$ **110.** $y = -\frac{1}{3}x - 1$ **111.** $y = \frac{3}{2}x + 6$

Solve the equation by graphing. Check your solution. *(Section 5.5)*

112. $-3x = -2x + 1$ **113.** $5x + 3 = \frac{3}{2}x - 4$ **114.** $4x - 3 = 8 - \frac{2}{5}x$ **115.** $6x - 1 = -4(x - 1)$

Mini-Assessment

Simplify the expression.

1. $\sqrt{144x^3}$ $12x\sqrt{x}$
2. $\sqrt[3]{-54x^3}$ $-3x\sqrt[3]{2}$
3. $\frac{7}{\sqrt{2}+3}$ $-\sqrt{2}+3$
4. $4\sqrt{5} - \sqrt{3} + 3\sqrt{5}$ $7\sqrt{5} - \sqrt{3}$
5. $\sqrt{3}(\sqrt{2} - \sqrt{8})$ $-\sqrt{6}$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

9.2 Solving Quadratic Equations by Graphing

Texas Essential Knowledge and Skills
A.7.A
A.8.B

Essential Question How can you use a graph to solve a quadratic equation in one variable?

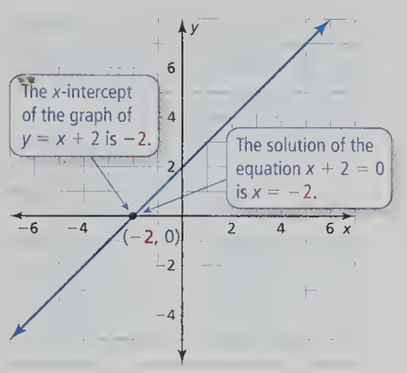
Based on what you learned about the x-intercepts of a graph in Section 3.4, it follows that the x-intercept of the graph of the linear equation

$y = ax + b$ 2 variables

is the same value as the solution of

$ax + b = 0$. 1 variable

You can use similar reasoning to solve *quadratic equations*.



EXPLORATION 1 Solving a Quadratic Equation by Graphing

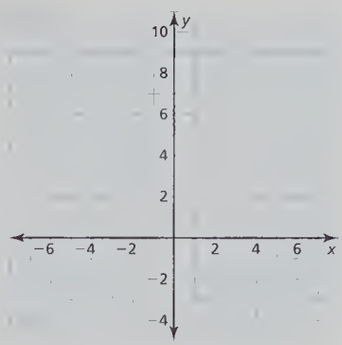
Work with a partner.

a. Sketch the graph of $y = x^2 - 2x$.

b. What is the definition of an x-intercept of a graph? How many x-intercepts does this graph have? What are they?

c. What is the definition of a solution of an equation in x? How many solutions does the equation $x^2 - 2x = 0$ have? What are they?

d. Explain how you can verify the solutions you found in part (c).



EXPLORATION 2 Solving Quadratic Equations by Graphing

Work with a partner. Solve each equation by graphing.

a. $x^2 - 4 = 0$
b. $x^2 + 3x = 0$
c. $-x^2 + 2x = 0$
d. $x^2 - 2x + 1 = 0$
e. $x^2 - 3x + 5 = 0$
f. $-x^2 + 3x - 6 = 0$

USING PROBLEM-SOLVING STRATEGIES

To be proficient in math, you need to check your answers to problems using a different method and continually ask yourself, "Does this make sense?"

Communicate Your Answer

3. How can you use a graph to solve a quadratic equation in one variable?
4. After you find a solution graphically, how can you check your result algebraically? Check your solutions for parts (a)–(d) in Exploration 2 algebraically.
5. How can you determine graphically that a quadratic equation has no solution?

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the Essential Question. Have students work in pairs to complete the explorations and Communicate Your Answer. Have them share their answers with the class.

Beginning/Advanced When sharing answers, Beginning students use simple phrases to provide details and Advanced students use multiple sentences.

Intermediate/Advanced High When sharing answers, Intermediate students use simple sentences to provide details and Advanced High students use multiple complex sentences.

ELPS 3.H.3 Explain with increasing specificity and detail as more English is acquired.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

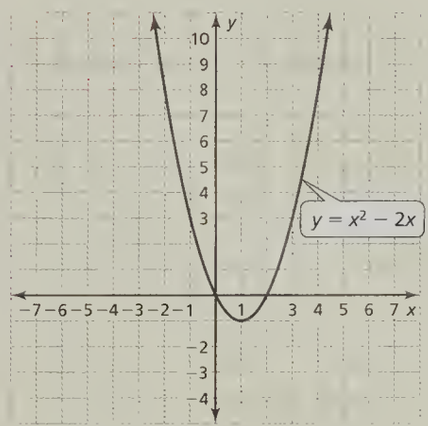
Texas Essential Knowledge and Skills

A.7.A The student is expected to graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

A.8.B The student is expected to write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

ANSWERS

1. a.



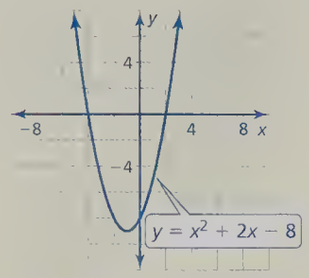
 b. the x-coordinate of a point where the graph crosses the x-axis; 2; 0, 2
 c. a value of x that makes the equation true; 2; 0, 2
 d. Substitute the solutions into the equation.

2. a. $x = 2, x = -2$
 b. $x = 0, x = -3$
 c. $x = 0, x = 2$
 d. $x = 1$
 e. no solution
 f. no solution

3. Write the equation in standard form $ax^2 + bx + c = 0$, graph the related function $y = ax^2 + bx + c$, and find the x-intercepts.
4. Substitute the solutions into the equation.
5. See Additional Answers.

Extra Example 1

Solve $x^2 + 2x = 8$ by graphing.



The solutions are $x = -4$ and $x = 2$.

MONITORING PROGRESS ANSWERS

1. $x = 2, x = -1$
2. $x = -5, x = -2$
3. $x = -4, x = 3$

9.2 Lesson

Core Vocabulary

quadratic equation, *p. 476*

Previous

x-intercept
root
zero of a function

What You Will Learn

- Solve quadratic equations by graphing.
- Use graphs to find and approximate the zeros of functions.
- Solve real-life problems using graphs of quadratic functions.

Solving Quadratic Equations by Graphing

A **quadratic equation** is a nonlinear equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

In Chapter 7, you solved quadratic equations by factoring. You can also solve quadratic equations by graphing.

Core Concept

Solving Quadratic Equations by Graphing

Step 1 Write the equation in standard form, $ax^2 + bx + c = 0$.

Step 2 Graph the related function $y = ax^2 + bx + c$.

Step 3 Find the *x*-intercepts, if any.

The solutions, or *roots*, of $ax^2 + bx + c = 0$ are the *x*-intercepts of the graph.

EXAMPLE 1 Solving a Quadratic Equation: Two Real Solutions

Solve $x^2 + 2x = 3$ by graphing.

SOLUTION

Step 1 Write the equation in standard form.

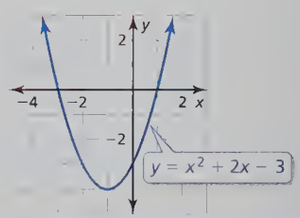
$x^2 + 2x = 3$ Write original equation.

$x^2 + 2x - 3 = 0$ Subtract 3 from each side.

Step 2 Graph the related function $y = x^2 + 2x - 3$.

Step 3 Find the *x*-intercepts.
The *x*-intercepts are -3 and 1.

So, the solutions are $x = -3$ and $x = 1$.



Check

$x^2 + 2x = 3$	Original equation	$x^2 + 2x = 3$
$(-3)^2 + 2(-3) \stackrel{?}{=} 3$	Substitute.	$1^2 + 2(1) \stackrel{?}{=} 3$
$3 = 3$ ✓	Simplify.	$3 = 3$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation by graphing. Check your solutions.

1. $x^2 - x - 2 = 0$
2. $x^2 + 7x = -10$
3. $x^2 + x = 12$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

Differentiated Instruction

Auditory

Have students verbally describe the steps they would use to graph a quadratic equation such as $y = 3x^2 - 3$. Ask them to explain how they can predict whether the graph will open up or down. Be sure students understand that this graph will open up because the coefficient of x^2 is greater than 0.

EXAMPLE 2 Solving a Quadratic Equation: One Real Solution

Solve $x^2 - 8x = -16$ by graphing.

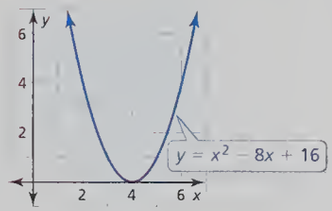
SOLUTION

Step 1 Write the equation in standard form.

$x^2 - 8x = -16$ Write original equation.

$x^2 - 8x + 16 = 0$ Add 16 to each side.

Step 2 Graph the related function $y = x^2 - 8x + 16$.



Step 3 Find the x-intercept. The only x-intercept is at the vertex, $(4, 0)$.

▶ So, the solution is $x = 4$.

ANOTHER WAY

You can also solve the equation in Example 2 by factoring.

$x^2 - 8x + 16 = 0$

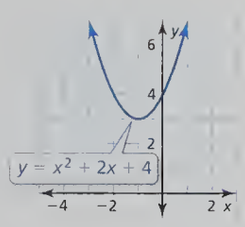
$(x - 4)(x - 4) = 0$

So, $x = 4$.

EXAMPLE 3 Solving a Quadratic Equation: No Real Solutions

Solve $-x^2 = 2x + 4$ by graphing.

SOLUTION



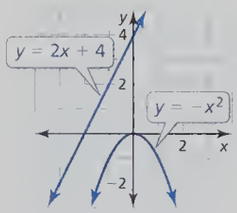
Method 1 Write the equation in standard form, $x^2 + 2x + 4 = 0$. Then graph the related function $y = x^2 + 2x + 4$, as shown at the left.

▶ There are no x-intercepts. So, $-x^2 = 2x + 4$ has no real solutions.

Method 2 Graph each side of the equation.

$y = -x^2$ Left side

$y = 2x + 4$ Right side



▶ The graphs do not intersect.
So, $-x^2 = 2x + 4$ has no real solutions.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation by graphing.

4. $x^2 + 36 = 12x$ **5.** $x^2 + 4x = 0$ **6.** $x^2 + 10x = -25$

7. $x^2 = 3x - 3$ **8.** $x^2 + 7x = -6$ **9.** $2x + 5 = -x^2$

Concept Summary

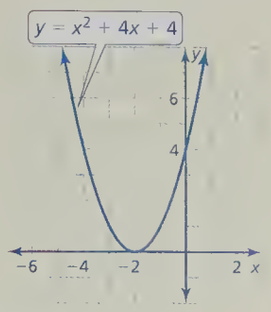
Number of Solutions of a Quadratic Equation

A quadratic equation has:

- two real solutions when the graph of its related function has two x-intercepts.
- one real solution when the graph of its related function has one x-intercept.
- no real solutions when the graph of its related function has no x-intercepts.

Extra Example 2

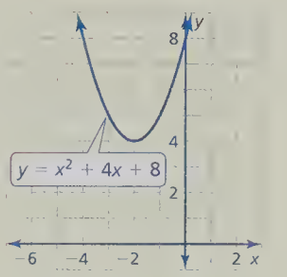
Solve $x^2 + 4x = -4$ by graphing.



The solution is $x = -2$.

Extra Example 3

Solve $-x^2 = 4x + 8$ by graphing.



$-x^2 = 4x + 8$ has no real solutions.

MONITORING PROGRESS ANSWERS

4. $x = 6$

5. $x = 0, x = -4$

6. $x = -5$

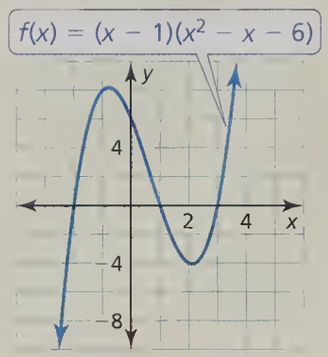
7. no solution

8. $x = -6, x = -1$

9. no solution

Extra Example 4

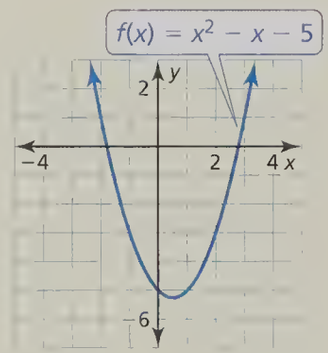
The graph of $f(x) = (x - 1)(x^2 - x - 6)$ is shown. Find the zeros of f.



The zeros of f are -2, 1, and 3.

Extra Example 5

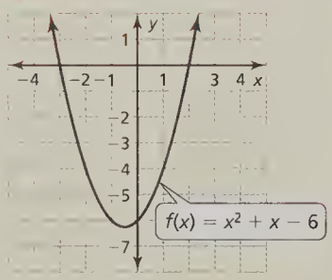
The graph of $f(x) = x^2 - x - 5$ is shown. Approximate the zeros of f to the nearest tenth.



The zeros of f are about -1.8 and 2.8.

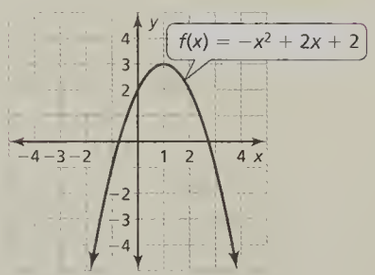
MONITORING PROGRESS ANSWERS

10.



-3, 2

11.

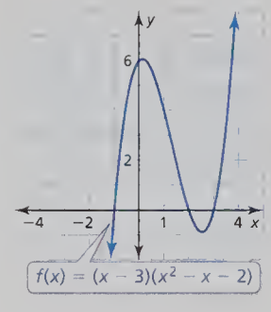


about -0.7, about 2.7

Finding Zeros of Functions

Recall that a zero of a function is an x-intercept of the graph of the function.

EXAMPLE 4 Finding the Zeros of a Function



The graph of $f(x) = (x - 3)(x^2 - x - 2)$ is shown. Find the zeros of f.

SOLUTION

The x-intercepts are -1, 2, and 3.

▶ So, the zeros of f are -1, 2, and 3.

Check

$f(-1) = (-1 - 3)[(-1)^2 - (-1) - 2] = 0$ ✓

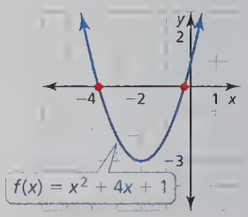
$f(2) = (2 - 3)(2^2 - 2 - 2) = 0$ ✓

$f(3) = (3 - 3)(3^2 - 3 - 2) = 0$ ✓

The zeros of a function are not necessarily integers. To approximate zeros, analyze the signs of function values. When two function values have different signs, a zero lies between the x-values that correspond to the function values.

EXAMPLE 5 Approximating the Zeros of a Function

The graph of $f(x) = x^2 + 4x + 1$ is shown. Approximate the zeros of f to the nearest tenth.



SOLUTION

There are two x-intercepts: one between -4 and -3, and another between -1 and 0.

Make tables using x-values between -4 and -3, and between -1 and 0. Use an increment of 0.1. Look for a change in the signs of the function values.

x	−3.9	−3.8	−3.7	−3.6	−3.5	−3.4	−3.3	−3.2	−3.1
$f(x)$	0.61	0.24	−0.11	−0.44	−0.75	−1.04	−1.31	−1.56	−1.79

change in signs

x	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1
$f(x)$	−1.79	−1.56	−1.31	−1.04	−0.75	−0.44	−0.11	0.24	0.61

change in signs

The function values that are closest to 0 correspond to x-values that best approximate the zeros of the function.

▶ In each table, the function value closest to 0 is -0.11. So, the zeros of f are about -3.7 and -0.3.

ANOTHER WAY

You could approximate one zero using a table and then use the axis of symmetry to find the other zero.

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10. Graph $f(x) = x^2 + x - 6$. Find the zeros of f.

11. Graph $f(x) = -x^2 + 2x + 2$. Approximate the zeros of f to the nearest tenth.



Solving Real-Life Problems

EXAMPLE 6 **Real-Life Application**

A football player kicks a football 2 feet above the ground with an initial vertical velocity of 75 feet per second. The function $h = -16t^2 + 75t + 2$ represents the height h (in feet) of the football after t seconds. (a) Find the height of the football each second after it is kicked. (b) Use the results of part (a) to estimate when the height of the football is 50 feet. (c) Using a graph, after how many seconds is the football 50 feet above the ground?

SOLUTION

a. Make a table of values starting with $t = 0$ seconds using an increment of 1. Continue the table until a function value is negative.

Seconds, t	Height, h
0	2
1	61
2	88
3	83
4	46
5	−23

▶ The height of the football is 61 feet after 1 second, 88 feet after 2 seconds, 83 feet after 3 seconds, and 46 feet after 4 seconds.

b. From part (a), you can estimate that the height of the football is 50 feet between 0 and 1 second and between 3 and 4 seconds.

▶ Based on the function values, it is reasonable to estimate that the height of the football is 50 feet slightly less than 1 second and slightly less than 4 seconds after it is kicked.

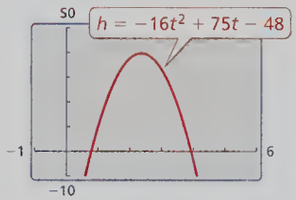
c. To determine when the football is 50 feet above the ground, find the t-values for which $h = 50$. So, solve the equation $-16t^2 + 75t + 2 = 50$ by graphing.

Step 1 Write the equation in standard form.

$-16t^2 + 75t + 2 = 50$ Write the equation.

$-16t^2 + 75t - 48 = 0$ Subtract 50 from each side.

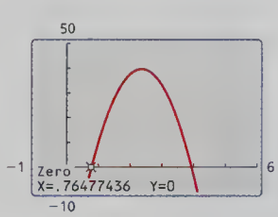
Step 2 Use a graphing calculator to graph the related function $h = -16t^2 + 75t - 48$.

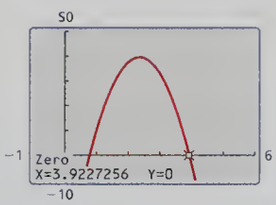


REMEMBER

Equations have *solutions*, or *roots*. Graphs have *x-intercepts*. Functions have *zeros*.

Step 3 Use the *zero* feature to find the zeros of the function.





▶ The football is 50 feet above the ground after about 0.8 second and about 3.9 seconds, which supports the estimates in part (b).

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12. WHAT IF? After how many seconds is the football 65 feet above the ground?

Extra Example 6

A soccer player kicks a soccer ball 2 feet above the ground with an initial vertical velocity of 60 feet per second. The function $h = -16t^2 + 60t + 2$ represents the height h (in feet) of the soccer ball after t seconds.

a. Find the height of the soccer ball each second after it is kicked. The height of the soccer ball is 46 feet after 1 second, 58 feet after 2 seconds, and 38 feet after 3 seconds.

b. Use the results of part (a) to estimate when the height of the soccer ball is 40 feet. The height of the soccer ball is 40 feet slightly less than 1 second and slightly less than 3 seconds after it is kicked.

c. Using a graph, after how many seconds is the soccer ball 40 feet above the ground? The soccer ball is 40 feet above the ground after about 0.8 second and about 2.9 seconds.

MONITORING PROGRESS ANSWERS

12. about 1.1 sec, about 3.6 sec

Extra Example 7

The table shows the recorded temperatures on one day.

Time	Temperature (°F)
6 A.M.	45
8 A.M.	51
10 A.M.	60
12 P.M.	68
2 P.M.	70
4 P.M.	64
6 P.M.	60

a. Use a graphing calculator to find a quadratic model for the data. $y = -0.41x^2 + 11.2x - 10$, where x represents the number of hours after midnight

b. At what time(s) during the day is the temperature 64°F? about 11.2, or 11:12 A.M., and about 16.1, or 4:06 P.M.

MONITORING PROGRESS ANSWERS

13. a. $y = 0.03x^2 - 0.1x + 2$; Yes, the graph of the equation passes through or is close to all of the data points.

b. 30 days

In Section 4.6, you used a graphing calculator to perform linear regression on a set of data to find a linear model for the data. You can also perform *quadratic regression*.

EXAMPLE 7 Finding a Quadratic Model Using Technology

The table shows the recorded temperatures (in degrees Fahrenheit) for a portion of a day. (a) Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit. (b) At what time(s) during the day is the temperature 77°F?

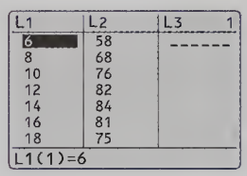
Time	Temperature (°F)
6 A.M.	58
8 A.M.	68
10 A.M.	76
12 P.M.	82
2 P.M.	84
4 P.M.	81
6 P.M.	75

STUDY TIP

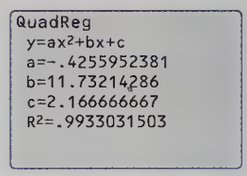
Notice that the graphing calculator does not calculate the correlation coefficient r, but it does calculate R^2, which is called the coefficient of determination. An R^2 value that is close to 1 also indicates that the model is a good fit for the data.

SOLUTION

a. Step 1 Enter the data from the table into two lists. Let x represent the number of hours after midnight.

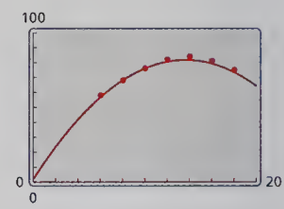


Step 2 Use the *quadratic regression* feature. The values in the equation can be rounded to obtain $y = -0.43x^2 + 11.7x + 2$.

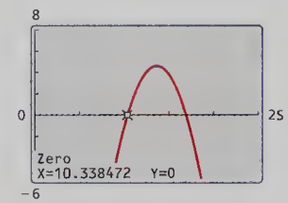


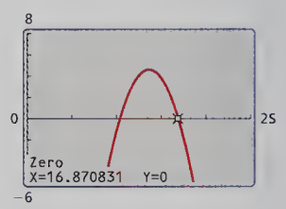
Step 3 Enter the equation $y = -0.43x^2 + 11.7x + 2$ into the calculator. Then plot the data and graph the equation in the same viewing window.

▶ The graph of the equation passes through or is close to all of the data points. So, the model is a good fit.



b. Find the x-values for which $y = 77$ by writing $-0.43x^2 + 11.7x + 2 = 77$ in standard form, graphing the related function $y = -0.43x^2 + 11.7x - 75$, and finding its zeros.





JUSTIFYING THE SOLUTION

From the table, you can estimate that the temperature is 77°F between 10 A.M. and 12 P.M. and between 4 P.M. and 6 P.M. So, your answers are reasonable.

▶ The temperature is 77°F at about 10.3, or 10:18 A.M., and at about 16.9, or 4:54 P.M.

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13. After a break, two students come to school with the flu. The table shows the total numbers of students infected with the flu x days after the break. (a) Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit. (b) How many days after the break are 26 students infected?

Days after break	0	7	14	21	28	35	42	49
Students with flu	2	3	5	8	18	30	42	55

Closure

- When a quadratic equation has two solutions, what do you know about the graph of its related function? It has two x-intercepts.

9.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What is a quadratic equation?
2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

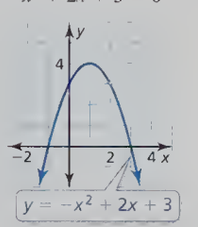
$x^2 + 5x = 20$ $\quad x^2 + x - 4 = 0$ $\quad x^2 - 6 = 4x$ $\quad 7x + 12 = x^2$

3. **WRITING** How can you use a graph to find the number of solutions of a quadratic equation?
4. **WRITING** How are solutions, roots, x-intercepts, and zeros related?

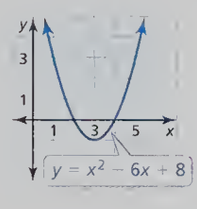
Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, use the graph to solve the equation.

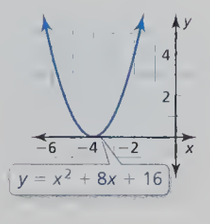
5. $-x^2 + 2x + 3 = 0$



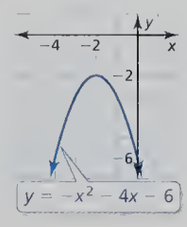
6. $x^2 - 6x + 8 = 0$



7. $x^2 + 8x + 16 = 0$



8. $-x^2 - 4x - 6 = 0$



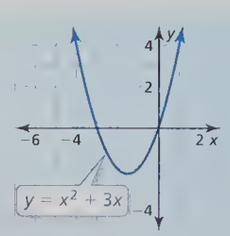
In Exercises 9–12, write the equation in standard form.

9. $4x^2 = 12$
10. $-x^2 = 15$
11. $2x - x^2 = 1$
12. $5 + x = 3x^2$

In Exercises 13–24, solve the equation by graphing. *(See Examples 1, 2, and 3.)*

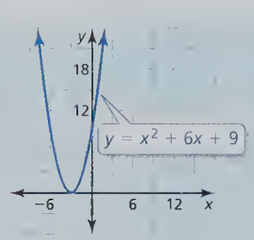
13. $x^2 - 5x = 0$
14. $x^2 - 4x + 4 = 0$
15. $x^2 - 2x + 5 = 0$
16. $x^2 - 6x - 7 = 0$
17. $x^2 = 6x - 9$
18. $-x^2 = 8x + 20$
19. $x^2 = -1 - 2x$
20. $x^2 = -x - 3$
21. $4x - 12 = -x^2$
22. $5x - 6 = x^2$
23. $x^2 - 2 = -x$
24. $16 - x^2 = -8x$

25. **ERROR ANALYSIS** Describe and correct the error in solving $x^2 + 3x = 18$ by graphing.



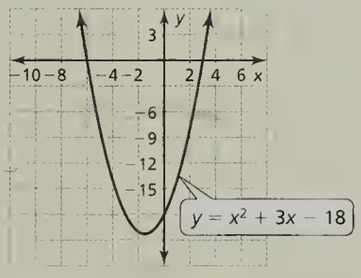
The solutions of the equation $x^2 + 3x = 18$ are $x = -3$ and $x = 0$.

26. **ERROR ANALYSIS** Describe and correct the error in solving $x^2 + 6x + 9 = 0$ by graphing.



The solution of the equation $x^2 + 6x + 9 = 0$ is $x = 9$.

25. The equation needs to be in standard form; $x^2 + 3x - 18 = 0$



The solutions are $x = -6$ and $x = 3$.

26. The solution is the x-intercept, not the y-intercept; The solution is $x = -3$.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–29 odd, 37–49 odd, 53, 55, 62, 66, 72–77

Average: 1–3, 4–62 even, 66, 72–77

Advanced: 1–4, 8, 12, 20–32 even, 40–62 even, 63–77

HOMEWORK CHECK

Basic: 13, 19, 37, 43, 53

Average: 14, 20, 38, 44, 54

Advanced: 20, 24, 40, 44, 54

ANSWERS

1. an equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$
2. $x^2 + x - 4 = 0$; It is the only equation written in standard form.
3. The number of x-intercepts is the number of solutions.
4. They are the same.
5. $x = 3, x = -1$
6. $x = 4, x = 2$
7. $x = -4$
8. no solution
9. $4x^2 - 12 = 0$ or $-4x^2 + 12 = 0$
10. $x^2 + 15 = 0$ or $-x^2 - 15 = 0$
11. $x^2 - 2x + 1 = 0$ or $-x^2 + 2x - 1 = 0$
12. $3x^2 - x - 5 = 0$ or $-3x^2 + x + 5 = 0$
13. $x = 0, x = 5$
14. $x = 2$
15. no solution
16. $x = 7, x = -1$
17. $x = 3$
18. no solution
19. $x = -1$
20. no solution
21. $x = -6, x = 2$
22. $x = 3, x = 2$
23. $x = -2, x = 1$
24. $x \approx 9.7$ or $x \approx -1.7$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

27. **a.** The x-intercepts are the distances at which the height is 0 yards.

b. 5 yd

28. **a.** no; One of the t-intercepts is negative.

b. 2 sec

29. $x = -5, x = 2$

30. no solution

31. no solution

32. $x = 5, x = 1$

33. $x = -10, x = -2$

34. $x = -9, x = 1$

35. no solution

36. $x = 2$

37. $2, 0, -1$

38. $-1, -4, -2$

39. $-3, 1$

40. 5

41. $-2, 2, -3, 1$

42. $-1, 2$

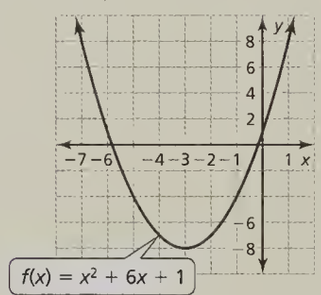
43. about 4.3, about 0.7

44. about 0.3, about -3.3

45. about 2.4, about -0.4

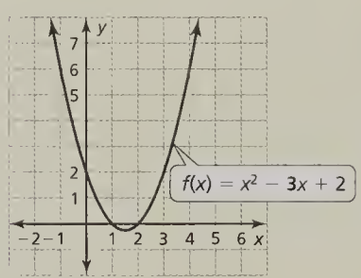
46. about 5.6, about 0.4

47.



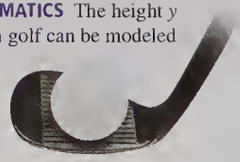
about -0.2, about -5.8

48.



1, 2

27. **MODELING WITH MATHEMATICS** The height y (in yards) of a flop shot in golf can be modeled by $y = -x^2 + 5x$, where x is the horizontal distance (in yards).



a. Interpret the x-intercepts of the graph of the equation.

b. How far away does the golf ball land?

28. **MODELING WITH MATHEMATICS** The height h (in feet) of an underhand volleyball serve can be modeled by $h = -16t^2 + 30t + 4$, where t is the time (in seconds).

a. Do both t-intercepts of the graph of the function have meaning in this situation? Explain.

b. No one receives the serve. After how many seconds does the volleyball hit the ground?

In Exercises 29–36, solve the equation by using Method 2 from Example 3.

29. $x^2 = 10 - 3x$

30. $2x - 3 = x^2$

31. $5x - 7 = x^2$

32. $x^2 = 6x - 5$

33. $x^2 + 12x = -20$

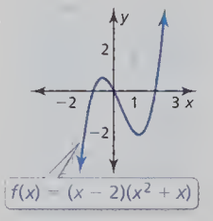
34. $x^2 + 8x = 9$

35. $-x^2 - 5 = -2x$

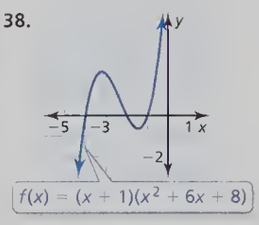
36. $-x^2 - 4 = -4x$

In Exercises 37–42, find the zero(s) of f. *(See Example 4.)*

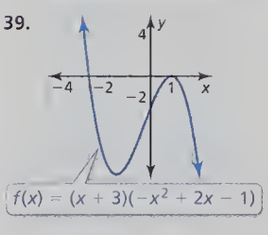
37.



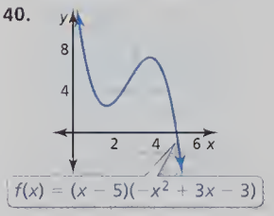
38.



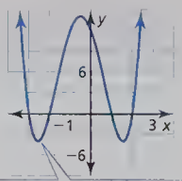
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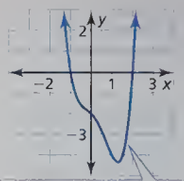
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41.

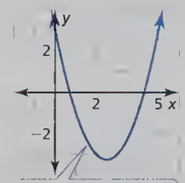


42.

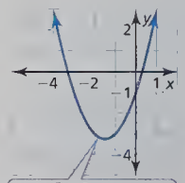


In Exercises 43–46, approximate the zeros of f to the nearest tenth. *(See Example 5.)*

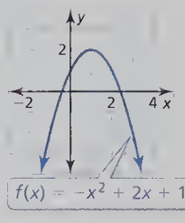
43.



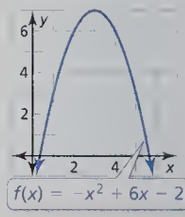
44.



45.



46.



In Exercises 47–52, graph the function. Approximate the zeros of the function to the nearest tenth, if necessary.

47. $f(x) = x^2 + 6x + 1$

48. $f(x) = x^2 - 3x + 2$

49. $y = -x^2 + 4x - 2$

50. $y = -x^2 + 9x - 6$

51. $f(x) = \frac{1}{2}x^2 + 2x - 5$

52. $f(x) = -3x^2 + 4x + 3$

53. **MODELING WITH MATHEMATICS** At a Civil War reenactment, a cannonball is fired into the air with an initial vertical velocity of 128 feet per second. The release point is 6 feet above the ground. The function $h = -16t^2 + 128t + 6$ represents the height h (in feet) of the cannonball after t seconds. *(See Example 6.)*

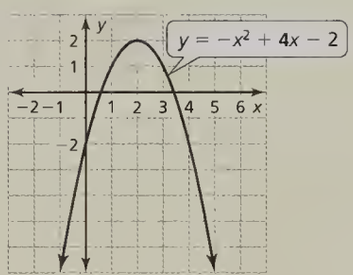


a. Find the height of the cannonball each second after it is fired.

b. Use the results of part (a) to estimate when the height of the cannonball is 150 feet.

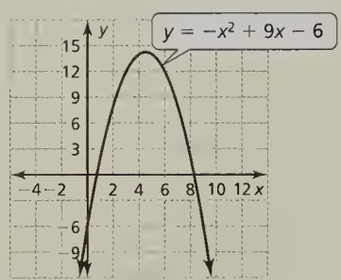
c. Using a graph, after how many seconds is the cannonball 150 feet above the ground?

49.



about 3.4, about 0.6

50.



about 8.3, about 0.7

51–53. See Additional Answers.

54. MODELING WITH MATHEMATICS You throw a softball straight up into the air with an initial vertical velocity of 40 feet per second. The release point is 5 feet above the ground. The function $h = -16t^2 + 40t + 5$ represents the height h (in feet) of the softball after t seconds.

a. Find the height of the softball each second after it is released.

b. Use the results of part (a) to estimate when the height of the softball is 15 feet.

c. Using a graph, after how many seconds is the softball 15 feet above the ground?

55. MODELING WITH MATHEMATICS The table shows the temperatures (in degrees Fahrenheit) of a cup of hot chocolate over time. (*See Example 7.*)

Time (minutes)	Temperature (°F)
0	200
10	157
20	128
30	109
40	99
50	92
60	90

a. Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit.

b. After how many minutes is the temperature of the hot chocolate 120°F? Round your answer to the nearest tenth.

c. Should you use the quadratic model you found in part (a) to predict the temperature of the hot chocolate after 60 minutes? Explain.

56. MODELING WITH MATHEMATICS The table shows the values (in dollars) of a car over time.

Age (years)	0	3	6	9	12
Value (dollars)	18,900	12,275	7972	5178	3363

a. Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit.

b. After how many years is the value of the car $10,000? Round your answer to the nearest tenth.

c. Should you use the quadratic model you found in part (a) to predict the value of the car after it is 12 years old? Explain your reasoning.

57. MATHEMATICAL CONNECTIONS The table shows the numbers of line segments that you can draw whose endpoints are chosen from x points, no three of which are collinear.

Number of points, x	2	3	4	5	6
Number of line segments, y	1	3			

a. Copy and complete the table. Use diagrams to support your answers.

b. Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit.

c. Predict the number of line segments that you can draw whose endpoints are chosen from 9 points.

d. How many points are chosen when you can draw 66 line segments? Explain how you found your answer.

58. MODELING WITH MATHEMATICS The table shows the numbers of cellular telephone sites (in thousands) in the U.S. for selected years from 1990 to 2012.

Year	Cellular sites (thousands)
1992	10.3
1996	30.0
2000	104.3
2004	175.7
2008	242.1
2012	301.8



a. Use a graphing calculator to find a linear model and a quadratic model for the data. Let $x = 0$ represent 1990. Is either model a better fit for the data? Explain.

b. Use each model in part (a) to determine in what year the number of cellular sites reached 200,000. Do you get the same result? Justify your answer.

c. Use each model in part (a) to predict in what year the number of cellular sites will reach 500,000. Do you get the same result? Justify your answer.

ANSWERS

54. **a.** 29 ft, 21 ft

b. about 0.5 sec, about 2.3 sec

c. about 0.3 sec, about 2.2 sec

55. **a.** $y = 0.04x^2 - 4.1x + 197$; Yes, the graph of the equation passes through or is close to all of the data points.

b. about 24.8 min

c. no; The quadratic model suggests that the temperature will increase.

56. **a.** $y = 88.33x^2 - 2332.3x + 18{,}762$; Yes, the graph of the equation passes through or is close to all of the data points.

b. about 4.5 years

c. no; The quadratic model suggests that the value will increase.

57. **a.** 6; 10; 15; Check students' work.

b. $y = 0.5x^2 - 0.5x$; Yes, the graph of the equation passes through all of the data points.

c. 36

d. 12; Set the equation from part (b) equal to 66. Write the equation in standard form and graph the equation. Find the zeros for $x > 0$.

58. **a.** linear: $y = 15.5x - 42$, quadratic: $y = 0.19x^2 + 11x - 23$; no; Both models pass through or are close to all of the data points.

b. linear: 2005, quadratic: 2005; yes; The models are very similar from 1992 to about 2010.

c. linear: 2024, quadratic: 2020; no; The quadratic model begins to increase faster as time goes on.

ANSWERS

59. about 3.7 ft

60. about 6.2 m

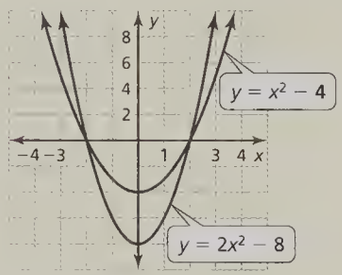
61. Graph the function to determine which integers the solutions are between. Then make tables using x-values between the integers with an interval of 0.1. Look for a change of sign in the function values, then select the value closest to zero.

62. a. 2; The graphs intersect in two places.

b. The x-intercepts are 1 and -4.

63. *Sample answer:* Method 1; Only one graph needs to be drawn.

64. infinitely many; *Sample answer:*



65. about 24.1 ft

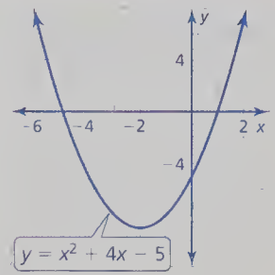
66. yes; When $x = 57$, $y = 26.313$.

67. sometimes; $y = -2x^2 + 1$ has two x-intercepts, but $y = -2x^2 + (-1)$ has no x-intercepts.

68–77. See Additional Answers.

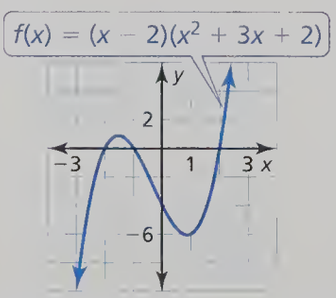
Mini-Assessment

1. Solve $x^2 + 4x = 5$ by graphing.



$x = -5$ and $x = 1$

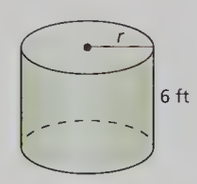
2. Use the graph to find the zeros of $f(x) = (x - 2)(x^2 + 3x + 2)$.



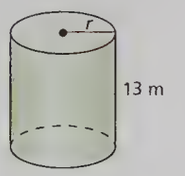
The zeros are -2, -1, and 2.

MATHEMATICAL CONNECTIONS **In Exercises 59 and 60, use the given surface area S of the cylinder to find the radius r to the nearest tenth.**

59. $S = 225$ ft^2

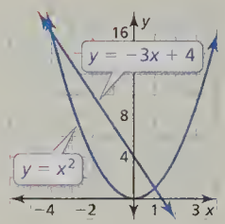


60. $S = 750$ m^2



61. WRITING Explain how to approximate zeros of a function when the zeros are not integers.

62. HOW DO YOU SEE IT? Consider the graph shown.



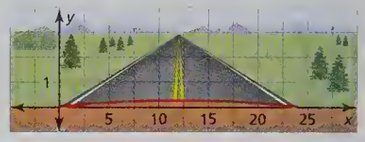
a. How many solutions does the quadratic equation $x^2 = -3x + 4$ have? Explain.

b. Without graphing, describe what you know about the graph of $y = x^2 + 3x - 4$.

63. COMPARING METHODS Example 3 shows two methods for solving a quadratic equation. Which method do you prefer? Explain your reasoning.

64. THOUGHT PROVOKING How many different parabolas have -2 and 2 as x-intercepts? Sketch examples of parabolas that have these two x-intercepts.

65. MODELING WITH MATHEMATICS To keep water off a road, the surface of the road is shaped like a parabola. A cross section of the road is shown in the diagram. The surface of the road can be modeled by $y = -0.0017x^2 + 0.041x$, where x and y are measured in feet. Find the width of the road to the nearest tenth of a foot.



66. MAKING AN ARGUMENT A stream of water from a fire hose can be modeled by $y = -0.003x^2 + 0.58x + 3$, where x and y are measured in feet. A firefighter is standing 57 feet from a building and is holding the hose 3 feet above the ground. The bottom of a window of the building is 26 feet above the ground. Your friend claims the stream of water will pass through the window. Is your friend correct? Explain.

REASONING **In Exercises 67–69, determine whether the statement is *always*, *sometimes*, or *never* true. Justify your answer.**

67. The graph of $y = ax^2 + c$ has two x-intercepts when a is negative.

68. The graph of $y = ax^2 + c$ has no x-intercepts when a and c have the same sign.

69. The graph of $y = ax^2 + bx + c$ has more than two x-intercepts when $a \neq 0$.

70. WRITING You want to find a model for a set of data. How do you determine whether to perform linear regression or quadratic regression on the set of data?

71. REASONING Show how you can use a system of equations to solve the problem in Example 7(b).

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain. *(Section 6.4)*

72.

x	−1	0	1	2
y	18	3	$\frac{1}{2}$	$\frac{1}{12}$

73.

x	0	1	2	3
y	2	8	32	128

Simplify the expression. *(Section 9.1)*

74. $\frac{5}{\sqrt{13}}$

75. $\frac{\sqrt{10}}{\sqrt{6x}}$

76. $\frac{8}{7 + \sqrt{5}}$

77. $\frac{3}{\sqrt{3} - 2}$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

9.3 Solving Quadratic Equations Using Square Roots

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.8.A

Essential Question How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

EXPLORATION 1 The Number of Solutions of $ax^2 + c = 0$

Work with a partner. Solve each equation by graphing. Explain how the number of solutions of $ax^2 + c = 0$ relates to the graph of $y = ax^2 + c$.

a. $x^2 - 4 = 0$

b. $2x^2 + 5 = 0$

c. $x^2 = 0$

d. $x^2 - 5 = 0$

EXPLORATION 2 Estimating Solutions

Work with a partner. Complete each table. Use the completed tables to estimate the solutions of $x^2 - 5 = 0$. Explain your reasoning.

a.

x	$x^2 - 5$
2.21	
2.22	
2.23	
2.24	
2.25	
2.26	

b.

x	$x^2 - 5$
−2.21	
−2.22	
−2.23	
−2.24	
−2.25	
−2.26	

USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to calculate accurately and express numerical answers with a level of precision appropriate for the problem's context.

EXPLORATION 3 Using Technology to Estimate Solutions

Work with a partner. Two equations are equivalent when they have the same solutions.

a. Are the equations $x^2 - 5 = 0$ and $x^2 = 5$ equivalent? Explain your reasoning.

b. Use the square root key on a calculator to estimate the solutions of $x^2 - 5 = 0$. Describe the accuracy of your estimates in Exploration 2.

c. Write the exact solutions of $x^2 - 5 = 0$.

Communicate Your Answer

4. How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

5. Write the exact solutions of each equation. Then use a calculator to estimate the solutions.

 a. $x^2 - 2 = 0$

 b. $3x^2 - 18 = 0$

 c. $x^2 = 8$

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.8.A The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

ANSWERS

1. a. $x = 2, x = -2$
 b. no solution
 c. $x = 0$
 d. $x \approx 2.2, x \approx -2.2$; The number of solutions is equal to the number of x-intercepts in the related graph.
2. a. $-0.1159, -0.0716, -0.0271, 0.0176, 0.0625, 0.1076$
 b. $-0.1159, -0.0716, -0.0271, 0.0176, 0.0625, 0.1076$
 $x \approx 2.24, x \approx -2.24$; The value 0.0176 is closest to 0.
3. a. yes; Adding 5 to each side of $x^2 - 5 = 0$ gives $x^2 = 5$.
 b. $x \approx 2.236, x \approx -2.236$; The estimates in Exploration 2 were accurate to the hundredths place.
 c. $x = \sqrt{5}, x = -\sqrt{5}$
4. Graph the related equation $y = ax^2 + c$. The number of solutions will be the same as the number of x-intercepts.
5. a. $x = \sqrt{2}, x = -\sqrt{2}; x \approx 1.41, x \approx -1.41$
 b. $x = \sqrt{6}, x = -\sqrt{6}; x \approx 2.45, x \approx -2.45$
 c. $x = \sqrt{8}, x = -\sqrt{8}; x \approx 2.83, x \approx -2.83$

Extra Example 1

a. Solve $2x^2 - 32 = 0$ using square roots. The solutions are $x = 4$ and $x = -4$.

b. Solve $x^2 - 8 = -8$ using square roots. The only solution is $x = 0$.

c. Solve $-2x^2 + 3 = 27$ using square roots. The equation has no real solutions.

9.3 Lesson

What You Will Learn

- Solve quadratic equations using square roots.
- Approximate the solutions of quadratic equations.

Core Vocabulary

Previous
square root
zero of a function

Solving Quadratic Equations Using Square Roots

Earlier in this chapter, you studied properties of square roots. Now you will use square roots to solve quadratic equations of the form $ax^2 + c = 0$. First isolate x^2 on one side of the equation to obtain $x^2 = d$. Then solve by taking the square root of each side.

Core Concept

Solutions of $x^2 = d$

- When $d > 0$, $x^2 = d$ has two real solutions, $x = \pm\sqrt{d}$.
- When $d = 0$, $x^2 = d$ has one real solution, $x = 0$.
- When $d < 0$, $x^2 = d$ has no real solutions.

ANOTHER WAY

You can also solve $3x^2 - 27 = 0$ by factoring.

$3(x^2 - 9) = 0$
$3(x - 3)(x + 3) = 0$
$x = 3$ or $x = -3$

EXAMPLE 1 Solving Quadratic Equations Using Square Roots

a. Solve $3x^2 - 27 = 0$ using square roots.

$3x^2 - 27 = 0$	Write the equation.
$3x^2 = 27$	Add 27 to each side.
$x^2 = 9$	Divide each side by 3.
$x = \pm\sqrt{9}$	Take the square root of each side.
$x = \pm 3$	Simplify.

▶ The solutions are $x = 3$ and $x = -3$.

b. Solve $x^2 - 10 = -10$ using square roots.

$x^2 - 10 = -10$	Write the equation.
$x^2 = 0$	Add 10 to each side.
$x = 0$	Take the square root of each side.

▶ The only solution is $x = 0$.

c. Solve $-5x^2 + 11 = 16$ using square roots.

$-5x^2 + 11 = 16$	Write the equation.
$-5x^2 = 5$	Subtract 11 from each side.
$x^2 = -1$	Divide each side by -5.

▶ The square of a real number cannot be negative. So, the equation has no real solutions.

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Ask students to read independently What You Will Learn, the first paragraph of Solving Quadratic Equations Using Square Roots, and the Core Concept. Have them take notes and share what they understand.

Beginning Use simple phrases to share what they understand.
Intermediate Use simple sentences to share what they understand.
Advanced/Advanced High Use complex sentences to share what they understand.

ELPS 4.G.4 Demonstrate comprehension of increasingly complex English by taking notes commensurate with content area and grade level needs.

STUDY TIP

Each side of the equation $(x - 1)^2 = 25$ is a square. So, you can still solve by taking the square root of each side.

EXAMPLE 2 **Solving a Quadratic Equation Using Square Roots**

Solve $(x - 1)^2 = 25$ using square roots.

SOLUTION

$(x - 1)^2 = 25$ Write the equation.

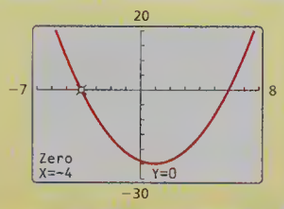
$x - 1 = \pm 5$ Take the square root of each side.

$x = 1 \pm 5$ Add 1 to each side.

▶ So, the solutions are $x = 1 + 5 = 6$ and $x = 1 - 5 = -4$.

Check

Use a graphing calculator to check your answer. Rewrite the equation as $(x - 1)^2 - 25 = 0$. Graph the related function $f(x) = (x - 1)^2 - 25$ and find the zeros of the function. The zeros are -4 and 6.



Monitoring Progress

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Solve the equation using square roots.

1. $-3x^2 = -75$
2. $x^2 + 12 = 10$
3. $4x^2 - 15 = -15$
4. $(x + 7)^2 = 0$
5. $4(x - 3)^2 = 9$
6. $(2x + 1)^2 = 36$

Approximating Solutions of Quadratic Equations

EXAMPLE 3 **Approximating Solutions of a Quadratic Equation**

Solve $4x^2 - 13 = 15$ using square roots. Round the solutions to the nearest hundredth.

SOLUTION

$4x^2 - 13 = 15$ Write the equation.

$4x^2 = 28$ Add 13 to each side.

$x^2 = 7$ Divide each side by 4.

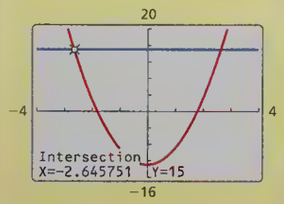
$x = \pm\sqrt{7}$ Take the square root of each side.

$x \approx \pm 2.65$ Use a calculator.

▶ The solutions are $x \approx -2.65$ and $x \approx 2.65$.

Check

Graph each side of the equation and find the points of intersection. The x-values of the points of intersection are about -2.65 and 2.65.



Monitoring Progress

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Solve the equation using square roots. Round your solutions to the nearest hundredth.

7. $x^2 + 8 = 19$
8. $5x^2 - 2 = 0$
9. $3x^2 - 30 = 4$

Extra Example 2

Solve $(x - 3)^2 = 16$ using square roots. The solutions are $x = 7$ and $x = -1$.

Extra Example 3

Solve $2x^2 + 10 = 32$ using square roots. Round the solutions to the nearest hundredth. The solutions are $x \approx -3.32$ and $x \approx 3.32$.

MONITORING PROGRESS ANSWERS

1. $x = 5, x = -5$
2. no real solutions
3. $x = 0$
4. $x = -7$
5. $x = \frac{9}{2}, x = \frac{3}{2}$
6. $x = \frac{5}{2}, x = -\frac{7}{2}$
7. $x \approx 3.32, x \approx -3.32$
8. $x \approx 0.63, x \approx -0.63$
9. $x \approx 3.37, x \approx -3.37$

English Language Learners

Pair Activity

Pair an English learner with an English speaker. One student solves a quadratic equation using square roots. The other student solves the same equation using a graphing calculator. Partners compare solutions and then take turns explaining their solutions to one another. Students should repeat the activity with a different quadratic equation and should swap solution methods.

Extra Example 4

A cardboard box has the shape of a rectangular prism. Its height is 10 inches. Its length is three times its width. The volume is 540 cubic inches. Find the length and width of the box.

width: $\sqrt{18} \approx 4.2$ inches;
length: $3\sqrt{18} \approx 12.7$ inches

Extra Example 5

The volume V of a cylinder with radius r and height h is given by the formula $V = \pi r^2 h$. Solve the formula for r. Then find the radius of a cylinder with a height of 15 centimeters and a volume of 750 cubic centimeters.

$r = \sqrt{\frac{V}{\pi h}}$; about 4 centimeters

MONITORING PROGRESS ANSWERS

10. width: about 5.9 ft, length: about 17.7 ft

11. $r = \sqrt{\frac{S}{4\pi}}$; about 8 in.

EXAMPLE 4 **Solving a Real-Life Problem**

A touch tank has a height of 3 feet. Its length is three times its width. The volume of the tank is 270 cubic feet. Find the length and width of the tank.



SOLUTION

The length ℓ is three times the width w, so $\ell = 3w$. Write an equation using the formula for the volume of a rectangular prism.

$V = \ell wh$	Write the formula.
$270 = 3w(w)(3)$	Substitute 270 for V, $3w$ for ℓ, and 3 for h.
$270 = 9w^2$	Multiply.
$30 = w^2$	Divide each side by 9.
$\pm\sqrt{30} = w$	Take the square root of each side.

The solutions are $\sqrt{30}$ and $-\sqrt{30}$. Use the positive solution.

▶ So, the width is $\sqrt{30} \approx 5.5$ feet and the length is $3\sqrt{30} \approx 16.4$ feet.

EXPLAINING MATHEMATICAL IDEAS

Use the positive square root because negative solutions do not make sense in this context. Length and width cannot be negative.

EXAMPLE 5 **Rearranging and Evaluating a Formula**

The area A of an equilateral triangle with side length s is given by the formula $A = \frac{\sqrt{3}}{4}s^2$. Solve the formula for s. Then approximate the side length of the traffic sign that has an area of 390 square inches.



SOLUTION

Step 1 Solve the formula for s.

$A = \frac{\sqrt{3}}{4}s^2$	Write the formula.
$\frac{4A}{\sqrt{3}} = s^2$	Multiply each side by $\frac{4}{\sqrt{3}}$.
$\sqrt{\frac{4A}{\sqrt{3}}} = s$	Take the positive square root of each side.

Step 2 Substitute 390 for A in the new formula and evaluate.

$$s = \sqrt{\frac{4A}{\sqrt{3}}} = \sqrt{\frac{4(390)}{\sqrt{3}}} = \sqrt{\frac{1560}{\sqrt{3}}} \approx 30$$ Use a calculator.

▶ The side length of the traffic sign is about 30 inches.

ANOTHER WAY

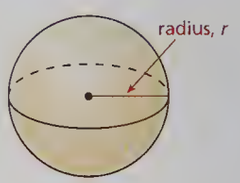
Notice that you can rewrite the formula as $s = \frac{2}{3^{1/4}}\sqrt{A}$, or $s \approx 1.52\sqrt{A}$. This can help you efficiently find the value of s for various values of A.

Monitoring Progress

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10. WHAT IF? In Example 4, the volume of the tank is 315 cubic feet. Find the length and width of the tank.

11. The surface area S of a sphere with radius r is given by the formula $S = 4\pi r^2$. Solve the formula for r. Then find the radius of a globe with a surface area of 804 square inches.



Closure

- **Exit Ticket:** State the number of solutions for each equation.

a. $2x^2 + 8 = 40$ 2 b. $2x^2 - 8 = -40$ 0 c. $2x^2 = 0$ 1

9.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The equation $x^2 = d$ has ____ real solutions when $d > 0$.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Solve $x^2 = 144$ using square roots.	Solve $x^2 - 144 = 0$ using square roots.
Solve $x^2 + 146 = 2$ using square roots.	Solve $x^2 + 2 = 146$ using square roots.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine the number of real solutions of the equation. Then solve the equation using square roots.

3. $x^2 = 25$
4. $x^2 = -36$
5. $x^2 = -21$
6. $x^2 = 400$
7. $x^2 = 0$
8. $x^2 = 169$

In Exercises 9–18, solve the equation using square roots. *(See Example 1.)*

9. $x^2 - 16 = 0$
10. $x^2 + 6 = 0$
11. $3x^2 + 12 = 0$
12. $x^2 - 55 = 26$
13. $2x^2 - 98 = 0$
14. $-x^2 + 9 = 9$
15. $-3x^2 - 5 = -5$
16. $4x^2 - 371 = 29$
17. $4x^2 + 10 = 11$
18. $9x^2 - 35 = 14$

In Exercises 19–24, solve the equation using square roots. *(See Example 2.)*

19. $(x + 3)^2 = 0$
20. $(x - 1)^2 = 4$
21. $(2x - 1)^2 = 81$
22. $(4x + 5)^2 = 9$
23. $9(x + 1)^2 = 16$
24. $4(x - 2)^2 = 25$

In Exercises 25–30, solve the equation using square roots. Round your solutions to the nearest hundredth. *(See Example 3.)*

25. $x^2 + 6 = 13$
26. $x^2 + 11 = 24$
27. $2x^2 - 9 = 11$
28. $5x^2 + 2 = 6$
29. $-21 = 15 - 2x^2$
30. $2 = 4x^2 - 5$

31. **ERROR ANALYSIS** Describe and correct the error in solving the equation $2x^2 - 33 = 39$ using square roots.

✗
$$2x^2 - 33 = 39$$
$$2x^2 = 72$$
$$x^2 = 36$$
$$x = 6$$
▶ The solution is $x = 6$.

32. **MODELING WITH MATHEMATICS** An in-ground pond has the shape of a rectangular prism. The pond has a depth of 24 inches and a volume of 72,000 cubic inches. The length of the pond is two times its width. Find the length and width of the pond. *(See Example 4.)*



33. **MODELING WITH MATHEMATICS** A person sitting in the top row of the bleachers at a sporting event drops a pair of sunglasses from a height of 24 feet. The function $h = -16x^2 + 24$ represents the height h (in feet) of the sunglasses after x seconds. How long does it take the sunglasses to hit the ground?

33. about 1.2 sec

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–33 odd, 34, 36, 40, 45–50

Average: 1, 2–30 even, 31–36, 40, 45–50

Advanced: 1, 2, 6, 16–30 even, 31, 32–36 even, 37–50

HOMEWORK CHECK

Basic: 9, 19, 25, 33, 36

Average: 10, 20, 26, 32, 36

Advanced: 16, 22, 28, 32, 36

ANSWERS

1. two
2. Solve $x^2 + 146 = 2$ using square roots; no real solutions; $x = 12, x = -12$
3. $2; x = 5, x = -5$
4. 0; no real solutions
5. 0; no real solutions
6. $2; x = 20, x = -20$
7. $1; x = 0$
8. $2; x = 13, x = -13$
9. $x = 4, x = -4$
10. no real solutions
11. no real solutions
12. $x = 9, x = -9$
13. $x = 7, x = -7$
14. $x = 0$
15. $x = 0$
16. $x = 10, x = -10$
17. $x = \frac{1}{2}, x = -\frac{1}{2}$
18. $x = \frac{7}{3}, x = -\frac{7}{3}$
19. $x = -3$
20. $x = 3, x = -1$
21. $x = 5, x = -4$
22. $x = -\frac{1}{2}, x = -2$
23. $x = \frac{1}{3}, x = -\frac{7}{3}$
24. $x = \frac{9}{2}, x = -\frac{1}{2}$
25. $x \approx 2.65, x \approx -2.65$
26. $x \approx 3.61, x \approx -3.61$
27. $x \approx 3.16, x \approx -3.16$
28. $x \approx 0.89, x \approx -0.89$
29. $x \approx 4.24, x \approx -4.24$
30. $x \approx 1.32, x \approx -1.32$
31. The number 36 has both a positive and negative square root; $x = \pm 6$
32. length: about 77.5 in., width: about 38.7 in.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

34. your cousin; Subtracting 4 from each side of the equation gives $x^2 = -4$, and a negative number has no real square roots.

35. 3 ft

36. a. $r = \sqrt{\frac{A}{\pi}}$

b. about 6 ft; about 24 in., about 13 m

c. The steps for solving only need to be completed once.

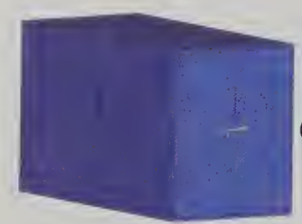
37. *Sample answer:* Use a calculator.

38–50. See Additional Answers.

Mini-Assessment

Solve the equation using square roots.

1. $3x^2 - 108 = 0$ $x = 6$ and $x = -6$
2. $(x - 5)^2 = 81$ $x = 14$ and $x = -4$
3. Solve $4x^2 - 3 = 17$ using square roots. Round the solutions to the nearest hundredth. $x \approx -2.24$ and $x \approx 2.24$
4. A storage container has the shape of a rectangular prism. Its height is 6 feet. Its length is two times its width. The volume is 288 cubic feet. Find the length and width of the container.



width: $\sqrt{24} \approx 4.9$ feet; length: $2\sqrt{24} \approx 9.8$ feet

5. The surface area S of a cube with side length a is given by the formula $S = 6a^2$. Solve the formula for a. Then find the side length of a cube with a surface area of 700 square inches.

$a = \sqrt{\frac{S}{6}}$; about 11 inches

34. MAKING AN ARGUMENT Your friend says that the solution of the equation $x^2 + 4 = 0$ is $x = 0$. Your cousin says that the equation has no real solutions. Who is correct? Explain your reasoning.

35. MODELING WITH MATHEMATICS The design of a square rug for your living room is shown. You want the area of the inner square to be 25% of the total area of the rug. Find the side length x of the inner square.



6 ft

36. MATHEMATICAL CONNECTIONS The area A of a circle with radius r is given by the formula $A = \pi r^2$. *(See Example 5.)*

a. Solve the formula for r.

b. Use the formula from part (a) to find the radius of each circle.



$A = 113$ ft^2 $A = 1810$ in.^{2} $A = 531$ m^2

c. Explain why it is beneficial to solve the formula for r before finding the radius.

37. WRITING How can you approximate the roots of a quadratic equation when the roots are not integers?

38. WRITING Given the equation $ax^2 + c = 0$, describe the values of a and c so the equation has the following number of solutions.

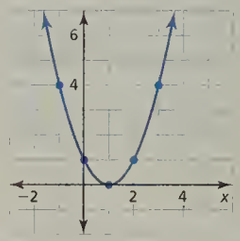
a. two real solutions

b. one real solution

c. no real solutions

39. REASONING Without graphing, where do the graphs of $y = x^2$ and $y = 9$ intersect? Explain.

40. HOW DO YOU SEE IT? The graph represents the function $f(x) = (x - 1)^2$. How many solutions does the equation $(x - 1)^2 = 0$ have? Explain.



41. REASONING Solve $x^2 = 1.44$ without using a calculator. Explain your reasoning.

42. THOUGHT PROVOKING The quadratic equation

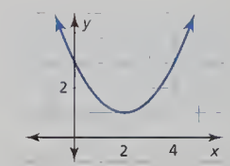
$$ax^2 + bx + c = 0$$

can be rewritten in the following form.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Use this form to write the solutions of the equation.

43. REASONING An equation of the graph shown is $y = \frac{1}{2}(x - 2)^2 + 1$. Two points on the parabola have y-coordinates of 9. Find the x-coordinates of these points.



44. CRITICAL THINKING Solve each equation without graphing.

a. $x^2 - 12x + 36 = 64$

b. $x^2 + 14x + 49 = 16$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the polynomial. *(Section 7.8)*

45. $x^2 + 8x + 16$ **46.** $x^2 - 4x + 4$ **47.** $x^2 - 14x + 49$

48. $x^2 + 18x + 81$ **49.** $x^2 + 12x + 36$ **50.** $x^2 - 22x + 121$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

9.1–9.3 What Did You Learn?

Core Vocabulary

counterexample, *p. 465*
radical expression, *p. 466*
simplest form, *p. 466*
rationalizing the denominator, *p. 468*
conjugates, *p. 468*
like radicals, *p. 470*
quadratic equation, *p. 476*

Core Concepts

Section 9.1

Product Property of Square Roots, *p. 466*
Quotient Property of Square Roots, *p. 466*
Rationalizing the Denominator, *p. 468*
Performing Operations with Radicals, *p. 470*

Section 9.2

Solving Quadratic Equations by Graphing, *p. 476*
Number of Solutions of a Quadratic Equation, *p. 477*
Finding Zeros of Functions, *p. 478*

Section 9.3

Solutions of $x^2 = d$, *p. 486*
Approximating Solutions of Quadratic Equations, *p. 487*

Mathematical Thinking

1. For each part of Exercise 100 on page 474 that is *sometimes* true, list all examples and counterexamples from the table that represent the sum or product being described.
2. Which Examples can you use to help you solve Exercise 54 on page 483?
3. Describe how solving a simpler equation can help you solve the equation in Exercise 41 on page 490.

Study Skills

Keeping a Positive Attitude

Do you ever feel frustrated or overwhelmed by math? You're not alone. Just take a deep breath and assess the situation. Try to find a productive study environment, review your notes and the examples in the textbook, and ask your teacher or friends for help.



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Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool
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ANSWERS

1. part (c) examples: $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$, $\sqrt{3} + \pi = \pi + \sqrt{3}$, $-\sqrt{3} + (-\sqrt{3}) = -2\sqrt{3}$, $-\sqrt{3} + \pi = \pi - \sqrt{3}$, $\pi + \pi = 2\pi$;
 part (c) counterexample: $\sqrt{3} + (-\sqrt{3}) = 0$,
 part (f) examples: $\sqrt{3} \cdot \pi = \pi\sqrt{3}$, $-\sqrt{3} \cdot \pi = -\pi\sqrt{3}$, $\pi \cdot \pi = \pi^2$;
 part (f) counterexamples: $\sqrt{3} \cdot \sqrt{3} = 3$, $\sqrt{3} \cdot (-\sqrt{3}) = -3$, $-\sqrt{3} \cdot (-\sqrt{3}) = 3$
2. Example 6 on page 493 or Example 5 on page 492
3. Solving the simpler equation $x^2 = 144$ helps because the solution can be found by moving the decimal point in the solution to the simpler equation.

ANSWERS

1. $4x\sqrt{7x}$
2. $\frac{\sqrt{2}}{3}$
3. $-5\sqrt[3]{5}$
4. $\frac{3\sqrt{2}}{2}$
5. $\frac{4\sqrt{11}}{11}$
6. $\frac{12\sqrt{13}}{13}$
7. $\frac{3x\sqrt[3]{2x}}{7y^2}$
8. $\frac{x\sqrt{7z}}{7y^2z^3}$
9. $\frac{15 - 3\sqrt{3}}{11}$
10. $-4\sqrt{5} + 7\sqrt{10}$
11. $-10\sqrt{2} - 5\sqrt{10}$
12. $30\sqrt{2}$
13. $x = -1, x = 3$
14. no real solutions
15. $x = -5$
16. $x = -7, x = -2$
17. $x = 8, x = -1$
18. no real solutions
19. $x = 4, x = -4$
20. no real solutions
21. $x = 9, x = 7$
22. Because $100 > 0$, the equation has two real solutions.
23. length: about 17.4 m, width: about 4.4 m
24. **a.** 0.5 sec, 1 sec
 b. about 1.65 sec

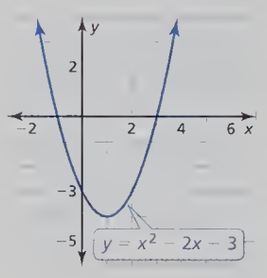
9.1–9.3 Quiz

Simplify the expression. *(Section 9.1)*

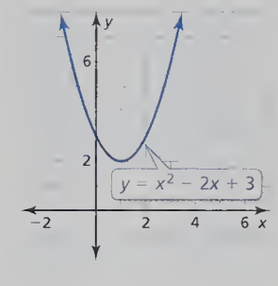
1. $\sqrt{112x^3}$
2. $\sqrt{\frac{18}{81}}$
3. $\sqrt[3]{-625}$
4. $\frac{12}{\sqrt{32}}$
5. $\frac{4}{\sqrt{11}}$
6. $\sqrt{\frac{144}{13}}$
7. $\sqrt[3]{\frac{54x^4}{343y^6}}$
8. $\sqrt{\frac{4x^2}{28y^4z^5}}$
9. $\frac{6}{5 + \sqrt{3}}$
10. $2\sqrt{5} + 7\sqrt{10} - 3\sqrt{20}$
11. $\frac{10}{\sqrt{8} - \sqrt{10}}$
12. $\sqrt{6}(7\sqrt{12} - 4\sqrt{3})$

Use the graph to solve the equation. *(Section 9.2)*

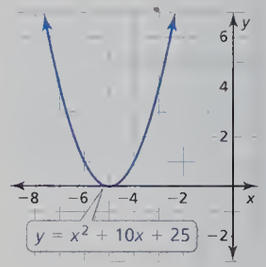
13. $x^2 - 2x - 3 = 0$



14. $x^2 - 2x + 3 = 0$



15. $x^2 + 10x + 25 = 0$



Solve the equation by graphing. *(Section 9.2)*

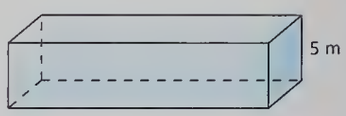
16. $x^2 + 9x + 14 = 0$
17. $x^2 - 7x = 8$
18. $x + 4 = -x^2$

Solve the equation using square roots. *(Section 9.3)*

19. $4x^2 = 64$
20. $-3x^2 + 6 = 10$
21. $(x - 8)^2 = 1$

22. Explain how to determine the number of real solutions of $x^2 = 100$ without solving. *(Section 9.3)*

23. The length of a rectangular prism is four times its width. The volume of the prism is 380 cubic meters. Find the length and width of the prism. *(Section 9.3)*





24. You cast a fishing lure into the water from a height of 4 feet above the water. The height h (in feet) of the fishing lure after t seconds can be modeled by the equation $h = -16t^2 + 24t + 4$. *(Section 9.2)*
 a. After how many seconds does the fishing lure reach a height of 12 feet?
 b. After how many seconds does the fishing lure hit the water?

9.4 Solving Quadratic Equations by Completing the Square

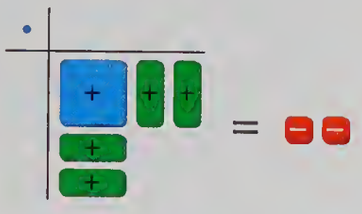
Texas Essential Knowledge and Skills
A.8.A

Essential Question How can you use "completing the square" to solve a quadratic equation?

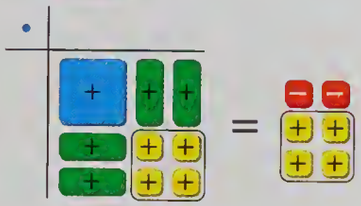
EXPLORATION 1 Solving by Completing the Square

Work with a partner.

a. Write the equation modeled by the algebra tiles. This is the equation to be solved.

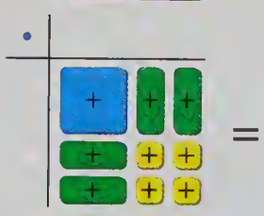


b. Four algebra tiles are added to the left side to "complete the square." Why are four algebra tiles also added to the right side?



c. Use algebra tiles to label the dimensions of the square on the left side and simplify on the right side.

d. Write the equation modeled by the algebra tiles so that the left side is the square of a binomial. Solve the equation using square roots.



USING PROBLEM-SOLVING STRATEGIES

To be proficient in math, you need to explain to yourself the meaning of a problem. After that, you need to look for entry points to its solution.

EXPLORATION 2 Solving by Completing the Square

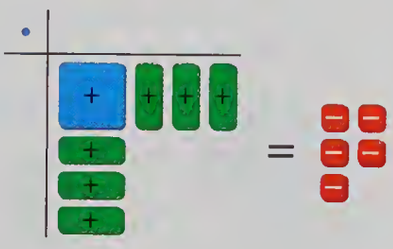
Work with a partner.

a. Write the equation modeled by the algebra tiles.

b. Use algebra tiles to "complete the square."

c. Write the solutions of the equation.

d. Check each solution in the original equation.



Communicate Your Answer

3. How can you use "completing the square" to solve a quadratic equation?

4. Solve each quadratic equation by completing the square.

a. $x^2 - 2x = 1$ **b.** $x^2 - 4x = -1$ **c.** $x^2 + 4x = -3$

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

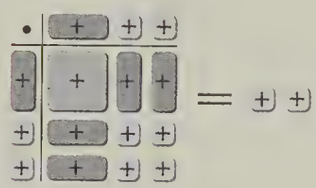
A.8.A The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

ANSWERS

1. a. $x^2 + 4x = -2$

b. To be equivalent equations, the same number must be added to each side.

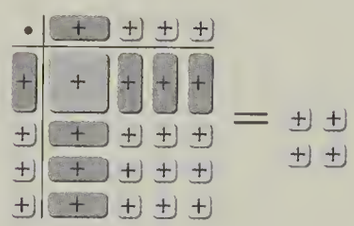
c.



d. $(x + 2)^2 = 2$; $x \approx -0.59$, $x \approx -3.41$

2. a. $x^2 + 6x = -5$

b.



c. $x = -1, x = -5$

d. $(-1)^2 + 6(-1) = -5$; $(-5)^2 + 6(-5) = -5$

3. Write the equation in the form $x^2 + bx = d$. Add $\left(\frac{b}{2}\right)^2$ to each side of the equation. Factor the resulting expression on the left side as the square of a binomial. Solve the resulting equation using square roots.

4. a. $x \approx 2.41, x \approx -0.41$

b. $x \approx 3.73, x \approx 0.27$

c. $x = -1, x = -3$

Extra Example 1

Complete the square for each expression. Then factor the trinomial.

a. $x^2 + 18x$ $x^2 + 18x + 81; (x + 9)^2$

b. $x^2 - 17x$ $x^2 - 17x + \frac{289}{4}; \left(x - \frac{17}{2}\right)^2$

MONITORING PROGRESS ANSWERS

1. $x^2 + 10x + 25; (x + 5)^2$
2. $x^2 - 4x + 4; (x - 2)^2$
3. $x^2 + 7x + \frac{49}{4}; \left(x + \frac{7}{2}\right)^2$

9.4 Lesson

What You Will Learn

- Complete the square for expressions of the form $x^2 + bx$.
- Solve quadratic equations by completing the square.
- Find and use maximum and minimum values.
- Solve real-life problems by completing the square.

Core Vocabulary

completing the square, *p. 494*

Previous

perfect square trinomial
coefficient
maximum value
minimum value
vertex form of a quadratic function

Completing the Square

For an expression of the form $x^2 + bx$, you can add a constant c to the expression so that $x^2 + bx + c$ is a perfect square trinomial. This process is called **completing the square**.

Core Concept

Completing the Square

Words To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1 Find one-half of b, the coefficient of x.

Step 2 Square the result from Step 1.

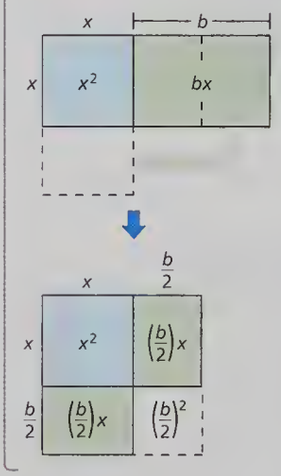
Step 3 Add the result from Step 2 to $x^2 + bx$.

Factor the resulting expression as the square of a binomial.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

JUSTIFYING STEPS

In each diagram below, the combined area of the shaded regions is $x^2 + bx$. Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.



EXAMPLE 1 Completing the Square

Complete the square for each expression. Then factor the trinomial.

a. $x^2 + 6x$ b. $x^2 - 9x$

SOLUTION

a. **Step 1** Find one-half of b. $\frac{b}{2} = \frac{6}{2} = 3$

Step 2 Square the result from Step 1. $3^2 = 9$

Step 3 Add the result from Step 2 to $x^2 + bx$. $x^2 + 6x + 9$

▶ $x^2 + 6x + 9 = (x + 3)^2$

b. **Step 1** Find one-half of b. $\frac{b}{2} = \frac{-9}{2}$

Step 2 Square the result from Step 1. $\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

Step 3 Add the result from Step 2 to $x^2 + bx$. $x^2 - 9x + \frac{81}{4}$

▶ $x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Complete the square for the expression. Then factor the trinomial.

1. $x^2 + 10x$ 2. $x^2 - 4x$ 3. $x^2 + 7x$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

SUPPORTING English Language Learners

As you present the material on page 494, ask students to write down what they understand without reading along. This should include the objectives from What You Will Learn and the main ideas from the topic Completing the Square. Have them share what they understand.

Beginning Use simple phrases to share what they understand.
Intermediate Use simple sentences to share what they understand.
Advanced/Advanced High Use complex sentences to share what they understand.

ELPS 2.I.5 Demonstrate listening comprehension of increasingly complex spoken English by taking notes commensurate with content and grade-level needs.

Solving Quadratic Equations by Completing the Square

The method of completing the square can be used to solve any quadratic equation. To solve a quadratic equation by completing the square, you must write the equation in the form $x^2 + bx = d$.

COMMON ERROR

When completing the square to solve an equation, be sure to add $\left(\frac{b}{2}\right)^2$ to each side of the equation.

EXAMPLE 2 Solving a Quadratic Equation: $x^2 + bx = d$

Solve $x^2 - 16x = -15$ by completing the square.

SOLUTION

$x^2 - 16x = -15$	Write the equation.
$x^2 - 16x + (-8)^2 = -15 + (-8)^2$	Complete the square by adding $\left(\frac{-16}{2}\right)^2$, or $(-8)^2$, to each side.
$(x - 8)^2 = 49$	Write the left side as the square of a binomial.
$x - 8 = \pm 7$	Take the square root of each side.
$x = 8 \pm 7$	Add 8 to each side.

▶ The solutions are $x = 8 + 7 = 15$ and $x = 8 - 7 = 1$.

Check

$x^2 - 16x = -15$	Original equation	$x^2 - 16x = -15$
$15^2 - 16(15) \stackrel{?}{=} -15$	Substitute.	$1^2 - 16(1) \stackrel{?}{=} -15$
$-15 = -15$ ✓	Simplify.	$-15 = -15$ ✓

EXAMPLE 3 Solving a Quadratic Equation: $ax^2 + bx + c = 0$

Solve $2x^2 + 20x - 8 = 0$ by completing the square.

COMMON ERROR

Before you complete the square, be sure that the coefficient of the x^2-term is 1.

SOLUTION

$2x^2 + 20x - 8 = 0$	Write the equation.
$2x^2 + 20x = 8$	Add 8 to each side.
$x^2 + 10x = 4$	Divide each side by 2.
$x^2 + 10x + 5^2 = 4 + 5^2$	Complete the square by adding $\left(\frac{10}{2}\right)^2$, or 5^2, to each side.
$(x + 5)^2 = 29$	Write the left side as the square of a binomial.
$x + 5 = \pm\sqrt{29}$	Take the square root of each side.
$x = -5 \pm \sqrt{29}$	Subtract 5 from each side.

▶ The solutions are $x = -5 + \sqrt{29} \approx 0.39$ and $x = -5 - \sqrt{29} \approx -10.39$.

Monitoring Progress

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Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

4. $x^2 - 2x = 3$
5. $m^2 + 12m = -8$
6. $3g^2 - 24g + 27 = 0$

Extra Example 2

Solve $x^2 - 18x = -17$ by completing the square.

The solutions are $x = 9 + 8 = 17$ and $x = 9 - 8 = 1$.

Extra Example 3

Solve $2x^2 + 12x - 10 = 0$ by completing the square.

The solutions are $x = -3 + \sqrt{14} \approx 0.74$ and $x = -3 - \sqrt{14} \approx -6.74$.

MONITORING PROGRESS ANSWERS

4. $x = 3, x = -1$
5. $m \approx -0.71, m \approx -11.29$
6. $g \approx 6.65, g \approx 1.35$

Differentiated Instruction

Inclusion

Have advanced students work in pairs to write and solve equations like those in Examples 2 and 3. Each student should write an equation, exchange equations with his or her partner, and solve the equation. After solving, students should exchange equations again and check the solutions.

Extra Example 4

Find the minimum value of $y = x^2 + 8x + 5$.

The function has a minimum value of -11.

Extra Example 5

Find the maximum value of $y = -x^2 + 4x + 2$.

The function has a maximum value of 6.

MONITORING PROGRESS ANSWERS

7. maximum value; 8
8. minimum value; 4
9. minimum value; -3

Finding and Using Maximum and Minimum Values

One way to find the maximum or minimum value of a quadratic function is to write the function in vertex form by completing the square. Recall that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where $a \neq 0$. The vertex of the graph is (h, k).

EXAMPLE 4 Finding a Minimum Value

Find the minimum value of $y = x^2 + 4x - 1$.

SOLUTION

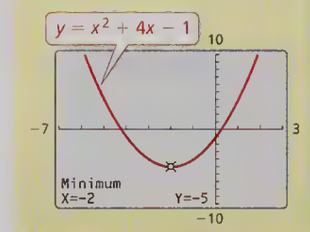
Write the function in vertex form.

$y = x^2 + 4x - 1$	Write the function.
$y + 1 = x^2 + 4x$	Add 1 to each side.
$y + 1 + 4 = x^2 + 4x + 4$	Complete the square for $x^2 + 4x$.
$y + 5 = x^2 + 4x + 4$	Simplify the left side.
$y + 5 = (x + 2)^2$	Write the right side as the square of a binomial.
$y = (x + 2)^2 - 5$	Write in vertex form.

The vertex is $(-2, -5)$. Because a is positive ($a = 1$), the parabola opens up and the y-coordinate of the vertex is the minimum value.

▶ So, the function has a minimum value of -5.

Check



EXAMPLE 5 Finding a Maximum Value

Find the maximum value of $y = -x^2 + 2x + 7$.

SOLUTION

Write the function in vertex form.

$y = -x^2 + 2x + 7$	Write the function.
$y - 7 = -x^2 + 2x$	Subtract 7 from each side.
$y - 7 = -(x^2 - 2x)$	Factor out -1.
$y - 7 - 1 = -(x^2 - 2x + 1)$	Complete the square for $x^2 - 2x$.
$y - 8 = -(x^2 - 2x + 1)$	Simplify the left side.
$y - 8 = -(x - 1)^2$	Write $x^2 - 2x + 1$ as the square of a binomial.
$y = -(x - 1)^2 + 8$	Write in vertex form.

The vertex is $(1, 8)$. Because a is negative ($a = -1$), the parabola opens down and the y-coordinate of the vertex is the maximum value.

▶ So, the function has a maximum value of 8.

STUDY TIP

Adding 1 inside the parentheses results in subtracting 1 from the right side of the equation.

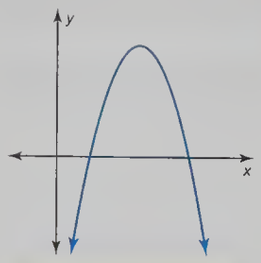
Monitoring Progress

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Determine whether the quadratic function has a maximum or minimum value. Then find the value.

7. $y = -x^2 - 4x + 4$
8. $y = x^2 + 12x + 40$
9. $y = x^2 - 2x - 2$

EXAMPLE 6 Interpreting Forms of Quadratic Functions



$f(x) = -\frac{1}{2}(x + 4)^2 + 8$

$g(x) = -(x - 5)^2 + 9$

$m(x) = (x - 3)(x - 12)$

$p(x) = -(x - 2)(x - 8)$

Which of the functions could be represented by the graph? Explain.

SOLUTION

You do not know the scale of either axis. To eliminate functions, consider the characteristics of the graph and information provided by the form of each function. The graph appears to be a parabola that opens down, which means the function has a maximum value. The vertex of the graph is in the first quadrant. Both x-intercepts are positive.

- The graph of f opens down because $a < 0$, which means f has a maximum value. However, the vertex $(-4, 8)$ of the graph of f is in the second quadrant. So, the graph does not represent f.
- The graph of g opens down because $a < 0$, which means g has a maximum value. The vertex $(5, 9)$ of the graph of g is in the first quadrant. By solving $0 = -(x - 5)^2 + 9$, you see that the x-intercepts of the graph of g are 2 and 8. So, the graph could represent g.
- The graph of m has two positive x-intercepts. However, its graph opens up because $a > 0$, which means m has a minimum value. So, the graph does not represent m.
- The graph of p has two positive x-intercepts, and its graph opens down because $a < 0$. This means that p has a maximum value and the vertex must be in the first quadrant. So, the graph could represent p.

▶ The graph could represent function g or function p.

EXAMPLE 7 Real-Life Application

The function $y = -16x^2 + 96x$ represents the height y (in feet) of a model rocket x seconds after it is launched. (a) Find the maximum height of the rocket. (b) Find and interpret the axis of symmetry.

STUDY TIP

Adding 9 inside the parentheses results in subtracting 144 from the right side of the equation.

SOLUTION

a. To find the maximum height, identify the maximum value of the function.

$y = -16x^2 + 96x$	Write the function.
$y = -16(x^2 - 6x)$	Factor out -16.
$y - 144 = -16(x^2 - 6x + 9)$	Complete the square for $x^2 - 6x$.
$y = -16(x - 3)^2 + 144$	Write in vertex form.

▶ Because the maximum value is 144, the model rocket reaches a maximum height of 144 feet.

b. The vertex is $(3, 144)$. So, the axis of symmetry is $x = 3$. On the left side of $x = 3$, the height increases as time increases. On the right side of $x = 3$, the height decreases as time increases.

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Determine whether the function could be represented by the graph in Example 6. Explain.

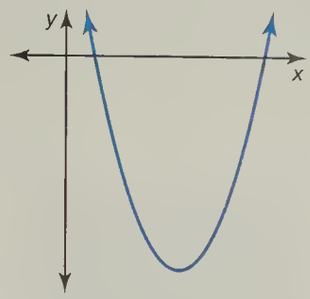
10. $h(x) = (x - 8)^2 + 10$

11. $n(x) = -2(x - 5)(x - 20)$

12. WHAT IF? Repeat Example 7 when the function is $y = -16x^2 + 128x$.

Extra Example 6

Which of the functions could be represented by the graph? Explain.



$f(x) = (x - 4)^2 + 9$

$g(x) = -(x - 3)^2 + 5$

$m(x) = -\frac{1}{4}(x + 2)(x + 6)$

$p(x) = (x - 1)(x - 7)$

The graph could represent function p.

The graph does not represent functions g or m because $a < 0$ and these graphs open down.

The graph does not represent f. The graph of f opens up because $a > 0$. The vertex $(4, 9)$ is in the first quadrant, so the graph of f does not cross the x-axis.

The graph of p opens up because $a > 0$. It has two positive x-intercepts. So, p has a minimum value and could represent the graph.

Extra Example 7

The function $y = -16x^2 + 160x$ represents the height y (in feet) of a model rocket x seconds after it is launched.

a. Find the maximum height of the rocket. 400 feet

b. Find and interpret the axis of symmetry. The axis of symmetry is $x = 5$. On the left side of $x = 5$, the height increases as time increases. On the right side of $x = 5$, the height decreases as time increases.

MONITORING PROGRESS ANSWERS

10. no; The graph opens up.

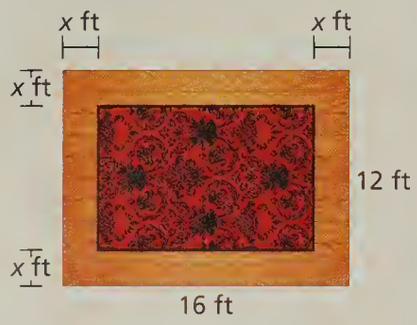
11. yes; The graph has two positive x-intercepts and opens down.

12. a. 256 ft

b. $x = 4$; On the left side of $x = 4$, the height increases as time increases. On the right side of $x = 4$, the height decreases as time increases.

Extra Example 8

You decide to put a rug on a floor. You want the rug to cover 100 square feet and to have a uniform border of floor visible, as shown. Find the width of the border to the nearest inch.



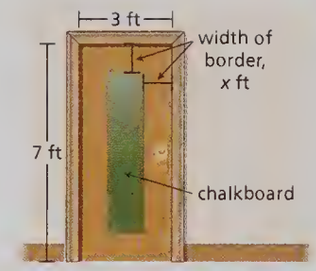
about 23 inches

MONITORING PROGRESS ANSWERS

13. about 13 in.

Solving Real-Life Problems

EXAMPLE 8 **Modeling with Mathematics**



You decide to use chalkboard paint to create a chalkboard on a door. You want the chalkboard to cover 6 square feet and to have a uniform border, as shown. Find the width of the border to the nearest inch.

SOLUTION

1. **Understand the Problem** You know the dimensions (in feet) of the door from the diagram. You also know the area (in square feet) of the chalkboard and that it will have a uniform border. You are asked to find the width of the border to the nearest inch.
2. **Make a Plan** Use a verbal model to write an equation that represents the area of the chalkboard. Then solve the equation.
3. **Solve the Problem**

Let x be the width (in feet) of the border, as shown in the diagram.

Area of chalkboard (square feet)	=	Length of chalkboard (feet)	•	Width of chalkboard (feet)
6	=	$(7 - 2x)$	•	$(3 - 2x)$

$6 = (7 - 2x)(3 - 2x)$ Write the equation.

$6 = 21 - 20x + 4x^2$ Multiply the binomials.

$-15 = 4x^2 - 20x$ Subtract 21 from each side.

$-\frac{15}{4} = x^2 - 5x$ Divide each side by 4.

$-\frac{15}{4} + \frac{25}{4} = x^2 - 5x + \frac{25}{4}$ Complete the square for $x^2 - 5x$.

$\frac{5}{2} = x^2 - 5x + \frac{25}{4}$ Simplify the left side.

$\frac{5}{2} = \left(x - \frac{5}{2}\right)^2$ Write the right side as the square of a binomial.

$\pm\sqrt{\frac{5}{2}} = x - \frac{5}{2}$ Take the square root of each side.

$\frac{5}{2} \pm \sqrt{\frac{5}{2}} = x$ Add $\frac{5}{2}$ to each side.

The solutions of the equation are $x = \frac{5}{2} + \sqrt{\frac{5}{2}} \approx 4.08$ and $x = \frac{5}{2} - \sqrt{\frac{5}{2}} \approx 0.92$.

It is not possible for the width of the border to be 4.08 feet because the width of the door is 3 feet. So, the width of the border is about 0.92 foot.

$0.92 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = 11.04 \text{ in.}$ Convert 0.92 foot to inches.

▶ The width of the border should be about 11 inches.

4. **Look Back** When the width of the border is slightly less than 1 foot, the length of the chalkboard is slightly more than 5 feet and the width of the chalkboard is slightly more than 1 foot. Multiplying these dimensions gives an area close to 6 square feet. So, an 11-inch border is reasonable.

Monitoring Progress

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13. WHAT IF? You want the chalkboard to cover 4 square feet. Find the width of the border to the nearest inch.

Closure

- **Point of Most Significance:** Ask students to identify, aloud or written, the most significant point in the lesson that aided their learning.

9.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The process of adding a constant c to the expression $x^2 + bx$ so that $x^2 + bx + c$ is a perfect square trinomial is called _____.
2. **VOCABULARY** Explain how to complete the square for an expression of the form $x^2 + bx$.
3. **WRITING** Is it more convenient to complete the square for $x^2 + bx$ when b is odd or when b is even? Explain.
4. **WRITING** Describe how you can use the process of completing the square to find the maximum or minimum value of a quadratic function.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, find the value of c that completes the square.

5. $x^2 - 8x + c$
6. $x^2 - 2x + c$
7. $x^2 + 4x + c$
8. $x^2 + 12x + c$
9. $x^2 - 15x + c$
10. $x^2 + 9x + c$

In Exercises 11–16, complete the square for the expression. Then factor the trinomial. *(See Example 1.)*

11. $x^2 - 10x$
12. $x^2 - 40x$
13. $x^2 + 16x$
14. $x^2 + 22x$
15. $x^2 + 5x$
16. $x^2 - 3x$

In Exercises 17–22, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary. *(See Example 2.)*

17. $x^2 + 14x = 15$
18. $x^2 - 6x = 16$
19. $x^2 - 4x = -2$
20. $x^2 + 2x = 5$
21. $x^2 - 5x = 8$
22. $x^2 + 11x = -10$

23. **MODELING WITH MATHEMATICS** The area of the patio is 216 square feet.



 a. Write an equation that represents the area of the patio.
 b. Find the dimensions of the patio by completing the square.

24. **MODELING WITH MATHEMATICS** Some sand art contains sand and water sealed in a glass case, similar to the one shown. When the art is turned upside down, the sand and water fall to create a new picture. The glass case has a depth of 1 centimeter and a volume of 768 cubic centimeters.



 a. Write an equation that represents the volume of the glass case.
 b. Find the dimensions of the glass case by completing the square.

In Exercises 25–32, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary. *(See Example 3.)*

25. $x^2 - 8x + 15 = 0$
26. $x^2 + 4x - 21 = 0$
27. $2x^2 + 20x + 44 = 0$
28. $3x^2 - 18x + 12 = 0$
29. $-3x^2 - 24x + 17 = -40$
30. $-5x^2 - 20x + 35 = 30$
31. $2x^2 - 14x + 10 = 26$
32. $4x^2 + 12x - 15 = 5$

31. $x = 8, x = -1$
32. $x \approx 1.19, x \approx -4.19$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–33 odd, 39–47 odd, 51–55 odd, 63, 66, 71, 75–80

Average: 1–3, 4–66 even, 71, 75–80

Advanced: 1–4, 10, 16, 22, 24, 30–38 even, 44–66 even, 67–80

HOMEWORK CHECK

Basic: 11, 17, 25, 41, 53

Average: 12, 20, 26, 42, 54

Advanced: 16, 22, 30, 44, 54

ANSWERS

1. completing the square
2. Add $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.
3. even; When b is even, $\frac{b}{2}$ is an integer.
4. Use completing the square to write the function in vertex form. The maximum or minimum is the y-coordinate of the vertex.
5. 16
6. 1
7. 4
8. 36
9. $\frac{225}{4}$
10. $\frac{81}{4}$
11. $x^2 - 10x + 25; (x - 5)^2$
12. $x^2 - 40x + 400; (x - 20)^2$
13. $x^2 + 16x + 64; (x + 8)^2$
14. $x^2 + 22x + 121; (x + 11)^2$
15. $x^2 + 5x + \frac{25}{4}; \left(x + \frac{5}{2}\right)^2$
16. $x^2 - 3x + \frac{9}{4}; \left(x - \frac{3}{2}\right)^2$
17. $x = 1, x = -15$
18. $x = 8, x = -2$
19. $x \approx 3.41, x \approx 0.59$
20. $x \approx 1.45, x \approx -3.45$
21. $x \approx 6.27, x \approx -1.27$
22. $x = -1, x = -10$
23. a. $x^2 + 6x = 216$
 b. width: 12 ft, length: 18 ft
24. a. $x^2 - 8x = 768$
 b. length: 32 cm, height: 24 cm
25. $x = 5, x = 3$
26. $x = 3, x = -7$
27. $x \approx -3.27, x \approx -6.73$
28. $x \approx 5.24, x \approx 0.76$
29. $x \approx 1.92, x \approx -9.92$
30. $x \approx 0.24, x \approx -4.24$

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

33. The number 16 should be added to each side of the equation; $x^2 + 8x + 16 = 10 + 16$; $x = -4 \pm \sqrt{26}$
34. The leading coefficient should be 1 before completing the square; $x^2 - x = 2$; $x^2 - x + \frac{1}{4} = 2 + \frac{1}{4}$
35. $b = 10, b = -10$; In a perfect square trinomial $c = \left(\frac{b}{2}\right)^2$, so $b = \pm 2\sqrt{c}$.
36. Divide each side of the equation by 3.
37. $y = (x + 3)^2 - 6$; D
38. $y = -(x + 4)^2 + 4$; A
39. $y = -(x + 2)^2 + 2$; B
40. $y = (x - 1)^2 + 3$; C
41. minimum value; -6
42. minimum value; 1
43. maximum value; -5
44. maximum value; 15
45. maximum value; -6
46. minimum value; -17
47. yes; The graph has two negative x-intercepts and it opens down.
48. no; The x-intercept is negative.
49. no; The x-intercepts are both positive.
50. yes; The graph has one positive x-intercept and one negative x-intercept, and it opens down.
51. f, m; The graph has two negative x-intercepts and it opens up.
52. r, n; The graph has one positive x-intercept and one negative x-intercept, and it opens down.
53. a. 36 ft
 b. $x = \frac{3}{2}$; On the left side of $x = \frac{3}{2}$, the height increases as time increases. On the right side of $x = \frac{3}{2}$, the height decreases as time increases.

33. ERROR ANALYSIS Describe and correct the error in solving $x^2 + 8x = 10$ by completing the square.

✗
$$x^2 + 8x = 10$$
$$x^2 + 8x + 16 = 10$$
$$(x + 4)^2 = 10$$
$$x + 4 = \pm\sqrt{10}$$
$$x = -4 \pm \sqrt{10}$$

34. ERROR ANALYSIS Describe and correct the error in the first two steps of solving $2x^2 - 2x - 4 = 0$ by completing the square.

✗
$$2x^2 - 2x - 4 = 0$$
$$2x^2 - 2x = 4$$
$$2x^2 - 2x + 1 = 4 + 1$$

35. NUMBER SENSE Find all values of b for which $x^2 + bx + 25$ is a perfect square trinomial. Explain how you found your answer.

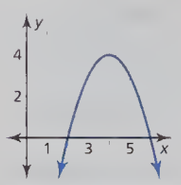
36. REASONING You are completing the square to solve $3x^2 + 6x = 12$. What is the first step?

In Exercises 37–40, write the function in vertex form by completing the square. Then match the function with its graph.

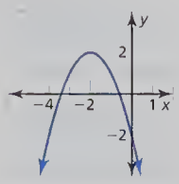
37. $y = x^2 + 6x + 3$ **38.** $y = -x^2 + 8x - 12$

39. $y = -x^2 - 4x - 2$ **40.** $y = x^2 - 2x + 4$

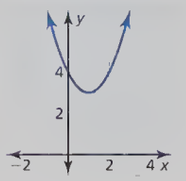
A.



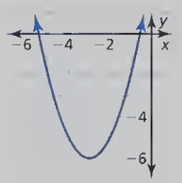
B.



C.



D.



In Exercises 41–46, determine whether the quadratic function has a maximum or minimum value. Then find the value. *(See Examples 4 and 5.)*

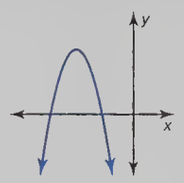
41. $y = x^2 - 4x - 2$ **42.** $y = x^2 + 6x + 10$

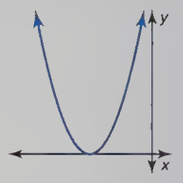
43. $y = -x^2 - 10x - 30$ **44.** $y = -x^2 + 14x - 34$

45. $f(x) = -3x^2 - 6x - 9$ **46.** $f(x) = 4x^2 - 28x + 32$

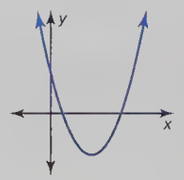
In Exercises 47–50, determine whether the graph could represent the function. Explain.

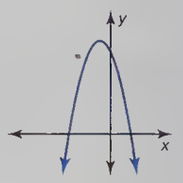
47. $y = -(x + 8)(x + 3)$ **48.** $y = (x - 5)^2$





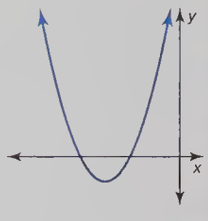
49. $y = \frac{1}{4}(x + 2)^2 - 4$ **50.** $y = -2(x - 1)(x + 2)$





In Exercises 51 and 52, determine which of the functions could be represented by the graph. Explain. *(See Example 6.)*

51.



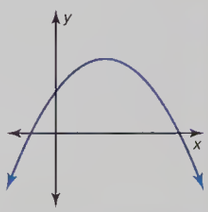
$h(x) = (x + 2)^2 + 3$

$f(x) = 2(x + 3)^2 - 2$

$g(x) = -\frac{1}{2}(x - 8)(x - 4)$

$m(x) = (x + 2)(x + 4)$

52.



$r(x) = -\frac{1}{3}(x - 5)(x + 1)$

$p(x) = -2(x - 2)(x - 6)$

$q(x) = (x + 1)^2 + 4$

$n(x) = -(x - 2)^2 + 9$

53. MODELING WITH MATHEMATICS The function $h = -16t^2 + 48t$ represents the height h (in feet) of a kickball t seconds after it is kicked from the ground. *(See Example 7.)*

a. Find the maximum height of the kickball.

b. Find and interpret the axis of symmetry.

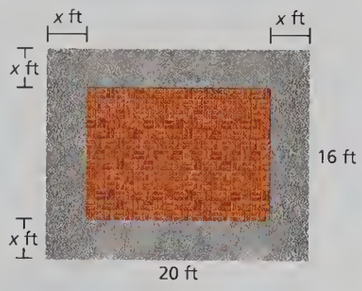
S4. MODELING WITH MATHEMATICS You throw a stone from a height of 16 feet with an initial vertical velocity of 32 feet per second. The function $h = -16t^2 + 32t + 16$ represents the height h (in feet) of the stone after t seconds.



a. Find the maximum height of the stone.

b. Find and interpret the axis of symmetry.

S5. MODELING WITH MATHEMATICS You are building a rectangular brick patio surrounded by a crushed stone border with a uniform width, as shown. You purchase patio bricks to cover 140 square feet. Find the width of the border. *(See Example 8.)*

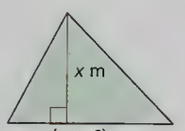


S6. MODELING WITH MATHEMATICS You are making a poster that will have a uniform border, as shown. The total area of the poster is 722 square inches. Find the width of the border to the nearest inch.

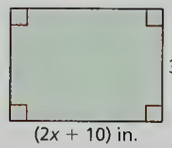


MATHEMATICAL CONNECTIONS **In Exercises 57 and 58, find the value of x. Round your answer to the nearest hundredth, if necessary.**

S7. $A = 108 \text{ m}^2$



S8. $A = 288 \text{ in.}^2$



In Exercises 59–62, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

S9. $0.5x^2 + x - 2 = 0$

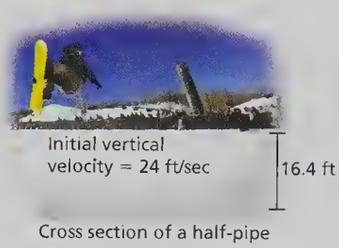
60. $0.75x^2 + 1.5x = 4$

61. $\frac{8}{3}x - \frac{2}{3}x^2 = -\frac{5}{6}$

62. $\frac{1}{4}x^2 + \frac{1}{2}x - \frac{5}{4} = 0$

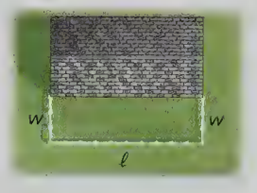
63. PROBLEM SOLVING The distance d (in feet) that it takes a car to come to a complete stop can be modeled by $d = 0.05s^2 + 2.2s$, where s is the speed of the car (in miles per hour). A car has 168 feet to come to a complete stop. Find the maximum speed at which the car can travel.

64. PROBLEM SOLVING During a "big air" competition, snowboarders launch themselves from a half-pipe, perform tricks in the air, and land back in the half-pipe. The height h (in feet) of a snowboarder above the bottom of the half-pipe can be modeled by $h = -16t^2 + 24t + 16.4$, where t is the time (in seconds) after the snowboarder launches into the air. The snowboarder lands 3.2 feet lower than the height of the launch. How long is the snowboarder in the air? Round your answer to the nearest tenth of a second.



Cross section of a half-pipe

65. PROBLEM SOLVING You have 80 feet of fencing to make a rectangular horse pasture that covers 750 square feet. A barn will be used as one side of the pasture, as shown.



a. Write equations for the amount of fencing to be used and the area enclosed by the fencing.

b. Use substitution to solve the system of equations from part (a). What are the possible dimensions of the pasture?

ANSWERS

54. a. 32 ft

b. $x = 1$; On the left side of $x = 1$, the height increases as time increases. On the right side of $x = 1$, the height decreases as time increases.

55. 3 ft

56. about 1 in.

57. 12

58. about 4.87

59. $x \approx 1.24, x \approx -3.24$

60. $x \approx 1.52, x \approx -3.52$

61. $x \approx 4.29, x \approx -0.29$

62. $x \approx 1.45, x \approx -3.45$

63. 40 mi/h

64. about 1.6 sec

65. a. $\ell + 2w = 80, \ell w = 750$

b. length: 30 ft, width: 25 ft; length: 50 ft, width: 15 ft

ANSWERS

66. a. 3, 1

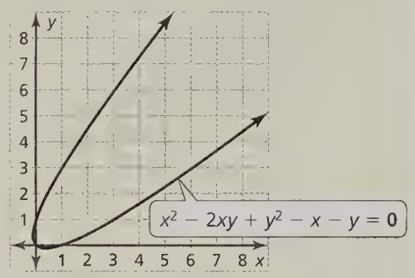
b. Substitute 3 for y in the equation, then solve the equation by completing the square.

67. a. $x \approx 0.8, x \approx -12.8$

b. $x \approx 0.78, x \approx -12.78$

c. *Sample answer:* completing the square; The result is more accurate.

68.



a parabola at an angle

69. $x(x + 2) = 48$, 6 and 8

70. $x(x + 2) = 195$; $x = -15$ and $x = -13$

71. yes; Substituting 23.50 for y in the model, the stock is worth \$23.50 ten days and thirty days after the stock is purchased.

72. Factoring does not work for this equation, and graphing does not produce an exact solution.

73. length: 66 in., width: 6 in.

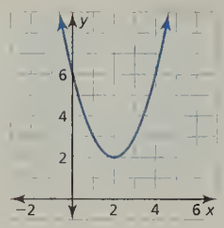
74. 0; If $c < -\left(\frac{b}{2}\right)^2$, then $\left(\frac{b}{2}\right)^2 + c < 0$, so there are no real solutions.

75–80. See Additional Answers.

Mini-Assessment

1. Complete the square for the expression $x^2 + 24x$. Then factor the trinomial. $x^2 + 24x + 144$; $(x + 12)^2$
2. Solve $x^2 - 12x = 45$ by completing the square. $x = 15$ and $x = -3$
3. Find the minimum value of $y = x^2 - 20x + 75$. -25
4. The function $y = -16x^2 + 32x$ represents the height y (in feet) of a football x seconds after it is kicked off the ground.
 a. Find the maximum height of the football. 16 feet
 b. Find and interpret the axis of symmetry. $x = 1$; The height increases when $x < 1$ and decreases when $x > 1$.

66. HOW DO YOU SEE IT? The graph represents the quadratic function $y = x^2 - 4x + 6$.



a. Use the graph to estimate the x-values for which $y = 3$.

b. Explain how you can use the method of completing the square to check your estimates in part (a).

67. COMPARING METHODS Consider the quadratic equation $x^2 + 12x + 2 = 12$.

a. Solve the equation by graphing.

b. Solve the equation by completing the square.

c. Compare the two methods. Which do you prefer? Explain.

68. THOUGHT PROVOKING Sketch the graph of the equation $x^2 - 2xy + y^2 - x - y = 0$. Identify the graph.

69. REASONING The product of two consecutive even integers that are positive is 48. Write and solve an equation to find the integers.

70. REASONING The product of two consecutive odd integers that are negative is 195. Write and solve an equation to find the integers.

71. MAKING AN ARGUMENT You purchase stock for \$16 per share. You sell the stock 30 days later for \$23.50 per share. The price y (in dollars) of a share during the 30-day period can be modeled by $y = -0.025x^2 + x + 16$, where x is the number of days after the stock is purchased. Your friend says you could have sold the stock earlier for \$23.50 per share. Is your friend correct? Explain.



72. REASONING You are solving the equation $x^2 + 9x = 18$. What are the advantages of solving the equation by completing the square instead of using other methods you have learned?

73. PROBLEM SOLVING You are knitting a rectangular scarf. The pattern results in a scarf that is 60 inches long and 4 inches wide. However, you have enough yarn to knit 396 square inches. You decide to increase the dimensions of the scarf so that you will use all your yarn. The increase in the length is three times the increase in the width. What are the dimensions of your scarf?



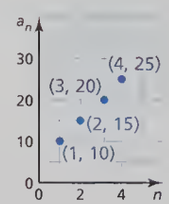
74. WRITING How many solutions does $x^2 + bx = c$ have when $c < -\left(\frac{b}{2}\right)^2$? Explain.

Maintaining Mathematical Proficiency

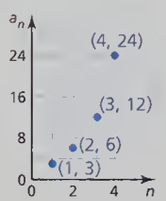
Reviewing what you learned in previous grades and lessons

Write a recursive rule for the sequence. *(Section 6.6)*

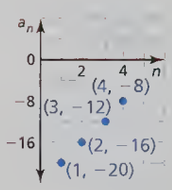
75.



76.



77.



Simplify the expression $\sqrt{b^2 - 4ac}$ for the given values. *(Section 9.1)*

78. $a = 3, b = -6, c = 2$ **79.** $a = -2, b = 4, c = 7$ **80.** $a = 1, b = 6, c = 4$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

9.5 Solving Quadratic Equations Using the Quadratic Formula

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.8.A

Essential Question How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

EXPLORATION 1 Deriving the Quadratic Formula

Work with a partner. The following steps show a method of solving $ax^2 + bx + c = 0$. Explain what was done in each step.

$$ax^2 + bx + c = 0 \quad \leftarrow \text{1. Write the equation.}$$

$$4a^2x^2 + 4abx + 4ac = 0 \quad \leftarrow \text{2. What was done?}$$

$$4a^2x^2 + 4abx + 4ac + b^2 = b^2 \quad \leftarrow \text{3. What was done?}$$

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac \quad \leftarrow \text{4. What was done?}$$

$$(2ax + b)^2 = b^2 - 4ac \quad \leftarrow \text{5. What was done?}$$

$$2ax + b = \pm\sqrt{b^2 - 4ac} \quad \leftarrow \text{6. What was done?}$$

$$2ax = -b \pm \sqrt{b^2 - 4ac} \quad \leftarrow \text{7. What was done?}$$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← 8. What was done?

EXPLORATION 2 Deriving the Quadratic Formula by Completing the Square

Work with a partner.

a. Solve $ax^2 + bx + c = 0$ by completing the square. (*Hint:* Subtract c from each side, divide each side by a, and then proceed by completing the square.)

b. Compare this method with the method in Exploration 1. Explain why you think $4a$ and b^2 were chosen in Steps 2 and 3 of Exploration 1.

Communicate Your Answer

SELECTING TOOLS
To be proficient in math, you need to identify relevant external mathematical resources.

3. How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?
4. Use the Quadratic Formula to solve each quadratic equation.
 a. $x^2 + 2x - 3 = 0$ b. $x^2 - 4x + 4 = 0$ c. $x^2 + 4x + 5 = 0$
5. Use the Internet to research *imaginary numbers*. How are they related to quadratic equations?

For a section overview and insights into this Exploration page, see Laurie's Notes at *BigIdeasMath.com*.

SUPPORTING English Language Learners

Present the Essential Question. Have students work in pairs to complete the explorations and Communicate Your Answer. Have them share their answers with the class.

Beginning/Advanced When sharing answers, Beginning students use simple phrases to provide details, and Advanced students use multiple sentences.

Intermediate/Advanced High When sharing answers, Intermediate students use simple sentences to provide details, and Advanced High students use multiple complex sentences.

ELPS 3.H.3 Explain with increasing specificity and detail as more English is acquired.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Lesson Planning Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

Texas Essential Knowledge and Skills

A.8.A The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

ANSWERS

1. 2. Multiply each side by $4a$.
 3. Add b^2 to each side.
 4. Subtract $4ac$ from each side.
 5. Write the left side in factored form.
 6. Take the square root of each side.
 7. Subtract b from each side.
 8. Divide each side by $2a$.
2. a. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 b. to obtain a perfect square trinomial on the left side of the equation
3. Complete the square of the general form of the quadratic equation, $ax^2 + bx + c = 0$.
4. a. $x = 1, x = -3$
 b. $x = 2$
 c. no real solution
5. The imaginary number, i, is $\sqrt{-1}$. Quadratic equations with no real solution have complex solutions, which include an imaginary part.

Extra Example 1

Solve $x^2 - 4x - 5 = 0$ using the Quadratic Formula.

The solutions are $x = \frac{4 + 6}{2} = 5$ and $x = \frac{4 - 6}{2} = -1$.

MONITORING PROGRESS ANSWERS

1. $x = 5, x = 1$
2. $x \approx 3.6, x \approx -5.6$
3. $x \approx -1.2, x \approx 1.9$
4. $x = \frac{1}{2}$

For classroom suggestions on teaching this lesson, see **Laurie's Notes** at *BigIdeasMath.com*.

9.5 Lesson

Core Vocabulary

Quadratic Formula, *p. 504*
discriminant, *p. 506*

What You Will Learn

- Solve quadratic equations using the Quadratic Formula.
- Interpret the discriminant.
- Choose efficient methods for solving quadratic equations.

Using the Quadratic Formula

By completing the square for the quadratic equation $ax^2 + bx + c = 0$, you can develop a formula that gives the solutions of any quadratic equation in standard form. This formula is called the **Quadratic Formula**.

Core Concept

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

where $a \neq 0$ and $b^2 - 4ac \geq 0$.

EXAMPLE 1 Using the Quadratic Formula

Solve $2x^2 - 5x + 3 = 0$ using the Quadratic Formula.

SOLUTION

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$ Substitute 2 for a, −5 for b, and 3 for c.

$$= \frac{5 \pm \sqrt{1}}{4}$$ Simplify.

$$= \frac{5 \pm 1}{4}$$ Evaluate the square root.

So, the solutions are $x = \frac{5 + 1}{4} = \frac{3}{2}$ and $x = \frac{5 - 1}{4} = 1$.

STUDY TIP

You can use the roots of a quadratic equation to factor the related expression. In Example 1, you can use 1 and $\frac{3}{2}$ to factor $2x^2 - 5x + 3$ as $(x - 1)(2x - 3)$.

Check

$2x^2 - 5x + 3 = 0$	Original equation	$2x^2 - 5x + 3 = 0$
$2\left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 3 \stackrel{?}{=} 0$	Substitute.	$2(1)^2 - 5(1) + 3 \stackrel{?}{=} 0$
$\frac{9}{2} - \frac{15}{2} + 3 \stackrel{?}{=} 0$	Simplify.	$2 - 5 + 3 \stackrel{?}{=} 0$
$0 = 0$ ✓	Simplify.	$0 = 0$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

1. $x^2 - 6x + 5 = 0$
2. $\frac{1}{2}x^2 + x - 10 = 0$
3. $-3x^2 + 2x + 7 = 0$
4. $4x^2 - 4x = -1$

Differentiated Instruction

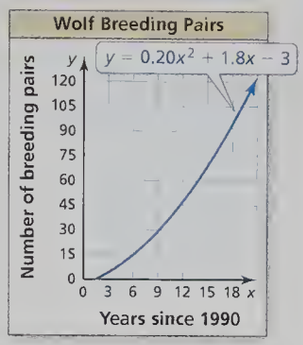
Organization/Inclusion

Some students may view the Quadratic Formula as complex and may not know what to do first when trying to use it. Suggest that they complete this table before they begin solving a quadratic equation in standard form.

After completing the table, students can substitute the appropriate values into the formula and simplify.

Coefficient	Value	Term	Value
a		$-b$	
b		b^2	
c		$4ac$	
		$2a$	

EXAMPLE 2 Modeling With Mathematics



The number y of Northern Rocky Mountain wolf breeding pairs x years since 1990 can be modeled by the function $y = 0.20x^2 + 1.8x - 3$. When were there about 35 breeding pairs?

SOLUTION

1. **Understand the Problem** You are given a quadratic function that represents the number of wolf breeding pairs for years after 1990. You need to use the model to determine when there were 35 wolf breeding pairs.

2. **Make a Plan** To determine when there were 35 wolf breeding pairs, find the x-values for which $y = 35$. So, solve the equation $35 = 0.20x^2 + 1.8x - 3$.

3. **Solve the Problem**

$35 = 0.20x^2 + 1.8x - 3$ Write the equation.

$0 = 0.20x^2 + 1.8x - 38$ Write in standard form.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \dfrac{-1.8 \pm \sqrt{1.8^2 - 4(0.2)(-38)}}{2(0.2)}$ Substitute 0.2 for a, 1.8 for b, and -38 for c.

$= \dfrac{-1.8 \pm \sqrt{33.64}}{0.4}$ Simplify.

$= \dfrac{-1.8 \pm 5.8}{0.4}$ Simplify.

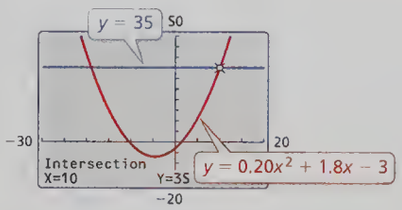
The solutions are $x = \dfrac{-1.8 + 5.8}{0.4} = 10$ and $x = \dfrac{-1.8 - 5.8}{0.4} = -19$.

EXPLAINING MATHEMATICAL IDEAS

You can ignore the solution $x = -19$ because -19 represents the year 1971, which is not in the given time period.

▶ Because x represents the number of years since 1990, x is greater than or equal to zero. So, there were about 35 breeding pairs 10 years after 1990, in 2000.

4. **Look Back** Use a graphing calculator to graph the equations $y = 0.20x^2 + 1.8x - 3$ and $y = 35$. Then use the *intersect* feature to find the point of intersection. The graphs intersect at (10, 35).



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

5. **WHAT IF?** When were there about 60 wolf breeding pairs?

6. The number y of bald eagle nesting pairs in a state x years since 2000 can be modeled by the function $y = 0.34x^2 + 13.1x + 51$.

 a. When were there about 160 bald eagle nesting pairs?

 b. How many bald eagle nesting pairs were there in 2000?

Extra Example 2

The number y of Northern Rocky Mountain wolf breeding pairs x years since 1990 can be modeled by the function $y = 0.20x^2 + 1.8x - 3$. When were there about 77 breeding pairs of wolves? There were about 77 breeding pairs 16 years after 1990, in 2006.

MONITORING PROGRESS ANSWERS

5. 2004

6. a. 2007

 b. 51 bald eagle nesting pairs

Extra Example 3

a. Determine the number of real solutions of $2x^2 - 3x - 1 = 0$.

The equation has two real solutions.

b. Determine the number of real solutions of $x^2 + 4 = x$.

The equation has no real solutions.

MONITORING PROGRESS ANSWERS

7. one real solution
8. no real solutions
9. two real solutions

Interpreting the Discriminant

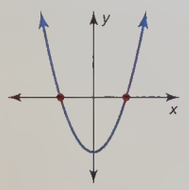
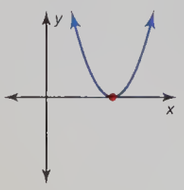
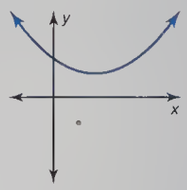
The expression $b^2 - 4ac$ in the Quadratic Formula is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ← discriminant

Because the discriminant is under the radical symbol, you can use the value of the discriminant to determine the number of real solutions of a quadratic equation and the number of x-intercepts of the graph of the related function.

Core Concept

Interpreting the Discriminant

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
		
• two real solutions • two x-intercepts	• one real solution • one x-intercept	• no real solutions • no x-intercepts

STUDY TIP

The solutions of a quadratic equation may be real numbers or *imaginary numbers*. You will study imaginary numbers in a future course.

EXAMPLE 3 Determining the Number of Real Solutions

a. Determine the number of real solutions of $x^2 + 8x - 3 = 0$.

$b^2 - 4ac = 8^2 - 4(1)(-3)$ Substitute 1 for a, 8 for b, and -3 for c.

$= 64 + 12$ Simplify.

$= 76$ Add.

▶ The discriminant is greater than 0. So, the equation has two real solutions.

b. Determine the number of real solutions of $9x^2 + 1 = 6x$.

Write the equation in standard form: $9x^2 - 6x + 1 = 0$.

$b^2 - 4ac = (-6)^2 - 4(9)(1)$ Substitute 9 for a, -6 for b, and 1 for c.

$= 36 - 36$ Simplify.

$= 0$ Subtract.

▶ The discriminant is 0. So, the equation has one real solution.

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Determine the number of real solutions of the equation.

7. $-x^2 + 4x - 4 = 0$
8. $6x^2 + 2x = -1$
9. $\frac{1}{2}x^2 = 7x - 1$

EXAMPLE 4 **Finding the Number of x-Intercepts of a Parabola**

Find the number of x-intercepts of the graph of $y = 2x^2 + 3x + 9$.

SOLUTION

Determine the number of real solutions of $0 = 2x^2 + 3x + 9$.

$b^2 - 4ac = 3^2 - 4(2)(9)$ Substitute 2 for a, 3 for b, and 9 for c.

$= 9 - 72$ Simplify.

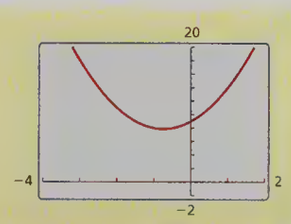
$= -63$ Subtract.

Because the discriminant is less than 0, the equation has no real solutions.

▶ So, the graph of $y = 2x^2 + 3x + 9$ has no x-intercepts.

Check

Use a graphing calculator to check your answer. Notice that the graph of $y = 2x^2 + 3x + 9$ has no x-intercepts.



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Find the number of x-intercepts of the graph of the function.

10. $y = -x^2 + x - 6$ **11.** $y = x^2 - x$ **12.** $f(x) = x^2 + 12x + 36$

Choosing an Efficient Method

The table shows five methods for solving quadratic equations. For a given equation, it may be more efficient to use one method instead of another. Some advantages and disadvantages of each method are shown.

Core Concept

Methods for Solving Quadratic Equations

Method	Advantages	Disadvantages
Factoring *(Lessons 7.6–7.9)*	• Straightforward when the equation can be factored easily	• Some equations are not factorable.
Graphing *(Lesson 9.2)*	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots *(Lesson 9.3)*	• Use to solve equations of the form $x^2 = d$.	• Can only be used for certain equations
Completing the Square *(Lesson 9.4)*	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula *(Lesson 9.5)*	• Can be used for any quadratic equation • Gives exact solutions	• Takes time to do calculations

Extra Example 4

Find the number of x-intercepts of the graph of $y = 2x^2 - 4x - 3$. The graph of $y = 2x^2 - 4x - 3$ has two x-intercepts.

MONITORING PROGRESS ANSWERS

10. no x-intercepts

11. two x-intercepts

12. one x-intercept

Extra Example 5

Solve the equation using any method. Explain your choice of method.

a. $x^2 - 4x - 7 = 0$

$x = 2 + \sqrt{11} \approx 5.3$ and $x = 2 - \sqrt{11} \approx -1.3$; The coefficient of the x^2-term is 1, and the coefficient of the x-term is an even number. So, solve by completing the square.

b. $(x - 2)^2 = 64$

$x = 10$ and $x = -6$; The equation has the form $x^2 = d$. So, solve by using square roots.

c. $10x^2 + 7x - 12 = 0$

$x = \frac{-7 + 23}{20} = \frac{4}{5}$ and $x = \frac{-7 - 23}{20} = -\frac{3}{2}$; The equation is not easily factorable, and the numbers are somewhat large. So, solve using the Quadratic Formula.

MONITORING PROGRESS ANSWERS

13. $x = -12, x = 1$; *Sample answer:* The equation is easily factorable, so solve by factoring.
14. $x = 1, x = -1$; *Sample answer:* The equation can be written in the form $x^2 = d$, so solve using square roots.
15. $x \approx 0.56, x \approx -0.36$; *Sample answer:* The equation is not factorable and the coefficient of the x^2-term is not 1, so solve using the quadratic formula.
16. no real solutions; *Sample answer:* The coefficient of the x^2-term is 1 and b is even, so solve by completing the square.

EXAMPLE 5 **Choosing a Method**

Solve the equation using any method. Explain your choice of method.

a. $x^2 - 10x = 1$ **b.** $2x^2 - 13x - 24 = 0$ **c.** $x^2 + 8x + 12 = 0$

SOLUTION

a. The coefficient of the x^2-term is 1, and the coefficient of the x-term is an even number. So, solve by completing the square.

$x^2 - 10x = 1$ Write the equation.

$x^2 - 10x + 25 = 1 + 25$ Complete the square for $x^2 - 10x$.

$(x - 5)^2 = 26$ Write the left side as the square of a binomial.

$x - 5 = \pm\sqrt{26}$ Take the square root of each side.

$x = 5 \pm \sqrt{26}$ Add 5 to each side.

▶ So, the solutions are $x = 5 + \sqrt{26} \approx 10.1$ and $x = 5 - \sqrt{26} \approx -0.1$.

b. The equation is not easily factorable, and the numbers are somewhat large. So, solve using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(-24)}}{2(2)}$ Substitute 2 for a, -13 for b, and -24 for c.

$= \frac{13 \pm \sqrt{361}}{4}$ Simplify.

$= \frac{13 \pm 19}{4}$ Evaluate the square root.

▶ So, the solutions are $x = \frac{13 + 19}{4} = 8$ and $x = \frac{13 - 19}{4} = -\frac{3}{2}$.

c. The equation is easily factorable. So, solve by factoring.

$x^2 + 8x + 12 = 0$ Write the equation.

$(x + 2)(x + 6) = 0$ Factor the polynomial.

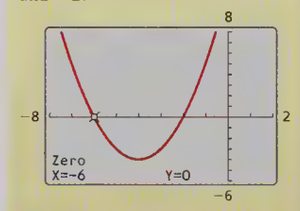
$x + 2 = 0$ or $x + 6 = 0$ Zero-Product Property

$x = -2$ or $x = -6$ Solve for x.

▶ The solutions are $x = -2$ and $x = -6$.

Check

Graph the related function $f(x) = x^2 + 8x + 12$ and find the zeros. The zeros are -6 and -2.



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Solve the equation using any method. Explain your choice of method.

13. $x^2 + 11x - 12 = 0$
14. $9x^2 - 5 = 4$
15. $5x^2 - x - 1 = 0$
16. $x^2 = 2x - 5$

Closure

- Write a quadratic equation that you would *not* solve using (a) square roots, (b) factoring, and (c) completing the square. *Sample answers:* (a) $x^2 + 12x + 8 = 0$; (b) $x^2 = 211$; (c) $x^2 - 7x + 12 = 0$

9.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What formula can you use to solve any quadratic equation? Write the formula.
2. **VOCABULARY** In the Quadratic Formula, what is the discriminant? What does the value of the discriminant determine?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write the equation in standard form. Then identify the values of a, b, and c that you would use to solve the equation using the Quadratic Formula.

3. $x^2 = 7x$
4. $x^2 - 4x = -12$
5. $-2x^2 + 1 = 5x$
6. $3x + 2 = 4x^2$
7. $4 - 3x = -x^2 + 3x$
8. $-8x - 1 = 3x^2 + 2$

In Exercises 9–22, solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary. *(See Example 1.)*

9. $x^2 - 12x + 36 = 0$
10. $x^2 + 7x + 16 = 0$
11. $x^2 - 10x - 11 = 0$
12. $2x^2 - x - 1 = 0$
13. $2x^2 - 6x + 5 = 0$
14. $9x^2 - 6x + 1 = 0$
15. $6x^2 - 13x = -6$
16. $-3x^2 + 6x = 4$
17. $1 - 8x = -16x^2$
18. $x^2 - 5x + 3 = 0$
19. $x^2 + 2x = 9$
20. $5x^2 - 2 = 4x$
21. $2x^2 + 9x + 7 = 3$
22. $8x^2 + 8 = 6 - 9x$

23. **MODELING WITH MATHEMATICS** A dolphin jumps out of the water, as shown in the diagram. The function $h = -16t^2 + 26t$ models the height h (in feet) of the dolphin after t seconds. After how many seconds is the dolphin at a height of 5 feet? *(See Example 2.)*



24. **MODELING WITH MATHEMATICS** The amount of trout y (in tons) caught in a lake from 1995 to 2014 can be modeled by the equation $y = -0.08x^2 + 1.6x + 10$, where x is the number of years since 1995.
 a. When were about 15 tons of trout caught in the lake?
 b. Do you think this model can be used to determine the amounts of trout caught in future years? Explain your reasoning.

In Exercises 25–30, determine the number of real solutions of the equation. *(See Example 3.)*

25. $x^2 - 6x + 10 = 0$
26. $x^2 - 5x - 3 = 0$
27. $2x^2 - 12x = -18$
28. $4x^2 = 4x - 1$
29. $-\frac{1}{4}x^2 + 4x = -2$
30. $-5x^2 + 8x = 9$

In Exercises 31–36, find the number of x-intercepts of the graph of the function. *(See Example 4.)*

31. $y = x^2 + 5x - 1$
32. $y = 4x^2 + 4x + 1$
33. $y = -6x^2 + 3x - 4$
34. $y = -x^2 + 5x + 13$
35. $f(x) = 4x^2 + 3x - 6$
36. $f(x) = 2x^2 + 8x + 8$

In Exercises 37–44, solve the equation using any method. Explain your choice of method. *(See Example 5.)*

37. $-10x^2 + 13x = 4$
38. $x^2 - 3x - 40 = 0$
39. $x^2 + 6x = 5$
40. $-5x^2 = -25$
41. $x^2 + x - 12 = 0$
42. $x^2 - 4x + 1 = 0$
43. $4x^2 - x = 17$
44. $x^2 + 6x + 9 = 16$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–49 odd, 71, 74, 83–86

Average: 1, 2–48 even, 49, 50–62 even, 72, 74, 83–86

Advanced: 1, 2, 8, 20–30 even, 34–48 even, 49, 50–76 even, 77–86

HOMEWORK CHECK

Basic: 9, 23, 25, 31, 37

Average: 10, 24, 26, 32, 38

Advanced: 20, 24, 28, 34, 38

ANSWERS

1. the quadratic formula; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2. $b^2 - 4ac$; the number of real solutions of a quadratic equation
3. $x^2 - 7x = 0$; $a = 1, b = -7, c = 0$ or $-x^2 + 7x = 0$; $a = -1, b = 7, c = 0$
4. $x^2 - 4x + 12 = 0$; $a = 1, b = -4, c = 12$ or $-x^2 + 4x - 12 = 0$; $a = -1, b = 4, c = -12$
5. $-2x^2 - 5x + 1 = 0$; $a = -2, b = -5, c = 1$ or $2x^2 + 5x - 1 = 0$; $a = 2, b = 5, c = -1$
6. $-4x^2 + 3x + 2 = 0$; $a = -4, b = 3, c = 2$ or $4x^2 - 3x - 2 = 0$; $a = 4, b = -3, c = -2$
7. $x^2 - 6x + 4 = 0$; $a = 1, b = -6, c = 4$ or $-x^2 + 6x - 4 = 0$; $a = -1, b = 6, c = -4$
8. $-3x^2 - 8x - 3 = 0$; $a = -3, b = -8, c = -3$ or $3x^2 + 8x + 3 = 0$; $a = 3, b = 8, c = 3$
9. $x = 6$
10. no real solutions
11. $x = 11, x = -1$
12. $x = 1, x = -\frac{1}{2}$
13. no real solutions
14. $x = \frac{1}{3}$
15. $x = \frac{3}{2}, x = \frac{2}{3}$
16. no real solutions
17. $x = \frac{1}{4}$
18. $x \approx 4.3, x \approx 0.7$
19. $x \approx 2.2, x \approx -4.2$
20. $x \approx 1.1, x \approx -0.3$
21. $x = -\frac{1}{2}, x = -4$
22. $x \approx -0.3, x \approx -0.8$
23. about 0.2 sec, about 1.4 sec
24. a. 1999, 2011
 b. no; According to the model, the amount of trout caught after 2020 is negative.
25. no real solutions
26. two real solutions
27. one real solution
28. one real solution
29. two real solutions
30. no real solutions
31. two x-intercepts
32. one x-intercept
33. no x-intercepts
34. two x-intercepts
35. two x-intercepts
36. one x-intercept
37. $x = \frac{1}{2}, x = \frac{4}{5}$; *Sample answer:* The equation is not easily factorable and $a \neq 1$, so solve using the quadratic formula.
38. $x = -5, x = 8$; *Sample answer:* The equation is easily factorable, so solve by factoring.
39. $x \approx 0.74, x \approx -6.74$; *Sample answer:* $a = 1$ and b is even, so solve by completing the square.
40–44. See Additional Answers.

Dynamic Teaching Tools

- Dynamic Assessment & Progress Monitoring Tool
- Interactive Whiteboard Lesson Library
- Dynamic Classroom with Dynamic Investigations

ANSWERS

45. $-b$ should be $-(-7)$, not -7;

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)};$$

$x = 3$ and $x = -\frac{2}{3}$

46. The equation needs to be in the form $ax^2 + bx + c = 0$, so $c = -4$ not 4;

$$x = \frac{-9 \pm \sqrt{9^2 - 4(-2)(-4)}}{2(-2)};$$

$x = \frac{1}{2}$ and $x = 4$

47. yes; about 42 ft, about 158 ft

48. **a.** no

b. yes; 2 days

49. no; The discriminant is -47, so the equation has no real solutions.

50. no; Substituting 4 for y in the model results in no real solutions.

51. 5; length: 13 m, width: 7 m

52. 4; length: 19 ft, width: 11 ft

53. **a–c.** $x = -2$

Sample answer: factoring; The equation is easily factorable.

54. **a–c.** $x = -\frac{2}{3}, x = -3$

Sample answer: quadratic formula; The equation is not easily factorable and one x-intercept is not an integer.

55. 2; When a and c have different signs, ac is negative, so the discriminant is positive.

56. rational; When the discriminant is a perfect square, the square root of the discriminant is an integer.

57. **a.** *Sample answer:* $\frac{1}{2}$

b. 1

c. *Sample answer:* 2

58. **a.** *Sample answer:* 1

b. 16

c. *Sample answer:* 17

59. **a.** *Sample answer:* 1

b. 9

c. *Sample answer:* 10

45. ERROR ANALYSIS Describe and correct the error in solving the equation $3x^2 - 7x - 6 = 0$ using the Quadratic Formula.

✗

$$x = \frac{-7 \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)}$$

$$= \frac{-7 \pm \sqrt{121}}{6}$$

$$x = \frac{2}{3} \text{ and } x = -3$$

46. ERROR ANALYSIS Describe and correct the error in solving the equation $-2x^2 + 9x = 4$ using the Quadratic Formula.

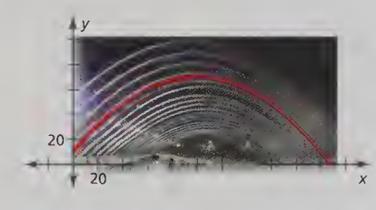
✗

$$x = \frac{-9 \pm \sqrt{9^2 - 4(-2)(4)}}{2(-2)}$$

$$= \frac{-9 \pm \sqrt{113}}{-4}$$

$$x \approx -0.41 \text{ and } x \approx 4.91$$

47. MODELING WITH MATHEMATICS A fountain shoots a water arc that can be modeled by the graph of the equation $y = -0.006x^2 + 1.2x + 10$, where x is the horizontal distance (in feet) from the river's north shore and y is the height (in feet) above the river. Does the water arc reach a height of 50 feet? If so, about how far from the north shore is the water arc 50 feet above the water?



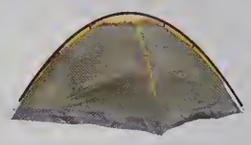
48. MODELING WITH MATHEMATICS Between the months of April and September, the number y of hours of daylight per day in Seattle, Washington, can be modeled by $y = -0.00046x^2 + 0.076x + 13$, where x is the number of days since April 1.

a. Do any of the days between April and September in Seattle have 17 hours of daylight? If so, how many?

b. Do any of the days between April and September in Seattle have 14 hours of daylight? If so, how many?

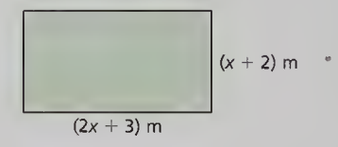
49. MAKING AN ARGUMENT Your friend uses the discriminant of the equation $2x^2 - 5x - 2 = -11$ and determines that the equation has two real solutions. Is your friend correct? Explain your reasoning.

50. MODELING WITH MATHEMATICS The frame of the tent shown is defined by a rectangular base and two parabolic arches that connect the opposite corners of the base. The graph of $y = -0.18x^2 + 1.6x$ models the height y (in feet) of one of the arches x feet along the diagonal of the base. Can a child who is 4 feet tall walk under one of the arches without having to bend over? Explain.

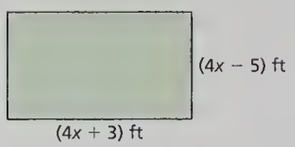


MATHEMATICAL CONNECTIONS **In Exercises 51 and 52, use the given area A of the rectangle to find the value of x. Then give the dimensions of the rectangle.**

51. $A = 91$ m^2



52. $A = 209$ ft^2



COMPARING METHODS **In Exercises 53 and 54, solve the equation by (a) graphing, (b) factoring, and (c) using the Quadratic Formula. Which method do you prefer? Explain your reasoning.**

53. $x^2 + 4x + 4 = 0$

54. $3x^2 + 11x + 6 = 0$

55. REASONING How many solutions does the equation $ax^2 + bx + c = 0$ have when a and c have different signs? Explain your reasoning.

56. REASONING When the discriminant is a perfect square, are the solutions of $ax^2 + bx + c = 0$ rational or irrational? (Assume a, b, and c are integers.) Explain your reasoning.

REASONING **In Exercises 57–59, give a value of c for which the equation has (a) two solutions, (b) one solution, and (c) no solutions.**

57. $x^2 - 2x + c = 0$

58. $x^2 - 8x + c = 0$

59. $4x^2 + 12x + c = 0$

60. REPEATED REASONING You use the Quadratic Formula to solve an equation.

a. You obtain solutions that are integers. Could you have used factoring to solve the equation? Explain your reasoning.

b. You obtain solutions that are fractions. Could you have used factoring to solve the equation? Explain your reasoning.

c. Make a generalization about quadratic equations with rational solutions.

61. MODELING WITH MATHEMATICS The fuel economy y (in miles per gallon) of a car can be modeled by the equation $y = -0.013x^2 + 1.25x + 5.6$, where $5 \le x \le 75$ and x is the speed (in miles per hour) of the car. Find the speed(s) at which you can travel and have a fuel economy of 32 miles per gallon.

62. MODELING WITH MATHEMATICS The depth d (in feet) of a river can be modeled by the equation $d = -0.25t^2 + 1.7t + 3.5$, where $0 \le t \le 7$ and t is the time (in hours) after a heavy rain begins. When is the river 6 feet deep?

ANALYZING EQUATIONS In Exercises 63–68, tell whether the vertex of the graph of the function lies above, below, or on the x-axis. Explain your reasoning without using a graph.

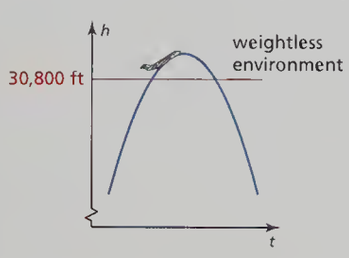
63. $y = x^2 - 3x + 2$ **64.** $y = 3x^2 - 6x + 3$

65. $y = 6x^2 - 2x + 4$ **66.** $y = -15x^2 + 10x - 25$

67. $f(x) = -3x^2 - 4x + 8$

68. $f(x) = 9x^2 - 24x + 16$

69. REASONING NASA creates a weightless environment by flying a plane in a series of parabolic paths. The height h (in feet) of a plane after t seconds in a parabolic flight path can be modeled by $h = -11t^2 + 700t + 21{,}000$. The passengers experience a weightless environment when the height of the plane is greater than or equal to 30,800 feet. For approximately how many seconds do passengers experience weightlessness on such a flight? Explain.

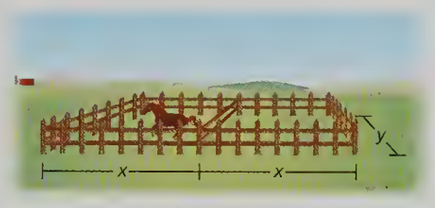


70. WRITING EQUATIONS Use the numbers to create a quadratic equation with the solutions $x = -1$ and $x = -\frac{1}{4}$.

$$__x^2 + __x + __ = 0$$

−5	−4	−3	−2	−1
1	2	3	4	5

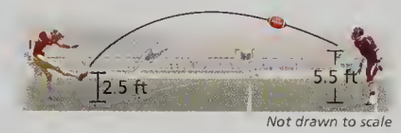
71. PROBLEM SOLVING A rancher constructs two rectangular horse pastures that share a side, as shown. The pastures are enclosed by 1050 feet of fencing. Each pasture has an area of 15,000 square feet.



a. Show that $y = 350 - \frac{4}{3}x$.

b. Find the possible lengths and widths of each pasture.

72. PROBLEM SOLVING A kicker punts a football from a height of 2.5 feet above the ground with an initial vertical velocity of 45 feet per second.



Not drawn to scale

a. Write an equation that models this situation using the function $h = -16t^2 + v_0t + s_0$, where h is the height (in feet) of the football, t is the time (in seconds) after the football is punted, v_0 is the initial vertical velocity (in feet per second), and s_0 is the initial height (in feet).

b. The football is caught 5.5 feet above the ground, as shown in the diagram. Find the amount of time that the football is in the air.

73. CRITICAL THINKING The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$. Find the mean of the solutions. How is the mean of the solutions related to the graph of $y = ax^2 + bx + c$? Explain.

ANSWERS

60. a. yes; The equation can be written in intercept form.

b. yes; The equation can be written in intercept form.

c. A quadratic equation with rational solutions can be solved by factoring.

61. about 31 mi/h, about 65 mi/h

62. after about 2.2 h, after about 4.6 h

63. below the x-axis; The discriminant is positive and $a > 0$.

64. on the x-axis; The discriminant is zero.

65. above the x-axis; The discriminant is negative and $a > 0$.

66. below the x-axis; The discriminant is negative and $a < 0$.

67. above the x-axis; The discriminant is positive and $a < 0$.

68. on the x-axis; The discriminant is zero.

69. about 22 sec; The height is 30,800 feet after about 20.8 seconds and after about 42.8 seconds.

70. 4; 5; 1 or −4; −5; −1

71. a. $4x + 3y = 1050$; $3y = 1050 - 4x$; $y = 350 - \frac{4}{3}x$

b. length: about 54 ft, width: about 278 ft; length: about 209 ft, width: about 72 ft

72. a. $h = -16t^2 + 45t + 2.5$

b. about 2.74 sec

73. $-\dfrac{b}{2a}$; The mean of the solutions is the x-coordinate of the vertex; The mean of the solutions is equal to the graph's axis of symmetry, which is where the vertex lies.

ANSWERS

74. **a.** C; The graph has two x-intercepts.

b. A; The graph has one x-intercept.

c. B; The graph has no x-intercepts.

75. about 24.7 ft/sec

76. $x = -\frac{b}{2a}$ is the axis of symmetry, and $\frac{\sqrt{b^2 - 4ac}}{2a}$ is the horizontal distance from the axis of symmetry to each x-intercept.

77. $\frac{-b}{a}$; $\frac{c}{a}$; *Sample answer:* $2x^2 - 4x + 1 = 0$

78. $x = \frac{-1 \pm \sqrt{1 - 4ac}}{2a}$; $x \approx -1.77$, $x \approx 2.27$

79. See Additional Answers.

80. **a.** $k < \frac{1}{24}$

b. $k = \frac{1}{24}$

c. $k > \frac{1}{24}$

81. **a.** $k < -3$ or $k > 3$

b. $k = -3$ or $k = 3$

c. $-3 < k < 3$

82–86. See Additional Answers.

Mini-Assessment

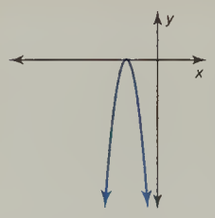
1. Solve $2x^2 - 5x + 3 = 0$ using the Quadratic Formula. $x = \frac{3}{2}$ and $x = 1$
2. Determine the number of real solutions of $-x^2 - 25 = -10x$. one real solution
3. Find the number of x-intercepts of the graph of $y = x^2 - 7x + 14$. no x-intercepts

Solve the equation. Explain your choice of method.

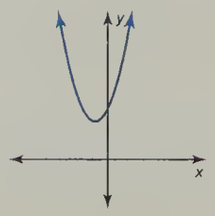
4. $x^2 - 6x = 4$
 $x = 3 + \sqrt{13} \approx 6.6$ and $x = 3 - \sqrt{13} \approx -0.6$; The coefficient of the x^2-term is 1, and the coefficient of the x-term is an even number. So, solve by completing the square.
5. $x^2 - 2x = 15$
 $x = -3$ and $x = 5$; The equation is easily factorable to $(x - 5)(x + 3) = 0$. So, solve by factoring.

74. HOW DO YOU SEE IT? Match each graph with its discriminant. Explain your reasoning.

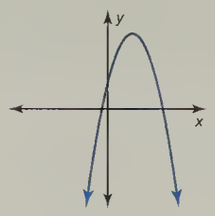
A.



B.



C.



a. $b^2 - 4ac > 0$

b. $b^2 - 4ac = 0$

c. $b^2 - 4ac < 0$

75. CRITICAL THINKING You are trying to hang a tire swing. To get the rope over a tree branch that is 15 feet high, you tie the rope to a weight and throw it over the branch. You release the weight at a height s_0 of 5.5 feet. What is the minimum initial vertical velocity v_0 needed to reach the branch? (*Hint*: Use the equation $h = -16t^2 + v_0t + s_0$.)

76. THOUGHT PROVOKING Consider the graph of the standard form of a quadratic function $y = ax^2 + bx + c$. Then consider the Quadratic Formula as given by

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$$

Write a graphical interpretation of the two parts of this formula.

77. ANALYZING RELATIONSHIPS Find the sum and product of $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$. Then write a quadratic equation whose solutions have a sum of 2 and a product of $\frac{1}{2}$.

78. WRITING A FORMULA Derive a formula that can be used to find solutions of equations that have the form $ax^2 + x + c = 0$. Use your formula to solve $-2x^2 + x + 8 = 0$.

79. MULTIPLE REPRESENTATIONS If p is a solution of a quadratic equation $ax^2 + bx + c = 0$, then $(x - p)$ is a factor of $ax^2 + bx + c$.

a. Copy and complete the table for each pair of solutions.

Solutions	Factors	Quadratic equation
3, 4	$(x - 3)$, $(x - 4)$	$x^2 - 7x + 12 = 0$
-1, 6		
0, 2		
$-\frac{1}{2}$, 5		

b. Graph the related function for each equation. Identify the zeros of the function.

CRITICAL THINKING **In Exercises 80–82, find all values of k for which the equation has (a) two solutions, (b) one solution, and (c) no solutions.**

80. $2x^2 + x + 3k = 0$

81. $x^2 - 4kx + 36 = 0$

82. $kx^2 + 5x - 16 = 0$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations using any method. Explain why you chose the method. *(Section 5.1, Section 5.2, and Section 5.3)*

83. $y = -x + 4$
$y = 2x - 8$

84. $x = 16 - 4y$
$3x + 4y = 8$

85. $2x - y = 7$
$2x + 7y = 31$

86. $3x - 2y = -20$
$x + 1.2y = 6.4$

If students need help...	If students got it...
Resources by Chapter • Practice A and Practice B • Puzzle Time	Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice	Start the *next* Section
Differentiating the Lesson Skills Review Handbook	

9.4–9.5 What Did You Learn?

Core Vocabulary

completing the square, *p. 494* Quadratic Formula, *p. 504* discriminant, *p. 506*

Core Concepts

Section 9.4

Completing the Square, *p. 494*

Section 9.5

Quadratic Formula, *p. 504*
Interpreting the Discriminant, *p. 506*

Mathematical Thinking

1. How does your answer to Exercise 74 on page 502 help create a shortcut when solving some quadratic equations by completing the square?
2. What logical progression led you to your answer in Exercise 55 on page 510?

Performance Task

Form Matters

Each form of a quadratic function has its pros and cons. Using one form, you can easily find the vertex, but the zeros are more difficult to find. Using another form, you can easily find the y-intercept, but the vertex is more difficult to find. Which form would you use in different situations? How can you convert one form into another?

To explore the answers to these questions and more, go to *BigIdeasMath.com*.



Dynamic Teaching Tools

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ANSWERS

1. It provides a quick way to check for equations with no solutions.
2. When a and c have different signs, ac is negative, so subtracting $-4ac$ is subtracting a negative, which is the same as adding a positive. So, $b^2 - 4ac$ is positive.

ANSWERS

1. $2\sqrt{31}$
2. $6p^3\sqrt{2p}$
3. $\frac{3\sqrt{35y}}{7y}$
4. $\frac{5x^3\sqrt[3]{2x^2}}{2}$
5. $\frac{\sqrt{6}}{2}$
6. $\frac{2\sqrt{6x}}{3x}$
7. $4\sqrt{6} - 8$
8. $14\sqrt{3}$
9. $2\sqrt{7} - \sqrt{5}$
10. $9\sqrt[3]{2}$
11. $88 + 30\sqrt{7}$
12. $10\sqrt{3}$

9 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

9.1 Properties of Radicals *(pp. 465–474)*

a. Simplify $\sqrt{\frac{19}{169}}$.

$$\sqrt{\frac{19}{169}} = \frac{\sqrt{19}}{\sqrt{169}}$$ Quotient Property of Square Roots

$$= \frac{\sqrt{19}}{13}$$ Simplify.

b. Simplify $\sqrt[3]{27x^{10}}$.

$$\sqrt[3]{27x^{10}} = \sqrt[3]{27 \cdot x^9 \cdot x}$$ Factor using the greatest perfect cube factors.

$$= \sqrt[3]{27} \cdot \sqrt[3]{x^9} \cdot \sqrt[3]{x}$$ Product Property of Cube Roots

$$= 3x^3\sqrt[3]{x}$$ Simplify.

c. Simplify $\frac{12}{3 + \sqrt{5}}$.

$$\frac{12}{3 + \sqrt{5}} = \frac{12}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$$ The conjugate of $3 + \sqrt{5}$ is $3 - \sqrt{5}$.

$$= \frac{12(3 - \sqrt{5})}{3^2 - (\sqrt{5})^2}$$ Sum and difference pattern

$$= \frac{36 - 12\sqrt{5}}{4}$$ Simplify.

$$= 9 - 3\sqrt{5}$$ Simplify.

d. Simplify $9\sqrt{6} + \sqrt{10} + 7\sqrt{6}$.

$$9\sqrt{6} + \sqrt{10} + 7\sqrt{6} = 9\sqrt{6} + 7\sqrt{6} + \sqrt{10}$$ Commutative Property of Addition

$$= (9 + 7)\sqrt{6} + \sqrt{10}$$ Distributive Property

$$= 16\sqrt{6} + \sqrt{10}$$ Add.

Simplify the expression.

1. $\sqrt{124}$
2. $\sqrt{72p^7}$
3. $\sqrt{\frac{45}{7y}}$
4. $\sqrt[3]{\frac{125x^{11}}{4}}$
5. $\frac{\sqrt{15}}{\sqrt{10}}$
6. $\frac{4}{\sqrt{6x}}$
7. $\frac{8}{\sqrt{6} + 2}$
8. $4\sqrt{3} + 5\sqrt{12}$
9. $\sqrt{5} + 2\sqrt{7} - 2\sqrt{5}$
10. $15\sqrt[3]{2} - 2\sqrt[3]{54}$
11. $(3\sqrt{7} + 5)^2$
12. $\sqrt{6}(\sqrt{18} + \sqrt{8})$

9.2 Solving Quadratic Equations by Graphing *(pp. 475–484)*

Solve $x^2 + 3x = 4$ by graphing.

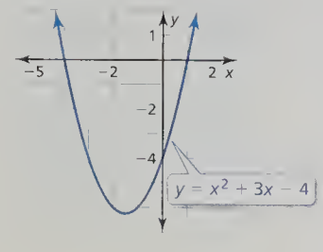
Step 1 Write the equation in standard form.

$x^2 + 3x = 4$ Write original equation.

$x^2 + 3x - 4 = 0$ Subtract 4 from each side.

Step 2 Graph the related function $y = x^2 + 3x - 4$.

Step 3 Find the x-intercepts. The x-intercepts are -4 and 1.



▶ So, the solutions are $x = -4$ and $x = 1$.

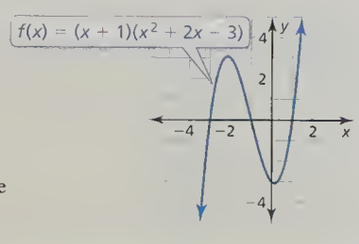
Solve the equation by graphing.

13. $x^2 - 9x + 18 = 0$

14. $x^2 - 2x = -4$

15. $-8x - 16 = x^2$

16. The graph of $f(x) = (x + 1)(x^2 + 2x - 3)$ is shown. Find the zeros of f.



17. Graph $f(x) = x^2 + 2x - 5$. Approximate the zeros of f to the nearest tenth.

9.3 Solving Quadratic Equations Using Square Roots *(pp. 485–490)*

A sprinkler sprays water that covers a circular region of 90π square feet. Find the diameter of the circle.

Write an equation using the formula for the area of a circle.

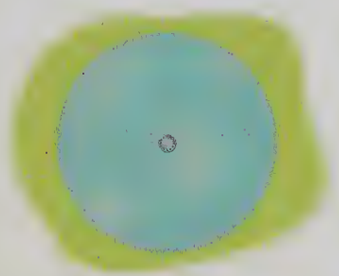
$A = \pi r^2$ Write the formula.

$90\pi = \pi r^2$ Substitute 90π for A.

$90 = r^2$ Divide each side by π.

$\pm\sqrt{90} = r$ Take the square root of each side.

$\pm 3\sqrt{10} = r$ Simplify.



A diameter cannot be negative, so use the positive square root. The diameter is twice the radius. So, the diameter is $6\sqrt{10}$.

▶ The diameter of the circle is $6\sqrt{10} \approx 19$ feet.

Solve the equation using square roots. Round your solutions to the nearest hundredth, if necessary.

18. $x^2 + 5 = 17$

19. $x^2 - 14 = -14$

20. $(x + 2)^2 = 64$

21. $4x^2 + 25 = -75$

22. $(x - 1)^2 = 0$

23. $19 = 30 - 5x^2$

ANSWERS

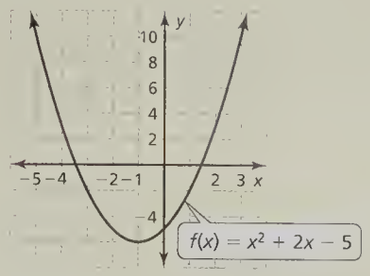
13. $x = 6, x = 3$

14. no solutions

15. $x = -4$

16. $-3, -1, 1$

17.



$-3.4, 1.4$

18. $x \approx 3.46, x \approx -3.46$

19. $x = 0$

20. $x = 6, x = -10$

21. no solutions

22. $x = 1$

23. $x \approx 1.48, x \approx -1.48$

ANSWERS

24. $x = 4, x = -10$
25. $x = -1$
26. $x \approx 3.45, x \approx -1.45$
27. maximum value; 8
28. minimum value; 7
29. minimum value; -33
30. 28 cm
31. $x = 3, x = -5$
32. $x \approx 2.3, x \approx -1.8$
33. $x = 1$
34. one x-intercept
35. no x-intercepts
36. two x-intercepts

9.4 Solving Quadratic Equations by Completing the Square *(pp. 493–502)*

Solve $x^2 - 6x + 4 = 11$ by completing the square.

$x^2 - 6x + 4 = 11$	Write the equation.
$x^2 - 6x = 7$	Subtract 4 from each side.
$x^2 - 6x + (-3)^2 = 7 + (-3)^2$	Complete the square by adding $\left(\frac{-6}{2}\right)^2$, or $(-3)^2$, to each side.
$(x - 3)^2 = 16$	Write the left side as the square of a binomial.
$x - 3 = \pm 4$	Take the square root of each side.
$x = 3 \pm 4$	Add 3 to each side.

▶ The solutions are $x = 3 + 4 = 7$ and $x = 3 - 4 = -1$.

Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

24. $x^2 + 6x - 40 = 0$ **25.** $x^2 + 2x + 5 = 4$ **26.** $2x^2 - 4x = 10$

Determine whether the quadratic function has a maximum or minimum value. Then find the value.

27. $y = -x^2 + 6x - 1$ **28.** $f(x) = x^2 + 4x + 11$ **29.** $y = 3x^2 - 24x + 15$

30. The width w of a credit card is 3 centimeters shorter than the length ℓ. The area is 46.75 square centimeters. Find the perimeter.

9.5 Solving Quadratic Equations Using the Quadratic Formula *(pp. 503–512)*

Solve $-3x^2 + x = -8$ using the Quadratic Formula.

$-3x^2 + x = -8$	Write the equation.
$-3x^2 + x + 8 = 0$	Write in standard form.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$x = \frac{-1 \pm \sqrt{1^2 - 4(-3)(8)}}{2(-3)}$	Substitute -3 for a, 1 for b, and 8 for c.
$x = \frac{-1 \pm \sqrt{97}}{-6}$	Simplify.

▶ So, the solutions are $x = \frac{-1 + \sqrt{97}}{-6} \approx -1.5$ and $x = \frac{-1 - \sqrt{97}}{-6} \approx 1.8$.

Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

31. $x^2 + 2x - 15 = 0$ **32.** $2x^2 - x + 8 = 16$ **33.** $-5x^2 + 10x = 5$

Find the number of x-intercepts of the graph of the function.

34. $y = -x^2 + 6x - 9$ **35.** $y = 2x^2 + 4x + 8$ **36.** $y = -\frac{1}{2}x^2 + 2x$

9 Chapter Test

Simplify the expression.

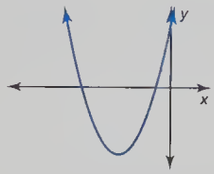
1. $-\sqrt{117}$
2. $\sqrt{98b^5}$
3. $\sqrt[3]{40w^3}$
4. $\sqrt[3]{\frac{27y^4}{1000z^3}}$
5. $\frac{10}{\sqrt{7}}$
6. $\frac{13}{\sqrt{3}-4}$
7. $(2\sqrt{5}-1)^2$
8. $12\sqrt{8}+3\sqrt{18}$

Solve the equation using any method. Explain your choice of method.

9. $x^2-121=0$
10. $x^2-6x=10$
11. $-2x^2+3x+7=0$
12. $x^2-7x+12=0$
13. $5x^2+x-4=0$
14. $(4x+3)^2=16$

15. Explain how you can determine, without graphing, whether the quadratic function $y=-x^2-6x-1$ has a maximum or minimum value. Then find the value.

16. Describe how you can use the method of completing the square to determine whether the function $f(x)=2x^2+4x-6$ can be represented by the graph shown.



17. The dimensions of a park form a golden rectangle. The shorter side of the park is 84 meters. What is the length of the longer side of the park?

18. A skier leaves an 8-foot-tall ramp with an initial vertical velocity of 28 feet per second. The function $h=-16t^2+28t+8$ represents the height h (in feet) of the skier after t seconds. The skier has a perfect landing. How many points does the skier earn?

Criteria	Scoring
Maximum height	1 point per foot
Time in air	5 points per second
Perfect landing	25 points

19. An amusement park ride lifts seated riders 265 feet above the ground. The riders are then dropped and experience free fall until the brakes are activated 105 feet above the ground. The function $h=-16t^2+265$ represents the height h (in feet) of the riders t seconds after they are dropped. How long do the riders experience free fall? Round your solution to the nearest hundredth.

20. Write an expression in simplest form that represents the area of the painting shown.



21. Explain how you can determine the number of times the graph of $y=5x^2-10x+5$ intersects the x-axis without graphing or solving an equation.

22. Consider the quadratic equation $ax^2+bx=c$. Find values of a, b, and c so that the graph of its related function has (a) two x-intercepts, (b) one x-intercept, and (c) no x-intercepts.

If students need help...	If students got it...
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Skills Review Handbook	Performance Task
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ANSWERS

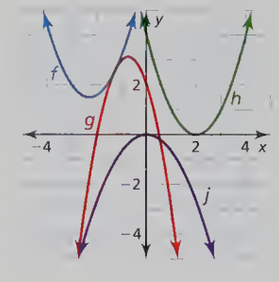
1. $-3\sqrt{13}$
2. $7b^2\sqrt{2b}$
3. $2w\sqrt[3]{5}$
4. $\frac{3y\sqrt[3]{y}}{10z}$
5. $\frac{10\sqrt{7}}{7}$
6. $-\sqrt{3}-4$
7. $21-4\sqrt{5}$
8. $33\sqrt{2}$
9. $x=11, x=-11$; *Sample answer:* The equation can be written in the form $x^2=d$, so solve using square roots.
10. $x\approx 7.36, x\approx -1.36$; *Sample answer:* $a=1$ and b is even, so solve by completing the square.
11. $x\approx -1.27, x\approx 2.77$; *Sample answer:* The equation is not factorable and $a\neq 1$, so solve using the quadratic formula.
12. $x=4, x=3$; *Sample answer:* The equation is easily factorable, so solve by factoring.
13. $x=\frac{4}{5}, x=-1$; *Sample answer:* The equation is not easily factorable and $a\neq 1$, so solve using the quadratic formula.
14. $x=\frac{1}{4}, x=-\frac{7}{4}$; *Sample answer:* The equation is in the form $x^2=d$ where x is a binomial, so solve using square roots.
15. Write the function in vertex form and find the value of a; maximum value: 8
16. The equation can be put in vertex form $y=2(x+1)^2-8$. The y-intercept is -8, so the function cannot be represented by the graph shown.
17. about 136 m
18. 55 points
19. about 3.16 sec
20. $36x^3\sqrt{10x}$ in.^{2}
21. Use the discriminant to determine the number of solutions.
22. **a.** *Sample answer:* $a=3, b=2, c=-1$
 b. *Sample answer:* $a=3, b=6, c=3$
 c. *Sample answer:* $a=2, b=3, c=4$

ANSWERS

1. A
2. J
3. D
4. H

9 Standards Assessment

1. The graphs of four quadratic functions are shown. Which equation has a negative discriminant? *(TEKS A.7.A)*



Ⓐ $f(x) = 0$

Ⓑ $g(x) = 0$

Ⓒ $h(x) = 0$

Ⓓ $j(x) = 0$

2. The population p (in thousands) of Phoenix, Arizona, can be modeled by the function $p(t) = 0.08t^2 + 17.4t + 421$, where $t = 0$ represents the year 1960. Using the model, what will the population of Phoenix be in the year 2030? *(TEKS A.8.B)*

Ⓕ 1015

Ⓖ 2031

Ⓗ 1,015,000

Ⓙ 2,031,000

3. The table represents the numbers of bottles of sports drink sold at a concession stand on days with different average temperatures. You find an equation of the line of best fit. Which of the following is a reasonable correlation coefficient? *(TEKS A.4.A)*

Temperature (°F), x	14	27	32	41	48	62	73
Bottles of sports drink, y	8	12	13	16	19	27	29

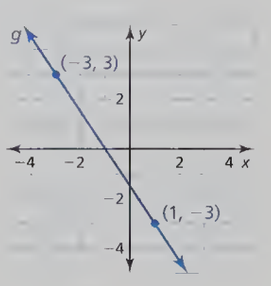
Ⓐ −0.99

Ⓑ −0.12

Ⓒ 0.12

Ⓓ 0.99

4. Which equation represents the line that passes through $(6, -1)$ and is perpendicular to line g? *(TEKS A.2.F)*



Ⓕ $y = -\frac{3}{2}x + \frac{9}{2}$

Ⓖ $y = -\frac{3}{2}x + 8$

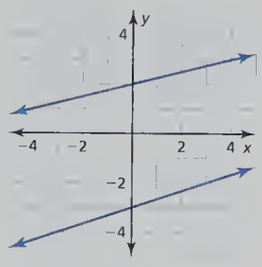
Ⓗ $y = \frac{2}{3}x - 5$

Ⓙ $y = \frac{2}{3}x + \frac{20}{3}$

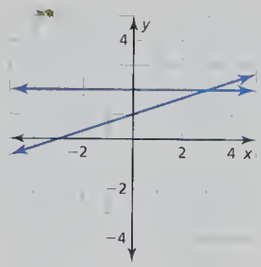
5. GRIDDED ANSWER What is the radius (in centimeters) of a cylinder with a height of 12 centimeters and a surface area of 320π square centimeters? *(TEKS A.8.A)*

6. Which graph represents a system of equations that has no solution? *(TEKS A.3.F)*

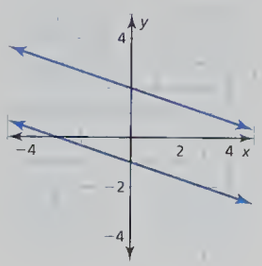
Ⓐ



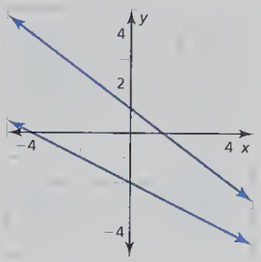
Ⓑ



Ⓒ



Ⓓ



7. Which of the following expressions are in simplest form? *(TEKS A.11.A)*

I. $x\sqrt{45x}$

II. $\sqrt[3]{\frac{4}{9}}$

III. $3x\sqrt{5x}$

IV. $2\sqrt[3]{x^2}$

Ⓕ I and II only

Ⓖ II and III only

Ⓗ I, III, and IV only

Ⓙ II, III, and IV only

8. The domain of the function $f(x) = 4x - 5$ is all integers in the interval $-3 < x \leq 3$. What is the range of the function? *(TEKS A.2.A)*

Ⓐ $-17, -13, -9, -5, -1, 3, 7$

Ⓑ $-13, -9, -5, -1, 3, 7$

Ⓒ $-17 < x \leq 7$

Ⓓ $-13 \leq x \leq 7$

ANSWERS

5. 8
6. C
7. J
8. B

Additional Answers

Chapter 1

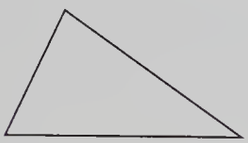
1.2 Explorations

1. e. $(5x + 2) + (3x + 5) + (8x + 8) + (4x + 15) + (5x + 10) = 540$; Write the equation.
 $25x + 40 = 540$; Combine like terms.
 $25x = 500$; Subtract 40 from each side.
 $x = 20$; Divide each side by 25.
 $102°, 65°, 168°, 95°, 110°$

 f. $(3x + 16) + (3x + 16) + (2x + 8) + (4x - 18) + (2x + 25) + (3x - 7) = 720$; Write the equation.
 $17x + 40 = 720$; Combine like terms.
 $17x = 680$; Subtract 40 from each side.
 $x = 40$; Divide each side by 17.
 $136°, 136°, 88°, 142°, 105°, 113°$

 Measure the angles with a protractor.

2. a. *Sample answer:*



 b. *Sample answer:* $80°, 35°, 65°$

 c. *Sample answer:* $x = 10$; $8x, 3x + 5, 6x + 5$

 d. Check students' work.

 e. *Sample answer:* $(16x + 4) + (6x + 4) + (23x + 2) + (13x + 2) = 360$, $x = 6$; $100°, 40°, 140°, 80°$; yes; The angles are close to those measured with a protractor.

3. Use multi-step equations to find the values of missing parts in an object with the shape of a geometric figure.

4. Connecting a vertex with each of the other vertices in a polygon creates $n - 2$ triangles, each of which has a total angle measure of $180°$.

5. 8; Solve the equation $180(n - 2) = 1080$ for n.

1.2 Monitoring Progress and Modeling with Mathematics

48. *Sample answer:*

Component	Student's score	Weight	Score × weight
Class Participation	92%	0.2	18.4%
Homework	95%	0.3	28.5%
Midterm Exam	88%	0.2	17.6%
Final Exam	85%	0.3	25.5%
Total		1	90.0%

The student would need a score of 85% on the final exam because $92(0.2) + 95(0.3) + 88(0.2) + f(0.3) = 90$
$f = 85$.

51. $x = -\frac{7}{b}$

52. $x = \frac{3}{4} - a$

53. $x = \frac{12.5 + b}{a}$

54. $x = \frac{c - b}{a}$

55. $x = -\frac{8}{b}$

56. $x = \frac{9b}{c}$

1.2 Maintaining Mathematical Proficiency

57. $m + 5$

58. $9 - 2b$

59. -2

60. a

61. b

62. b

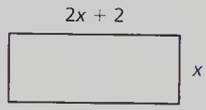
63. a

64. b

65. a

1.3 Monitoring Progress and Modeling with Mathematics

40. *Sample answer:*



The perimeters are both $6x + 4$.

1.3 Maintaining Mathematical Proficiency

41. $-4, |2|, |-4|, 5, 9$

42. $-|21|, -16, |-10|, 22, |-32|$

43. $-19, -18, |-18|, |22|, |-24|$

44. $-|-3|, -2, -1, |0|, |2|$

1.4 Maintaining Mathematical Proficiency

47. 35

48. 34

49. 47

50. -29

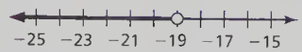
51. $x = 9$

52. $m = 10$

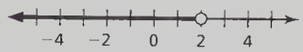
Chapter 2

2.2 Monitoring Progress and Modeling with Mathematics

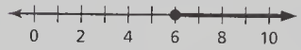
16. $t < -19$



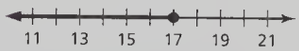
17. $j < 2$



18. $y \geq 6$



19. $p \leq 17$



20. $z > -9$



21. $n + 8 > 11$; $n > 3$

22. $n - 3 \geq -5$; $n \geq -2$

23. $n - 9 < 4$; $n < 13$

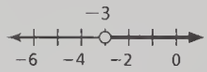
24. $6 \leq n + 15$; $n \geq -9$

25. a. $38 + w \leq 50$; $w \leq 12$

 b. no; The total being added is 14 pounds, which is not a solution of the inequality found in part (a).

26. $19.76 + x \geq 25$; $x \geq 5.24$

27. The graph is going in the wrong direction.



28. In the second step, 10 needs to be added to each side; $-10 + 10 + x \geq -9 + 10$; $x \geq 1$



29. 33 or more goals

36. a. no; It is also possible that the number of students with brown eyes is greater than the number of students with brown hair.

b. no; It is also possible that the number of students with brown eyes is greater than 10 more than the number of students with brown hair.

c. yes; All students with both characteristics must have brown hair.

d. yes; If H is greater than a number, then $H + 10$ must also be greater than the number.

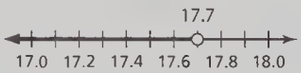
e. no; If all students with brown eyes also have brown hair, then the values would be equal.

f. yes; If H is greater than a number, then $H + 10$ must also be greater than the number.

37. a. $x \geq 6$



b. $x < 17.7$



38. b, a, d, c

2.2 Maintaining Mathematical Proficiency

39. -63
40. 132
41. 9
42. -4
43. $x = 4$
44. $y = 6$
45. $s = -104$
46. $n = -29.2$

2.3 Monitoring Progress and Modeling with Mathematics

27. The inequality should not be reversed when multiplying each side by $\frac{3}{2}$; $\frac{3}{2} \cdot (-6) > \frac{3}{2} \cdot \frac{2}{3}x$; $-\frac{18}{2} > x$; $-9 > x$; $x < -9$; The solution is $x < -9$.

28. The inequality should be reversed when dividing by -4; $\frac{-4y}{-4} \geq \frac{-32}{-4}$; $y \geq 8$; The solution is $y \geq 8$.

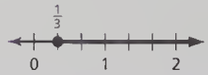
29. $(14 \cdot 14)c \leq 700$; ft, ft, dollars;

$196c \leq 700$; ft^2, dollars;

$c \leq 3.57$; dollars/ft^2

38. $\frac{d}{0.75} < 18$, $d < 13.5$

39. $36p \geq 12$, $p \geq \frac{1}{3}$



2.3 Maintaining Mathematical Proficiency

40. $x = 2$
41. $y = -4$
42. $n = 1$
43. $z = 6$
44. 85%
45. $\frac{16}{30}$
46. 120%
47. $\frac{2}{3}$

2.4 Monitoring Progress and Modeling with Mathematics

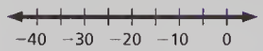
39. $a = 4$
40. $a = 3$

2.4 Maintaining Mathematical Proficiency

41. $6y \leq 10$
42. $p + 7 > 24$
43. $\frac{r}{7} \leq 18$

2.5 Monitoring Progress and Modeling with Mathematics

29. all real numbers



30. no solution

31. $\geq$

32. *Sample answer:* You are biking on a trail. The spot where you start on the trail is the 6-mile mark. You bike 4 miles on the trail, and then leave the trail for a different path. The graph represents where you bike on the trail.

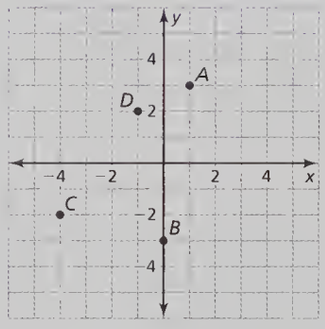
33. $7 + 5 > x$; $x < 12$; $7 + x > 5$; $x > -2$; $5 + x > 7$; $x > 2$; no; A value of 1 does not make the inequality $x > 2$ true.

34. a. 50 million $\leq P \leq$ 90 million

b. no; Solving the inequality $50 \leq R - 125 \leq 90$ indicates that the revenue must be between \$175 million and \$215 million.

2.5 Maintaining Mathematical Proficiency

35–38.



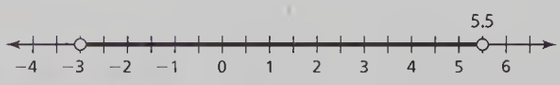
Point A is one unit left and 3 units up from the origin. Point B is 3 units down from the origin. Point C is 4 units left and 2 units down from the origin. Point D is 1 unit left and 2 units up from the origin.

39. 4; The data values are clustered close together.

40. 2.8; The data values are clustered close together.

Chapter 2 Test

10. $-3 < c < 5.5$



Chapter 3

3.1 Monitoring Progress and Modeling with Mathematics

33. a. Each letter-number combination is paired with exactly one food or drink item.

b. The food item is the dependent variable and the letter-number combination is the independent variable.

c. domain: A1, A2, A3, B1, B2, B3, B4, C1, C2, C3, C4; range: popcorn, nuts, pretzels, protein bar, granola bar, cereal, energy bar, orange juice, water, milk

34. a. No vertical line can be drawn through more than one point on the graph.

b. *Sample answer:* 20 ft; 29 ft

c. *Sample answer:* $0 \le t \le 2.6$

d. no; There are many values of h that have two corresponding values of t.

35. no; A vertical line does not represent a function.

36. *Sample answer:* the score and letter grade on a 10-point quiz; The letter grade is the dependent variable and the score is the independent variable; domain: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; range: A, B, C, D, F

37. no; Items that cost the same to make could be sold for different prices.

38. yes; Each possible selling price would have the same amount of tax charged.

39. yes; Each student has exactly one homeroom teacher.

40. no; Each chaperone has more than one student assigned.

41. true

42. false; A relation is not a function when an input value has more than one output value.

43. false; More than one input can have the same output in a function, so reversing the values may not produce a function.

44. false; Because more than one input can be paired with the same output, the outputs could be selected from a finite list.

45. **a.** $P = h + 23$

b. P is the dependent variable and h is the independent variable.

c. domain: $3 < h < 23$; range: $26 < P < 46$

46. domain: all real numbers; range: $y \ge 0$

47. domain: all real numbers; range: $y \le 0$

48. domain: all real numbers; range: $y \ge -6$

49. domain: all real numbers; range: $y \le 4$

3.1 Maintaining Mathematical Proficiency

50. $y < 16$

51. $3 \ge x$

52. $7 \le \frac{d}{-5}$

53. $w + 4 > 12$

54. 121

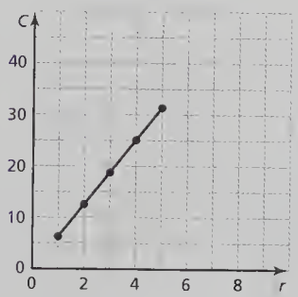
55. 81

56. -25

57. 32

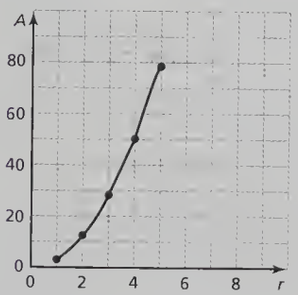
3.2 Explorations

1. **c.** 2π or about 6.3, 4π or about 12.6, 6π or about 18.8, 8π or about 25.1, 10π or about 31.4;



linear; The graph is a line.

d. π or about 3.1, 4π or about 12.6, 9π or about 28.3, 16π or about 50.3, 25π or about 78.5;



nonlinear; The graph is not a line.

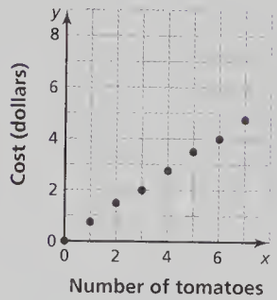
2. No vertical line can be drawn through more than one point on any of the graphs.

3. Sketch the graph of the function. When the graph is a line, the pattern is linear. When the graph is not a line, the pattern is not linear.

4. *Sample answer:* Distance traveled during a time interval at constant speed is linear. Distance traveled during a time interval while accelerating is nonlinear.

3.2 Monitoring Progress and Modeling with Mathematics

50. *Sample answer:*



The tomatoes are sold for \$0.75 each or 3 for \$2.00.

51. linear; As x increases by 1, y increases by 4. The rate of change is constant

52. **a.** Your friend ran farther in the same amount of time; You run at a constant rate and your friend does not; Your graph is a line, and your friend's graph is not a line; A person may not run at a constant rate because of fatigue.

b. The domain for both functions is $0 \le x \le 60$; You both run for a total of 60 minutes.

53. *Sample answer:* how long it takes an ice cube to melt as a function of its temperature in degrees Celsius

54. *Sample answer:* profit (or loss) on the sale of x units at \$1 each that cost \$0.75 to make, plus \$50 in fixed costs

3.2 Maintaining Mathematical Proficiency

55. 20

56. 14

57. -24

58. 4

59. $x = 3$

60. $p = -1$

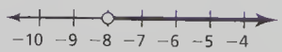
61. $t = 8$

62. $y = -\frac{19}{2}$

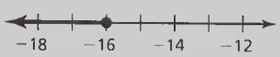
63. $x \ge \frac{1}{2}$



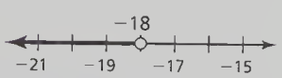
64. $t > -8$



65. $r \le -16$

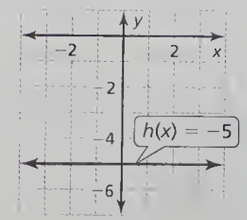


66. $a < -18$

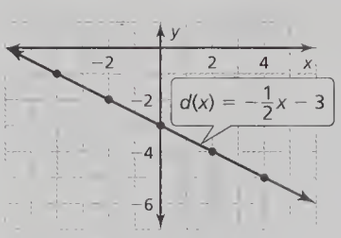


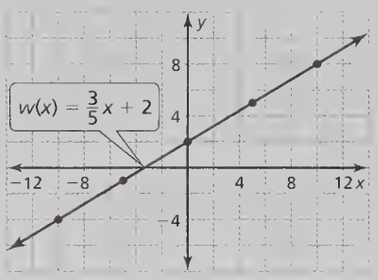
3.3 Monitoring Progress and Modeling with Mathematics

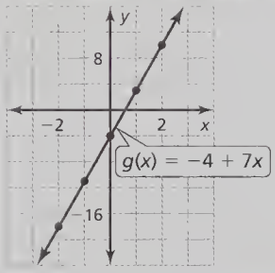
24.

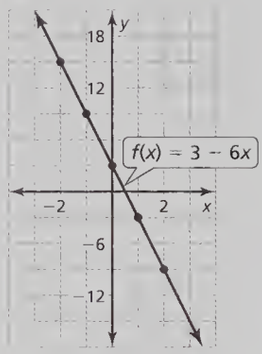


Additional Answers

25. 

26. 

27. 

28. 

30. Master Remodeling; Using the function, the cost for Certified Remodeling is \$250. The table shows an increase of \$30 for every increase of 2 hours, so the cost for Master Remodeling is \$220.

31. no; Because the function rule is unknown, it is unknown whether an increase in x would result in an increase or decrease in the value of the function.

32. *Sample answer:* You receive a paycheck between days 0 and 2, and then spend some of it between days 2 and 4.

33. a. $d(r) = 2r$; 10; The diameter of the circle is 10 feet.
 b. $A(r) = \pi r^2$; 25π or about 78.5; The area of the circle is 25π square feet.
 c. $C(r) = 2\pi r$; 10π or about 31.4; The circumference of the circle is 10π feet.

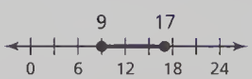
34. a. It decreases rapidly, then slowly increases.
 b. *Sample answer:* 425; The school attendance was about 425 students 13 weeks after the flu outbreak.
 c. *Sample answer:* 1, 10; The school attendance was about 400 students 1 week and 10 weeks after the flu outbreak.
 d. *Sample answer:* about 350 students; about 4 weeks after the flu outbreak
 e. *Sample answer:* about 460 students; The graph does not show any attendance values greater than 460.

35. a. (5, 9)
 b. (5, −3)

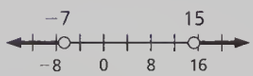
36. false; The function could include operations that would not give the same result. For example if $f(x) = x^2$, $f(3 + 4) \neq f(3) + f(4)$.

3.3 Maintaining Mathematical Proficiency

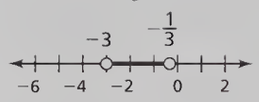
37. $9 \leq x \leq 17$;



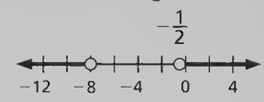
38. $a < -7$ *or* $a > 15$;



39. $-3 < k < -\frac{1}{3}$;



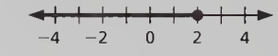
40. $d < -8$ *or* $d > -\frac{1}{2}$;



41. $-1 \leq y < 3$;



42. $v \leq 2$;



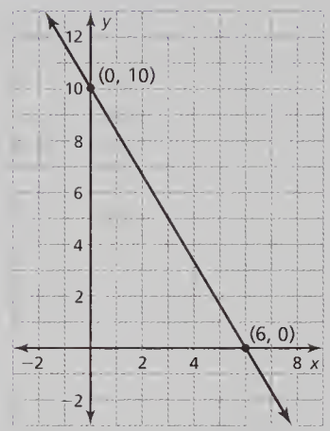
3.4 Explorations

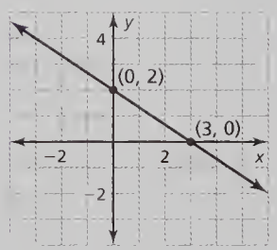
2. e. The intercepts represent the maximum number of pounds of each type of cheese you can buy if you only buy one type of cheese.

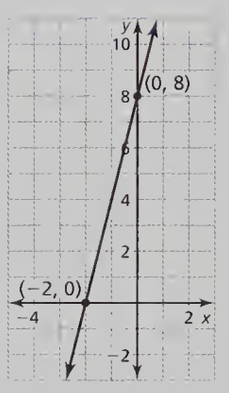
3. a straight line

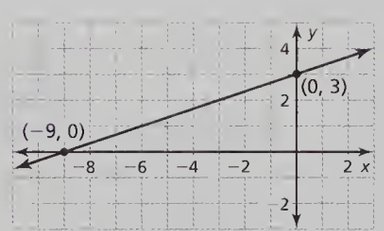
4. *Sample answer:* You sold a total of \$375 worth of DVDs. You forgot to record how many of each type you sold. TV season DVDs sold for \$30 each and movie DVDs sold for \$15 each.

3.4 Monitoring Progress and Modeling with Mathematics

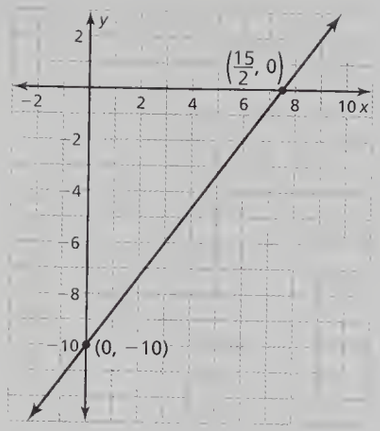
19. 

20. 

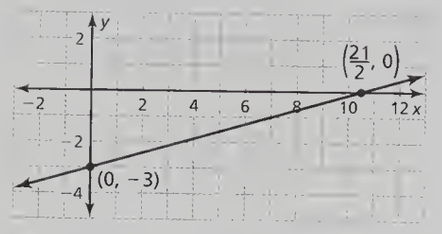
21. 

22. 

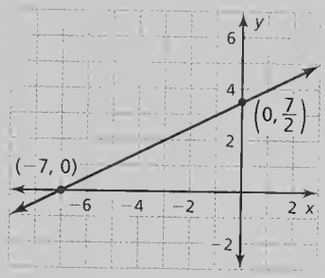
23.



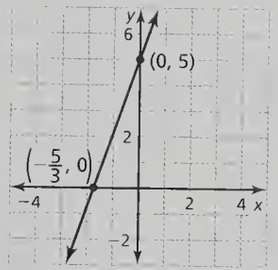
24.



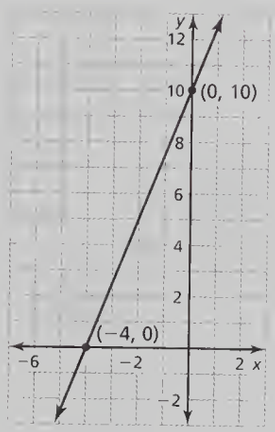
25.



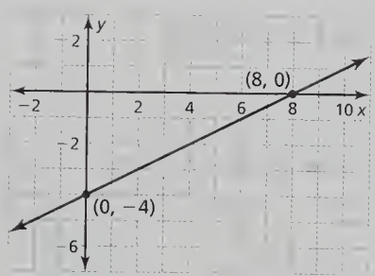
26.



27.



28.



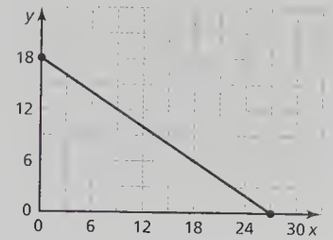
29. The equations locate 2 separate intercepts, not 1; The intercepts are 8 and 2.

30. The x- and y-coordinates of the intercepts are reversed; The x-intercept is at (5, 0) and the y-intercept is at (0, 2).

52. a. 27; 18; The x-intercept indicates 27 two-point baskets were made if there were no three-point baskets. The y-intercept indicates 18 three-point baskets were made if there were no two-point baskets.

b. no; If y is odd, then $3y$ is odd and $54 - 3y$ is odd. Dividing this result by 2 to find x does not produce a whole number.

c.



Sample answer: 24 two-point baskets and 2 three-point baskets; 9 two-point baskets and 12 three-point baskets

58. a. The x-intercept indicates 90 students can go if none of them choose the meal plan. The y-intercept indicates 60 students can go if all of them choose the meal plan.

b. domain: 0 to 90 students not choosing the meal plan; range: 0 to 60 students choosing the meal plan

59. −3; 6

60. *Sample answer:* $2x + 3y = 18$; 18 is divisible by both 2 and 3.

61. yes; $x = a$ can be written as $1x + 0y = a$, and $y = b$ can be written as $0x + 1y = b$.

62. They are multiples of 15; For the intercepts to be integers, k must be divisible by both 3 and 5.

3.4 Maintaining Mathematical Proficiency

63. $\frac{1}{2}$

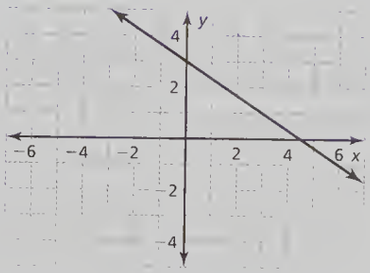
64. 2

65. $-\frac{4}{5}$

66. $\frac{5}{3}$

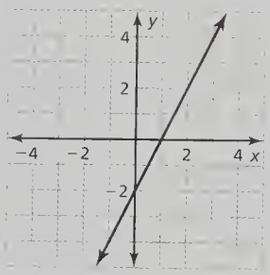
3.5 Explorations

2. a.



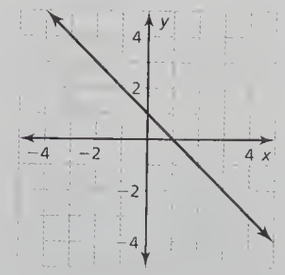
line; $-\frac{2}{3}$; 3

b.



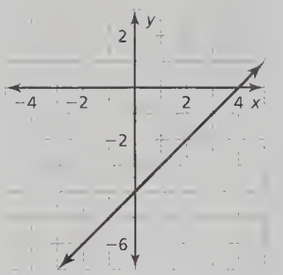
line; 2; −2

c.



line; −1; 1

d.



line; 1; −4

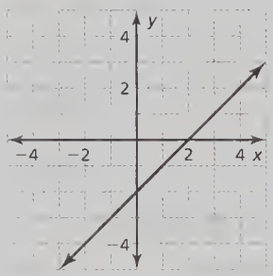
In the graph of $y = mx + b$, m is the slope of the line and b is the y-intercept of the line.

3. a line with slope m and y-intercept b

a. The value of m affects the steepness of the graph and whether the graph rises or falls from left to right.

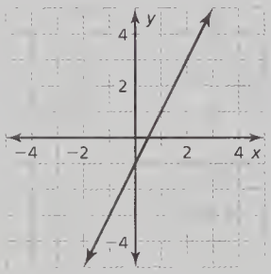
b. The value of b affects the location of the y-intercept.

c. **(1)** *Sample answer:*



$y = x - 2$; slope: 1; y-intercept: −2

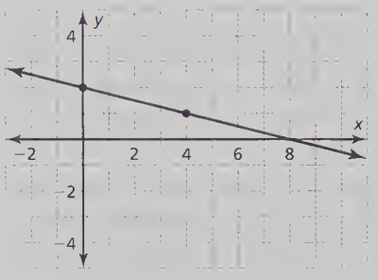
(2) *Sample answer:*



$y = 2x - 1$; slope: 2; y-intercept: −1

3.5 Monitoring Progress and Modeling with Mathematics

30.



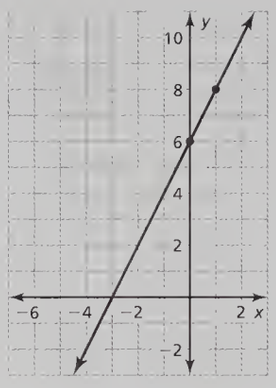
x-intercept: 8

31.



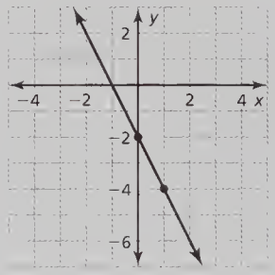
x-intercept: 0

32.



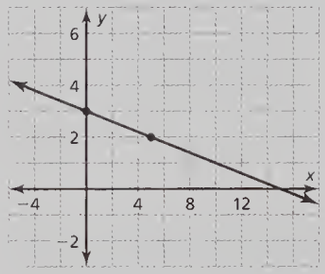
x-intercept: −3

33.



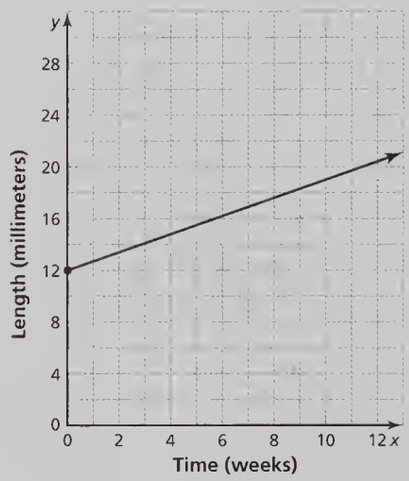
slope: −2; y-intercept: −2; x-intercept: −1

34.



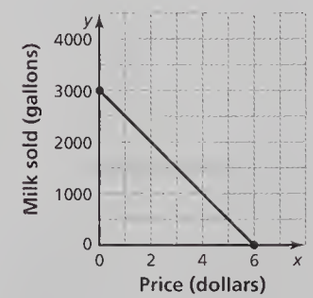
slope: $-\frac{1}{5}$; y-intercept: 3; x-intercept: 15

35.



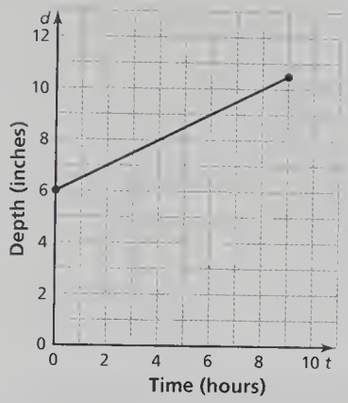
slope: $\frac{7}{10}$; y-intercept: 12; So, the right index fingernail is initially 12 millimeters long.

36.



slope: −500; x-intercept: 6; So, no milk is sold when the price reaches $6 per gallon; y-intercept: 3000; So, the maximum amount of milk sold is 3000 gallons.

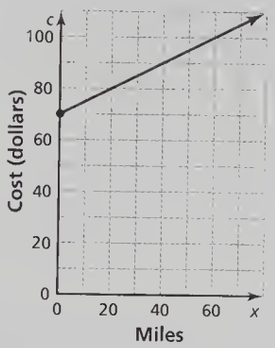
37. a.



domain: $0 \le t \le 9$; range: $6 \le d \le 10\frac{1}{2}$

b. slope: $\frac{1}{2}$; So, $\frac{1}{2}$ inch of snow falls every hour during the storm; d-intercept: 6; So, there were 6 inches of snow already on the ground at the start of the storm.

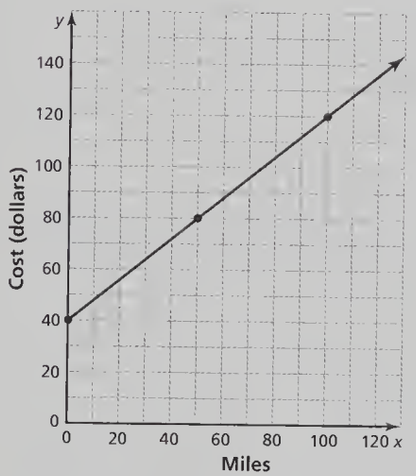
38. a.



domain: $x \ge 0$; range: $c \ge 70$

b. slope: 0.5; So, the mileage rate is \$0.50 per mile; c-intercept: 70; So, the basic rental fee is \$70.

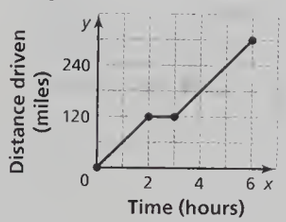
39.



The slope of $\frac{4}{5}$ is greater than the slope in Exercise 38. The y-intercept of 40 is less than the c-intercept in Exercise 38.

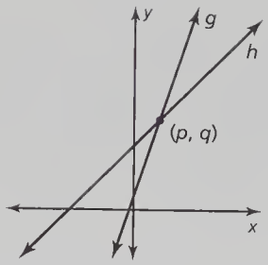
47. *Sample answer:* the ratio of the rise to the run; the ratio of the change in y to the change in x

48. *Sample answer:*



The first line on the graph represents your family driving 60 miles per hour for 2 hours. The second line represents your family not driving at all for 1 hour. The third line represents your family driving 60 miles per hour for 3 hours to complete your trip to the beach.

49. a.



b. a is the y-intercept of g; b is the y-intercept of h

c. 8 units greater

50. a. You commute to school by walking $\frac{1}{2}$ mile to the bus stop in 10 minutes, then waiting 4 minutes for the bus, then riding the bus for 2 miles in 4 minutes.

b. The slope of the first part is 0.05. So, you walk at an average speed of 0.05 mile per minute; The slope of the second part is 0. So, you do not move while waiting for the bus; The slope of the last part is 0.5. So, the bus travels at an average speed of 0.5 mile per minute.

51. $k = \frac{1}{8}$

52. $k = -12$

53.
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(mx_2 + b) - (mx_1 + b)}{x_2 - x_1}$$
$$= \frac{mx_2 + b - mx_1 - b}{x_2 - x_1}$$
$$= \frac{mx_2 - mx_1}{x_2 - x_1}$$
$$= \frac{m(x_2 - x_1)}{x_2 - x_1}$$
$$= m$$

3.5 Maintaining Mathematical Proficiency

54. $a \le \frac{5}{2}$

55. no solution

56. all real numbers

57. $x > -\frac{3}{4}$

58. nonlinear; The graph is not a line.

59. linear; The graph is a line.

60. nonlinear; As x increases by 1, y increases by different amounts. The rate of change is not constant.

61. linear; As x increases by 2, y decreases by 1. The rate of change is constant.

3.6 Monitoring Progress and Modeling with Mathematics

31. a. yes; Distance varies directly with time and the constant of variation is 6.

b. yes; Distance varies directly with rate and the constant of variation is 50.

c. no; The graph of $300 = rt$ is not a line.

32. a. The line passes through the origin.

b. yes; The scale of the x-axis does not affect the equation of the line.

33. yes; The constants of variation are reciprocals of each other.

3.6 Maintaining Mathematical Proficiency

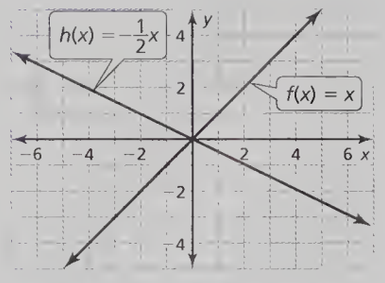
34. $A'(-3, 0)$, $B'(-1, 0)$, $C'(-1, -4)$, $D'(-3, -4)$

35. $X'(0, 0)$, $Y'(4, 2)$, $Z'(4, 0)$

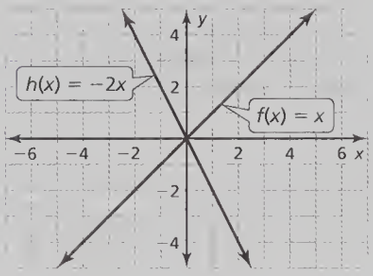
36. $Q'(4, 4)$, $R'(2, 2)$, $S'(2, -1)$, $T'(4, -3)$

3.7 Explorations

2. c.



d.

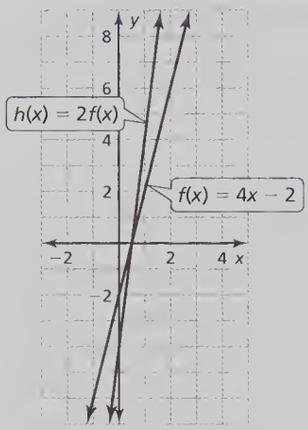


Multiplying a constant by a function causes the graph to get closer to or further from the axis.

3. a. C; The graph is shifted down 4 units and further from the x-axis.
 b. A; The graph is shifted up 2 units and further from the x-axis.
 c. D; The graph is shifted up 4 units and closer to the x-axis.
 d. B; The graph is shifted down 2 units and closer to the x-axis.
4. The graph of $g(x) = f(x) + c$ is shifted vertically from $f(x) = x$; The graph of $h(x) = f(cx)$ is closer to or further from the x-axis than $f(x) = x$.

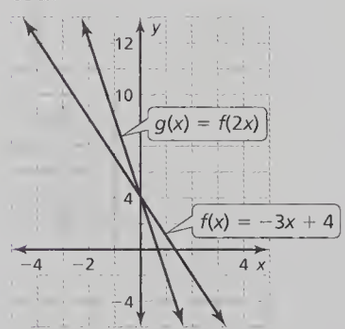
3.7 Lesson Monitoring Progress

3. b.



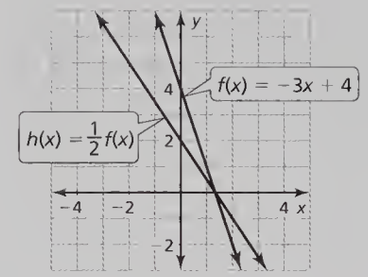
The graph of h is a vertical stretch of the graph of f by a factor of 2.

4. a.



The graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{2}$.

b.

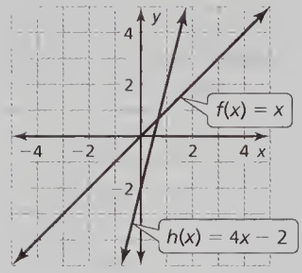


The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{2}$.

3.7 Monitoring Progress and Modeling with Mathematics

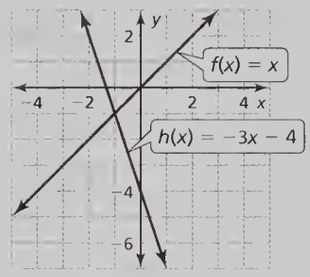
13. The graph of h is a reflection in the x-axis of the graph of f.
14. The graph of h is a reflection in the y-axis of the graph of f.
15. The graph of h is a reflection in the y-axis of the graph of f.
16. The graph of h is a reflection in the x-axis of the graph of f.

42.



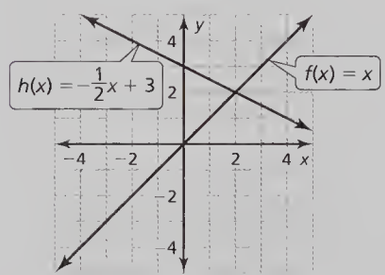
The transformations are a vertical stretch by a factor of 4 then a vertical translation 2 units down.

43.



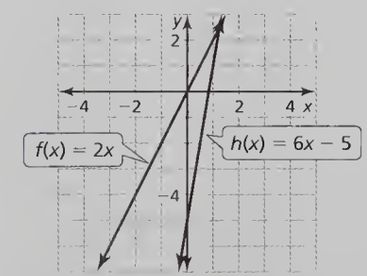
The transformations are a vertical stretch by a factor of 3, then a reflection in the x-axis, then a vertical translation 4 units down.

44.



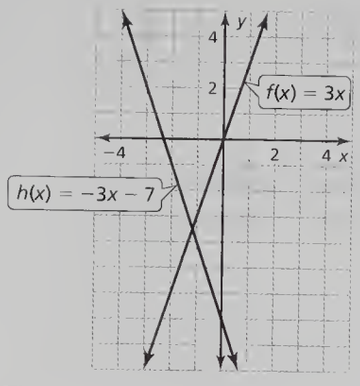
The transformations are a vertical shrink by a factor of $\frac{1}{2}$, then a reflection in the x-axis, then a vertical translation 3 units up.

45.



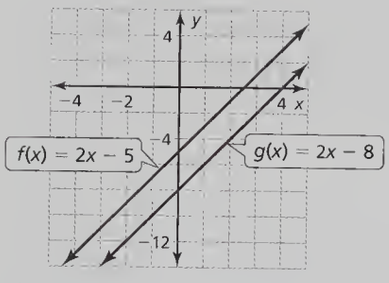
The transformations are a vertical stretch by a factor of 3 then a vertical translation 5 units down.

46.



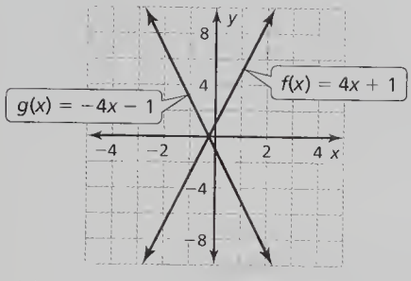
The transformations are a reflection in the x-axis then a vertical translation 7 units down.

55.



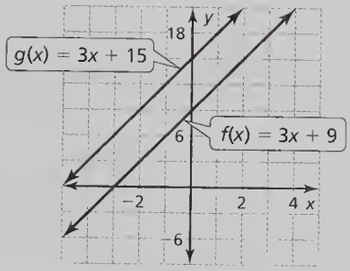
$g(x) = f(x) - 3$; The graph of g is a vertical translation 3 units down of the graph of f.

56.



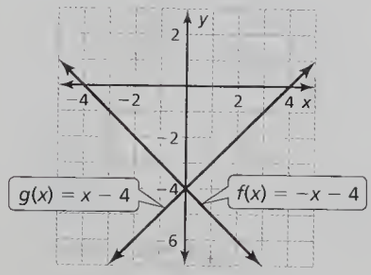
$g(x) = -f(x)$; The graph of g is a reflection in the x-axis of the graph of f.

57.



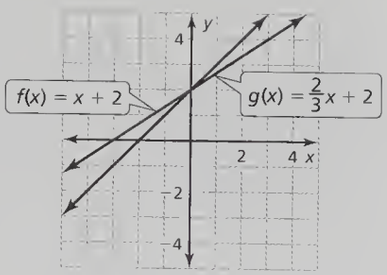
$g(x) = f(x) + 6$; The graph of g is a vertical translation 6 units up of the graph of f.

58.



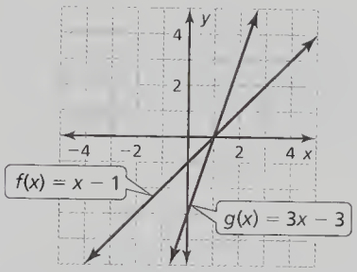
$g(x) = f(-x)$; The graph of g is a reflection in the y-axis of the graph of f.

59.



$g(x) = f\left(\frac{2}{3}x\right)$; The graph of g is a horizontal stretch of the graph of f by a factor of $\frac{3}{2}$.

60.



$g(x) = 3f(x)$; The graph of g is a vertical stretch of the graph of f by a factor of 3.

61. Translate the graph of $f(x) = x$ horizontally 5 units left.

62. **a.** A; reflection in the y-axis

b. D; vertical translation down

c. B; vertical translation up

d. C; horizontal shrink

63. $r = 2$

64. $r = \frac{1}{3}$

65. $r = 2$

66. $r = -8$

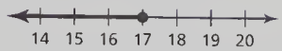
67. when the slope is 1; A slope of 1 occurs when the ratio of the vertical change to the horizontal change is 1, meaning the vertical change and horizontal change are the same.

3.7 Maintaining Mathematical Proficiency

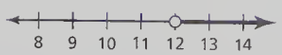
68. $h = \frac{V}{\pi r^2}$

69. $w = \frac{P - 2\ell}{2}$ or $w = \frac{1}{2}P - \ell$

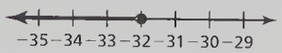
70. $x \leq 17$



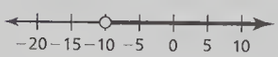
71. $x > 12$



72. $x \leq -32$

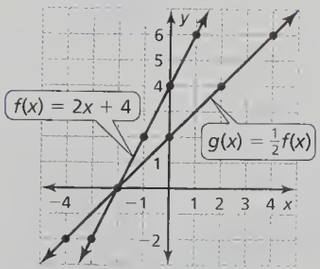


73. $x > -10$



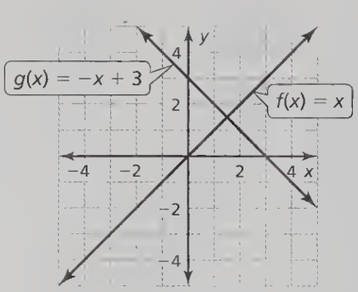
Chapter 3 Test

9.



The transformation is a vertical shrink by a factor of $\frac{1}{2}$.

10.



The transformations are a reflection in the x-axis then a vertical translation 3 units up.

11. 3

12. $-\frac{1}{4}$

13. The domain of A is discrete because the number of quarters must be a whole number, and the domain of B is continuous because the time can be any value greater than or equal to 0.

14. **a.** slope: 125; So, the climber ascends at a rate of 125 feet per hour; y-intercept: 50; So, the climber began the ascent at 50 feet above sea level.

b. $f(3)$ can be found by reading the point off the graph or by substituting 3 into the function definition; $f(3) = 425$; So, the climber is 425 feet above sea level after climbing for 3 hours.

c. 3.6 h; At the top, $f(x) = 500$. Setting 500 equal to $125x + 50$ and solving for x gives the time it takes to reach the top.

15. The slope of g is twice the slope of f. Both functions have the same y-intercept; The x-intercept of g is half the x-intercept of f.

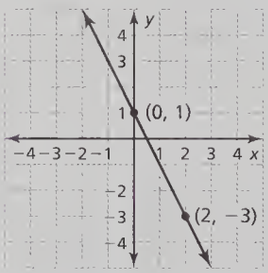
16. 7 c

Chapter 4

4.1 Explorations

3. Use the graph to find the slope and the y-intercept, and substitute these values into the slope-intercept form.

4. *Sample answer:*



$y = -2x + 1$

4.1 Monitoring Progress and Modeling with Mathematics

33. $y = -\frac{A}{B}x + \frac{C}{B}$; slope is $\frac{6}{5}$; y-intercept is $\frac{9}{5}$

34. your friend; If f is a function, then the line is not vertical.

35. $y = \frac{5}{3}x - 1$

36. **a.** *Sample answer:* about 0.22; 8

b. *Sample answer:* The approximate U.S. box office revenue increased by about \$0.22 billion each year from 2000 to 2012 and was \$8 billion in 2000.

c. Write a linear model in slope-intercept form, then substitute 18 for x and solve for y.

37. slope $= \frac{(b + m) - b}{1 - 0} = m$; y-intercept $= b$; $y = mx + b$;

Substitute -1 for x in $y = mx + b$ and verify that $y = b - m$.

4.1 Maintaining Mathematical Proficiency

38. $x = 28$

39. no solution

40. $n = 0$

41. yes; -6

42. no

43. no

4.2 Monitoring Progress and Modeling with Mathematics

37. **a.** $C = 80n + 145$

b. \$865

38. **a.** yes; The total cost increases at a constant rate.

b. \$42; \$102

c. 11 days

39. *Sample answer:* Plot the point (4, 1), then use the slope of $\frac{2}{3}$ to find a second point on the graph and draw a line through the points; Rewrite the equation in slope-intercept form, then use the y-intercept (0, 5) and the slope of $\frac{2}{3}$ to graph the equation.

40. *Sample answer:* $y + 5 = \frac{2}{5}(x - 12)$; $y = \frac{2}{5}x - \frac{49}{5}$

41. *Sample answer:* point-slope form; The value of the y-intercept is unknown.

42. **a.** *Sample answer:* negative; It appears the line through the two points will extend below the x-axis before crossing the y-axis.

b. *Sample answer:* (8, 4); $\left(4, \frac{5}{3}\right)$; Use the two points to find the slope of the line. Substitute the slope and one of the points into $y - y_1 = m(x - x_1)$. Rewrite the equation in slope-intercept form and determine the y-intercept from the rewritten equation.

43. *Sample answer:* The graph of $y - k_1 = m(x - h)$ is a translation h units to the right and k units upward of the graph of $y = mx$.

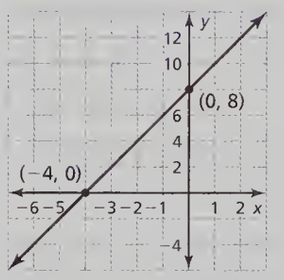
44. **a.** B; A

b. B; A

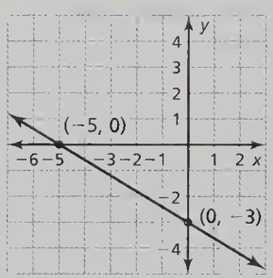
c. C; A

4.2 Maintaining Mathematical Proficiency

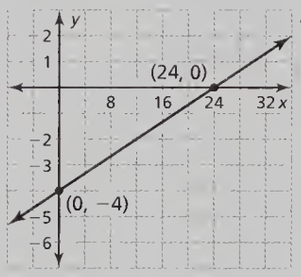
45.



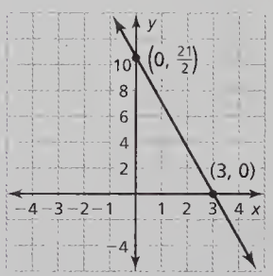
46.



47.



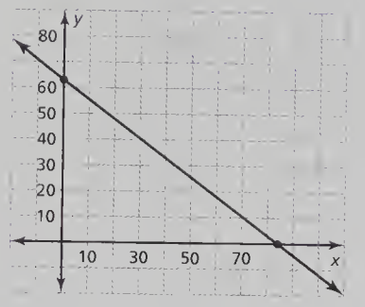
48.



4.3 Monitoring Progress and Modeling with Mathematics

30. a. $0.75x + y = 63$

b.



c. 18; Substitute 60 for x in the equation from part (a) and solve for y.

31. a. *Sample answer:* slope-intercept form; The slope and y-intercept can be easily determined and used to graph the equation.

b. *Sample answer:* standard form; Substituting 0 for y allows you to solve the equation for x easily.

c. *Sample answer:* point-slope form; One point can easily be substituted into an equation of this form.

32. a. *Sample answer:* 2 nights and 10 dog treats, 3 nights and 5 dog treats

b. If you do not keep your dog in the kennel overnight, you can buy 20 dog treats. If you keep your dog in the kennel for 4 nights, you cannot buy any dog treats.

35. $2\ell + 2w = 60$; *Sample answer:*

Length	Width
28	2
26	4
25	5
22	8
18	12

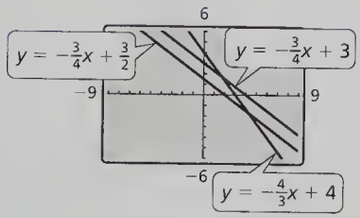
4.3 Maintaining Mathematical Proficiency

36. $\frac{1}{5}$ **37.** $-\frac{1}{8}$

38. $-\frac{7}{2}$ **39.** $\frac{2}{3}$

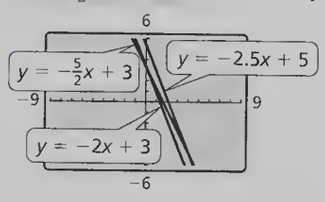
4.4 Explorations

1. a. $y = -\frac{3}{4}x + \frac{3}{2}$; $y = -\frac{3}{4}x + 3$; $y = -\frac{4}{3}x + 4$



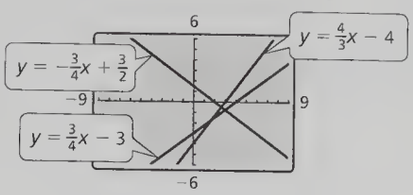
$3x + 4y = 6$ and $3x + 4y = 12$; *Sample answer:* The lines are always the same distance apart.

b. $y = -\frac{5}{2}x + 3$; $y = -2x + 3$; $y = -2.5x + 5$;



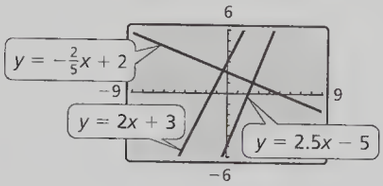
$5x + 2y = 6$ and $2.5x + y = 5$; *Sample answer:* The lines are always the same distance apart.

2. a. $y = -\frac{3}{4}x + \frac{3}{2}$; $y = \frac{3}{4}x - 3$; $y = \frac{4}{3}x - 4$;



$3x + 4y = 6$ and $4x - 3y = 12$; *Sample answer:* The lines intersect at a right angle.

b. $y = -\frac{2}{5}x + 2$; $y = 2x + 3$; $y = 2.5x - 5$;



$2x + 5y = 10$ and $2.5x - y = 5$; *Sample answer:* The lines intersect at a right angle.

4.4 Monitoring Progress and Modeling with Mathematics

34. a. yes; Student B paid a greater registration fee.

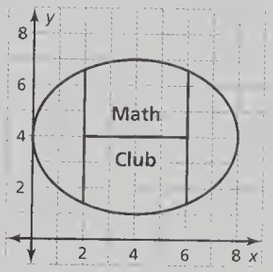
b. no; Both graphs have the same slope.

35. never; Perpendicular lines have opposite reciprocal slopes, so one must be positive and the other must be negative.

36. always; The y-axis is a vertical line, and all vertical lines are parallel.

37. sometimes; They are perpendicular when the slopes are negative reciprocals, otherwise they will not be perpendicular.

38. *Sample answer:*



$y = 4$; $x = 2$; $x = 6$

4.4 Maintaining Mathematical Proficiency

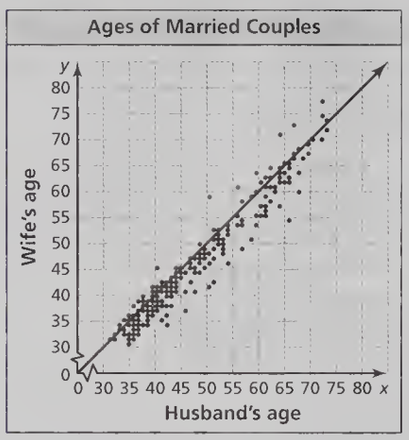
39. function; Each input value is paired with exactly one output value.

40. not a function; The input values of -1 and 1 are each paired with more than one output value.

Additional Answers

4.5 Explorations

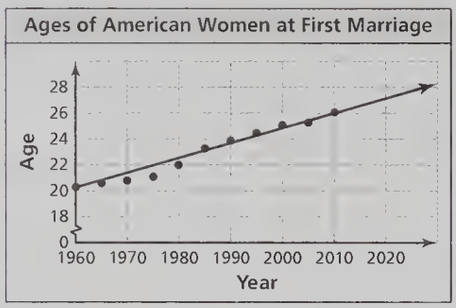
1. a. *Sample answer:*



$y = x$; Estimate a line through the middle of the plotted data, then use two points on the line to find the equation.

b. *Sample answer:* The ages of the husband and wife in a married couple tend to be about the same; The equation shows they are equal.

2. a. *Sample answer:*



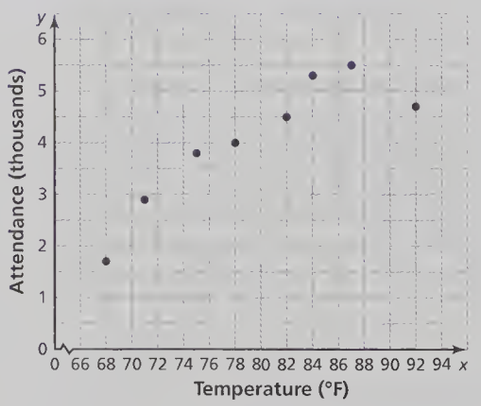
$y = 0.1x + 20.1$; Estimate a line through the middle of the plotted data, then use two points on that line to find the equation.

b. *Sample answer:* The median age at which American women first marry increases about 1 year every 10 years.

c. *Sample answer:* about 26.1 years old

4.5 Lesson Monitoring Progress

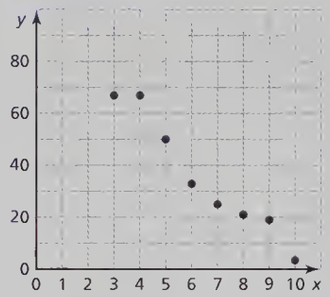
3.



positive

4.5 Monitoring Progress and Modeling with Mathematics

14.



negative

15. a. *Sample answer:* $y = -0.3x + 35$

b. *Sample answer:* The slope of -0.3 means the birthrate is decreasing by about 3 births per 1000 people every 10 years. The y-intercept of 35 means in 1960 the birth rate was about 35 births per 1000 people.

16. a. *Sample answer:* $y = 20x$

b. *Sample answer:* The slope of 20 means the server's earnings increase by about \$20 per hour. The y-intercept of 0 means the server makes \$0 if he doesn't work.

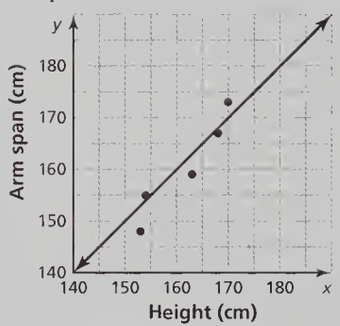
17. *Sample answer:*

Weight of car (pounds), x	2400	2500	2900	3000
Gas mileage (mpg), y	39	38	25	32

Weight of car (pounds), x	3400	3500	3700	5100
Gas mileage (mpg), y	30	24	21	16

18. no; Because both variables are decreasing, the data show a positive correlation.

19. a. *Sample answer:*



b. *Sample answer:* The slope of 1 means a person's arm span increases by about 1 centimeter for every 1 centimeter increase in height. The y-intercept of 0 has no meaning in this context because the height cannot be 0.

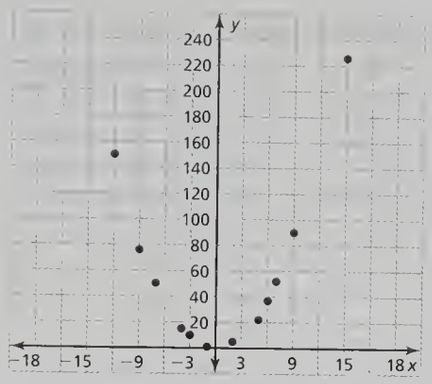
20. *Sample answer:* The number of students y in a school club x years after the club was founded.

21. *Sample answer:* When the data are from two sets such as age and time.

22. *Sample answer:* (2, 5), (9, 57), (12, 99), (15, 102)

23. no; The data points do not have a linear trend.

24.



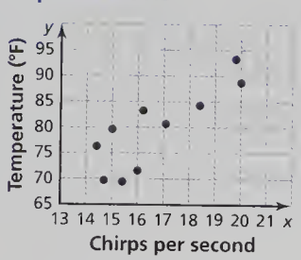
no correlation; no; The data points do not have a linear trend.

4.5 Maintaining Mathematical Proficiency

25. −18, 0, 24
26. 30, 0, −40
27. −23, −8, 12
28. 23, 14, 2

4.6 Explorations

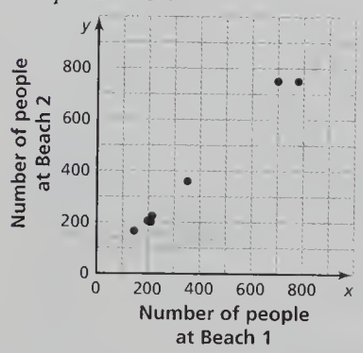
3.



$y = 3.27x + 25.0$; about 87.1°F

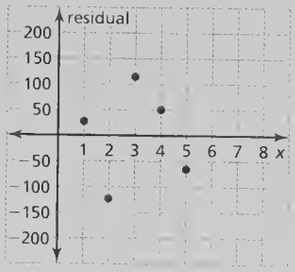
4.6 Monitoring Progress and Modeling with Mathematics

26. a. B; The points appear linear with a positive slope.
 b. C; The points appear linear with a negative slope.
 c. A; The points are scattered.
 d. D; The points appear somewhat linear with a positive slope.
27. a. $y = -0.08x + 3.8$; $r = -0.965$; strong negative correlation
 b. The slope of −0.08 means the GPA decreases by about 0.08 for every hour spent watching television in a week. The y-intercept of 3.8 means that a student who watches no television has a GPA of about 3.8.
 c. 2.7
 d. *Sample answer:* no; The time spent watching television does not determine GPA.
28. yes; This point is an outlier.
29. a. 2863 people; 5149 people
 b. relatively accurate at 9 min, but not accurate at 15 min.
30. *Sample answer:*



As the number of people at Beach 1 increases, the number of people at Beach 2 also increases; possible $r = 0.99$; There is not a causal relationship; Warm weather and holidays will increase the number of people at both beaches, but the number of people at Beach 1 does not determine the number of people at Beach 2.

31. a. $y = 513.5x - 298$; $r = 0.986$; strong positive correlation
 b. no; The year does not determine the number of text messages sent.
 c. 25.5; −128; 117.5; 50; −63.5



The equation $y = 513.5x - 298$ is a good fit.

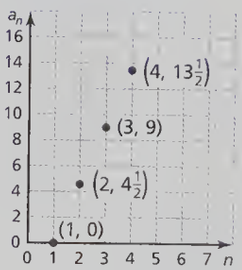
 d. *Sample answer:* part (a); The correlation coefficient is a single value, which is easily interpreted where as interpreting the scatter plot of the residuals is more subjective.

4.6 Maintaining Mathematical Proficiency

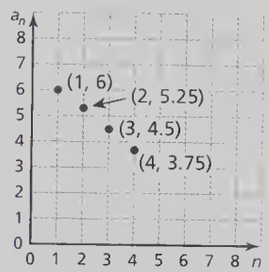
32. nonlinear; The rate of change is not constant.
33. linear; The rate of change is constant.

4.7 Monitoring Progress and Modeling with Mathematics

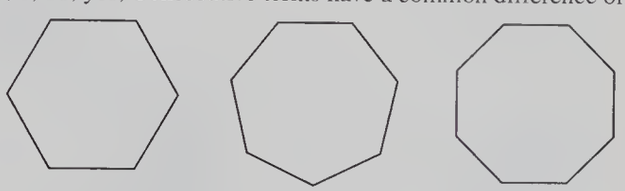
21.



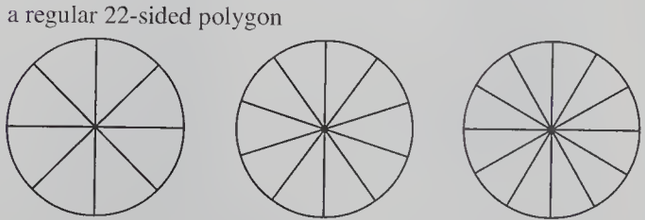
22.



23. not an arithmetic sequence; Consecutive terms do not have a common difference.
24. arithmetic sequence; Consecutive terms have a common difference of 7.
25. arithmetic sequence; Consecutive terms have a common difference of −15.
26. not an arithmetic sequence; Consecutive terms do not have a common difference.
27. arithmetic sequence; 13
28. not an arithmetic sequence
29. not an arithmetic sequence
30. arithmetic sequence; −6
31. 4, 6, 8, 10; yes; Consecutive terms have a common difference of 2.
43. a.



 b. a regular 22-sided polygon
44. a.



 b. a circle divided into 40 equal sections
57. a. arithmetic sequence; Consecutive terms have a common difference of $2x$.
 b. not an arithmetic sequence; Consecutive terms do not have a common difference.

Additional Answers

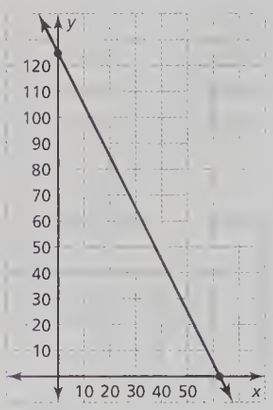
4.7 Maintaining Mathematical Proficiency

58. $\frac{5}{2}$ 59. $\frac{1}{3}$

60. $-\frac{5}{4}$

Chapter 4 Test

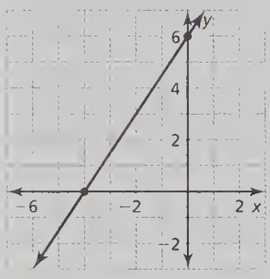
15. a. $0.08x + 0.04y = 5$

b. 

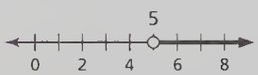
c. *Sample answer:* 20 yards of organza ribbon and 85 yards of satin ribbon, 30 yards of organza ribbon and 65 yards of satin ribbon, 40 yards of organza ribbon and 45 yards of satin ribbon, and 50 yards of organza ribbon and 25 yards of satin ribbon

Chapter 5

Chapter 5 Maintaining Mathematical Proficiency

4. 

5. $m > 5$



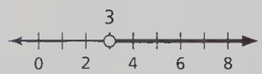
6. $t \leq -4$



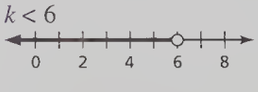
7. $a \leq 9$



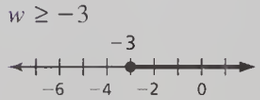
8. $z > 3$



9. $k < 6$



10. $w \geq -3$



11. The two lines intersect.

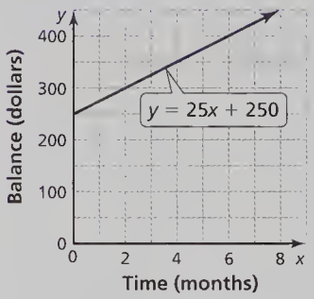
5.1 Explorations

2. a.

x (nights)	0	1	2	3	4	5
C (dollars)	600	615	630	645	660	675
R (dollars)	0	75	150	225	300	375

x (nights)	6	7	8	9	10	11
C (dollars)	690	705	720	735	750	765
R (dollars)	450	525	600	675	750	825

5.1 Monitoring Progress and Modeling with Mathematics

30. 

Sample answer: $y = 20x + 280$; The slope of the line is 20. This means you deposit \$20 each month. The y-intercept is 280. This means you opened the account with \$280.

31. a. $x = 3$

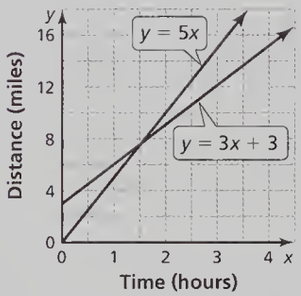
b. (3, 5)

c. The equation in part (a) shows that the 2 values of y in the system in part (b) are equal. The x-coordinate of the solution of the system in part (b) is the solution of the equation in part (a).

32. a. 20, 35, 50; Two of the lines intersect at the points where the number of binders is each of these values.

b. Each of these values is the x-coordinate of the solutions of the systems of linear equations formed by each pair of equations.

33. a. $y = 5x$ and $y = 3x + 3$



b. no; The point of intersection does not occur at $x = 1$.

5.2 Explorations

4. d. (3, 2); *Sample answer:* The second equation can be solved for x easily.

e. (1, −3); *Sample answer:* The second equation can be solved for x easily.

f. (−1, 3); *Sample answer:* The first equation can be solved for y easily.

5.2 Monitoring Progress and Modeling with Mathematics

31. *Sample answer:* $y = -3x + 4$ and $x + y = 6$

32. a. (4, 5)

b. yes; Write an equation of each line. Then solve this system of linear equations using substitution.

33. 144 pop songs, 48 rock songs, 80 hip-hop songs

34. *Sample answer:* $x + y = 13$ and $25x + 10y = 265$, x = number of quarters and y = number of dimes

35. 47

5.2 Maintaining Mathematical Proficiency

36. $3x - 11$ 37. -13

38. -23 39. $3d + 5$

40. $22m - 4$ 41. $64v$

5.3 Monitoring Progress and Modeling with Mathematics

32. Equation 1 can be rewritten as Equation 3 by adding $2(x + y)$ to the left and 2(6) to the right, and Equation 3 can be rewritten as Equation 1 by subtracting $2(x + y)$ from the left and 2(6) from the right. Because either system can be rewritten as the other, System 1 and System 2 have the same solution.

33. 4.5 qt of 100% fruit juice, 1.5 qt of 20% fruit juice

34. 5 mi/h

35. $(-5, 4, 2)$; *Sample answer:* Subtract Equation 2 from Equation 1. The resulting equation only has 1 variable, y, so use it to solve for y. Substitute this result in Equation 3 and solve for x. Substitute the values of x and y in Equation 2 and solve for z.

5.3 Maintaining Mathematical Proficiency

36. no solution

37. $t = \frac{3}{8}$; one solution

38. $n = -8$; one solution

39. all real numbers; infinitely many solutions

40. $y = -2x + 7$

41. $y = 5x + 6$

42. $y = \frac{2}{3}x + \frac{4}{3}$

5.4 Explorations

1. a. $C = 450 + 20x$; $R = 20x$

x (skateboards)	0	1	2	3	4
C (dollars)	450	470	490	510	530
R (dollars)	0	20	40	60	80

x (skateboards)	5	6	7	8	9	10
C (dollars)	550	570	590	610	630	650
R (dollars)	100	120	140	160	180	200

b. never; Both cost and revenue increase at the same rate, but have different initial values.

5.4 Monitoring Progress and Modeling with Mathematics

27. yes; The system of equations $150c + 80b = 22{,}860$ and $170c + 100b = 27{,}280$ has a solution.

28. *Sample answer:* $y = 2x$, $y = \frac{1}{2}x$, and $y = -x + 6$

29. one; The lines have different slopes, so they will intersect at exactly one point.

30. a. *Sample answer:* 40 m

b. yes; The lines that represent Team A and Team C's runners have different slopes, so they will intersect.

c. no; The lines that represent Team A and Team B's runners have the same slope, so they will never intersect.

31. a. never; The y-intercepts are different, so they can never be equations for the same line.

b. sometimes; When $a = b$, the lines are parallel and there is no solution.

c. always; When $a < b$, the slopes are different and the lines intersect at one point.

32. no; The system $3x + 2y = 38$ and $15x + 10y = 190$ has infinitely many solutions.

5.4 Maintaining Mathematical Proficiency

33. (8, 2)

34. $(-2, -1)$

35. (4, −2)

5.5 Monitoring Progress and Modeling with Mathematics

37. $x = 10$

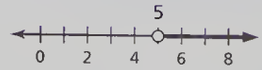
38. *Sample answer:* initial value \$22,500, decrease in value at a rate of \$2000 per year

39. a. negative

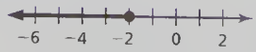
b. positive

5.5 Maintaining Mathematical Proficiency

40.



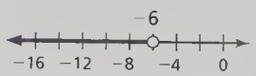
41.



42.



43.



44. The graph of g is a horizontal translation 2 units left of the graph of f.

45. The graph of g is a reflection in the x-axis of the graph of f.

46. The graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{4}$.

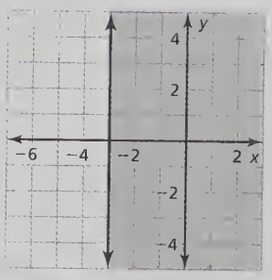
47. The graph of g is a horizontal translation 1 unit right of the graph of f.

5.6 Explorations

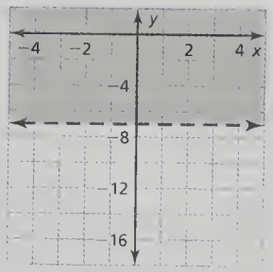
5. *Sample answer:* You want to spend no more than \$15 at the deli for bologna at \$2.99 per pound and cheese at \$1.99 per pound. How many pounds of each can you purchase?

5.6 Monitoring Progress and Modeling with Mathematics

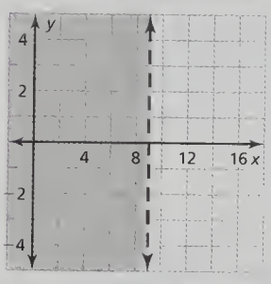
22.



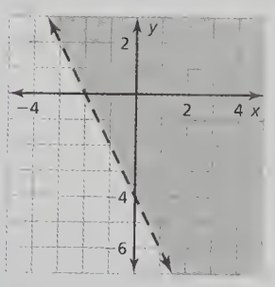
23.



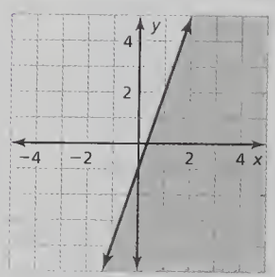
24.



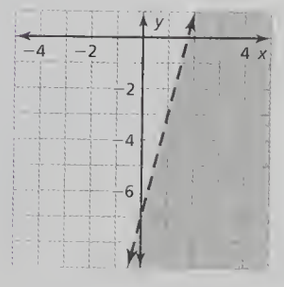
25.



26.

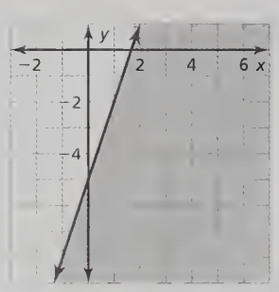


27.

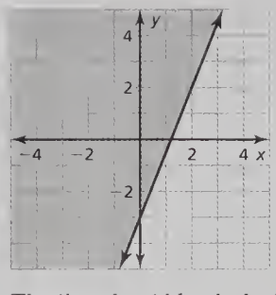


Additional Answers

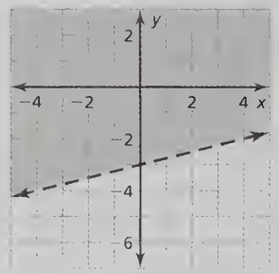
28.



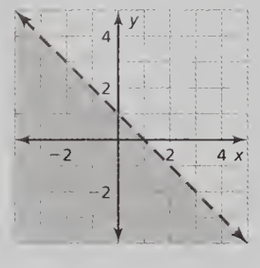
29.



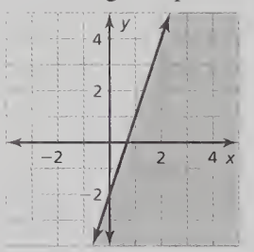
30.



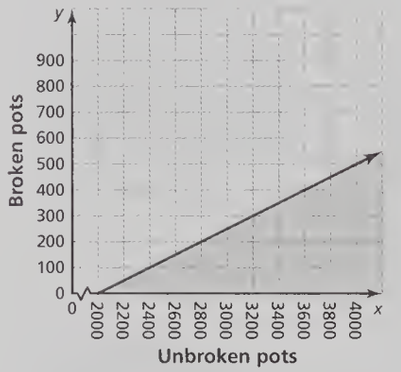
31. The line should be dashed.



32. The wrong half-plane is shaded.

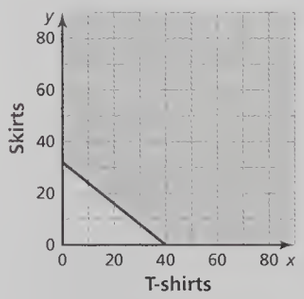


46. $x - 4y \geq 2000$



Sample answer: (2400, 100), The company can deliver 2400 unbroken pots and 100 broken pots; (3000, 200), The company can deliver 3000 unbroken pots and 200 broken pots.

47. $20x + 25y \geq 800$



Sample answer: (40, 10), The store can sell 40 T-shirts and 10 skirts; (20, 30), The store can sell 20 T-shirts and 30 skirts.

48. a. $75x + 40y \leq 1800$



b. *Sample answer:* The number of boxes must be a positive whole number. It is also unlikely for 1 person to carry 45 boxes onto an elevator.

49. no; You cannot use (0, 0) as a test point when it is on the boundary line.

50. a. C

b. A

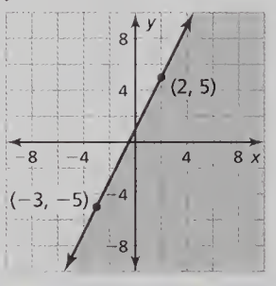
c. D

d. B

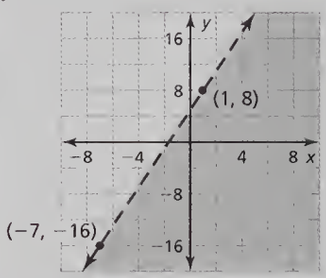
51. A test point on the boundary line is not in either half-plane, so it will not indicate which half-plane to shade.

52. *Sample answer:* $y > -x + 1$

53. $y \leq 2x + 1$



54. $y < 3x + 5$



5.6 Maintaining Mathematical Proficiency

55. 40, 48, 56

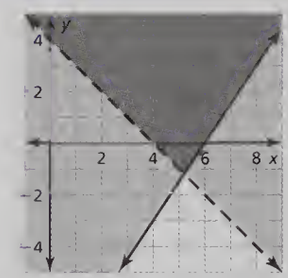
56. −20, −23, −26

57. −0.4, −1.2, −2

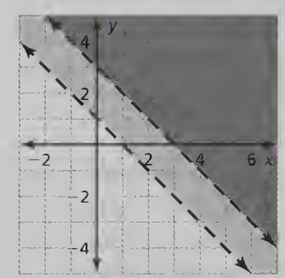
58. $\frac{7}{2}, \frac{9}{2}, \frac{11}{2}$

5.7 Monitoring Progress and Modeling with Mathematics

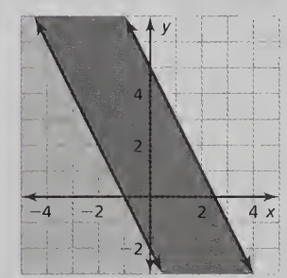
16.



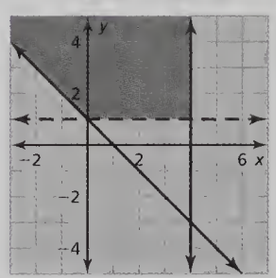
17.



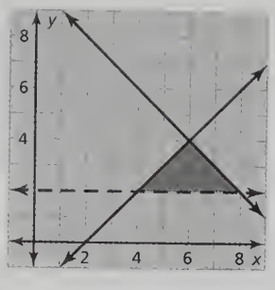
18.



19.



20.



21. $x \geq -1$ and $y < 3$

22. $x > 2$ and $x < 4$

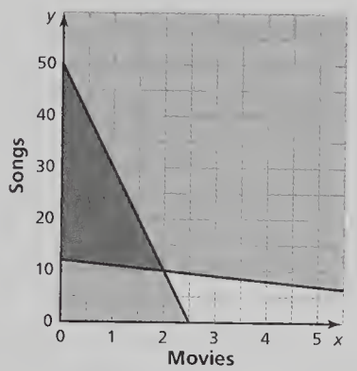
23. $y \geq \frac{2}{3}x - 2$ and $y \geq -3x + 2$

24. $y \leq 5x + 1$ and $y > x - 2$

25. $y > -2x - 1$ and $y < -2x - 3$

26. $y \geq \frac{1}{3}x$ and $y \geq \frac{1}{3}x - 2$

39. *Sample answer:* You receive a $50 gift card to an online retail store. Movies cost $20 each and songs cost $1 each. You want to purchase at least 12 items. How many of each item can you buy?; $20x + y \leq 50$ and $x + y \geq 12$



40. no; The half-planes could overlap depending on which side of the boundaries they are on.

41. no; The solution of each inequality is a half-plane, and so the intersection can be at most a half-plane.

42. *Sample answer:* $x > 2$ and $y > 3$

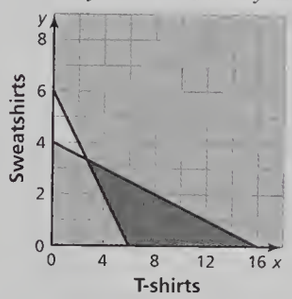
43. *Sample answer:* $x > 2$ and $y < -4$

44. *Sample answer:* $y > 2$ and $y < -2$

45. **a.** *Sample answer:* $-4x + 2y < 6$

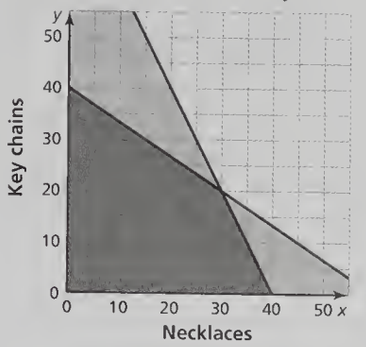
b. *Sample answer:* $-2x + y > 3$

46. *Sample answer:* T-shirts cost $5, sweatshirts cost $20, gift certificate is worth $80, want to buy at least 6 shirts; $5x + 20y \leq 80$ and $x + y \geq 6$



47. *Sample answer:* $x \geq 4$, $x \leq 4$, $y \geq 5$, and $y \leq 5$

48. **a.** $0.5x + 0.25y \leq 20$, $2x + 3y \leq 120$, $x \geq 0$, and $y \geq 0$



b. (0, 0); (0, 40); (30, 20); (40, 0)

c. $0, $320, $460, $400; (30, 20)

5.7 Maintaining Mathematical Proficiency

49. 4^5

50. $(-13)^3$

51. x^6

52. $y = x - 6$

53. $y = -3x + 5$

54. $y = -\frac{1}{4}x - 1$

55. $y = \frac{4}{3}x$

Chapter 5 Test

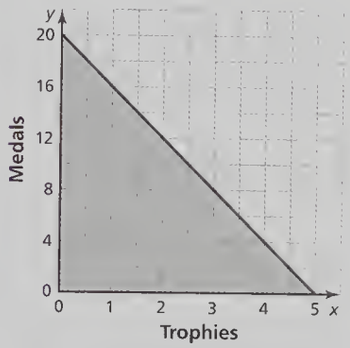
12. **a.** no; There are infinitely many solutions for the system of linear equations that represents this situation.

b. yes; The equation representing the receipt and either of the equations from part (a) form a linear system that has a single solution.

c. $3.55/gallon of gas, $5/quart of oil

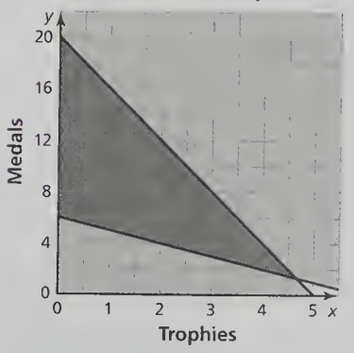
13. can be quick when an estimate is sufficient; does not always give exact answers

14. **a.** $12x + 3y \leq 60$



Sample answer: (2, 8); You can buy 2 trophies and 8 medals.

b. $12x + 3y \leq 60$ and $x + y \geq 6$



Sample answer: 2 trophies, 6 medals

15. no solution; The lines have the same slope but different y-intercepts, so they are parallel.

Chapter 6

6.1 Monitoring Progress and Modeling with Mathematics

69. **a.** When $a > 1$ and $n < 0$, $a^n < a^{-n}$ because a^n will be less than 1 and a^{-n} will be greater than 1. When $a > 1$ and $n = 0$, $a^n = a^{-n} = 1$, because any number to the zero power is 1. When $a > 1$ and $n > 0$, $a^n > a^{-n}$ because a^n will be greater than 1 and a^{-n} will be less than 1.

b. When $0 < a < 1$ and $n < 0$, $a^n > a^{-n}$, because a^n will be greater than 1 and a^{-n} will be less than 1. When $0 < a < 1$ and $n = 0$, $a^n = a^{-n} = 1$, because any number to the zero power is 1. When $0 < a < 1$ and $n > 0$, $a^n < a^{-n}$ because a^n will be less than 1 and a^{-n} will be greater than 1.

6.1 Maintaining Mathematical Proficiency

70. 5 **71.** -10

72. $\pm\frac{1}{8}$

73. natural number, whole number, integer, rational number, real number

74. rational number, real number

75. irrational number, real number

6.2 Explorations

2. **d.** *Sample answer:* 4.0; E; 65 is between $4^3 = 64$ and $5^3 = 125$, and E is the only point on the graph between 4 and 5.

e. *Sample answer:* 3.8; D; 55 is between $3^3 = 27$ and $4^3 = 64$, and D is the only point on the graph between 3 and 4.

f. *Sample answer:* 5.2; F; 20,000 is between $5^6 = 15{,}625$ and $6^6 = 46{,}656$, and F is the only point on the graph between 5 and 6.

3. Find what real number multiplied by itself n times gives you that number. If that is not possible, determine which nth powers the number is between and estimate the decimal part.

4. about 512.7 mm

6.2 Monitoring Progress and Modeling with Mathematics

60. sometimes; true when $x = 0$, $x = -1$, or $x = 1$, otherwise false

61. sometimes; false when $x = 0$ because division by 0 is undefined

62. sometimes: true when $x = 0$ or $x = 1$, otherwise false

6.2 Maintaining Mathematical Proficiency

63. $f(-3) = -16$; $f(0) = -10$; $f(8) = 6$

64. $w(-3) = 14$; $w(0) = -1$; $w(8) = -41$

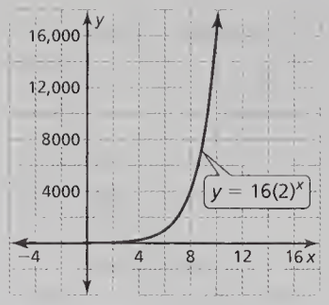
65. $h(-3) = 16$; $h(0) = 13$; $h(8) = 5$

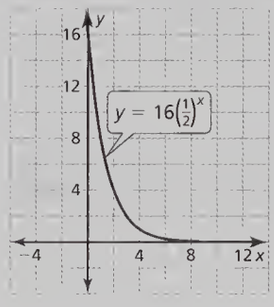
66. $g(-3) = -8$; $g(0) = 16$; $g(8) = 80$

6.3 Explorations

2. 16, 8, 4, 2, 1, $\frac{1}{2}$; 16, 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$; yes; As the exponent increases by a constant amount, the base is multiplied by itself the same number of additional times.

3.

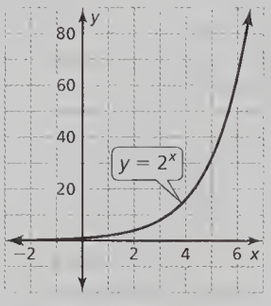




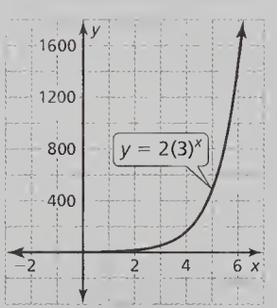
Both are curved and do not intersect the x-axis; The graph from Exploration 1 is increasing, the graph from Exploration 2 is decreasing.

4. *Sample answer:* curved shape, does not intersect the x-axis

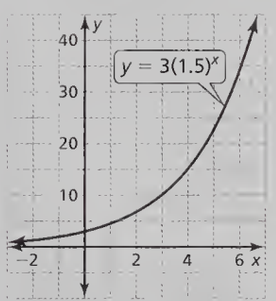
5. **a.**



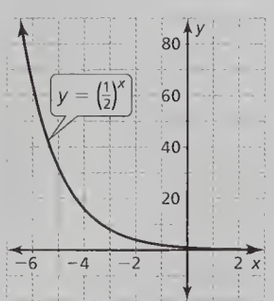
b.



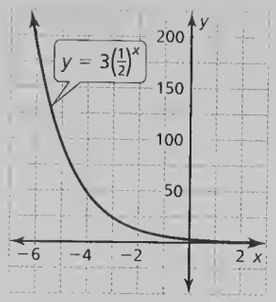
c.



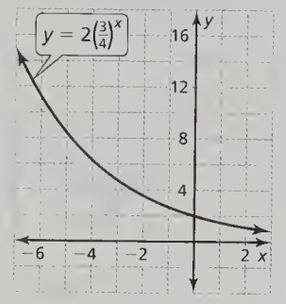
d.



e.



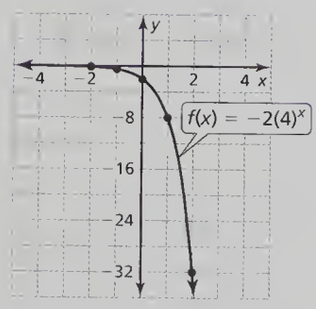
f.



yes; They have the same general curved shape, and they do not intersect the x-axis.

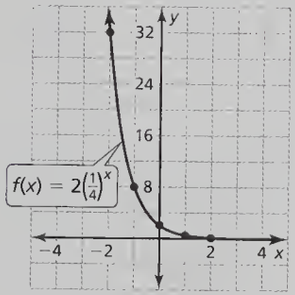
6.3 Lesson Monitoring Progress

5.



The graph of f is a vertical stretch by a factor of 2 and a reflection in the x-axis of the graph of $g(x) = 4^x$. The y-intercept of the graph of f, -2, is below the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y < 0$

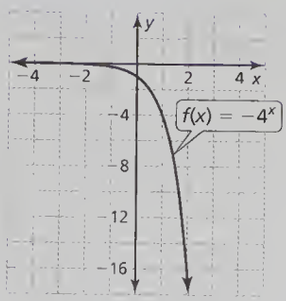
6.



The graph of f is a vertical stretch by a factor of 2 of the graph of $g(x) = \left(\frac{1}{4}\right)^x$. The y-intercept of the graph of f, 2, is above the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y > 0$

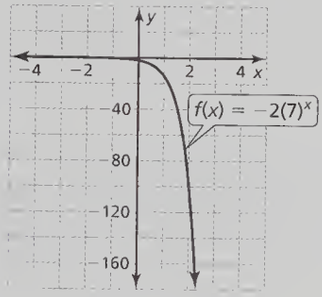
6.3 Monitoring Progress and Modeling with Mathematics

26.



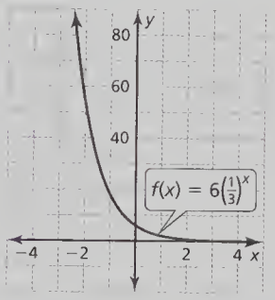
The graph of f is a reflection in the x-axis of the graph of $g(x) = 4^x$. The y-intercept of the graph of f, -1, is below the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y < 0$

27.



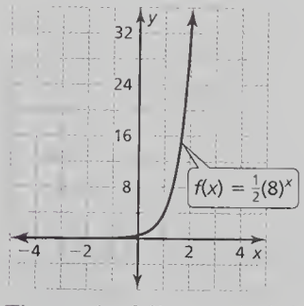
The graph of f is a vertical stretch by a factor of 2 and a reflection in the x-axis of the graph of $g(x) = 7^x$. The y-intercept of the graph of f, -2, is below the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y < 0$

28.



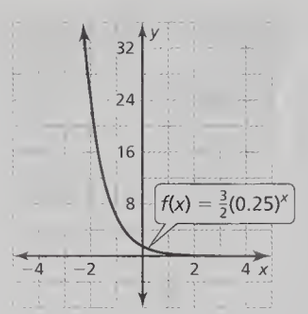
The graph of f is a vertical stretch by a factor of 6 of the graph of $g(x) = \left(\frac{1}{3}\right)^x$. The y-intercept of the graph of f, 6, is above the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y > 0$

29.



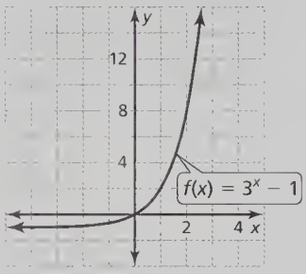
The graph of f is a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $g(x) = 8^x$. The y-intercept of the graph of f, $\frac{1}{2}$, is below the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y > 0$

30.



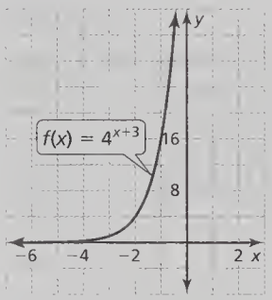
The graph of f is a vertical stretch by a factor of $\frac{3}{2}$ of the graph of $g(x) = 0.25^x$. The y-intercept of the graph of f, $\frac{3}{2}$, is above the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g; domain: all real numbers, range: $y > 0$

31.



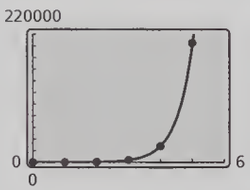
asymptote: $y = -1$; domain: all real numbers, range: $y > -1$

32.



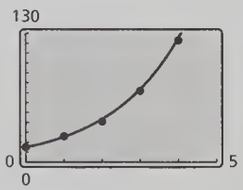
asymptote: $y = 0$; domain: all real numbers, range: $y > 0$

45. a. $y = 11(7.1)^x$



b. about 10.1 million views

46. a. $y = 15(1.7)^x$



b. about 362 coyotes

51. When a is positive, it causes a vertical stretch or shrink on the graph. When a is negative, it causes a vertical stretch or shrink and a reflection in the x-axis on the graph.

52. *Sample answer:* $f(x) = 4^{x-2}$

53. $g(x) = 5^{x-3} + 4$

54. a. 2 weeks

b. The stock price drops \$30.

55. −192

56. *Sample answer:* The number of bacteria, y, in a Petri dish after x hours is modeled by $y = 100(16)^x$; The constant 100 indicates that there were 100 bacteria in the Petri dish at the beginning, and the constant 16 indicates the number of bacteria is 16 times greater every hour.

57. $\frac{f(x+h)}{f(x)} = \frac{ab^{x+h}}{ab^x} = \frac{b^{x+h}}{b^x} = b^{(x+h)-x} = b^h$

58. $g(x) = 5(2)^x$

59. *Sample answer:* $y = 8(2)^x$

6.3 Maintaining Mathematical Proficiency

60. 0.04

61. 0.35

62. 1.28

63. 2.5

6.4 Explorations

2. b.

Time (h)	0	1	2	3
Temperature difference (°F)	38.6	34.8	31.4	28.3
Body temperature (°F)	98.6	94.8	91.4	88.3

Time (h)	4	5	6
Temperature difference (°F)	25.5	23.0	20.7
Body temperature (°F)	85.5	83.0	80.7

about 6 P.M.

6.4 Monitoring Progress and Modeling with Mathematics

61. b. All three functions indicate the initial amount of ibuprofen in a person's bloodstream is 800 mg. The first function indicates the amount of ibuprofen in a person's bloodstream decreases by about 29% each hour. The second function indicates the amount of ibuprofen in a person's bloodstream decreases by about 0.57% each minute. The third function indicates the amount of ibuprofen in a person's bloodstream decreases by about 15.7% each half-hour.

65. *Sample answer:* $y = 5(1)^x$; Any power of one is 1, so this is a constant function equivalent to $y = 5$.

66. *Sample answer:* account 1: 4% compounded annually, account 2: 8% compounded quarterly; $y = 1000(1.04)^t$; $y = 1000(1.02)^{4t}$; account 2; Graphing both functions shows the balance is always greater in account 2.

67. no; The discount is 20% of the preceding day's price, not always the original price, so the amount of the discount is less each day.

68. a. As t increases by 1, y is multiplied by 2; 100%

b. The graph of g is a vertical stretch of the graph of f; $k = 4$

c. $r = 2$

6.4 Maintaining Mathematical Proficiency

69. $x = -3$

70. $t = -2$

71. $r = 5$

72. arithmetic sequence; Consecutive terms have a common difference of −6.

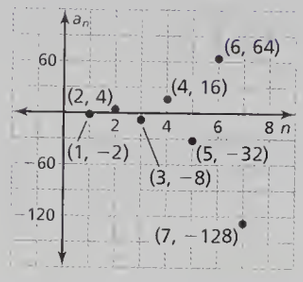
73. not an arithmetic sequence; Consecutive terms do not have a common difference.

74. not an arithmetic sequence; Consecutive terms do not have a common difference.

75. arithmetic sequence; Consecutive terms have a common difference of 10.

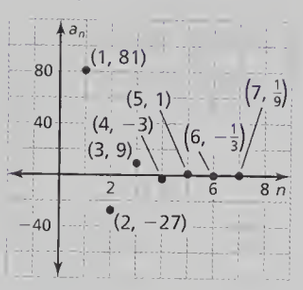
6.5 Lesson Monitoring Progress

7. $-32, 64, -128$

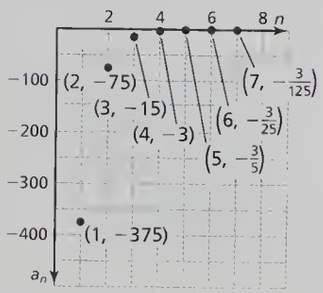


6.5 Monitoring Progress and Modeling with Mathematics

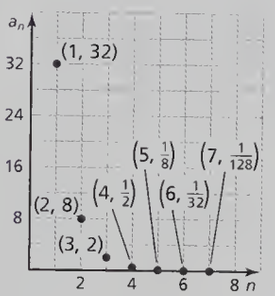
21. $1, -\frac{1}{3}, \frac{1}{9}$



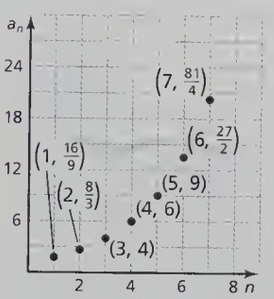
22. $-\frac{3}{5}, -\frac{3}{25}, -\frac{3}{125}$



23. $\frac{1}{8}, \frac{1}{32}, \frac{1}{128}$



24. $9, \frac{27}{2}, \frac{81}{4}$



25. $a_n = 2(4)^{n-1}$; 2048

26. $a_n = 0.6(-5)^{n-1}$; −1875

27. $a_n = -\frac{1}{8}(2)^{n-1}$; −4

28. $a_n = 0.1(9)^{n-1}$; 5904.9

29. $a_n = 7640(0.1)^{n-1}$; 0.0764

30. $a_n = -192\left(-\frac{1}{4}\right)^{n-1}$; $\frac{3}{16}$

31. $a_n = 0.5(-6)^{n-1}$; −3888

32. $a_n = 224\left(\frac{1}{2}\right)^{n-1}$; 7

33. 16 teams; 8 teams; 4 teams

50. **a.** $a_n = 0.01(2)^{n-1}$

b. \$167,772.16

c. live on campus; By the end of the month, she will be paying over \$5 million.

51. **a.** 2048, 1536, 1152, 864, 648

b. $a_n = 2048\left(\frac{3}{4}\right)^{n-1}$

c. never; An exponential function approaches 0, but never actually equals 0.

52. 2; Putting the values in a spreadsheet and calculating the sum after each term is added shows that the sum approaches 2;

Sample answer: $\frac{1}{2}, \frac{3}{8}, \frac{9}{32}, \frac{27}{128}, \ldots, \frac{1}{2}\left(\frac{3}{4}\right)^{n-1}, \ldots$

53. *Sample answer:* $1, -2, 4, -8, \ldots$

54. $a_n = 2(3)^{n-1}$; $b_n = 5^{n-1}$

a. no

b. yes; It is the quotient of the common ratios of each sequence.

6.5 Maintaining Mathematical Proficiency

55. yes; The residual points are evenly dispersed about the horizontal axis.

56. no; The residual points are not evenly dispersed about the horizontal axis.

6.6 Monitoring Progress and Modeling with Mathematics

13. $a_1 = 7, a_n = a_{n-1} + 9$

14. $a_1 = 8, a_n = 3a_{n-1}$

15. $a_1 = 243, a_n = \frac{1}{3}a_{n-1}$

16. $a_1 = 3, a_n = a_{n-1} + 8$

17. $a_1 = 0, a_n = a_{n-1} - 3$

18. $a_1 = 5, a_n = -4a_{n-1}$

19. $a_1 = -1, a_n = 4a_{n-1}$

20. $a_1 = 35, a_n = a_{n-1} - 11$

21. $a_1 = 1$ cell, $a_n = (2a_{n-1})$ cells

22. $a_1 = 4\frac{1}{2}$ in., $a_n = \left(a_{n-1} + \frac{1}{4}\right)$ in.

47. 10; 31; 66

48. −6; −1296; −10,077,696

49. −8; 8; −8

50. 5; 23; 254

51. 15; −15; −10

52. **a.** $a_1 = 2.5$ cm, $a_2 = 3.5$ cm, $a_n = (a_{n-2} + a_{n-1})$ cm

b. *Sample answer:* 2.7 cm, 2.4 cm, 2.5 cm, 8.5 cm; no; They do not follow a pattern.

56. Each row begins and ends with 1, and each term is the sum of the two terms above it in the previous row; $a_{1,n} = 1$, $a_{n,n} = 1$, $a_{m,n} = a_{m-1,n-1} + a_{m,n-1}$

57. **a.** Substituting $n - 1$ for n in the explicit rule that defines an arithmetic sequence results in this expression.

b. Write the equation; Identity Property of Addition; Additive Inverse Property; Associative Property of Addition; Distributive Property; Substitution Property of Equality

58. no; The sequence is a geometric sequence with a common factor of −1, so it can be represented by $a_1 = -5, a_n = -a_{n-1}$.

59. $a_1 = 3, a_n = a_{n-1} + 2^n$

6.6 Maintaining Mathematical Proficiency

60. $17x$

61. $-6y - 5$

62. $-6d - 7$

63. $8m + 3$

64. $f(x) = 3x$

65. $f(x) = -\frac{1}{2}x - 1$

66. $f(x) = 5$

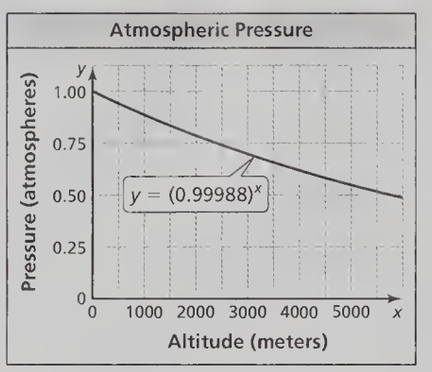
67. $f(x) = 2x - 7$

Additional Answers

Chapter 6 Test

15. -75

16. a. 1; 0.99988; 0.012%

b. 

about 0.549 atmosphere

17. a. $a_n = 1.2^{n-1}$; $a_1 = 1$, $a_n = 1.2a_{n-1}$

b. Day 7

Chapter 7

7.1 Monitoring Progress and Modeling with Mathematics

59. a. no; The product of two negative integers is always a positive integer.

b. yes; The sum of two whole numbers is always a whole number.

60. a. $x^2 - 2x$; $x^2 - 12x$; $2x^2 - 14x$

b. 520 ft^2

c. 1.3 gal; Divide the total area of the deck by the area that can be covered by 1 gallon of sealant to get the amount of sealant needed.

61. a. $19x^2 - 12x$

b. \$14,310

7.1 Maintaining Mathematical Proficiency

62. $5x + 4$

63. $34y - 34$

64. $22r - 1$

7.2 Explorations

3. Multiply each term in one polynomial by each term in the other polynomial, then combine like terms.

4. *Sample answer:* $(x + 1)(x - 2) = x^2 - x - 2$



7.3 Monitoring Progress and Modeling with Mathematics

44. *Sample answer:* Increase both dimensions by 2, which can be modeled by $A = (x + 2)^2$; Decrease both dimensions by 3, which can be modeled by $A = (x - 3)^2$; Increase one dimension by 4, and decrease the other dimension by 4, which can be modeled by $A = (x + 4)(x - 4)$.

45. $k = 64$

46. $x^3 + 3x^2 + 3x + 1$; $x^3 + 6x^2 + 12x + 8$; $a^3 + 3a^2b + 3ab^2 + b^3$

47. *Sample answer:* $a = 3$, $b = 4$

7.3 Maintaining Mathematical Proficiency

48. $\frac{1}{10^3}$

49. $\frac{1}{x^2}$

50. $\frac{1}{27z^{18}}$

51. $\frac{1}{4y^2}$

7.4 Vocabulary and Core Concept Check

1. Multiply the quotient by the divisor and add the remainder. The result should be the dividend; *Sample answer:* Check the result of Example 5:

$$(3x - 4)(x - 2) + 12 = (3x^2 - 10x + 8) + 12$$
$$= 3x^2 - 10x + 20 \quad \checkmark$$

7.4 Monitoring Progress and Modeling with Mathematics

25. $5 + \frac{-8x + 17}{x^2 - 3}$

26. $3 + \frac{3x - 14}{3x^2 + 2}$

27. $-4 + \frac{-6x + 8}{5x^2 - 3x + 2}$

28. $-2 + \frac{6x - 17}{-2x^2 + 3x}$

29. $8x + 12 + \frac{-7}{x - 1}$

30. $2x - 7 + \frac{-20}{x - 4}$

31. $3x + 2 + \frac{5}{x + 5}$

32. $9x - 2 + \frac{2}{x + 3}$

33. $15x + 30 + \frac{12}{x - 2}$

34. $2x + 1 + \frac{3}{x - 3}$

35. $-x + 3 - \frac{9}{x + 4}$

36. $-7x + 7 + \frac{5}{x + 1}$

45. $2x + 2 + \frac{-10}{3x + 6}$; $6x + 6 + \frac{-10}{x + 2}$; The result of the long division problem is $\frac{1}{3}$ of the result of the synthetic division problem; The divisor of the long division problem, $3x + 6$, is 3 times the divisor of the synthetic division problem, $x + 2$.

46. $3x^2 - 6x + 12 + \frac{-35}{x + 4}$; yes; $3x^3 + 6x^2 - 12x + 13$ is a polynomial and $x + 4$ is a binomial of the form $x - k$, where $k = -4$.

47. 9

7.4 Maintaining Mathematical Proficiency

48. $6(2y - 3)$

49. $9(r + 3)$

50. $7(7s + 5t)$

51. $5(3x - 2y)$

52. $p = 9$

53. $z = -17$

54. $c = -42$

55. $k = 0$

7.5 Explorations

2.

$x =$	1	2	3	4	5	6
a. $(x - 1)(x - 2) = 0$	T	T	F	F	F	F
b. $(x - 2)(x - 3) = 0$	F	T	T	F	F	F
c. $(x - 3)(x - 4) = 0$	F	F	T	T	F	F
d. $(x - 4)(x - 5) = 0$	F	F	F	T	T	F
e. $(x - 5)(x - 6) = 0$	F	F	F	F	T	T
f. $(x - 6)(x - 1) = 0$	T	F	F	F	F	T

If the product of two values is 0, then at least one of the values must be 0. If $(x - a)$ is a factor of an equation, then $x = a$ is a solution.

5. b; It describes what happens when you have a product that is equal to zero; It is used to solve polynomial equations by setting each polynomial factor equal to 0. It is important because it provides an easy way to solve polynomial equations.

7.5 Monitoring Progress and Modeling with Mathematics

45. no; Roots will occur if $x^2 + 3 = 0$ or $x^4 + 1 = 0$. However, solving these equations results in $x^2 = -3$ or $x^4 = -1$, and even powers of any number cannot be negative.

46. *Sample answer:* $(x - 1)^2(x - 2)(x - 3)$

47. **a.** $x = -y, x = \frac{1}{2}y$

b. $x = \pm y, x = -4y$

7.5 Maintaining Mathematical Proficiency

48. 1, 10; 2, 5

49. 1, 18; 2, 9; 3, 6

50. 1, 30; 2, 15; 3, 10; 5, 6

51. 1, 48; 2, 24; 3, 16; 4, 12; 6, 8

52. 1, 36; 2, 18; 3, 12; 4, 9; 6, 6

53. 1, 52; 2, 26, 4, 13

7.6 Maintaining Mathematical Proficiency

54. *Sample answer:* $10x - 6y = -14, 15x - 9y = -21$

55. *Sample answer:* $-x + 3y = 5, -4x + 12y = 20$

7.7 Monitoring Progress and Modeling with Mathematics

29. $x = -\frac{7}{2}, x = 5$

30. $x = -3, x = \frac{1}{4}$

31. $x = -1, x = \frac{5}{7}$

32. $x = -\frac{1}{3}, x = 5$

33. **a.** $(5x - 2)$ ft

b. Substitute 3 for x into the expression for the area $15x^2 - x - 2$, then simplify; Substitute 3 for x into the expressions for the length $(5x - 2)$ and width $(3x + 1)$, simplify each, then multiply these two numbers.

42. **a.**



$2x^2 + 5x - 3 = (x + 3)(2x - 1)$

b.



$3x^2 - 2x - 1 = (x - 1)(3x + 1)$

46. $(2x - y)(3x + 4y)$

47. $-(a - 2b)(6a - 7b)$

48. $3m(2m + 5n)(3m - n)$

7.7 Maintaining Mathematical Proficiency

49. ± 8

50. 2

51. -15

52. ± 9

53. $(-1, -4)$

54. $(4, 6)$

55. $(0, -7)$

56. $(-5, 3)$

7.8 Explorations

1. **a.** $(2x + 1)(2x - 1)$; yes, sum and difference pattern



b. $(2x - 1)^2$; yes, square of a binomial pattern



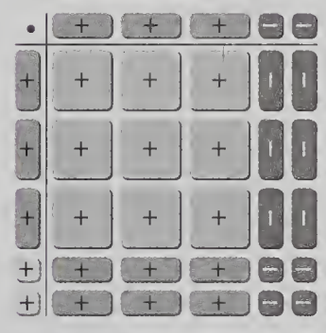
c. $(2x + 1)^2$; yes, square of a binomial pattern



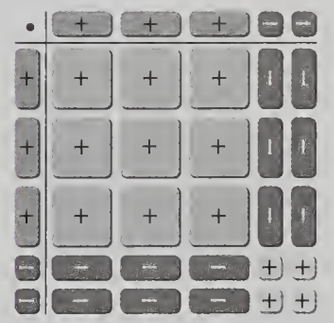
d. $(2x - 2)(2x - 1)$; no, not a special pattern



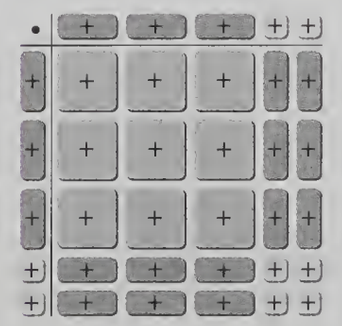
2.



$9x^2 - 4; (3x + 2)(3x - 2)$



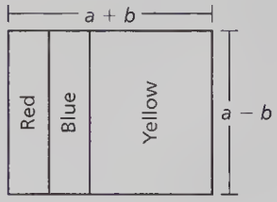
$9x^2 - 12x + 4; (3x - 2)^2$



$9x^2 + 12x + 4; (3x + 2)^2$

7.8 Monitoring Progress and Modeling with Mathematics

46. a. the blue, yellow, and red regions; *Sample answer:*



b. $(a - b)^2$ is represented by the yellow region. a^2 is the entire square. Subtracting $2ab$ requires removing 2 regions that each represent ab, which would be the blue and purple regions, and the red and purple regions. There is only one purple region, and subtracting $2ab$ required removing it twice, so one purple region needs to be replaced, which adds b^2.

47. a. $(9x^2 - 144)$ in.^{2}

b. 5 in.; Setting the polynomial in part (a) equal to 81 and solving results in $x = -5$ or $x = 5$. Length cannot be negative, so 5 is the solution.

48. a. $(x^3 + 16x)$ in.^{3}

b. $x = 3$; Setting the polynomial in part (a) equal to $25x$ and solving results in $x = 0$, $x = -3$, or $x = 3$. Because x is also a side length, and length cannot be negative or zero, 3 is the solution.

7.8 Maintaining Mathematical Proficiency

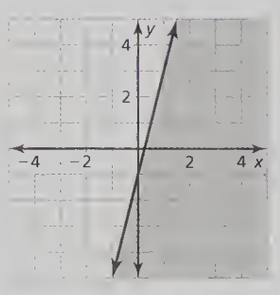
49. $2 \cdot 5^2$

50. $2^2 \cdot 11$

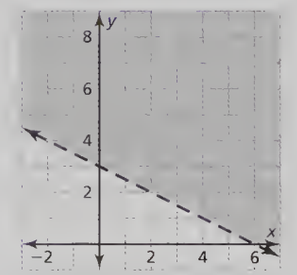
51. $5 \cdot 17$

52. $2^5 \cdot 3$

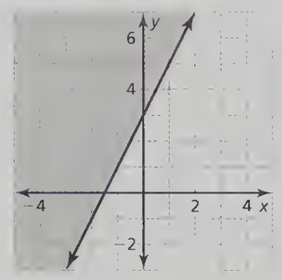
53.



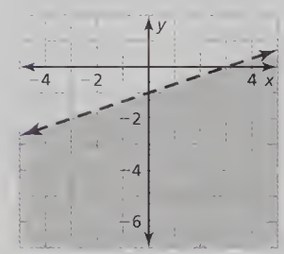
54.



55.



56.



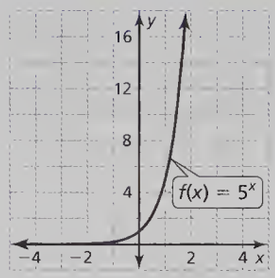
7.9 Monitoring Progress and Modeling with Mathematics

49. a. *Sample answer:* $w = 40$; When $w = 40$, factoring out $5x$ will leave a perfect square trinomial, so there will be 2 factors.

b. *Sample answer:* $w = 50$; When $w = 50$, factoring out $5x$ will leave a factorable trinomial that is not a perfect square, so there will be 3 factors.

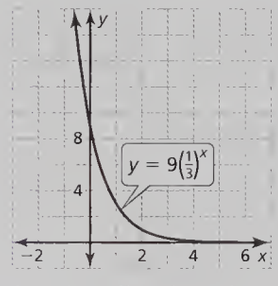
7.9 Maintaining Mathematical Proficiency

54.



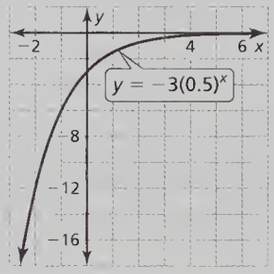
domain: all real numbers, range: $y > 0$

55.



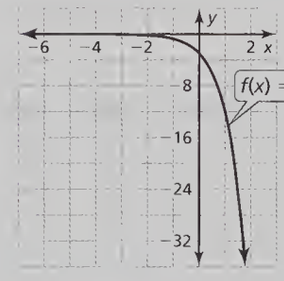
domain: all real numbers, range: $y > 0$

56.



domain: all real numbers, range: $y < 0$

57.



domain: all real numbers, range: $y < 0$

Chapter 8

Chapter 8 Maintaining Mathematical Proficiency

5. 11

6. 8

7. −11

8. −7

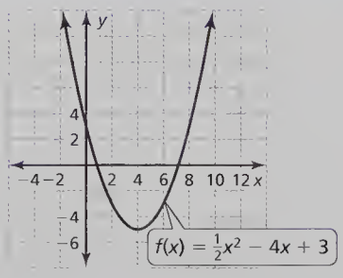
9. 3

10. −26

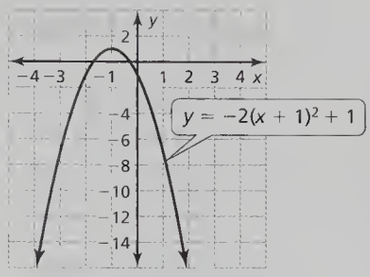
11. a, $4a$, $9a$, $16a$, $25a$; *Sample answer:* The coefficient of each difference is the next consecutive odd integer; $36a$

Chapter 8 Mathematical Thinking

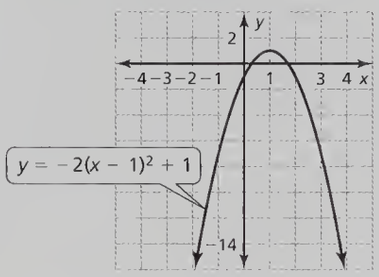
6.



The graph opens up and the lowest point is at $(4, -5)$.

7. 

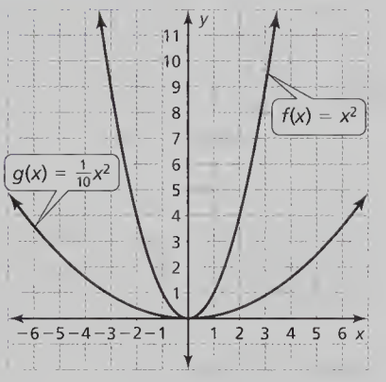
The graph opens down and the highest point is at $(-1, 1)$.

8. 

The graph opens down and the highest point is at $(1, 1)$.

9. *Sample answer:* They are all U-shaped; Some open up and some open down.

8.1 Explorations

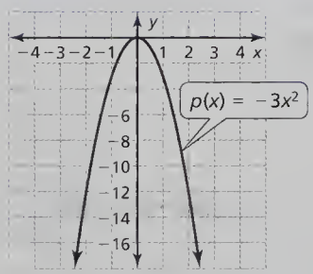
1. **d.** 

The graph of g is a vertical shrink by a factor of $\frac{1}{10}$ of the graph of f.

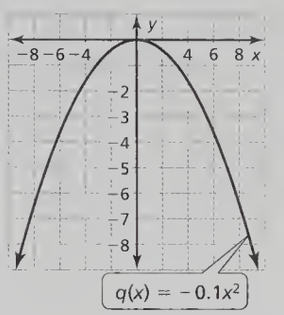
3. *Sample answer:* When $0 < a < 1$ the graph of $f(x) = ax^2$ is a vertical shrink of the graph of $f(x) = x^2$, when $a > 1$ the graph of $f(x) = ax^2$ is a vertical stretch of the graph of $f(x) = x^2$, when $-1 < a < 0$ the graph of $f(x) = ax^2$ is a vertical shrink and a reflection in the x-axis of the graph of $f(x) = x^2$, and when $a < -1$ the graph of $f(x) = ax^2$ is a vertical stretch and a reflection in the x-axis of the graph of $f(x) = x^2$.

4. $0 < a < 1$; *Sample answer:* The graph is a vertical shrink of the graph of $f(x) = x^2$.

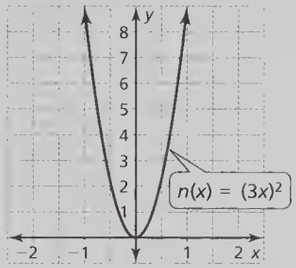
8.1 Lesson Monitoring Progress

5. 

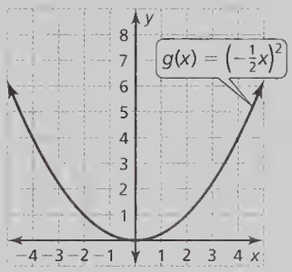
The graph of p is a vertical stretch by a factor of 3 and a reflection in the x-axis of the graph of f.

6. 

The graph of q is a vertical shrink by a factor of 0.1 and a reflection in the x-axis of the graph of f.

7. 

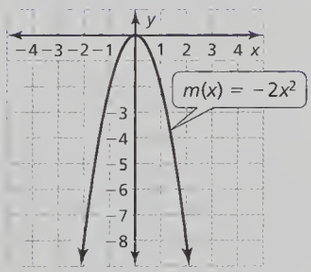
The graph of n is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of f.

8. 

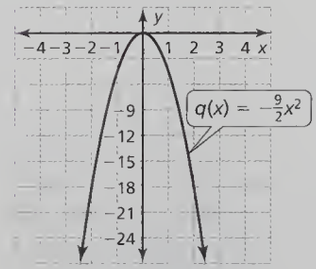
The graph of g is a horizontal stretch by a factor of 2 of the graph of f.

9. 4 inches wide, 2 inches deep

8.1 Monitoring Progress and Modeling with Mathematics

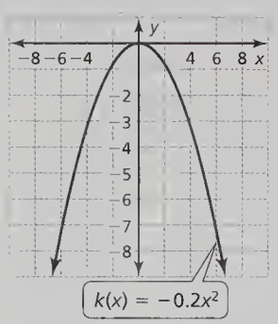
9. 

The graph of m is a vertical stretch by a factor of 2 and a reflection in the x-axis of the graph of f.

10. 

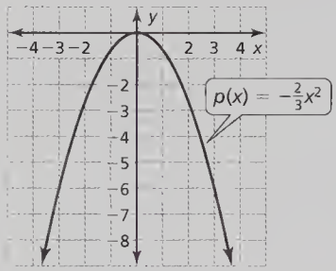
The graph of q is a vertical stretch by a factor of $\frac{9}{2}$ and a reflection in the x-axis of the graph of f.

11.



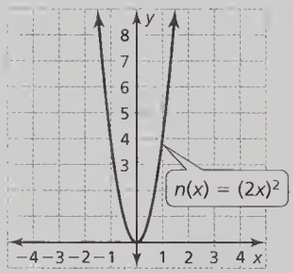
The graph of k is a vertical shrink by a factor of 0.2 and a reflection in the x-axis of the graph of f.

12.



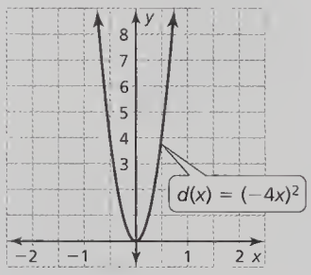
The graph of p is a vertical shrink by a factor of $\frac{2}{3}$ and a reflection in the x-axis of the graph of f.

13.



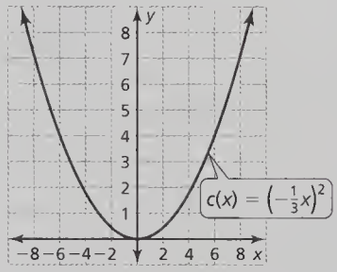
The graph of n is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f.

14.



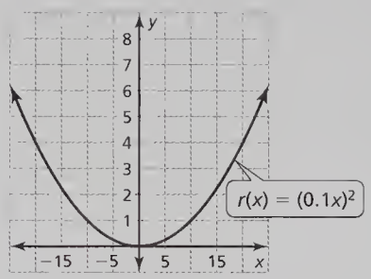
The graph of d is a horizontal shrink by a factor of $\frac{1}{4}$ of the graph of f.

15.



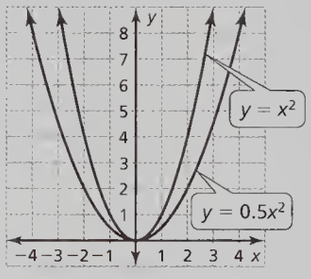
The graph of c is a horizontal stretch by a factor of 3 of the graph of f.

16.



The graph of r is a horizontal stretch by a factor of 10 of the graph of f.

17. The graph of $y = 0.5x^2$ should be wider than the graph of $y = x^2$.

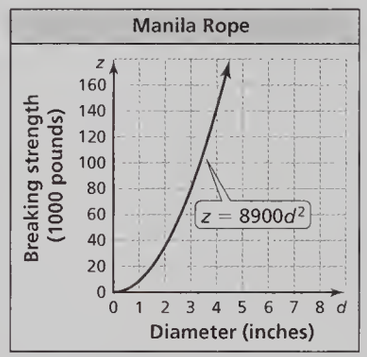


The graphs have the same vertex and the same axis of symmetry. The graph of $y = 0.5x^2$ is wider than the graph of $y = x^2$.

18. 300 feet high, 1000 feet wide

19. **a.** domain: $d \geq 0$, range: $z \geq 0$

b.



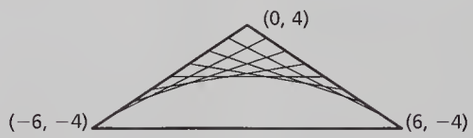
c. no; *Sample answer:* The relationship is quadratic, so a rope with 4 times the diameter will have $4^2 = 16$ times the breaking strength.

27. always; *Sample answer:* When $|a| > 1$, the graph of f will be a vertical stretch of the graph of g, so it will be narrower.

28. always; *Sample answer:* When $0 < |a| < 1$, the graph of f will be a vertical shrink of the graph of g, so it will be wider.

29. never; *Sample answer:* When $|a| > |d|$, the graph of f will be a vertical stretch of the graph of g, so it will be narrower, not wider.

30.



$y = -\frac{1}{9}x^2$

31. **a.** 8 cm

b. no; *Sample answer:* A faster rotational speed would increase the depth. The diagram shown has a depth of 3.2 centimeters. A model of $y = 0.1x^2$ would only have a depth of 1.6 centimeters, so it would have a slower rotational speed.

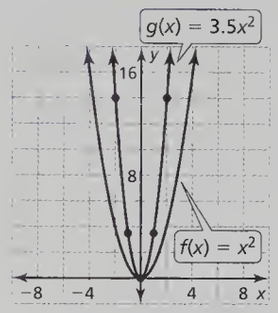
8.1 Maintaining Mathematical Proficiency

32. 14 **33.** 3

34. −25 **35.** 11

8.1 Mini-Assessment

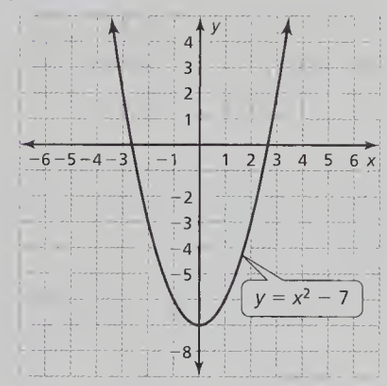
2.



Both graphs open up and have the same vertex, (0, 0), and axis of symmetry, $x = 0$. The graph of g is a vertical stretch by a factor of 3.5 of the graph of f.

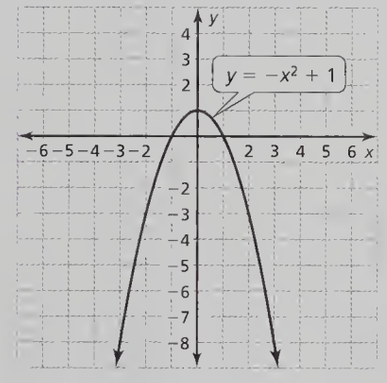
8.2 Explorations

2. a.



$x = \sqrt{7} \approx 2.6, x = -\sqrt{7} \approx -2.6$

b.



$x = 1, x = -1$

Sample answer: Estimate the points where the graph intersects the x-axis.

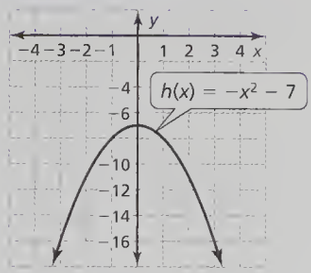
3. It causes a vertical shift of c units. When $c > 0$, the graph is translated up. When $c < 0$, the graph is translated down.

4. Check students' work.

5. *Sample answer:* $a > 1, c = 1$; The graph is narrower than $y = x^2$, so it is a vertical stretch. The vertex is 1 unit above the origin, so the graph is a vertical translation 1 unit up of the graph of $y = x^2$.

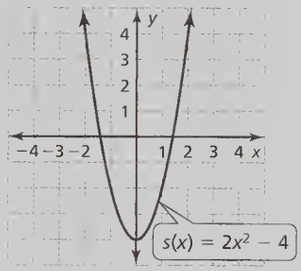
8.2 Monitoring Progress and Modeling with Mathematics

8.



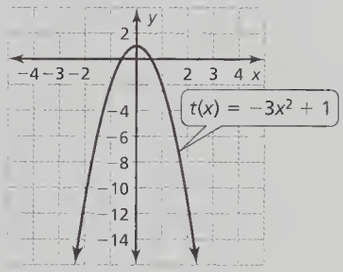
The graph of h is a reflection in the x-axis, and a vertical translation 7 units down of the graph of f.

9.



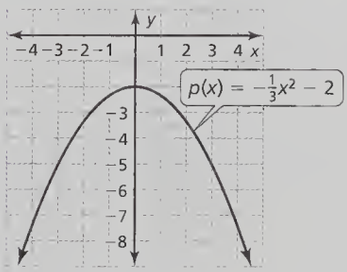
The graph of s is a vertical stretch by a factor of 2 and a vertical translation 4 units down of the graph of f.

10.



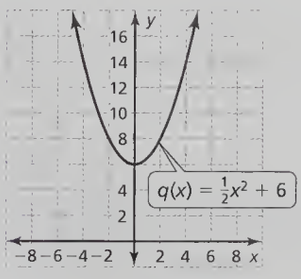
The graph of t is a vertical stretch by a factor of 3, a reflection in the x-axis, and a vertical translation 1 unit up of the graph of f.

11.



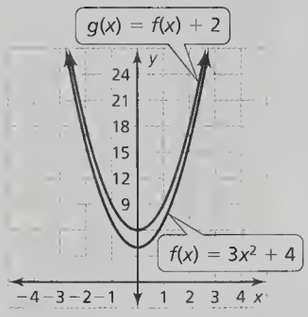
The graph of p is a vertical shrink by a factor of $\frac{1}{3}$, a reflection in the x-axis, and a vertical translation 2 units down of the graph of f.

12.



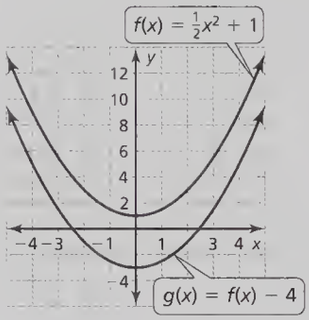
The graph of q is a vertical shrink by a factor of $\frac{1}{2}$ and a vertical translation 6 units up of the graph of f.

13. The graph of g is a vertical translation 2 units up of the graph of f.



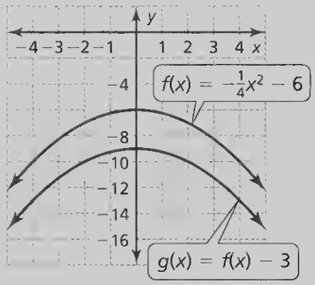
$g(x) = 3x^2 + 6$

14. The graph of g is a vertical translation 4 units down of the graph of f.



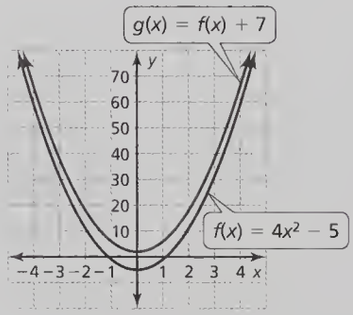
$g(x) = \frac{1}{2}x^2 - 3$

15. The graph of g is a vertical translation 3 units down of the graph of f.



$g(x) = -\frac{1}{4}x^2 - 9$

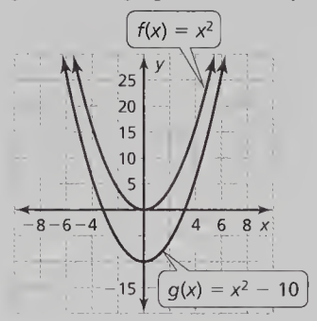
16. The graph of g is a vertical translation 7 units up of the graph of f.



$g(x) = 4x^2 + 2$

17. The graph of $y = 3x^2 + 2$ is narrower, so it should be a stretch not a shrink; The graph of $y = 3x^2 + 2$ is a vertical stretch by a factor of 3 and a translation 2 units up of the graph of $y = x^2$.

18. $g(x)$ is not graphed correctly.



Both graphs open up and have the same axis of symmetry. However, the vertex of the graph of g, $(0, -10)$, is 10 units below the vertex of the graph of f, $(0, 0)$.

19. $x = 1, x = -1$

20. $x = 6, x = -6$

21. $x = 5, x = -5$

22. $x = 7, x = -7$

23. $x = 2, x = -2$

24. $x = 3, x = -3$

25. $x = \frac{1}{2}, x = -\frac{1}{2}$

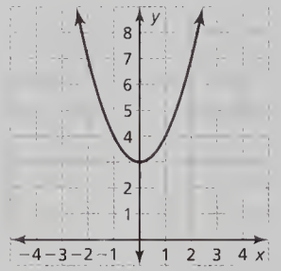
26. $x = \frac{7}{2}, x = -\frac{7}{2}$

27. **a.** 3 sec

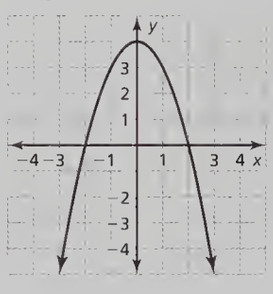
b. When $k > 0$, the water balloon will take more than 3 seconds to hit the ground. When $k < 0$, the water balloon will take less than 3 seconds to hit the ground.

28. The x-intercept is 1.5. This means the apple hits the ground after 1.5 seconds. The y-intercept is 36. This means the apple fell from a height of 36 feet.

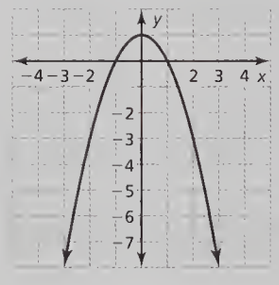
29. *Sample answer:*



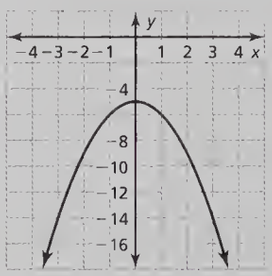
30. *Sample answer:*



31. *Sample answer:*

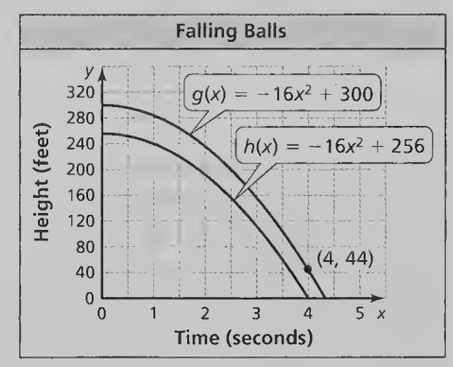


32. *Sample answer:*



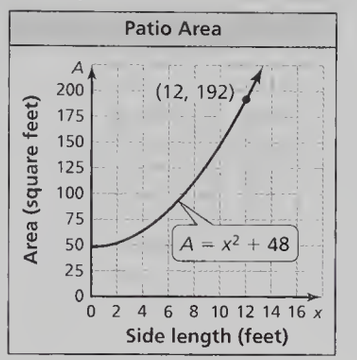
33. **a.** $T(x) = -44$; the distance between the balls

b. 44 ft



34. no; *Sample answer:* Changing a causes a vertical stretch or shrink, which does not change the vertex.

35. 12 m by 12 m



36. A

37. Graph the function and determine the x-intercepts; Set the function equal to 0, factor $-16t^2 + 400$, and apply the Zero-Product Property.

38. **a.** waterfall 1

b. waterfall 2

c. waterfall 1

39. **a.** $h = -16t^2 + 45$; $h = -16t^2 + 32$

b. The graph of $h = -16t^2 + 32$ is a vertical translation 13 units down of the graph of $h = -16t^2 + 45$.

40. *Sample answer:* 8; $A = 2\left(\frac{1}{2}bh\right) = bh = 2(4) = 8$

41. (0, 5.8); *Sample answer:* The outer edges are located 40 feet from the center. Substituting this into $y = 0.012x^2$ indicates they are 19.2 feet above the ground. To be 25 feet above the ground, they must be vertically translated up 5.8 feet.

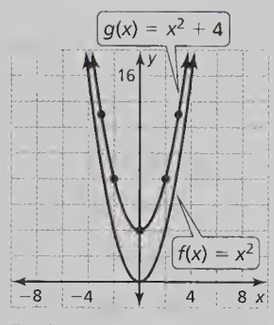
8.2 Maintaining Mathematical Proficiency

42. $\frac{1}{3}$ **43.** $\frac{3}{8}$

44. $\frac{7}{9}$ **45.** $\frac{5}{12}$

8.2 Mini-Assessment

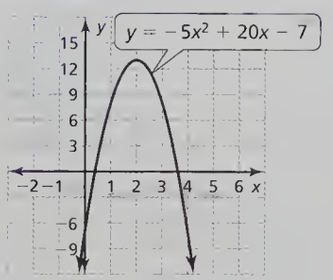
1.



Both graphs open up and have the same axis of symmetry. The graph of g is a vertical translation 4 units up of the graph of f.

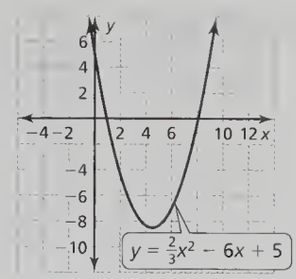
8.3 Monitoring Progress and Modeling with Mathematics

16.



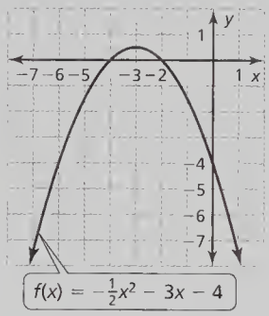
domain: all real numbers, range: $y \leq 13$

17.



domain: all real numbers, range: $y \geq -\frac{17}{2}$

18.



domain: all real numbers, range: $y \leq \frac{1}{2}$

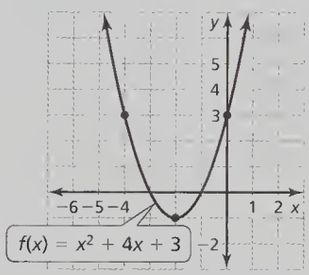
19. There should be two negatives in the substitution, one from the formula and one because b is -12;

$x = -\frac{b}{2a} = -\frac{-12}{2(3)} = 2$; The axis of symmetry is $x = 2$.

20. The formula for the axis of symmetry has a negative sign;

The axis of symmetry is $x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$;

$f(-2) = (-2)^2 + 4(-2) + 3 = -1$; So, the vertex is $(-2, -1)$; The y-intercept is 3. So, the points (0, 3) and $(-4, 3)$ lie on the graph.



21. minimum value; -12 **22.** maximum value; 12

23. maximum value; -1

46. It is not possible to identify characteristics of parabola A with only two points; The vertex of parabola B is $(3, -4)$ because 3 is halfway between 1 and 5, which are the x-coordinates of points that have the same y-coordinates. Parabola B opens up because -4, the y-coordinate of the vertex, is the minimum value.

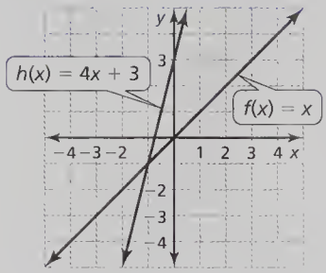
47. 14 ft

48. *Sample answer:* 2; Sketch a graph of $y = x^2$ and a tangent line through (1, 1). Then select another point on the tangent line, $(0, -1)$, and substitute these points into the slope formula to calculate a slope of 2.

49. $\frac{k^2}{8}$ ft^2

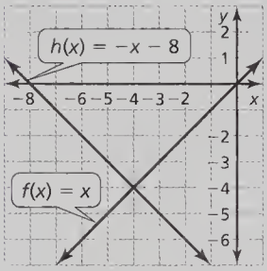
8.3 Maintaining Mathematical Proficiency

50.



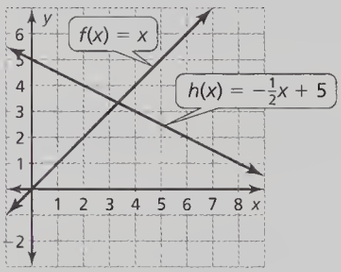
The graph of h is a vertical stretch by a factor of 4 and a vertical translation 3 units up of the graph of f.

51.



The graph of h is a reflection in the x-axis and a vertical translation 8 units down of the graph of f.

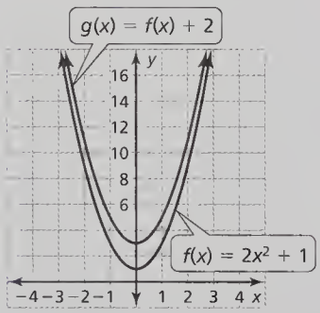
52.



The graph of h is a vertical shrink by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a vertical translation 5 units up of the graph of f.

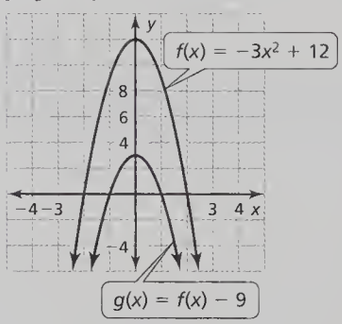
8.1–8.3 Quiz

9. The graph of g is a vertical translation 2 units up of the graph of f.



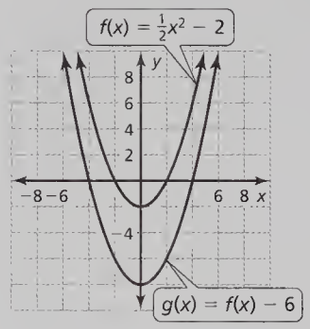
$g(x) = 2x^2 + 3$

10. The graph of g is a vertical translation 9 units down of the graph of f.



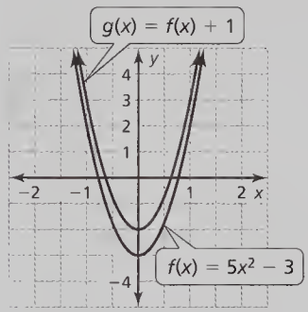
$g(x) = -3x^2 + 3$

11. The graph of g is a vertical translation 6 units down of the graph of f.



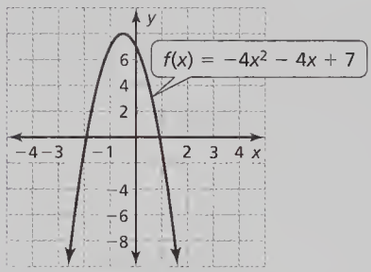
$g(x) = \frac{1}{2}x^2 - 8$

12. The graph of g is a vertical translation 1 unit up of the graph of f.



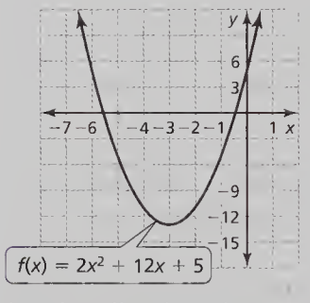
$g(x) = 5x^2 - 2$

13.



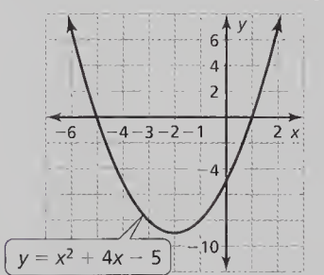
domain: all real numbers, range: $y \leq 8$

14.

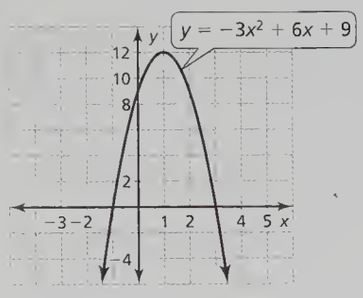


domain: all real numbers, range: $y \geq -13$

15.



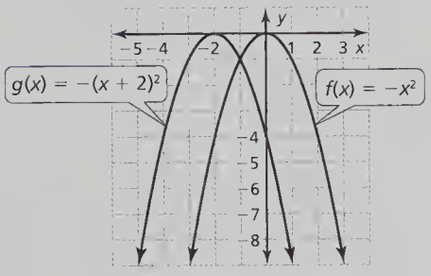
domain: all real numbers, range: $y \geq -9$

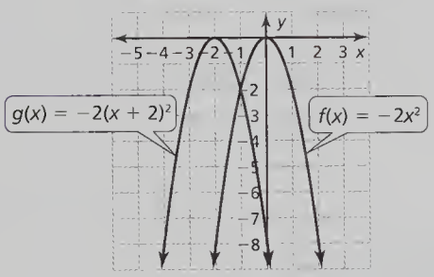
16. 

domain: all real numbers, range: $y \leq 12$

17. minimum value; -8
18. maximum value; 18
19. maximum value; 16
20. minimum value; -5
21. 2 sec
22. **a.** 1.25 sec
 b. the first pinecone; *Sample answer:* The second pinecone will take 1.5 seconds to fall, which is longer than the first.
23. domain: $0 \leq t \leq 2$, range: $0 \leq h \leq 18$; 18 ft

8.4 Explorations

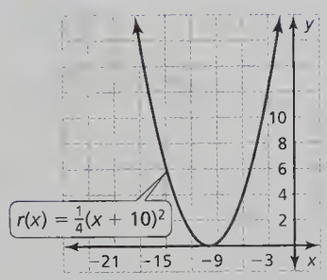
2. **a.** 

b. 

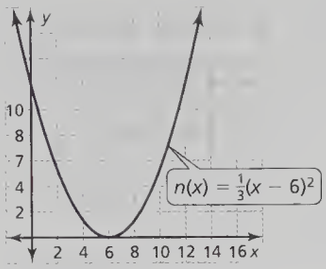
Sample answer: The value of h causes a horizontal translation of the graph of $y = a(x - h)^2$ from the graph of $y = ax^2$.

3. When $h > 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units to the right of the graph of $f(x) = ax^2$. When $h < 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units to the left of the graph of $f(x) = ax^2$.
4. **a.** The graph of $y = (x - 3)^2$ is a horizontal translation 3 units right of the graph of $y = x^2$.
 b. The graph of $y = (x + 3)^2$ is a horizontal translation 3 units left of the graph of $y = x^2$.
 c. The graph of $y = -(x - 3)^2$ is a horizontal translation 3 units right and a reflection in the x-axis of the graph of $y = x^2$.

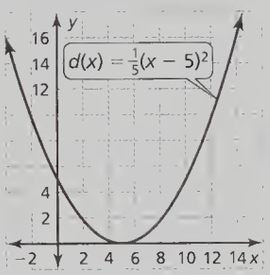
8.4 Monitoring Progress and Modeling with Mathematics

25. 

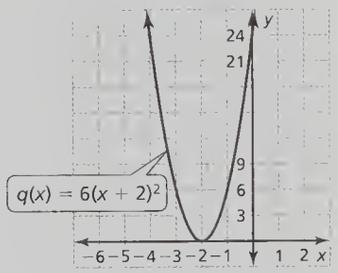
The graph of r is a horizontal translation 10 units left and a vertical shrink by a factor of $\frac{1}{4}$ of the graph of f.

26. 

The graph of n is a horizontal translation 6 units right and a vertical shrink by a factor of $\frac{1}{3}$ of the graph of f.

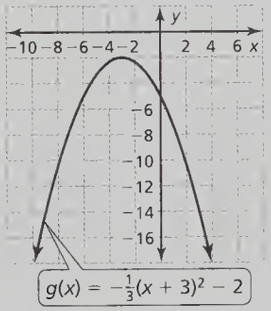
27. 

The graph of d is a horizontal translation 5 units right and a vertical shrink by a factor of $\frac{1}{5}$ of the graph of f.

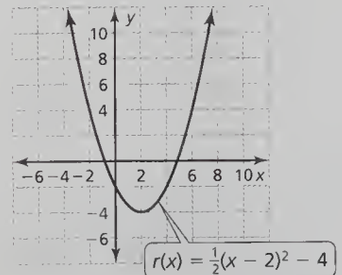
28. 

The graph of q is a horizontal translation 2 units left and a vertical stretch by a factor of 6 of the graph of f.

29. If $f(-x) = f(x)$ the function is even; So, $f(x)$ is an even function.
30. h is the x-coordinate of the vertex, not the y-coordinate; Because $h = -8$, the vertex is $(-8, 0)$.

43. 

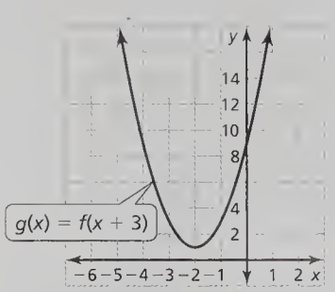
The graph of g is a vertical shrink by a factor of $\frac{1}{3}$, a reflection in the x-axis, and a translation 3 units left and 2 units down of the graph of f.

44. 

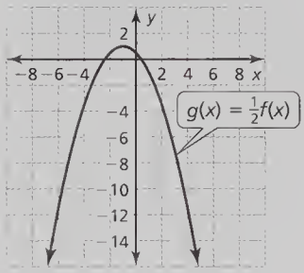
The graph of r is a vertical shrink by a factor of $\frac{1}{2}$, and a translation 2 units right and 4 units down of the graph of f.

45. A
46. C
47. B
48. D

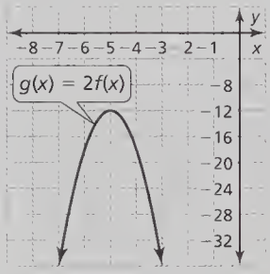
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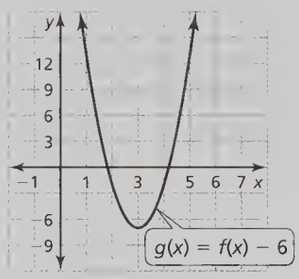
50.



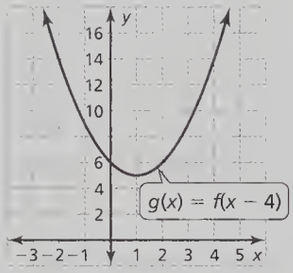
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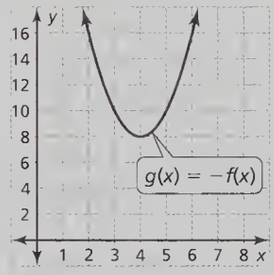
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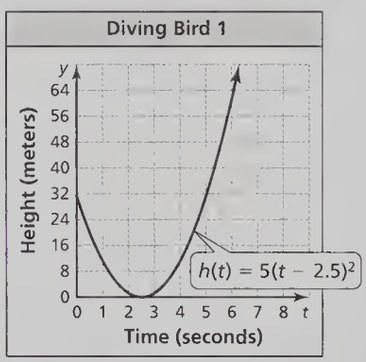
53.



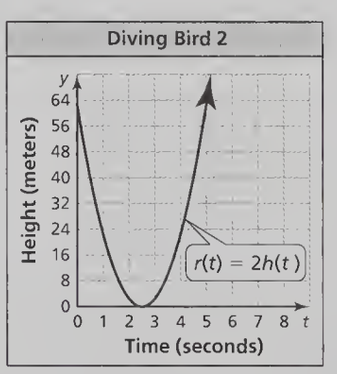
54.



55. a.

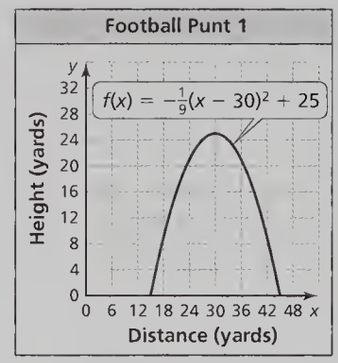


b.



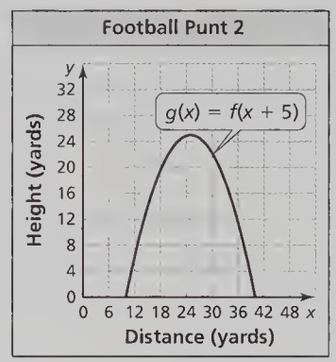
c. The graph of r is a vertical stretch by a factor of 2 of the graph of h; the second bird; *Sample answer:* Because $r(t)$ is twice $h(t)$, the second bird starts at a height twice as high as the first bird.

56. a.



domain: $15 \le x \le 45$, range: $0 \le y \le 25$

b.



domain: $10 \le x \le 40$, range: $0 \le y \le 25$

c. The graph of g is a horizontal translation 5 units left of the graph of f; second possession; *Sample answer:* On the first possession, the ball is punted 15 yards from the kicker's goal line. On the second possession, the ball is punted 10 yards from the kicker's goal line.

57. $f(x) = 2(x - 1)^2 + 2$

58. $f(x) = -\frac{19}{9}(x + 3)^2 + 5$

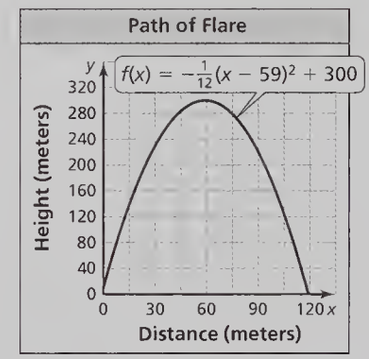
59. $f(x) = -2(x + 2)^2 - 4$

60. $f(x) = (x - 1)^2 + 8$

61. $f(x) = \frac{1}{2}(x - 5)^2 - 2$

62. $f(x) = \frac{1}{3}(x + 5)^2 - 1$

64. $f(x) = -\frac{1}{12}(x - 59)^2 + 300$



65. $y = 2(x - 2)^2 - 4$

66. $y = 3(x + 1)^2 - 4$

67. $f(x) = -5(x - 1)^2 + 8$

68. $f(x) = -(x + 2)^2 + 6$

69. no; *Sample answer:* The graph would not pass the vertical line test.

70. $+$; $-$; *Sample answer:* The vertex is $(-2, -3)$, so in the vertex form $h = -2$ and $k = -3$.

71. The graph of h is a vertical translation 4 units up of the graph of f; $h(x) = -(x + 1)^2 + 2$

72. The graph of h is a horizontal translation 5 units right of the graph of f; $h(x) = 2(x - 6)^2 + 1$

73. The graph of h is a vertical stretch by a factor of 2 of the graph of f; $h(x) = 8(x - 2)^2 + 6$

74. The graph of h is a vertical shrink by a factor of $\frac{1}{3}$ of the graph of f; $h(x) = -\frac{1}{3}(x + 5)^2 - 2$

75. $y = (x - 2)^2 - 5$; $y = x^2 - 4x - 1$; *Sample answer:* The vertex, $(2, -5)$, can be quickly determined from the vertex form; The y-intercept, -1, can be quickly determined from the standard form.

76. a. true; *Sample answer:* If $g(x) = af(x)$, then $g(-x) = af(-x)$. Because f is even, $g(-x) = af(x) = g(x)$, so g is even.

b. true; *Sample answer:* If $g(x) = af(x)$, then $g(-x) = af(-x)$. Because f is odd, $g(-x) = a(-f(x)) = -af(x) = -g(x)$, so g is odd.

c. true; *Sample answer:* If $h(x) = f(x) + g(x)$, then $h(-x) = f(-x) + g(-x)$. Because f and g are even, $h(-x) = f(x) + g(x) = h(x)$, so h is even.

d. true; *Sample answer:* If $h(x) = f(x) + g(x)$, then $h(-x) = f(-x) + g(-x)$. Because f and g are odd, $h(-x) = -f(x) - g(x) = -(f(x) + g(x)) = -h(x)$, so h is odd.

e. false; *Sample answer:* A counterexample is illustrated by adding $f(x) = x^2 + 4$, an even function, to $g(x) = 3x$, an odd function. The sum $h(x) = x^2 + 3x + 4$ is neither even nor odd.

77. a. the second birdbath; *Sample answer:* The first birdbath has a depth of 4 inches and the second birdbath has a depth of 6 inches.

b. the first birdbath; *Sample answer:* The first birdbath has a width of 36 inches and the second birdbath has a width of 30 inches.

78. The graph of $y = 2x^2 + 8x + 8$ is a vertical stretch by a factor of 2 and a horizontal translation 2 units left of the graph of $y = x^2$; *Sample answer:* Factoring the right side of $y = 2x^2 + 8x + 8$ changes the equation to vertex form, $y = 2(x + 2)^2$, from which the transformations can be easily determined.

8.4 Maintaining Mathematical Proficiency

79. $x = 0, x = 1$ **80.** $x = -3, x = 8$

81. $x = 3, x = -3$

8.5 Explorations

1. f. $y = -x(x - 3)$; *Sample answer:* The graph opens down and the x-intercepts are 0 and 3.

g. $y = (x + 1)(x - 2)$; *Sample answer:* The graph opens up and the x-intercepts are -1 and 2.

h. $y = -(x + 1)(x - 2)$; *Sample answer:* The graph opens down and the x-intercepts are -1 and 2.

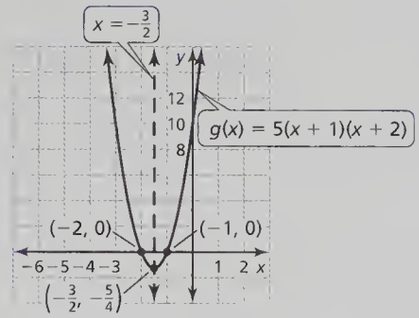
2. *Sample answer:* It is a parabola with x-intercepts p and q.

3. a. no; yes; *Sample answer:* The x-intercepts are still p and q; Changing the sign of a will also change the sign of the y-intercept when the y-intercept is not 0.

b. yes; yes; *Sample answer:* One of the x-intercepts will change from the old value of p to the new value of p. The y-intercept is apq, so changing the value of p will change the value of the y-intercept when the y-intercept is not 0.

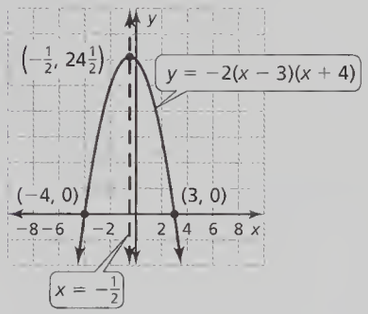
8.5 Monitoring Progress and Modeling with Mathematics

11.



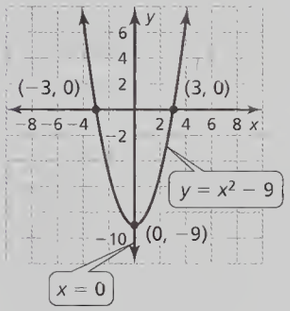
domain: all real numbers, range: $y \geq -\frac{5}{4}$

12.



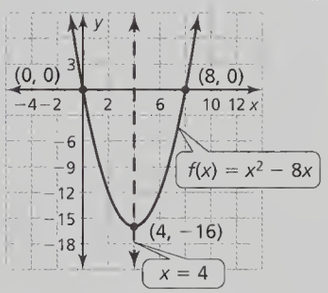
domain: all real numbers, range: $y \leq 24\frac{1}{2}$

13.



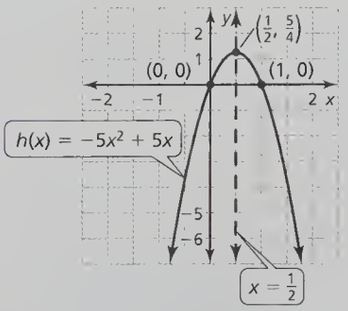
domain: all real numbers, range: $y \geq -9$

14.



domain: all real numbers, range: $y \geq -16$

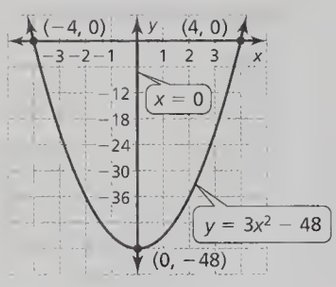
15.



domain: all real numbers, range: $y \leq \frac{5}{4}$

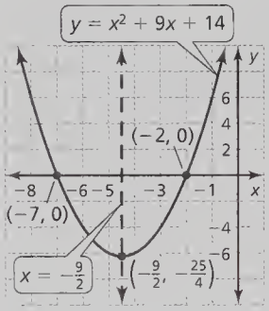
Additional Answers

16.



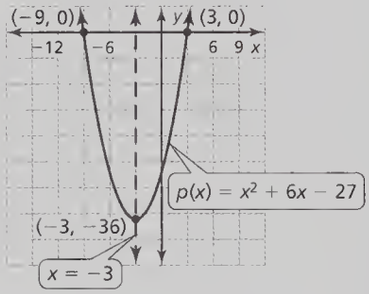
domain: all real numbers, range: $y \geq -48$

17.



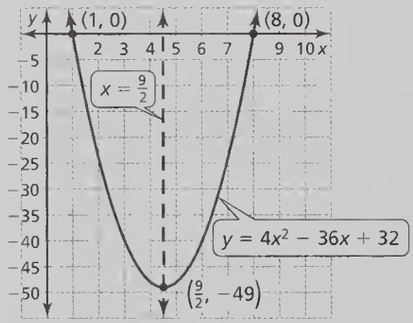
domain: all real numbers, range: $y \geq -\frac{25}{4}$

18.



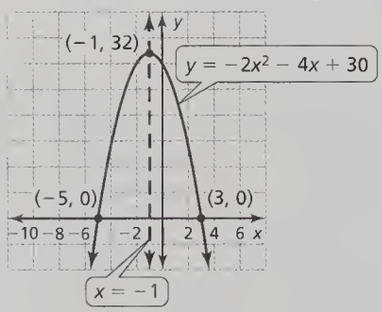
domain: all real numbers, range: $y \geq -36$

19.



domain: all real numbers, range: $y \geq -49$

20.



domain: all real numbers, range: $y \leq 32$

21. 2, 10

22. −5, 1

23. −8, 3

24. 4, 13

25. −2, 7

26. −1

27. −5, −2, 2

28. −6, 6, 11

29. −7, 0, 7

30. −3, 1, 3

31. D

32. F

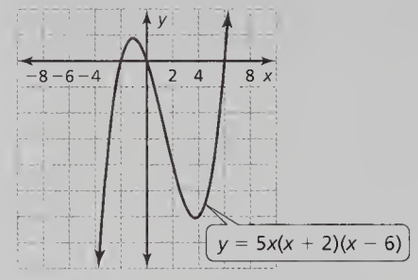
33. C

34. B

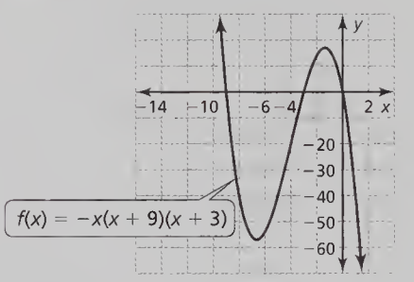
35. A

36. E

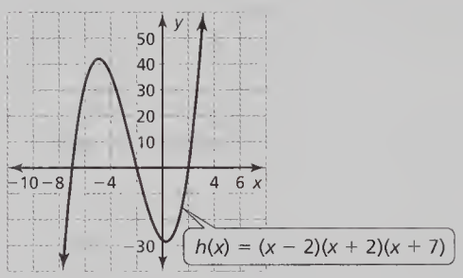
61.



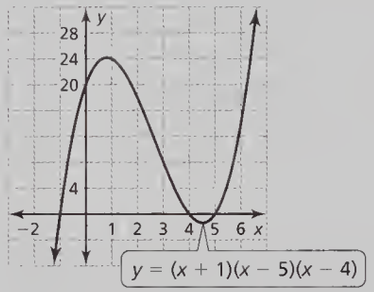
62.



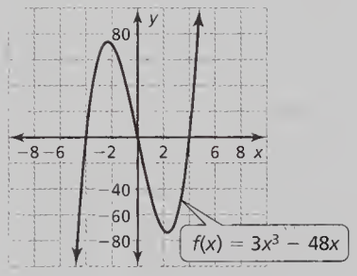
63.



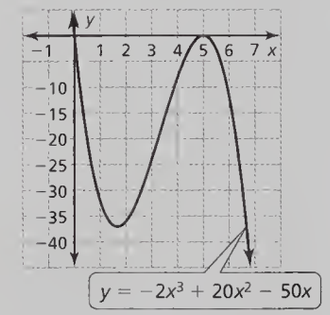
64.



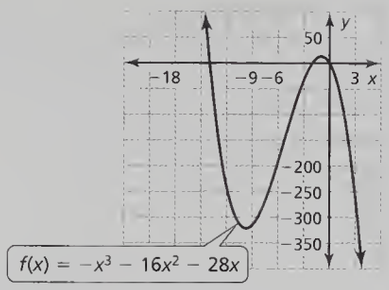
65.



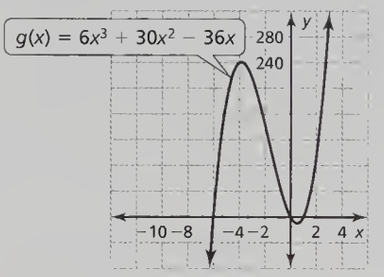
66.



67.



68.



69. $f(x) = 4x^3 - 12x^2 - 16x$

70. $f(x) = -6x^3 - 6x^2 + 36x$

71. $f(x) = -2x^3 - 22x^2 - 56x$

72. $f(x) = 5x^3 - 50x^2 + 135x + 90$

73. *Sample answer:* $f(x) = x^3 - 9x^2 + 2x + 48$

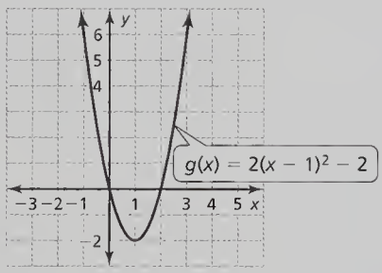
74. *Sample answer:* $f(x) = x^3 + 12x^2 + 35x$

75. *Sample answer:* $f(x) = x^3 - 8x^2 + 7x$

76. *Sample answer:* $f(x) = x^3 + 6$

94. a. $g(x) = 2x(x - 2)$

b.



Sample answer: Plot the vertex $(1, -2)$, which can be determined from the vertex form. Then plot the x-intercepts $(0, 0)$ and $(2, 0)$, which can be determined from the intercept form. Draw a smooth curve through these points.

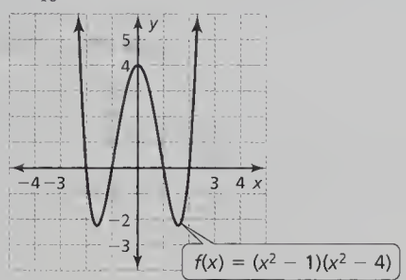
95. yes; *Sample answer:* If a quadratic function has exactly one real zero, then the vertex must lie on the x-axis. This means the zero is one of the intercepts and the x-coordinate of the vertex.

$x = \frac{p + q}{2}$ can only be true if $p = q$, so the function can be written in intercept form with p and q having the same values.

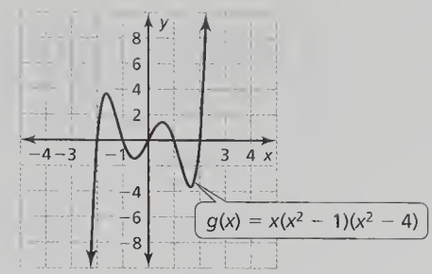
96. yes; *Sample answer:* In standard form, $f(x) = ax^2 + bx + c$, missing terms can be included by using 0 for b or c. In vertex form, $f(x) = a(x - h)^2 + k$, missing values can be included by using 0 for h or k.

97. $f(x) = \frac{1}{10}(x + 5)(x + 2)(x - 1)(x - 4)(x - 8)$

98. a.



b.



Sample answer: Factor the functions into intercept form. Plot the intercepts. Calculate the values for several additional points, then plot them to get an idea of the general shape of the curve. Draw a smooth curve through the plotted points.

99. $-k, 2k$

100. *Sample answer:* $y = (x + 4)(x - 2)$ and $y = 2(x + 4)(x - 2)$; The two given points are x-intercepts, so the graphs of any two quadratic equations having these two intercepts would intersect at these points.

101. *Sample answer:* $y = (x - 5)^2 + 2$ and $y = -(x - 5)^2 + 10$; The two given points have the same y-coordinate, so the axis of symmetry of any parabola passing through them would be halfway between, which is $x = 5$. The vertex form can be used with $h = 5$ and any selected value of k, where $k \neq 6$, to find quadratic equations that would pass through these points. Any two of these equations would form a system with these two points as solutions.

8.5 Maintaining Mathematical Proficiency

102. 300 calories

103. 29 g

104. increases

105. neither; There is no common difference or common ratio.

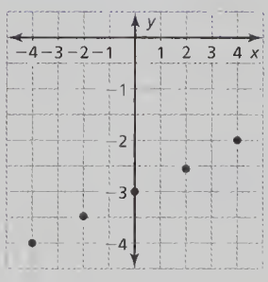
106. geometric; There is a common ratio of 3.

107. arithmetic; There is a common difference of -8.

108. neither; There is no common difference or common ratio.

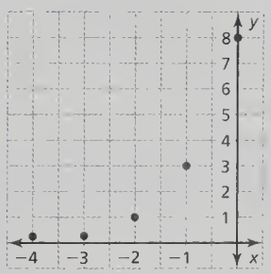
8.6 Monitoring Progress and Modeling with Mathematics

13.



linear

14.



exponential

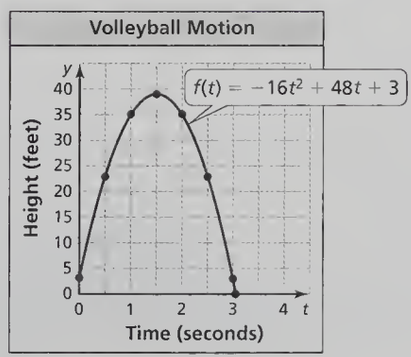
15. linear

16. exponential

17. exponential

18. quadratic

31. b.



33. **a.** exponential; linear; quadratic

b. Organization A: 3, 12, 48, 192, 768, 3072; Organization B: 4, 4, 4, 4, 4, 4; Organization C: 4, 12, 20, 28, 36, 44

c. Organization A; Organization A will have the most donations, followed by Organization C, then Organization B.

34. **a.** 9 nights

b. yes; Because the cost at Sea Breeze Resort is increasing exponentially, it will eventually be greater than the cost at Blue Water Resort, which is increasing linearly.

c. no; Because the cost at Sea Breeze Resort is increasing exponentially, it will remain greater than the cost at Blue Water Resort, which is increasing linearly.

35. The average rate of change of a linear function is constant because the dependent variable of a linear function increases by the same amount for each constant change in the independent variable. The average rate of change of a quadratic or exponential function is not constant because the dependent variable of a quadratic or exponential function changes by a different amount for each constant change in the independent variable.

36. **a.** D; A line is the graph of a linear function.

b. C; A constantly decreasing curve is the graph of an exponential function with a base between 0 and 1.

c. B; A constantly increasing curve is the graph of an exponential function with a base greater than 1.

d. A; A parabola is the graph of a quadratic function.

37. quadratic; The second differences have a constant value of $9n - 5$.

38. *Sample answer:* $y = 1.5x^2$

39. no; *Sample answer:* There may not be enough points to clearly determine the shape of the graph.

40. *Sample answer:* the number of new branches, the length of new branches, the total length of all branches, the total number of branches; exponential, exponential, linear, exponential; $y = (2)^{n-1}$, $y = \left(\frac{1}{2}\right)^{n-1}$, $y = n$, $y = 2^n - 1$

41. no; The graph of an exponential function will always eventually have greater y-values than the graph of a quadratic function.

42. *Sample answer:* $y = -0.4x^2 + 6.2x + 47.0$; Plot the points in a computer graphing program, then use the interpolation function; about \$70.8 billion

8.6 Maintaining Mathematical Proficiency

43. 11 **44.** 5

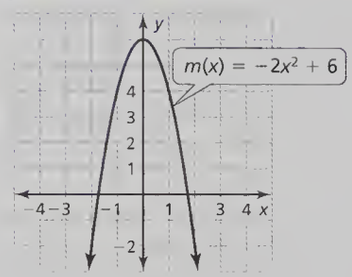
45. 8 **46.** 3

47. $x^2 - 64$ **48.** $16y^2 - 4$

49. $9a^2 - 25b^2$ **50.** $4r^2 - 36s^2$

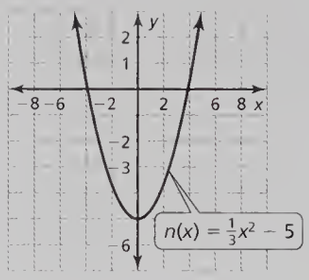
Chapter 8 Review

8.



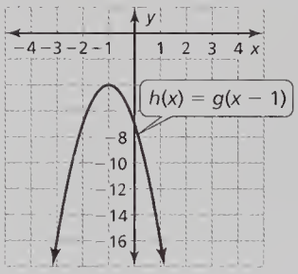
The graph of m is a vertical stretch by a factor of 2, a reflection in the x-axis, and a vertical translation 6 units up of the graph of f.

9.



The graph of n is a vertical shrink by a factor of $\frac{1}{3}$ and a vertical translation 5 units down of the graph of f.

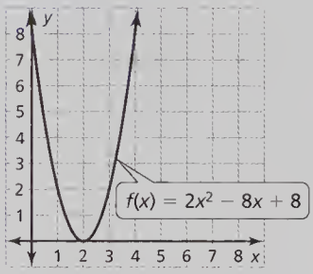
20.



21. $f(x) = 5(x - 3)^2 + 2$

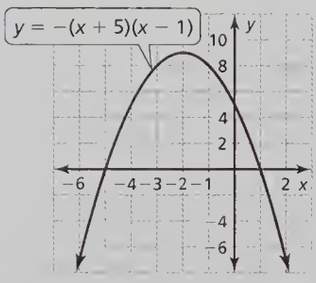
Chapter 8 Test

5.



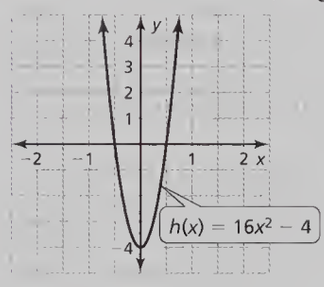
domain: all real numbers, range: $y \geq 0$

6.



domain: all real numbers, range: $y \leq 9$

7.



domain: all real numbers, range: $y \geq -4$

8. exponential; The y-values increase by a constant factor; $y = 8(2)^x$

9. quadratic; The second differences increase by a constant amount; $y = -2x^2$

10. $f(x) = -\frac{1}{2}x^2 - 5x - 8$; *Sample answer:* Use the intercepts to write an equation in intercept form. Substitute the third point into the equation to find a. Write the equation in intercept form with the value of a. Simplify the equation to put it in standard form.

11. $f(x) = 3x^2 - 30x$; *Sample answer:* Use the intercepts to write an equation in intercept form. Substitute the third point into the equation to find a. Write the equation in intercept form with the value of a. Simplify the equation to put it in standard form.

12. *Sample answer:* $f(x) = x^2 + 3$; To be even, the vertex needs to be on the y-axis, so it must be in the form $ax^2 + c$. Because the range is $y \geq 3$, a must be positive and $c = 3$. Any positive value of a is acceptable.

13. *Sample answer:* $f(x) = x^2 - 8x + 16$; Use the x-intercept as the vertex. Use the intercept to write an equation in intercept form. Substitute the second point into the equation to find a. Write the equation in intercept form with the value of a. Simplify the equation to put it in standard form.

14. linear; The first differences are constant; $y = 19x$

15. **a.** 4.445 ft

b. yes; *Sample answer:* After traveling a horizontal distance of 30 feet, the height is 3.6 feet, so the ball will be 0.6 feet above the net.

16. **a.** *Sample answer:* $a = 1, b = 0, c = 0$

b. *Sample answer:* $a = 0, b = 1, c = 0$

c. *Sample answer:* $a = 1, b = 2, c = 3$

17. 1, 3, 5; They are increasing by a constant amount.

Chapter 9

9.1 Monitoring Progress and Modeling with Mathematics

99. **a.** $4, 2\frac{1}{4}, 2, 2 + \sqrt{3}, 2 - \sqrt{3}, 2 + \pi$;

$2\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4} + \sqrt{3}, \frac{1}{4} - \sqrt{3}, \frac{1}{4} + \pi$;

$2, \frac{1}{4}, 0, \sqrt{3}, -\sqrt{3}, \pi$;

$2 + \sqrt{3}, \frac{1}{4} + \sqrt{3}, \sqrt{3}, 2\sqrt{3}, 0, \pi + \sqrt{3}$;

$2 - \sqrt{3}, \frac{1}{4} - \sqrt{3}, -\sqrt{3}, 0, -2\sqrt{3}, \pi - \sqrt{3}$;

$2 + \pi, \frac{1}{4} + \pi, \pi, \pi + \sqrt{3}, \pi - \sqrt{3}, 2\pi$

b. $4, \frac{1}{2}, 0, 2\sqrt{3}, -2\sqrt{3}, 2\pi$;

$\frac{1}{2}, \frac{1}{16}, 0, \frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}, \frac{\pi}{4}$;

$0, 0, 0, 0, 0, 0$;

$2\sqrt{3}, \frac{\sqrt{3}}{4}, 0, 3, -3, \pi\sqrt{3}$;

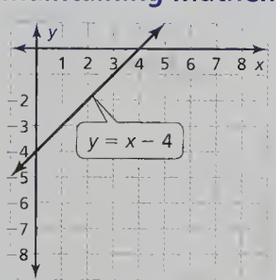
$-2\sqrt{3}, -\frac{\sqrt{3}}{4}, 0, -3, 3, -\pi\sqrt{3}$;

$2\pi, \frac{\pi}{4}, 0, \pi\sqrt{3}, -\pi\sqrt{3}, \pi^2$

107. $\frac{2\sqrt[3]{x^2} - 2\sqrt[3]{x} + 2}{x + 1}$; Multiplying the numerator and denominator by $\sqrt[3]{x^2} - \sqrt[3]{x} + 1$ rationalizes the denominator.

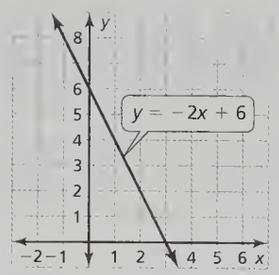
9.1 Maintaining Mathematical Proficiency

108.



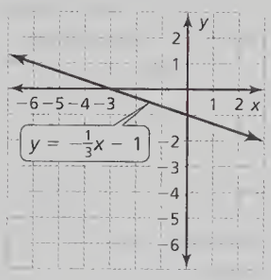
4

109.



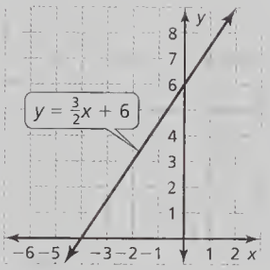
3

110.



−3

111.



−4

112. $x = -1$

113. $x = -2$

114. $x = \frac{5}{2}$

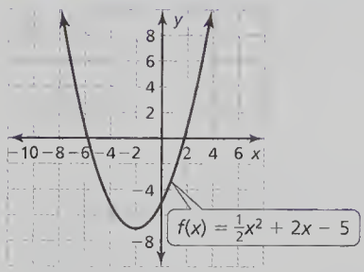
115. $x = \frac{1}{2}$

9.2 Explorations

5. The related graph will have no x-intercepts.

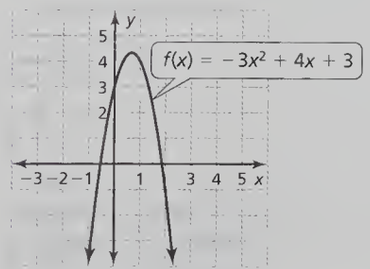
9.2 Monitoring Progress and Modeling with Mathematics

51.



about 1.7, about −5.7

52.



about 1.9, about −0.5

53. **a.** 118 ft, 198 ft, 246 ft, 262 ft, 246 ft, 198 ft, 118 ft, 6 ft

b. about 1.5 sec, about 6.5 sec

c. about 1.4 sec, about 6.6 sec

68. always; The sign of y does not change.

69. never; The graph of $y = ax^2 + bx + c$ has at most two x-intercepts.

70. *Sample answer:* Plot the data points. Find both the linear model and the quadratic model. Graph both models and see if one model is closer to the data points than the others. If neither is closer, compare r and R^2.

71. Graph the system of equations $y = -0.43x^2 + 11.7x + 2$ and $y = 77$. Find the points of intersection, which are about (10.3, 77) and about (16.9, 77).

9.2 Maintaining Mathematical Proficiency

72. exponential decay function; As x increases by 1, y is multiplied by $\frac{1}{6}$.

73. exponential growth function; As x increases by 1, y is multiplied by 4.

74. $\frac{5\sqrt{13}}{13}$

75. $\frac{\sqrt{15x}}{3x}$

76. $\frac{14 - 2\sqrt{5}}{11}$

77. $-3\sqrt{3} - 6$

9.3 Monitoring Progress and Modeling with Mathematics

38. **a.** a and c have opposite signs.
 b. a is not 0, and c is 0.
 c. a and c have the same sign.

39. (3, 9), (−3, 9); When $y = 9$, $x = \pm 3$.

40. 1; The graph has one x-intercept.

41. $x = 1.2$, $x = -1.2$; $1.2^2 = 1.44$

42. $x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$ or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

43. $x = 6$, $x = -2$

44. **a.** $x = 14$, $x = -2$
 b. $x = -3$, $x = -11$

9.3 Maintaining Mathematical Proficiency

45. $(x + 4)^2$

46. $(x - 2)^2$

47. $(x - 7)^2$

48. $(x + 9)^2$

49. $(x + 6)^2$

50. $(x - 11)^2$

9.4 Maintaining Mathematical Proficiency

75. $a_1 = 10$, $a_n = a_{n+1} + 5$

76. $a_1 = 3$, $a_n = 2a_{n-1}$

77. $a_1 = -20$, $a_n = a_{n-1} + 4$

78. $2\sqrt{3}$

79. $6\sqrt{2}$

80. $2\sqrt{5}$

9.5 Monitoring Progress and Modeling with Mathematics

40. $x \approx 2.24$, $x \approx -2.24$; *Sample answer:* The equation can be written in the form $x^2 = d$, so solve using square roots.

41. $x = -4$, $x = 3$; *Sample answer:* The equation is easily factorable, so solve by factoring.

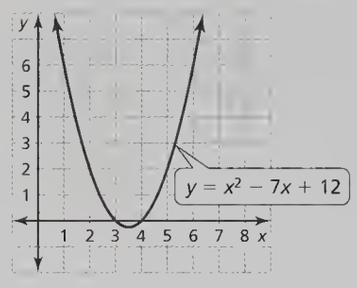
42. $x \approx 3.73$, $x \approx 0.27$; *Sample answer:* $a = 1$ and b is even, so solve by completing the square.

43. $x \approx 2.19$, $x \approx -1.94$; *Sample answer:* The equation cannot be factored and $a \neq 1$, so solve using the quadratic formula.

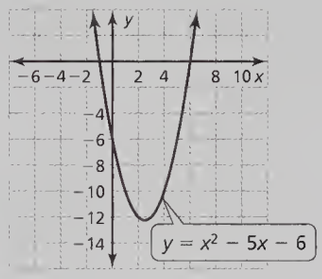
44. $x = 1$, $x = -7$; *Sample answer:* The left side is a perfect square trinomial, so solve using square roots.

79. **a.** $(x + 1)$, $(x - 6)$; $x^2 - 5x - 6 = 0$; x, $(x - 2)$; $x^2 - 2x = 0$; $\left(x + \frac{1}{2}\right)$, $(x - 5)$; $x^2 - \frac{9}{2}x - \frac{5}{2} = 0$

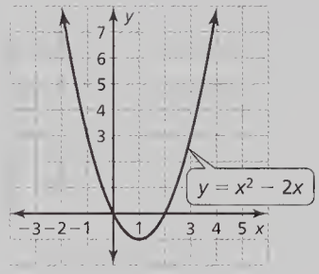
b.



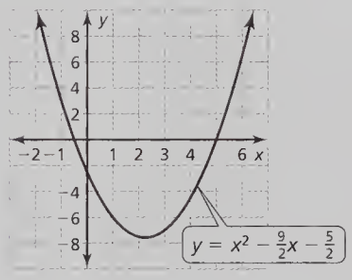
3, 4



−1, 6



0, 2



$-\frac{1}{2}$, 5

82. **a.** $k > -\frac{25}{64}$
 b. $k = -\frac{25}{64}$
 c. $k < -\frac{25}{64}$

9.5 Maintaining Mathematical Proficiency

83. (4, 0); *Sample answer:* substitution because both equations are solved for y

84. (−4, 5); *Sample answer:* substitution because one equation is solved for x

85. (5, 3); *Sample answer:* elimination because one pair of like terms has the same coefficient

86. (−2, 7); *Sample answer:* substitution because one of the variables has a coefficient of 1

English-Spanish Glossary

English	Spanish
A	
arithmetic sequence *(p. 202)* An ordered list of numbers in which the difference between each pair of consecutive terms is the same	**secuencia aritmética** *(p. 202)* Una lista ordenada en donde la diferencia entre cada par de términos consecutivos es la misma
asymptote *(p. 291)* A line that a graph approaches but never intersects	**asíntota** *(p. 291)* Una recta a la que se aproxima una gráfica, pero nunca la interseca
average rate of change *(p. 448)* The slope of the line through $(a, f(a))$ and $(b, f(b))$ of a function $y = f(x)$ between $x = a$ and $x = b$	**tasa de variación media** *(p. 448)* La pendiente de la línea a través de $(a, f(a))$ y $(b, f(b))$ de una función $y = f(x)$ entre $x = a$ y $x = b$
axis of symmetry *(p. 406)* The vertical line that divides a parabola into two symmetric parts	**eje de simetría** *(p. 406)* La línea vertical que divide una parábola en dos partes simétricas
B	
binomial *(p. 339)* A polynomial with two terms	**binomio** *(p. 339)* Un polinomio con dos términos
C	
causation *(p. 197)* When a change in one variable causes a change in another variable	**causalidad** *(p. 197)* Cuando un cambio en una variable causa un cambio en otra variable
closed *(p. 340)* When an operation performed on any two numbers in the set results in a number that is also in the set	**operación interna** *(p. 340)* Cuando una operación efectuada en dos números cualesquiera del conjunto da como resultado un número que también está en el conjunto
common difference *(p. 202)* The difference between each pair of consecutive terms in an arithmetic sequence	**diferencia común** *(p. 202)* La diferencia entre cada par de términos consecutivos en una secuencia aritmética
common ratio *(p. 310)* The ratio between each pair of consecutive terms in a geometric sequence	**razón común** *(p. 310)* La razón entre cada par de términos consecutivos en una secuencia geométrica
completing the square *(p. 494)* To add a constant c to an expression of the form $x^2 + bx$ so that $x^2 + bx + c$ is a perfect square trinomial	**completando el cuadrado** *(p. 494)* Agregar una constante c a una expresión de la forma $x^2 + bx$ para que $x^2 + bx + c$ sea un trinomio de cuadrado perfecto
compound inequality *(p. 74)* An inequality formed by joining two inequalities with the word "and" or the word "or"	**desigualdad compuesta** *(p. 74)* Una desigualdad formada por dos desigualdades con la palabra "y" o la palabra "o"
compound interest *(p. 301)* The interest earned on the principal and on previously earned interest	**interés compuesto** *(p. 301)* El interés obtenido en el principal y en intereses previamente obtenidos
conjecture *(p. 3)* An unproven statement about a general mathematical concept	**conjetura** *(p. 3)* Una afirmación no comprobada sobre un concepto matemático general

conjugates *(p. 468)* Binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers

conjugados *(p. 468)* Binomios de la forma $a\sqrt{b} + c\sqrt{d}$ y $a\sqrt{b} - c\sqrt{d}$, donde a, b, c y d son números racionales

constant function *(p. 126)* A linear equation written in the form $y = 0x + b$ or $y = b$

función constante *(p. 126)* Una ecuación lineal escrita bajo la forma $y = 0x + b$ o $y = b$

constant of variation *(p. 134)* The constant a in the direct variation equation $y = ax$, where $a \neq 0$

constante de variación *(p. 134)* La constante a en la ecuación de varición directa $y = ax$, donde $a \neq 0$

continuous domain *(p. 100)* A set of input values that consists of all numbers in an interval

dominio continuo *(p. 100)* Un conjunto de valores que consiste en todos los números en un intervalo

correlation *(p. 189)* A relationship between data sets

correlación *(p. 189)* Una relación entre conjuntos de datos

correlation coefficient *(p. 195)* A value r that tells how closely the equation of the line of best fit models the data

coeficiente de correlación *(p. 195)* Un valor r que indica cuán fielmente la ecuación de la línea de mejor ajuste modela los datos

counterexample *(p. 465)* An example that proves that a general statement is not true

contraejemplo *(p. 465)* Un ejemplo que prueba que una afirmación general no es verdadera

D

degree of a monomial *(p. 338)* The sum of the exponents of the variables in the monomial

grado de un monomio *(p. 338)* La suma de los exponentes de las variables en el monomio

degree of a polynomial *(p. 339)* The greatest degree of the terms in a polynomial

grado de un polinomio *(p. 339)* El grado mayor de los términos en un polinomio

dependent variable *(p. 93)* The variable that represents the output values of a function

variable dependiente *(p. 93)* La variable que representa los resultados de una función

direct variation *(p. 134)* An equation of the form $y = ax$, where $a \neq 0$ and y is said to vary directly with x

variacion directa *(p. 134)* Una ecuación de la forma $y = ax$, donde $a \neq 0$ y se dice que y varia directamente con x

discrete domain *(p. 100)* A set of input values that consists of only certain numbers in an interval

dominio discreto *(p. 100)* Un conjunto de valores que consiste en solamente ciertos números en un intervalo

discriminant *(p. 506)* The expression $b^2 - 4ac$ in the Quadratic Formula

discriminante *(p. 506)* La expresión $b^2 - 4ac$ en la Fórmula Cuadrática

domain *(p. 92)* The set of all possible input values of a function

dominio *(p. 92)* El conjunto de todos los valores posibles de una función

E

equation *(p. 4)* A statement that two expressions are equal

ecuación *(p. 4)* Una afirmación de que dos expresiones son iguales

equivalent equations *(p. 4)* Equations that have the same solution(s)

ecuaciones equivalentes *(p. 4)* Ecuaciones que tiene(n) la(s) misma(s) solución(es)

equivalent inequalities *(p. 54)* Inequalities that have the same solutions

desigualdades equivalentes *(p. 54)* Desigualdades que tiene(n) la(s) misma(s) solución(es)

even function *(p. 428)* A function $y = f(x)$ is even when $f(-x) = f(x)$ for each x in the domain of f.

explicit rule *(p. 318)* A rule to define arithmetic and geometric sequences that gives a_n as a function of the term's position number n in the sequence

exponential decay *(p. 299)* When a quantity decreases by the same factor over equal intervals of time

exponential decay function *(p. 299)* A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$

exponential function *(p. 290)* A nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$

exponential growth *(p. 298)* When a quantity increases by the same factor over equal intervals of time

exponential growth function *(p. 298)* A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$

extrapolation *(p. 197)* To predict a value outside the range of known values using a graph or its equation

par función *(p. 428)* Una función $y = f(x)$ es par cuando $f(-x) = f(x)$ para cada x en el dominio de f.

regla explícita *(p. 318)* Una regla para definir secuencias aritméticas y geométricas que da a_n como función del número de posición del término n en la secuencia

decaimiento exponencial *(p. 299)* Cuando una cantidad disminuye por el mismo factor sobre intervalos iguales de tiempo

función de decaimiento exponencial *(p. 299)* Una función de la forma $y = a(1 - r)^t$, donde $a > 0$ y $0 < r < 1$

función exponencial *(p. 290)* Una función no lineal de la forma $y = ab^x$, donde $a \neq 0$, $b \neq 1$, y $b > 0$

crecimiento exponencial *(p. 298)* Cuando una cantidad se incrementa por el mismo factor sobre intervalos iguales de tiempo

función de crecimiento exponencial *(p. 298)* Una función de la forma $y = a(1 + r)^t$, donde $a > 0$ y $r > 0$

extrapolación *(p. 197)* Predecir un valor fuera del rango de valores conocidos usando un gráfico o su ecuación

F

factored completely *(p. 390)* A polynomial that is written as a product of unfactorable polynomials with integer coefficients

factored form *(p. 364)* A polynomial that is written as a product of factors

factoring by grouping *(p. 390)* To use the Distributive Property to factor a polynomial with four terms

family of functions *(p. 140)* A group of functions with similar characteristics

FOIL Method *(p. 347)* A shortcut for multiplying two binomials by finding the sum of the products of the first terms, outer terms, inner terms, and last terms

formula *(p. 29)* A literal equation that shows how one variable is related to one or more other variables

factorizado completamente *(p. 390)* Un polinomio que se escribe como un producto de polinomios no factorizables con coeficientes de números enteros

forma factorizada *(p. 364)* Un polinomio que se escribe como un producto de factores

factorización por agrupamiento *(p. 390)* Uso de la propiedad distributiva para factorizar un polinomio con cuatro términos

familia de funciones *(p. 140)* Un grupo de funciones con características similares

método FOIL o de multiplicación de binomios *(p. 347)* Un método rápido para multiplicar dos binomios encontrando la suma de los productos de los primeros términos, los segundos términos, los terceros términos y los últimos términos

fórmula *(p. 29)* Una ecuación literal que muestra cómo una variable está relacionada con una o más variables

function *(p. 90)* A relation that pairs each input with exactly one output

función *(p. 90)* Una relación que asocia cada valor exactamente con un resultado

function notation *(p. 108)* Another name for y denoted as $f(x)$ and read as "the value of f at x"

notación de función *(p. 108)* Otro nombre para y que se denota como $f(x)$ y que se lee como "el valor de f en x"

G

geometric sequence *(p. 310)* An ordered list of numbers in which the ratio between each pair of consecutive terms is the same

secuencia geométrica *(p. 310)* Una lista ordenada en donde la razón entre cada par de términos consecutivos es la misma

graph of an inequality *(p. 48)* A graph that shows the solution set of an inequality on a number line

gráfico de una desigualdad *(p. 48)* Un gráfico que muestra el conjunto de soluciones de una desigualdad en una línea numérica

graph of a linear inequality *(p. 252)* The graph in two variables that shows all the solutions of the inequality in a coordinate plane

gráfico de una desigualdad lineal *(p. 252)* El gráfico en dos variables que muestra todas las soluciones de la desigualdad en un plano coordenado

graph of a system of linear inequalities *(p. 261)* The graph of all the solutions of the system of linear inequalities

gráfico de un sistema de desigualdades lineales *(p. 261)* El gráfico de todas las soluciones del sistema de las desigualdades lineales

H

half-planes *(p. 252)* Two regions of the coordinate plane divided by a boundary line

semiplanos *(p. 252)* Dos regiones del plano coordenado divididas por una línea limítrofe

horizontal shrink *(p. 142)* A transformation that causes the graph of a function to shrink toward the y-axis when all the x-coordinates are multiplied by a factor a, where $a > 1$

reducción horizontal *(p. 142)* Una transformación que hace que el gráfico de una función se reduzca hacia el eje y cuando todas las coordenadas x se multiplican por un factor a, donde $a > 1$

horizontal stretch *(p. 142)* A transformation that causes the graph of a function to stretch away from the y-axis when all the x-coordinates are multiplied by a factor a, where $0 < a < 1$

ampliación horizontal *(p. 142)* Una transformación que hace que el gráfico de una función se amplíe desde el eje y cuando todas las coordenadas x se multiplican por un factor a, donde $0 < a < 1$

I

identity *(p. 23)* An equation that is true for all values of the variable

identidad *(p. 23)* Una ecuación que es verdadera para todos los valores de la variable

independent variable *(p. 93)* The variable that represents the input values of a function

variable independiente *(p. 93)* La variable que representa los valores de una función

index of a radical *(p. 284)* The value of n in the radical $\sqrt[n]{a}$

índice de un radical *(p. 284)* El valor de n en el radical $\sqrt[n]{a}$

inequality *(p. 46)* A mathematical sentence that compares expressions

desigualdad *(p. 46)* Una oración matemática que compara expresiones

intercept form *(p. 436)* A quadratic function written in the form $f(x) = a(x - p)(x - q)$, where $a \neq 0$

forma intersección *(p. 436)* Una ecuación cuadrática escrita en la forma $f(x) = a(x - p)(x - q)$, donde $a \neq 0$

interpolation *(p. 197)* To approximate a value between two known values using a graph or its equation

interpolación *(p. 197)* Aproximar un valor entre dos valores conocidos usando un gráfico o su ecuación

inverse operations *(p. 4)* Two operations that undo each other, such as addition and subtraction

operaciones inversas *(p. 4)* Dos operaciones que se anulan entre sí, como la suma y la resta

L

leading coefficient *(p. 339)* The coefficient of the first term of a polynomial written in standard form

coeficiente principal *(p. 339)* El coeficiente del primer término de un polinomio escrito en forma estándar

line of best fit *(p. 195)* A line that best models a set of data

línea de mejor ajuste *(p. 195)* Una línea que mejor modela un conjunto de datos

line of fit *(p. 190)* A line drawn on a scatter plot that is close to most of the data points

línea de ajuste *(p. 190)* Una línea dibujada en un diagrama de dispersión que está cerca a la mayoría de los puntos de datos

linear equation in one variable *(p. 4)* An equation that can be written in the form $ax + b = 0$, where a and b are constants and $a \neq 0$

ecuación lineal en una variable *(p. 4)* Una ecuación que puede escribirse en la forma $ax + b = 0$, donde a y b son constantes y $a \neq 0$

linear equation in two variables *(p. 98)* An equation that can be written in the form $y = mx + b$, where m and b are constants

ecuación lineal en dos variables *(p. 98)* Una ecuación que puede escribirse en la forma $y = mx + b$, donde m y b son constantes

linear function *(p. 98)* A function whose graph is a non-vertical line

función lineal *(p. 98)* Una función cuyo gráfico es una línea no vertical

linear inequality in two variables *(p. 252)* An inequality written in the form $ax + by < c$, $ax + by \leq c$, $ax + by > c$, or $ax + by \geq c$, where a, b, and c are real numbers

desigualdad lineal en dos variables *(p. 252)* Una desigualdad escrita en la forma $ax + by < c$, $ax + by \leq c$, $ax + by > c$, o $ax + by \geq c$, donde a, b y c son números reales

linear model *(p. 164)* A linear function that models a real-life situation

modelo lineal *(p. 164)* Una función lineal que modela una situación de la vida real

linear regression *(p. 195)* A method that graphing calculators use to find a precise line of fit that models a set of data

regresión lineal *(p. 195)* Un método que usan las calculadoras gráficas para encontrar una línea precisa de ajuste que modela un conjunto de datos

literal equation *(p. 28)* An equation that has two or more variables

ecuación literal *(p. 28)* Una ecuación que tiene dos o más variables

M

maximum value *(p. 419)* The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$, where $a < 0$

valor máximo *(p. 419)* La coordenada y del vértice del gráfico de $f(x) = ax^2 + bx + c$, donde $a < 0$

minimum value *(p. 419)* The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$, where $a > 0$

valor mínimo *(p. 419)* La coordenada y del vértice del gráfico de $f(x) = ax^2 + bx + c$, donde $a > 0$

monomial *(p. 338)* A number, a variable, or the product of a number and one or more variables with whole number exponents

monomio *(p. 338)* Un número, una variable o el producto de un número y una o más variables con exponentes en números enteros

N

nonlinear function *(p. 98)* A function that does not have a constant rate of change and whose graph is not a line

función no lineal *(p. 98)* Una función que no tiene una tasa constante de cambio y cuyo gráfico no es una línea

n*th root of *a *(p. 284)* For an integer n greater than 1, if $b^n = a$, then b is an nth root of a.

raíz de orden *n* de *a* *(p. 284)* Para un número entero n mayor que 1, si $b^n = a$, entonces b es una raíz de orden n de a.

O

odd function *(p. 428)* A function $y = f(x)$ is odd when $f(-x) = -f(x)$ for each x in the domain of f.

impar funcíon *(p. 428)* Una función $y = f(x)$ es impar cuando $f(-x) = -f(x)$ para cada x en el dominio de f.

P

parabola *(p. 406)* The U-shaped graph of a quadratic function

parábola *(p. 406)* El gráfico en forma de U de una función cuadrática

parallel lines *(p. 180)* Two lines in the same plane that never intersect

líneas paralelas *(p. 180)* Dos líneas en el mismo plano que nunca se intersectan

parent function *(p. 140)* The most basic function in a family of functions

función principal *(p. 140)* La función más básica en una familia de funciones

perpendicular lines *(p. 181)* Two lines in the same plane that intersect to form right angles

líneas perpendiculares *(p. 181)* Dos líneas en el mismo plano que se intersectan para formar ángulos rectos

point-slope form *(p. 168)* A linear equation written in the form $y - y_1 = m(x - x_1)$

forma punto-pendiente *(p. 168)* Una ecuación lineal escrita en la forma $y - y_1 = m(x - x_1)$

polynomial *(p. 339)* A monomial or a sum of monomials

polinomio *(p. 339)* Un monomio o una suma de monomios

polynomial long division *(p. 358)* A method to divide a polynomial $f(x)$ by a nonzero divisor $d(x)$ to yield a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$

división larga de polinomios *(p. 358)* Un método para dividir un polinomio $f(x)$ por un divisor distinto de cero $d(x)$ para obtener un polinomio de cociente $q(x)$ y un polinomio de resto $r(x)$

Q

quadratic equation *(p. 476)* A nonlinear equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$

Quadratic Formula *(p. 504)* The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a \neq 0$ and $b^2 - 4ac \geq 0$.

quadratic function *(p. 406)* A nonlinear function that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$

ecuación cuadrática *(p. 476)* Una ecuación no lineal que puede escribirse en la forma estándar $ax^2 + bx + c = 0$, donde $a \neq 0$

fórmula cuadrática *(p. 504)* Las soluciones reales de la ecuación cuadrática $ax^2 + bx + c = 0$ son

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, donde $a \neq 0$ y $b^2 - 4ac \geq 0$.

función cuadrática *(p. 406)* Una ecuación no lineal que puede escribirse en la forma estándar $y = ax^2 + bx + c$, donde $a \neq 0$

R

radical *(p. 284)* An expression of the form $\sqrt[n]{a}$

radical expression *(p. 466)* An expression that contains a radical

range *(p. 92)* The set of all possible output values of a function

rationalizing the denominator *(p. 468)* To eliminate a radical from the denominator of a fraction by multiplying by an appropriate form of 1

recursive rule *(p. 318)* A rule to define arithmetic and geometric sequences that gives the beginning term(s) of a sequence and a recursive equation that tells how a_n is related to one or more preceding terms

reflection *(p. 141)* A transformation that flips a graph over a line called the line of reflection

relation *(p. 90)* A pairing of inputs with outputs

repeated roots *(p. 365)* Two or more roots of an equation that are the same number

residual *(p. 194)* The difference of the y-value of a data point and the corresponding y-value found using the line of fit

rise *(p. 124)* The change in y of the slope of a nonvertical line

roots *(p. 364)* The solutions of a polynomial equation

radical *(p. 284)* Una expresión de la forma $\sqrt[n]{a}$

expresión radical *(p. 466)* Una expresión que contiene un radical

rango *(p. 92)* El conjunto de todos los resultados posibles de una función

racionalización del denominador *(p. 468)* Eliminar un radical del denominador de una fracción multiplicando por una forma apropiada de 1

regla recursiva *(p. 318)* Una regla para definir las secuencias aritméticas y geométricas que da el(los) primer(os) término(s) de una secuencia y una ecuación recursiva que indica cómo se relaciona a_n a uno o más términos precedentes

reflexión *(p. 141)* Una transformación que voltea un gráfico sobre una línea llamada la línea de reflexión

relación *(p. 90)* Una pareja de valores con resultados

raíces repetidas *(p. 365)* Dos o más raíces de una ecuación que son el mismo número

residual *(p. 194)* La diferencia del valor y de un punto de datos y el valor y correspondiente usando la línea de ajuste

desplazamiento vertical *(p. 124)* El cambio en y de la pendiente de una línea no vertical

raíces *(p. 364)* Las soluciones deuna ecuación de polinomios

rule *(p. 3)* A proven statement about a general mathematical concept

regla *(p. 3)* Una afirmación comprobada sobre un concepto matemático general

run *(p. 124)* The change in x of the slope of a nonvertical line

desplazamiento horizontal *(p. 124)* El cambio en x de la pendiente de una línea no vertical

S

scatter plot *(p. 188)* A graph that shows the relationship between two data sets

diagrama de dispersión *(p. 188)* Un gráfico que muestra la relación entre dos conjuntos de datos

sequence *(p. 202)* An ordered list of numbers

secuencia *(p. 202)* Una lista ordenada de números

simplest form *(p. 466)* A radical that has no radicands with perfect nth powers as factors other than 1, no radicands that contain fractions, and no radicals that appear in the denominator of a fraction

mínima expresión *(p. 466)* Un radical que no tiene radicandos con potencias perfectas de orden n como factores distintos a 1, que no tiene radicandos que contengan fracciones y que no tiene radicales que aparezcan en el denominador de una fracción

slope *(p. 124)* The rate of change between any two points on a line

pendiente *(p. 124)* La tasa de cambio entre dos puntos cualesquiera en una línea

slope-intercept form *(p. 126)* A linear equation written in the form $y = mx + b$

forma intersección-pendiente *(p. 126)* Una ecuación lineal escrita en la forma $y = mx + b$

solution of an equation *(p. 4)* A value that makes an equation true

solución de una ecuación *(p. 4)* Un valor que hace que una ecuación sea verdadera

solution of an inequality *(p. 47)* A value that makes an inequality true

solución de una desigualdad *(p. 47)* Un valor que hace que una desigualdad sea verdadera

solution of a linear equation in two variables *(p. 100)* An ordered pair (x, y) that makes an equation true

solución de una ecuación lineal en dos variables *(p. 100)* Un par ordenado (x, y) que hace que una ecuación sea verdadera

solution of a linear inequality in two variables *(p. 252)* An ordered pair (x, y) that makes the inequality true

solución de una desigualdad lineal en dos variables *(p. 252)* Un par ordenado (x, y) que hace que la desigualdad sea verdadera

solution set *(p. 47)* The set of all solutions of an inequality

conjunto solución *(p. 47)* El conjunto de todas las soluciones de una desigualdad

solution of a system of linear equations *(p. 220)* An ordered pair that is a solution of each equation in the system

solución de un sistema de ecuaciones lineales *(p. 220)* Un par ordenado que es una solución de cada ecuación en el sistema

solution of a system of linear inequalities *(p. 260)* An ordered pair that is a solution of each inequality in the system

solución de un sistema de desigualdades lineales *(p. 260)* Un par ordenado que es una solución de cada desigualdad en el sistema

standard form of a linear equation *(p. 116)* A linear equation written in the form $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero

forma estándar de una ecuación lineal *(p. 116)* Una ecuación lineal escrita en la forma $Ax + By = C$, donde A, B y C son números reales y ni A ni B son cero

standard form of a polynomial *(p. 339)* A polynomial in one variable written with the exponents of the terms decreasing from left to right

forma estándar de un polinomio *(p. 339)* Un polinomio en una variable escrita con los exponentes de los términos decreciendo de izquierda a derecha

synthetic division *(p. 360)* A shortcut method to divide a polynomial by a binomial of the form $x - k$

división sintética *(p. 360)* Un método abreviado para dividir un polinomio por un binomio de la forma $x - k$

system of linear equations *(p. 220)* A set of two or more linear equations in the same variables

sistema de ecuaciones lineales *(p. 220)* Un conjunto de dos o más ecuaciones lineales en las mismas variables

system of linear inequalities *(p. 260)* A set of two or more linear inequalities in the same variables

sistema de desigualdades lineales *(p. 260)* Un conjunto de dos o más desigualdades lineales en las mismas variables

T

term of a sequence *(p. 202)* Each number in a sequence

término de una sucesión *(p. 202)* Cada número en una secuencia

theorem *(p. 3)* A proven statement about a general mathematical concept

teorema *(p. 3)* Una declaración comprobada acerca de un concepto matemático general

transformation *(p. 140)* A change in the size, shape, position, or orientation of a graph

transformación *(p. 140)* Un cambio en el tamaño, forma, posición u orientación de un gráfico

translation *(p. 140)* A transformation that shifts a graph horizontally and/or vertically but does not change the size, shape, or orientation of the graph

traslación *(p. 140)* Una transformación que desplaza un gráfico horizontal y/o verticalmente pero no cambia el tamaño, forma u orientación del gráfico

trinomial *(p. 339)* A polynomial with three terms

trinomio *(p. 339)* Un polinomio con tres términos

V

vertex *(p. 406)* The lowest point on a parabola that opens up or the highest point on a parabola that opens down

vértice *(p. 406)* El punto más bajo de una parábola que se abre hacia arriba o el punto más alto de una parábola que se abre hacia abajo

vertex form *(p. 430)* A quadratic function written in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$

forma vértice *(p. 430)* Una función cuadrática escrita en la forma $f(x) = a(x - h)^2 + k$, donde $a \neq 0$

vertical shrink *(p. 142)* A transformation that causes the graph of a function to shrink toward the x-axis when all the y-coordinates are multiplied by a factor a, where $0 < a < 1$

reducción vertical *(p. 142)* Una transformación que hace que el gráfico de una función se reduzca hacia el eje x cuando todas las coordenadas y se multiplican por un factor a, donde $0 < a < 1$

vertical stretch *(p. 142)* A transformation that causes the graph of a function to stretch away from the x-axis when all the y-coordinates are multiplied by a factor a, where $a > 1$

ampliación vertical *(p. 142)* Una transformación que hace que el gráfico de una función se amplíe desde el eje x cuando todas las coordenadas y se multiplican por un factor a, donde $a > 1$

X

x-intercept *(p. 117)* The x-coordinate of a point where the graph crosses the x-axis

intersección x *(p. 117)* La coordenada x de un punto donde el gráfico cruza el eje x

Y

y-intercept *(p. 117)* The y-coordinate of a point where the graph crosses the y-axis

intersección y *(p. 117)* La coordenada y de un punto donde el gráfico cruza el eje y

Z

zero of a function *(p. 118)* An x-value of a function f for which $f(x) = 0$

cero de una función *(p. 118)* Un valor x de una función f para el cual $f(x) = 0$

Zero-Product Property *(p. 364)* If the product of two real numbers is 0, then at least one of the numbers is 0.

propiedad de producto cero *(p. 364)* Si el producto de dos números reales es 0, entonces al menos uno de los números es 0.

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Subtraction Property of Equality
If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality
If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality
If $a = b$, then $a \div c = b \div c$, $c \neq 0$.

Properties of Inequality

Addition Property of Inequality
If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

Subtraction Property of Inequality
If $a > b$, then $a - c > b - c$.
If $a < b$, then $a - c < b - c$.

Multiplication Property of Inequality ($c > 0$)

If $a > b$ and $c > 0$, then $ac > bc$.

If $a < b$ and $c > 0$, then $ac < bc$.

Division Property of Inequality ($c > 0$)

If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

Multiplication Property of Inequality ($c < 0$)

If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Division Property of Inequality ($c < 0$)

If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

* The Properties of Inequality are also true for $\geq$ and $\leq$.

Properties of Exponents

Zero Exponent
$a^0 = 1$, where $a \neq 0$

Negative Exponent
$a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

Product of Powers Property
$a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property
$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property
$(a^m)^n = a^{mn}$

Power of a Product Property
$(ab)^n = a^n b^n$

Power of a Quotient Property
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

Rational Exponents
$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

Properties of Radicals

Product Property of Square Roots
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

Quotient Property of Square Roots
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

Other Properties

Zero-Product Property
If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

Patterns

Square of a Binomial Pattern
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

Sum and Difference Pattern
$(a + b)(a - b) = a^2 - b^2$

Difference of Two Squares Pattern
$a^2 - b^2 = (a + b)(a - b)$

Perfect Square Trinomial Pattern
$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

Formulas

Slope
$m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept form
$y = mx + b$

Point-slope form
$y - y_1 = m(x - x_1)$

Standard form of a linear equation
$Ax + By = C$, where A and B are not both 0

Standard form of a quadratic function
$f(x) = ax^2 + bx + c$, where $a \neq 0$

Vertex form of a quadratic function
$f(x) = a(x - h)^2 + k$, where $a \neq 0$

Intercept form of a quadratic function
$f(x) = a(x - p)(x - q)$, where $a \neq 0$

Axis of Symmetry

$x = \frac{-b}{2a}$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a \neq 0$ and $b^2 - 4ac \geq 0$

Exponential growth
$y = a(1 + r)^t$, where $a > 0$ and $r > 0$

Exponential decay
$y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$

Explicit rule for an arithmetic sequence
$a_n = a_1 + (n - 1)d$

Explicit rule for a geometric sequence
$a_n = a_1 r^{n-1}$

Recursive equation for an arithmetic sequence
$a_n = a_{n-1} + d$

Recursive equation for a geometric sequence

$a_n = r \cdot a_{n-1}$

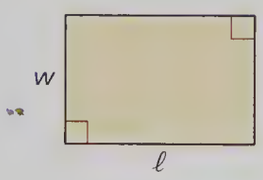
Perimeter, Area, and Volume Formulas

Square



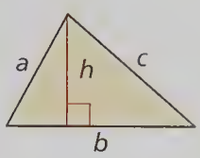
$P = 4s$
$A = s^2$

Rectangle



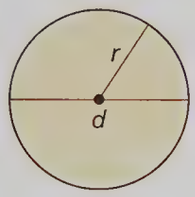
$P = 2\ell + 2w$
$A = \ell w$

Triangle



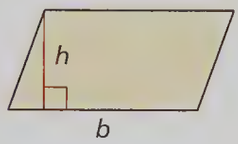
$P = a + b + c$
$A = \frac{1}{2}bh$

Circle



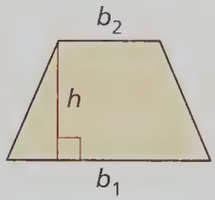
$C = \pi d$ or $C = 2\pi r$
$A = \pi r^2$

Parallelogram



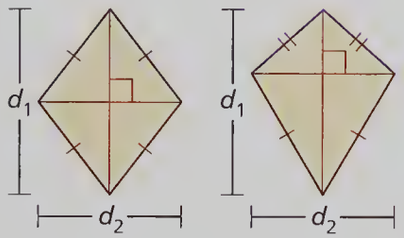
$A = bh$

Trapezoid



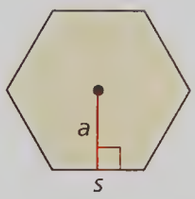
$A = \frac{1}{2}h(b_1 + b_2)$

Rhombus/Kite



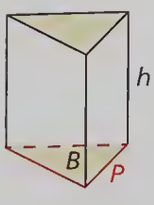
$A = \frac{1}{2}d_1d_2$

Regular *n*-gon



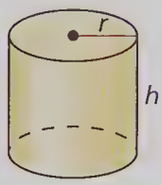
$A = \frac{1}{2}aP$ or $A = \frac{1}{2}a \cdot ns$

Prism



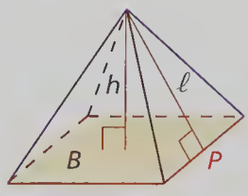
$L = Ph$
$S = 2B + Ph$
$V = Bh$

Cylinder



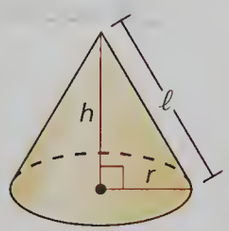
$L = 2\pi rh$
$S = 2\pi r^2 + 2\pi rh$
$V = \pi r^2h$

Pyramid



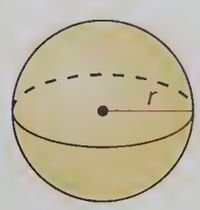
$L = \frac{1}{2}P\ell$
$S = B + \frac{1}{2}P\ell$
$V = \frac{1}{3}Bh$

Cone



$L = \pi r\ell$
$S = \pi r^2 + \pi r\ell$
$V = \frac{1}{3}\pi r^2h$

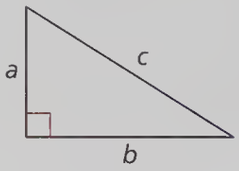
Sphere



$S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Other Formulas

Pythagorean Theorem
$a^2 + b^2 = c^2$



Simple Interest
$I = Prt$

Compound Interest
$y = P\left(1 + \frac{r}{n}\right)^{nt}$

Distance
$d = rt$

Conversions

U.S. Customary
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5280 feet
1 mile = 1760 yards
1 acre = 43,560 square feet
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 pound = 16 ounces
1 ton = 2000 pounds

U.S. Customary to Metric
1 inch = 2.54 centimeters
1 foot ≈ 0.3 meter
1 mile ≈ 1.61 kilometers
1 quart ≈ 0.95 liter
1 gallon ≈ 3.79 liters
1 cup ≈ 237 milliliters
1 pound ≈ 0.45 kilogram
1 ounce ≈ 28.3 grams
1 gallon ≈ 3785 cubic centimeters

Time
1 minute = 60 seconds
1 hour = 60 minutes
1 hour = 3600 seconds
1 year = 52 weeks

Temperature
$C = \frac{5}{9}(F - 32)$
$F = \frac{9}{5}C + 32$

Metric
1 centimeter = 10 millimeters
1 meter = 100 centimeters
1 kilometer = 1000 meters
1 liter = 1000 milliliters
1 kiloliter = 1000 liters
1 milliliter = 1 cubic centimeter
1 liter = 1000 cubic centimeters
1 cubic millimeter = 0.001 milliliter
1 gram = 1000 milligrams
1 kilogram = 1000 grams

Metric to U.S. Customary
1 centimeter ≈ 0.39 inch
1 meter ≈ 3.28 feet
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile
1 liter ≈ 1.06 quarts
1 liter ≈ 0.26 gallon
1 kilogram ≈ 2.2 pounds
1 gram ≈ 0.035 ounce
1 cubic meter ≈ 264 gallons

Credits

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- Apply mathematics to problems arising in everyday life, society, and the workplace.
- Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.
- Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.
- Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.
- Create and use representations to organize, record, and communicate mathematical ideas.
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